Racah Inst. of Physics, The Hebrew University
Computational Physics of Complex Systems: 77732
given by Prof. Ofer Biham
Note: The solutions can be submitted by groups of up to three students. It is sufficient to formally submit solutions to four of the seven problems, using the rest of them as examples to use when you study for the final exam.

Date: June 11, 2006

Problem set number 4

Problem 1

A random variable X is distributed according to $P(x) = A\exp(-\alpha x)$. Consider the following procedure: you draw N independent numbers from this distribution and order them in a monotonically decreasing list, x_1, x_2,$\ldots$,x_N. You then create a Zipf plot of x_n vs. n, where $n = 1,\ldots,N$ (not necessarily on a logarithmic scale). What function would fit the resulting curve?

Problem 2

Consider a random walker (RW) on a finite lattice of L sites, indexed from 1 to L. The lattice has a closed boundary on the left (to the left of site 1) such that when the RW is in site 1 it can move only to the right. It has an open boundary on the right hand size of site L. The time is discrete and the RW makes one move each time step. Calculate the *average* number of steps that is required for a RW starting from site 1 to exit the system (try it first with small systems such as $L = 3$, etc.). This is an example of a large class of problems called first passage processes.

Problem 3

A random variable X is distributed according to the log-normal distribution if it can be expressed as $X = \log Y$, where Y follows the Normal distribution. (a) Suppose that Y is distributed according to the Normal distribution with an average μ and standard deviation σ; Calculate the average and standard deviation of X. (b) Show how to generate random numbers distributed according to the log-normal distribution if you are equipped with a random number generator that provides numbers uniformly distributed in the interval $[0, 1)$.

Problem 4

Consider a random walker on an undirected graph. The graph includes N links indexed by $i = 1, \ldots, N$. Sites i and j are connected by a link if the matrix element $M(i, j) = 1$ and are not connected if $M(i, j) = 0$. The time is discrete and at each step the walker moves from its current site (in one of the nodes of the graph) by randomly choosing one of the links connected to this site and moving to the site at the other end of this link. Find an expression for the frequency in which each site is visited in terms of the numbers of links. Is it possible to extend this result to the case of undirected graph?

Problem 5

Propose a way to modify the dynamical rules of the Bak-Tang-Wiesenfeld sendpile model such that the modified model will be: (a) stochastic but Abelian; (b) deterministic and non-Abelian.

Problem 6

Consider the following example of the Cantor set in one dimension: Starting from the [0,1] interval, at each stage the 1/5th of each interval on the right and the 1/5th of each interval on the left are maintained while the middle 3/5 is removed. Calculate the fractal dimension of the resulting set.

Problem 7

Construct a diffusion limited aggregation cluster on a two dimensional lattice. This is done by setting one particle as a seed and then bringing random walkers from "infinity" one by one. Each random walker that visits a site adjacent to the cluster sticks immediately and joins the cluster. Find ways to improve the efficiency of the construction process. Calculate the fractal dimension of the resulting cluster using the box-counting method.