Intrinsic Nonlinear Scale Governs Oscillations in Rapid Fracture

Tamar Goldman,1 Roi Harpaz,2 Eran Bouchbinder,2 and Jay Fineberg1
1The Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Israel
2Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel
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When branching is suppressed, rapid cracks undergo a dynamic instability from a straight to an oscillatory path at a critical velocity \(v_c\). In a systematic experimental study using a wide range of different brittle materials, we first show how the opening profiles of straight cracks scale with the size \(\ell_{nl}\) of the nonlinear zone surrounding a crack’s tip. We then show, for all materials tested, that \(v_c\) is both a fixed fraction of the shear speed and, moreover, that the instability wavelength is proportional to \(\ell_{nl}\). These findings directly verify recent theoretical predictions and suggest that the nonlinear zone is not passive, but rather is closely linked to rapid crack instabilities.

Since their discovery, a fundamental understanding of the origin of rapid crack instabilities [1–5] has proven to be very elusive. The dynamics of single straight cracks are well described [6–8] by linear elastic fracture mechanics (LEFM) [9,10]. This theoretical framework predicts singular crack tip fields and describes a crack’s dynamics as a balance between the elastic energy flux into the tip region and the energy dissipated at the tip. LEFM, however, cannot explain rapid crack instabilities and accompanying nontrivial crack patterns, without additional assumptions or physical insights about the near-tip region where linear elasticity breaks down.

There have been a number of notable attempts to describe crack instabilities by supplementing or extending LEFM in various ways. These include phase-field models [11–16], cohesive-zone models [17–19], models based on the “principle of local symmetry” [20–22], energy conservation bounds on crack branching [23,24] and models based on nonlinear constitutive behavior near the crack tip [25,26]. Although many of these models are qualitatively consistent with both experimental and numerical observations, decisive quantitative experiments that are able to differentiate between them are lacking. Classically, the near-tip region has been considered as a passive region that both regularizes the singular fields predicted by LEFM and accounts for the dissipative processes at the tip. Understanding crack instabilities, however, may require the introduction of fundamentally new physics in which the near-tip region plays a more active role.

Here we focus on the oscillatory instability in rapid brittle fracture in which a straight crack becomes unstable to sinusoidal path oscillations [4]. The onset of these oscillations was observed at a critical velocity \(v_c\) of about 90% of the shear wave speed \(c_s\), when the microbranching instability [1,2] was suppressed. This instability is particularly intriguing since it involves a finite instability wavelength at onset that is independent of either system geometry or loading conditions. This suggests the existence of an intrinsic scale that cannot exist in linear elastic solutions for cracks, which are scale-free.

Recently, a theory describing this instability was proposed [26]. This theory is based on the existence of a dynamic nonlinear length scale \(\ell_{nl}\), where linear elasticity breaks down and material nonlinearities become significant due to the large deformation near a crack’s tip [27–30]. The basic idea behind this approach is that in the presence of a finite \(\ell_{nl}\), causality implies that the singular LEFM fields lag behind the actual tip location with a delay of \(\tau_d \propto \ell_{nl}\). This led to a high-velocity oscillatory instability with the following properties: (i) the scaled critical velocity for the onset of oscillations \(v_c/c_s\) is material independent, and (ii) the oscillation wavelength \(\lambda_{osc}\) is proportional to \(\ell_{nl}\).

In this Letter we investigate the rapid fracture of a variety of different brittle gels, whose mechanical properties vary over a wide range. We first demonstrate that the opening profiles of straight cracks collapse onto a single velocity-dependent form, when scaled by the size of the nonlinear elastic zone, as predicted by [29]. We then show that the oscillatory instability is triggered in each material at the same scaled value of \(v_c/c_s\) and, moreover, that the instability wavelength indeed scales with \(\ell_{nl}\), confirming the theoretical predictions of [26].

Our experiments were performed using polyacrylamide gels which are transparent, homogeneous, brittle, incompressible elastomers. The dynamics of rapid cracks in these neo-Hookean materials are identical to those observed in other brittle amorphous materials (e.g., glass, PMMA). Because of the low elastic moduli of these soft materials, the wave speeds and corresponding crack velocities are nearly 3 orders of magnitude [31] lower than in conventional materials. This enables us to slow down the fracture process while obtaining detailed measurements of rapid cracks at unprecedented scaled velocities.

We control the gels’ physical properties by varying their chemical composition [32]. We varied the total monomer
concentration (by weight) between 14.2%–32.4%, cross-linker concentration between 2.7%–4.6% and polymer initiators in the range 0.03%–0.06%. In what follows we will label each gel by its shear modulus, $\mu$ ($33 < \mu < 187$ kPa) and fracture energy at the critical velocity, $\Gamma(u_c)$ ($24 < \Gamma(u_c) < 60$ J/m$^2$). $\Gamma$ is defined as the amount of energy dissipated per unit crack extension and sample thickness. The details of these gel compositions are provided in [33]. Typical dimensions of the gels used were $(x \times y \times z)$ $(130 \times 130 \times 0.2)$ mm and $(200 \times 200 \times 0.2)$ mm, where $x$, $y$, and $z$ are, respectively, the propagation, loading, and thickness directions.

