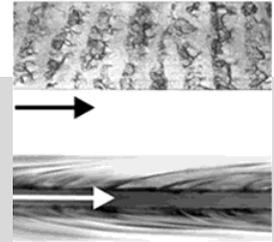


The Dynamics of Fast Fracture**

By Eran Sharon* and Jay Fineberg

A new instability-dominated picture of dynamic fracture in brittle materials has arisen as a result of intensive research over the past few years, and is based on new high-resolution measurements of crack dynamics. This may have impact on the design of new energy-absorbing materials.



What determines the strength of the materials around us? We know that the chemical bonds connecting the molecules that compose a given material hold it together, but to what degree do the bond strengths themselves determine the strength of a given structure? In order for a structure to separate, one must at least supply enough energy to break the bonds along a plane that cuts through it. The cumulative energy of these bonds provides an energy barrier that ensures the structure's mechanical stability. The existence of a crack in an otherwise perfect solid is Nature's way to efficiently supply the energy to overcome this barrier. In a material under stress, a nearly singular stress field occurs at the tip of a crack. This large stress concentration at the tip of a crack serves as an energy "funnel" that can enable a crack to propagate by efficiently severing bonds in only the immediate vicinity of its tip. Thus, the ultimate strength of any material is governed by the existence of the cracks within it.

We can distinguish between three main stages in crack evolution:

- Crack nucleation, where an initial crack is formed in a body.
- Quasistatic crack growth, where the crack is either static or propagates at low velocities compared to the sound velocity in the material.
- Dynamic growth, where a crack propagates at velocities of the order of the sound velocity in the material.

The importance of the first two stages is clear. In most applications we wish to avoid or control fracture formation. Indeed, there have been many successes in material design that have dramatically improved the resistance of the materials to fracture.^[1] The practical importance of the dynamic stage of fracture is less obvious; once a crack starts to run close to the sound speed in an aircraft wing, the precise speed that it achieves is a rather academic point. As we will see, however, understanding dynamic fracture may be of more than academic interest.

As a result of intensive research over the past few years, a new instability-dominated picture of dynamic fracture in brittle materials has arisen. Much of our insight into these processes is based on new high-resolution measurements (see Fig. 1) of a crack's dynamics.^[2,3] In this picture a dynamic crack

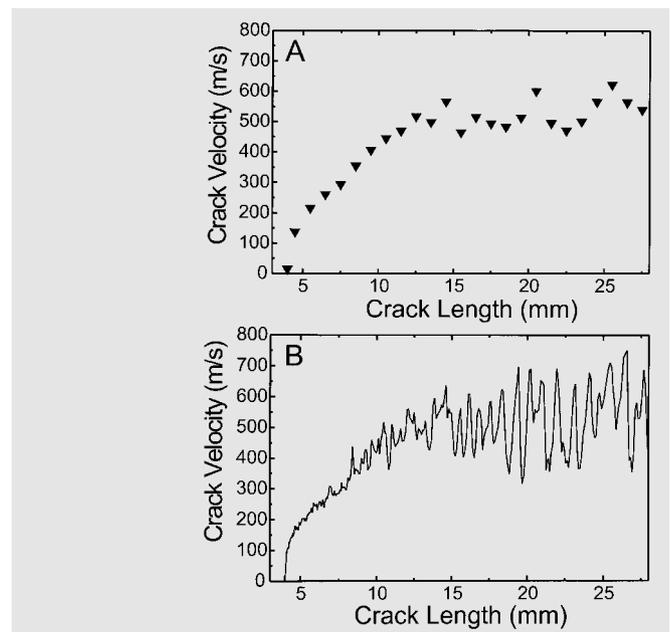


Fig. 1. A measurement of a typical propagating crack in a Plexiglas (PMMA) plate with a) a spatial resolution of 1 mm and b) a spatial resolution of 0.1 mm. Both (a) and (b) are taken for the same experiment. The data in (a) are representative of early measurements, which had suggested that a crack accelerates smoothly until attaining a terminal velocity of about 550 m/s. The high-resolution measurement in (b) reveals that a crack's dynamics are much richer than previously supposed. A crack's dynamics are far from smooth. It develops large amplitude velocity oscillations with instantaneous velocities beyond 700 m/s.

