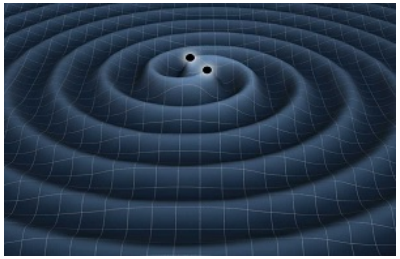
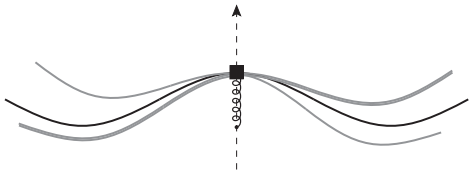
Higher Dimensions & Higher Orders in EFT for Gravitational Waves



Ofek Birnholtz



OB, Shahar Hadar & Barak Kol, Phys. Rev. D 88, 104037 (2013)
OB & Shahar Hadar, Phys. Rev. D 89, 045003 (2014)
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14th Marcel Grossmann Meeting, La Sapienza, Roma

14 July 2015

Outline

1. Motivation
2. Plan
3. Math
4. Results
5. Higher Order interactions
6. Summary & Future

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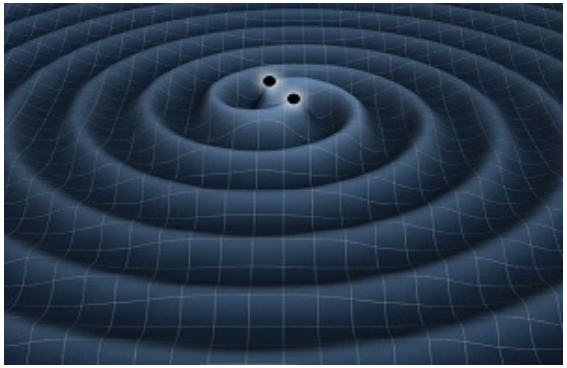
Gravitational Waves from a 2-body system



$$S = \frac{1}{16\pi G_d}\int\sqrt{-g}\,R\,d^dx - \frac{1}{2}\int h_{\mu\nu}T^{\mu\nu}d^dx\,,$$

$$\nabla_\mu T^{\mu\nu} = 0\ ,\ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$(\ c = 1,\ \text{linearized source}\ )$$

Issues:

- Metric fields are numerous, mixed, of different behaviours
- Some degrees of freedom are pure gauge
- Number of d.o.f (real and gauge) change with dimension
- Radiation and system zones have different symmetries
- Problem part-conservative, part dissipative
- Solution senstive to new effects & increasing order

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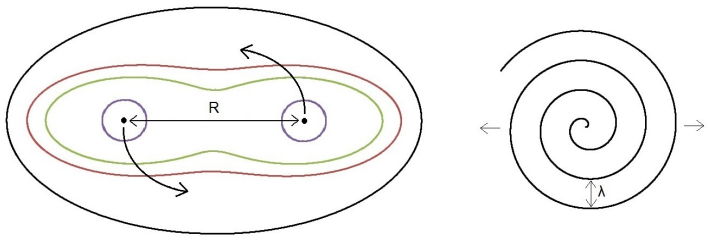
"Everything should be made as simple as possible, but not simpler" - Albert Einstein

- Zone Separation ($\lambda \propto \frac{R}{v} \gg R$)
  - System Zone: $\sim$ stationarity, NRG fields
  - Radiation Zone: $\sim$ (hyper-)spherical symmetry



- Rad. Z. : Dimensional Reduction + Gauge Invariance
  $\Longrightarrow$ Master-Fields $\mathfrak{h}_S$, $\mathfrak{h}_V$, $\mathfrak{h}_T$
- Sys. Z. : Match Master Sources $\mathcal{T}_S$, $\mathcal{T}_V$, $\mathcal{T}_T$