Experiments were performed as in [4] by imposing uniaxial tensile loading via constant displacement in the vertical ($y$) direction. Once a desired strain $\epsilon$ was reached, a scalpel was used to initiate fracture at the sample’s edge, midway between the vertical boundaries. The influence of stress waves generated at crack initiation and reflected back to the crack tip by the sample boundaries was negated by selecting applied strain levels large enough to ensure that all cracks reached the onset of the oscillatory instability before the arrival of any reflected waves. For experimentally feasible system sizes, this entails strains $\epsilon$ in the range 6%–18%. The crack tip opening displacement (CTOD) of the moving crack was measured with a high speed camera focused on an ($\text{CTOD}$) area of $60 \times 9.5 - 19$ mm with $1280 \times 200 - 400$ pixel resolution. Successive photographs were taken at between 2490/15 000 frames/s with a $2 \mu s$ exposure time. Multiple exposures were utilized, when needed. The microbranching instability was suppressed (as in [4]) by setting the gel thickness to $160$–$220 \mu m$. In all experiments analyzed, no micro-branches occurred in regions that could influence the instability onset. Post-fracture $xy$ profiles were measured via an optical scanner with $300$dpi resolution.

Let us now consider a simple straight crack moving at velocity $v$ under constant tensile loading, prior to any instability. According to LEFM, the CTOD has a parabolic form that is inversely proportional to the instantaneous value of $\Gamma(v)$ [9]. This characteristic parabolic form is indeed experimentally measured at points that are at a distance not too close to the crack tip. Sufficiently near the crack tip, regions of very high strain are encountered. The resulting nonlinear elastic effects shift the actual crack tip by a distance $\delta$ from the apex of the parabolic form defined by LEFM [27]. The dissipative zone adjacent to the tip is also contained within $\delta$. In gels, the dissipative zone is significantly smaller than the size of the nonlinear elastic deformation zone [28].

The strain levels imposed in our measurements suggest that the CTOD predicted by LEFM should be calculated with respect to the background strain $\epsilon$. To this end, we consider the energy functional describing our incompressible gels under plane stress conditions [34]
We now turn to the oscillatory instability. We consider the first additional scale could be related to either the strongly material-dependent, varying by over a factor of 2.5 in different materials [Fig. 3(b)]. In each material there is a well-defined velocity $v_\text{f}$ for the onset of the instability. As predicted by [26], Fig. 3(c) shows that $v_\text{f}$, when scaled by $c_\text{s}$, has the nearly constant value of $0.9 c_\text{s}$, in each of the 6 materials studied.

What is the origin of the instability wavelength? Figure 3 confirms that $\lambda_{\text{osc}}$ is not related to details of the experimental system. In experiments with identical conditions, $\lambda_{\text{osc}}$ varied widely with the material used. In [26], $\lambda_{\text{osc}}$ was predicted to be proportional to the size of the nonlinear zone $\ell_{\text{nl}}$. Here we use $\delta(v = v_\text{c})$ to estimate $\ell_{\text{nl}}$ at the critical velocity $v_\text{c}$ for different materials. The obvious advantage of doing this is that $\delta(v)$ is directly measurable, and hence the theoretical prediction of [26] can be recast as a relation between two directly measurable quantities, $\lambda_{\text{osc}}$ and $\delta(v = v_\text{c})$. In Fig. 4 we plot $\lambda_{\text{osc}}$ vs $\delta(v = v_\text{c})$ for the 6 materials used. We indeed find that $\delta$ is directly proportional to $\lambda_{\text{osc}}$, as predicted in [26]. Moreover, the constant of proportionality between $\lambda_{\text{osc}}$ and $\delta(v = v_\text{c})$ in Fig. 4 is consistent with the analysis of [26,33].

We note that the weakly nonlinear estimate of $\ell_{\text{nl}} \propto \Gamma/\mu$ [29] is linearly related to $\delta$. In contrast to Fig. 4, however, this linear relation involves an offset corresponding to
This scale is also apparent in the imperfect data collapse in Fig. 2(b), suggesting that \( \delta \) includes length scales such as the strongly nonlinear contributions to the nonlinear elastic zone and/or the scale of the “dissipative zone” at the crack tip which are beyond the perturbative estimate of Eq. (1) used in [29]. This offset is consistent with previous experimental estimates of the size of the strongly nonlinear and dissipative zones given in [28].

In conclusion, our results conclusively demonstrate that the oscillatory instability of fast brittle cracks indeed involves an intrinsic scale that is governed, in a large part, by the nonlinear elastic zone surrounding the crack tip. The size of this zone quantitatively agrees with the predictions of [26]. These results indicate that the nonlinear (and dissipative) zones surrounding the tip of a moving crack are not “passive” objects that are simply “dragged along” by the crack tip. Instead, as suggested by [4,25,26], this region may play an active role in destabilizing crack motion. The demonstration of this presented in this work is, therefore, an important step in obtaining a fundamental understanding of the origin of instabilities in dynamic fracture. These ideas are as general as the singular behavior that occurs at the tip of a moving crack. It is therefore conceivable that dynamics of the near-tip zone could play an important role in unraveling the physical mechanism driving other instabilities of rapid cracks [1–5]. The effects of crack instabilities are not simply academic; they are both significant and easily visible at macroscopic scales. Examples include limiting the mean propagation velocities of rapid cracks and giving rise to significant increases in fracture-related dissipation.

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[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.108.104303 for details of the gel compositions used in the experiments; the derivation of Eqs. (2) and (3) in the main text together with general comments on the size of the nonlinear scale; a description of the predicted constant of proportionality between the experimental and predicted nonlinear scales; a detailed comparison between the oscillation wavelength and the two possible nonlinear scales mentioned in the text.