is completely different than a quasistatic one. As the speed of a crack increases beyond a critical speed, $v_c \sim 0.4V_R$ (where the Rayleigh wave speed, V_R , is the speed at which a wave travels across a free surface), a dynamic instability occurs that entirely

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changes a crack's behavior. The evolution of the instability is summarized in Figure 2. Below v_c a single crack exists that creates in its wake a smooth fracture surface. Beyond v_c a crack will change its topology as it sprouts short-lived side branches (microbranches). Microbranch dynamics are interrelated with those of the main crack. As a function of the crack's mean velocity (v , or equivalently the energy flux into the crack tip per unit surface, G), microbranches increase in length as the mean dynamics of a crack change dramatically. Because a crack is driven by the energy flux (G) to its tip, when a new microbranch is formed it decreases the amount of energy flowing into the main crack, thereby reducing its velocity. Conversely, the death of a microbranch causes the main crack to accelerate, as more energy is available to drive it. This results in fluctuations in the velocity of crack "front", with a time scale characteristic of a microbranch lifetime [about 1 μ s in Plexiglas (poly(methyl methacrylate), PMMA)]. The instability was first experimentally observed in Plexiglas and was later seen in other brittle amorphous materials such as soda-lime glass.^[4,5]

Although there have been many notable attempts to describe the instability in the framework of an isotropic continuum theory,^[6-9] they have not yet been clearly successful.^[10] Surprisingly, however, results that are markedly similar to these? experiments have been observed in models of "crystalline" materials. Analytical^[11,12] and numerical^[11-14] work on models of ideal brittle crystals, molecular-dynamic simulations of crystals with different potentials,^[15-18] and finite elements^[19-21] have all clearly established the universal nature of this instability.

The existence and dynamics of the instability explain a number of phenomena long known to be related to fracture, but not previously understood. Fracture surfaces have long been known to have a characteristic morphology known as "mirror mist hackle", where the initially "mirror-smooth" surface gradually roughens, appearing "misty" at first and, as the crack further develops, giving rise to structure on many scales. This leads to an increasingly rough "hacked" appearance. This structure may be naturally understood in view of the microbranching instability. Studies in brittle plastic^[3,22] have

shown that the onset of surface roughness coincides with the onset of microbranching. Both the mean fracture surface amplitude and mean microbranch length are well-defined functions of the mean crack velocity; however, the microbranches are two orders of magnitude larger in size than the root mean square (rms) amplitude of the surface structure. Thus, the surface structure is simply the "tip of the iceberg" of the subsurface microbranch structure.

Another direct consequence of the instability is that the total amount of surface area formed, and hence the amount of energy dissipated by the fracture process via the creation of the fracture surface, increases substantially once the microbranching instability occurs. (In Plexiglas the surface area increases by nearly one order of magnitude over a 50% change in the mean crack velocity.) Studies^[23] have shown that even in a microscopically complex polymer, such as PMMA, the energy cost per unit surface formed is nearly constant. The sharp rise in energy dissipation with mean crack velocity, ob-