$\Longrightarrow$ Master Action & Master Equation for outgoing waves

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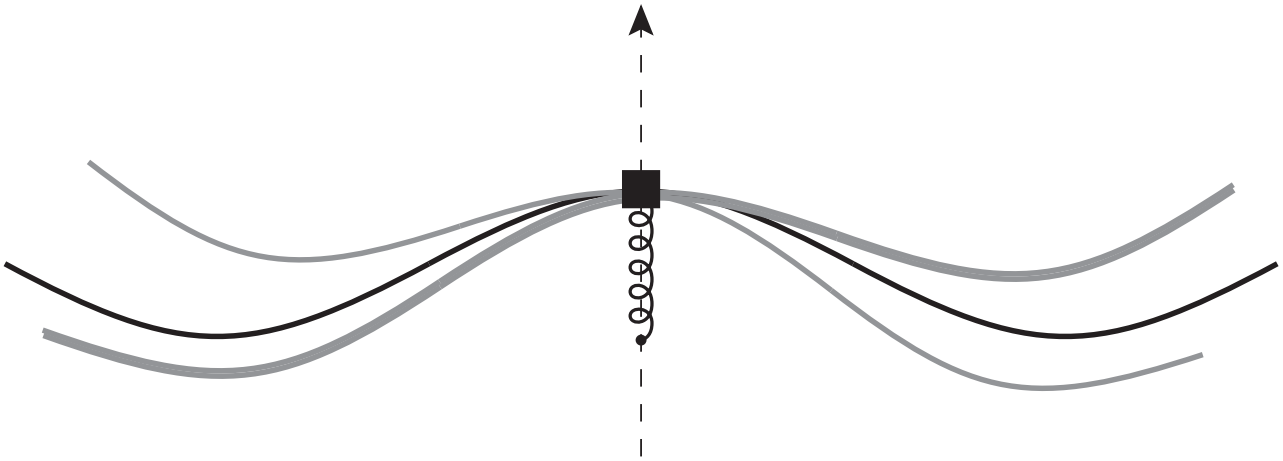

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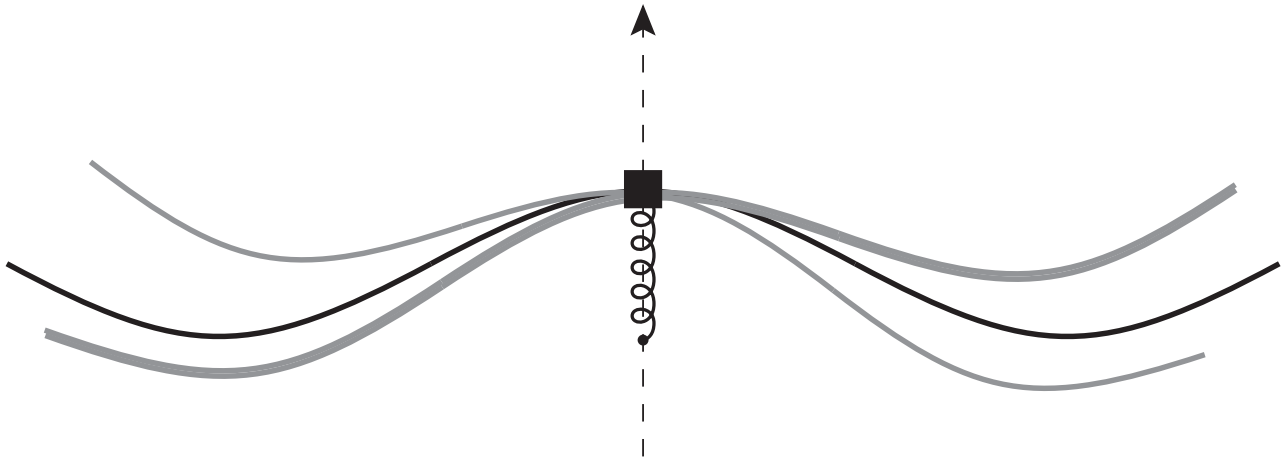

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$\Longrightarrow$ Master Action & Master Equation for outgoing waves
$\Longrightarrow$ Field Doubling for Radiation-Reaction Effective Action

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Finding the G.I. Master Fields