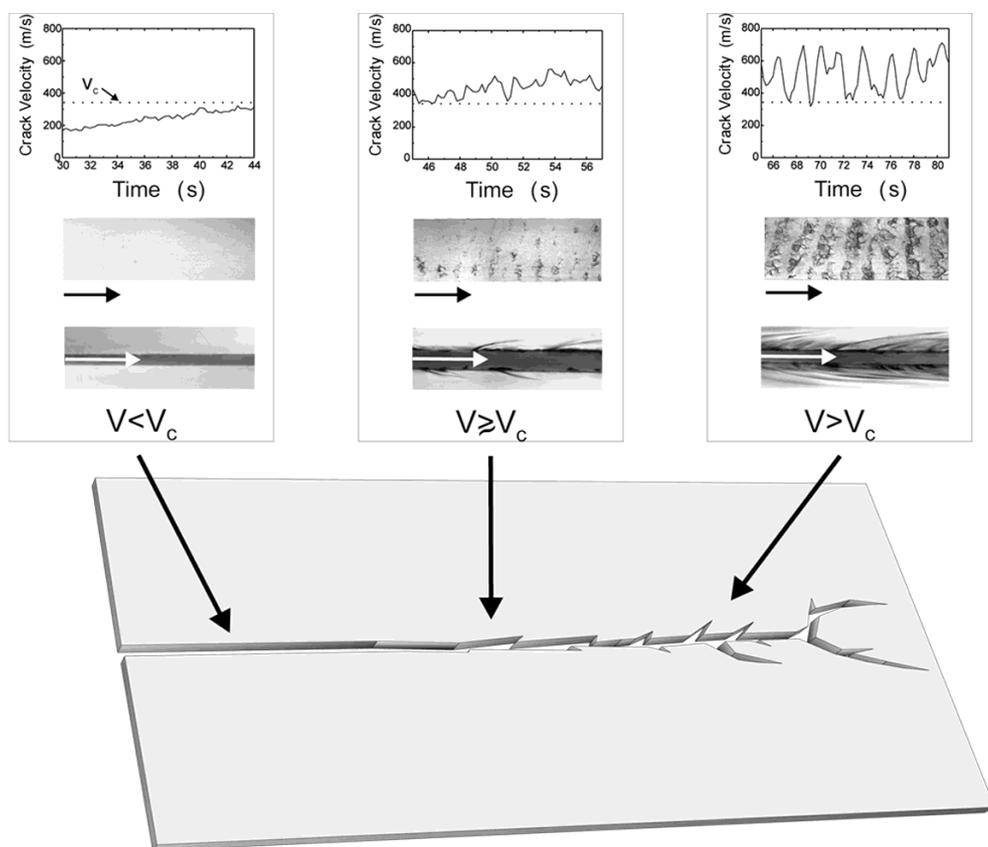


Fig. 2. The evolution of the microbranching instability in a crack that propagates from left to right (x direction) is demonstrated in three different types of measurements. Top: crack velocity measurement; the dotted line marks v_c . Middle: fracture surface morphology, demonstrated by a (x - z plane) photograph of the fracture surface created by a dynamic crack. The arrow of length 3 mm indicates the (x) direction of propagation direction. Bottom: views in the x - y plane of the two halves of the fractured plate. The white arrow of length 0.5 mm lies along the path of the main crack. Left: for $v < v_c$, the velocity of the crack is a smooth function of time, the fracture surface is mirrorlike, and a single dynamic crack is observed. Center: as the mean velocity increases slightly beyond v_c , fluctuations in the crack velocity appear, a structure is formed on the fracture surface, and short-lived microbranches bifurcate from the main crack. Right: at higher velocities the oscillations in the crack velocity increase in amplitude, and the structure on the fracture surface grows in both amplitude and width as the microbranches that create the fracture surface grow in size. For a large enough energy flux into the crack tip, the microbranches may turn into macrobranches and propagate independently of the main crack (see illustration).

served in many materials, is entirely explained by the corresponding increase in the amount of total new surface created by the main crack together with its microbranches.^[20] Additional phenomena, such as the existence and observed threshold for crack branching on large scales, may now be understood as consequences of the evolution of the same instability.^[22,24]

If controlled these properties, which are unique to dynamic fracture, can be important in a variety of applications. For example, suppression of the instability could result in the formation of smooth, featureless fracture surfaces. This could be used to our advantage in applications such as gem cutting or crystal cleavage in the microelectronics industry, where such surfaces are desirable. Conversely, enhancing the instability could be desirable in many applications. For example, it is a common practice to increase the yield of an oil well by either explosively or hydraulically pressurizing it. The object of this is to induce subterranean fracture, thereby connecting additional pockets of oil to the well. This process generally results in the formation of a single crack. If crack branching could be induced in this process, significantly enhanced oil yields would result.

An interesting potential application that would result from induced microbranching is in the design of energy-absorbing materials. As we mentioned above, the effective fracture energy of a material can be increased by over one order of magnitude by the microbranching process. If this instability were enhanced, perhaps by intrinsic material design, the ability of a material to absorb energy on impact would be greatly extended. This could result in more effective armor, containment vessels, or crack arresters in critical structures (such as airplane wings).

Finally, in exploring the instability we have seen that although dynamic fracture can lead to complicated cracking patterns, the dynamic fracture process has a number of characteristic signatures. For example, microbranches appear to have a universal (power-law) shape,^[3] and the amount of fracture surface created is proportional to the energy flux supplied by the event that precipitated the fracture. This may be important in analyzing geological patterns. Complex systems of cracks, such as presented in Figure 3, are not necessarily the result of quasistatic fracture under a complex loading, as commonly assumed in the analysis of geological patterns. The observed patterns may well be the signature of a single dynamic event. Quantitative information about this event may be gleaned from pattern analysis millions of years after it occurred.

As the above examples imply, a fundamental understanding of dynamic fracture may have both practical as well as academic importance. Theories describing the dynamics of a single crack, however, have been in apparent contradiction with experimental measurements. In particular, the theory of linear elasticity, which is very successful in describing quasistatic cracks, seems to fail in predicting dynamic crack motion. Recent, highly precise measurements of a crack's dynamics have revealed the basis for these apparent discrepancies.

The equation of motion of the crack is based on local energy conservation arguments. We may view crack formation as a

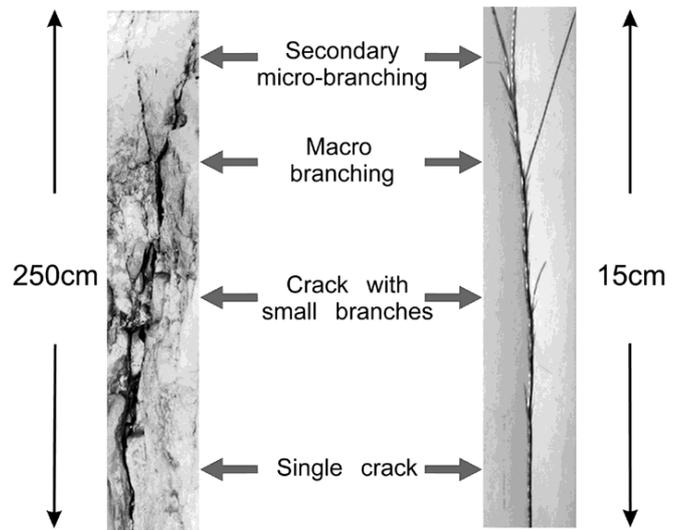


Fig. 3. Mode I (tensile) cracks in a thin Plexiglas (PMMA) plate (right) and in a Dolomite layer found near the Dead Sea (left). Despite the significant differences in the properties, scale, and degree of homogeneity of the two materials, both cracks include the same evolutionary stages typical of the microbranching instability: as the initial single crack propagates from bottom to top, it develops small side branches. As the crack grows further, some of the branches turn into macrobranches that may, themselves, contain additional generations of side branches. A quantitative understanding of the microbranching instability may provide information about both the history and conditions at which geological fracture occurred millions of years ago. In addition, the "instantaneous" record of fracture events provided by comparison to laboratory experiments may provide valuable quantitative information that may enable detailed comparison between the past and present. (Left photo was taken from Sagy and Reches.^[28])

process that converts the elastic energy stored in a loaded body into new fracture surface, with an energy cost of Γ (defined as the fracture energy) per unit surface formed. In a linearly elastic material, the stress field is dominated by a square root singularity throughout a small zone surrounding the crack tip, called the "singular zone". Equating the energy flux through a loop, surrounding the tip and enclosed entirely within the "singular zone", with Γ leads to the equation of motion for a single crack (Eq. 1),^[25] where $G(l)$ is the amount of energy per unit area present at the tip of a static crack of length l . $G(l)$ contains all of the effects of applied stresses and specimen geometry. Equation 1 predicts that a crack should smoothly accelerate to the limiting velocity, V_R . The behavior predicted by Equation 1 has not been in accord with the measurements of a crack's dynamics in a variety of brittle amorphous materials. Even the rather robust prediction for the limiting velocity has not been supported by experiment. The maximal observed velocities of a crack typically reach only about half of the predicted value of V_R .^[26]

$$v(l) = V_R \cdot \left(1 - \frac{\Gamma}{G(l)}\right) \quad (1)$$

These discrepancies have been resolved by the results of recent experiments.^[27] The new experiments have shown that Equation 1 provides, in fact, an excellent description of a crack's dynamics as long as the crack can be considered a "single crack". Below v_c , when only a single crack exists, the theory works well for both PMMA and soda-lime glass, two

materials whose microstructures are entirely different. Above v_c , as the microbranches grow beyond the singular zone, they can no longer be considered "small" and a multibrack rather than a single-crack state exists. At this stage, the conditions for which Equation 1 was derived are no longer valid. A theory for the propagation of the "ensemble" of propagating cracks is now needed. Hence, the microbranches are the cause for the apparent failure of the theory.

Beyond v_c , however, there are times when, momentarily, the crack ensemble is in a single-crack state. This occurs immediately after the "death" of a microbranch and before a new one is generated. At these times, all of the energy that had driven both the main crack and microbranches now flows into the tip of the main crack. The highest peaks in the instantaneous crack velocity correspond to these times. For these instantaneous "single-crack states" Equation 1 is (momentarily) valid. Because Equation 1 has no inertial term, a crack will behave as a "massless" particle and will instantaneously accelerate to the velocity predicted by the single-crack theory. This is shown in Figure 4, where we see that the highest-measured peaks in the crack velocity are indeed described accurately by the theoretical curve predicted by Equation 1. In contrast to the mean velocity measurements, at high values of $G(l)$ peak velocities in excess of $0.9V_R$ are observed

This work confirms that our understanding of single-crack states in brittle, amorphous materials is fundamentally correct. It provides a firm basis for further work in understanding, and perhaps controlling, the dynamics of the multibrack state and the many interesting and diverse phenomena arising from dynamic fracture.

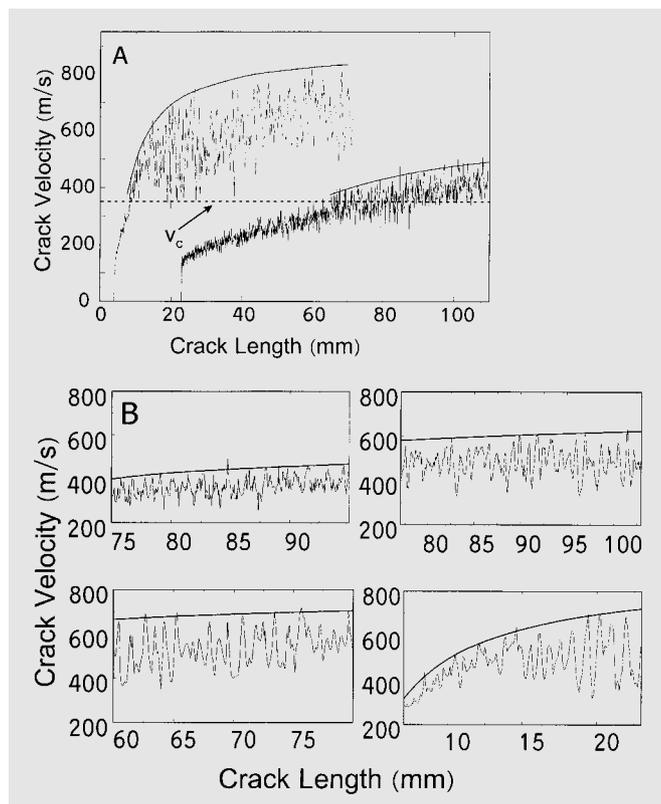


Fig. 4. A comparison of theory with measurements in PMMA for $v > v_c$. (a) Full and (b) expanded views of the instantaneous velocities (thin lines) under different loading conditions are compared to the single-crack predictions of Equation 1 (bold lines) using a single measured value of the fracture energy, $\Gamma = 3000 \text{ J/m}^2$, obtained by Sharon et al.^[23] With no adjustable parameters, the velocity peaks agree remarkably well with the theoretical curve at velocities beyond 90% of the asymptotic crack speed, V_R (data taken from Sharon and Fineberg^[27]).

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