Starting from

$$h_{\alpha\beta} = \sum\!\!\!\!\!\!\int \begin{pmatrix} h_{tt} n_L & h_{tr} n_L & h_t \partial_\Omega n_L + h_{t\aleph} n^L_{\aleph\Omega} \\ \cdots & h_{rr} n_L & h_r \partial_\Omega n_L + h_{r\aleph} n^L_{\aleph\Omega} \\ \cdots & \cdots & h_S n^L_{\Omega\Omega'} + \tilde{h}_S \tilde{n}^L_{\Omega\Omega'} + h_{\aleph\aleph} n^L_{\aleph\,\Omega\Omega'} + h_{\aleph\beth} n^L_{\aleph\beth\Omega\Omega'} \end{pmatrix} e^{-i\omega t}$$

we find the tensors Master fields

$$\mathfrak{h}_{\aleph\beth} = r^{-(\ell+2)} h_{\aleph\beth} \ .$$

Using 1 algebraic equation we find the vector Master fields

$$\mathfrak{h}_{V\aleph} = -\frac{\ell(\ell+\hat{d}+1)}{4\, r^\ell} \left(\omega^2 - \frac{\hat{c}_s}{r^2}\right)^{-1} \left(\frac{2h_{tV\aleph} + i\omega h_{V\aleph}}{r^2}\right)'$$

Using 3 algebraic equations we find the scalars $\Phi := h_{tt} + 2i\omega h_t - \omega^2 \tilde{h}_S$, canonically transformed to $\tilde{\Phi}$ with $F\left[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*\right] = -\hat{c}_s r^{\frac{\hat{d}-1}{2}} \left(\tilde{\Phi}^* \Phi + \tilde{\Phi} \Phi^*\right)$, and finally the scalar Master fields

$$\mathfrak{h}_S = -\frac{\hat{d}(\ell+\hat{d}+1)}{2\ell} r^{-(\ell+\frac{\hat{d}+1}{2})} \tilde{\Phi}$$

1D Master Action & Master Equation

Master Action:

$$S^{\epsilon} = \frac{1}{2} \sum\!\!\!\!\!\!\int_{L\omega} dr \left[\frac{N_{\ell,\hat{d}}}{G_d R^{\epsilon}_{\ell,\hat{d}}} r^{2\ell+\hat{d}+1} \mathfrak{h}^*_{\epsilon} \mathfrak{L} \mathfrak{h}^{\epsilon} - (\mathfrak{h}^*_{\epsilon} \mathcal{T}^{\epsilon} + c.c.) \right]$$

with

$$N_{\ell,\hat{d}} = \frac{\Gamma(1+\hat{d}/2)}{2^{\ell}\,\Gamma(1+\alpha)} = \frac{\hat{d}!!}{(2\ell+\hat{d})!!} \quad , \quad \alpha = \ell + \frac{\hat{d}}{2} \quad , \quad M_{\ell,\hat{d}} = \frac{\pi}{2^{2\alpha+1} N_{\ell,\hat{d}}\, \Gamma^2(\alpha+1)}$$

$$R^{S}_{\ell,\hat{d}} = \frac{\hat{d}(\ell+\hat{d}+1)(\ell+\hat{d})}{(\ell-1)\,\ell} \quad , \quad R^{V}_{\ell,\hat{d}} = \frac{(\hat{d}+1)(\ell+\hat{d}+1)\ell}{2(\ell-1)(\ell+\hat{d})} \quad , \quad R^{T}_{\ell,\hat{d}} = \frac{8(\hat{d}+1)}{\hat{d}^2 c_s (c_s - \hat{d})}$$

Master Equation:

$$0 = \frac{\delta S}{\delta \mathfrak{h}^{L\omega *}_{\epsilon}} = \frac{N_{\ell,\hat{d}}}{G_d R^{\epsilon}_{\ell,\hat{d}}} r^{2\ell+\hat{d}+1} \left(\omega^2 + \partial_r^2 + \frac{2\ell+\hat{d}+1}{r} \partial_r \right) \mathfrak{h}^{\epsilon}_{L\omega} - \mathcal{T}^{\epsilon}_{L\omega}$$

Field doubling

Double the field and the source (Schwinger, Galley) :

- These reflect directed propagation
- Interpretation: "pulling" mirror / radiation "sink"

$$\mathfrak{h} \to \hat{\mathfrak{h}} \quad , \quad Q \to \hat{Q} = \frac{\delta Q}{\delta x}\,\hat{x}$$

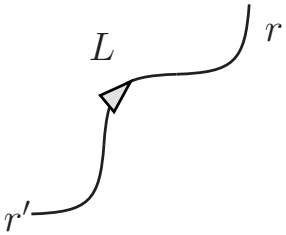
Doubled action:

$$\hat{S}\left[\mathfrak{h}, \hat{\mathfrak{h}}; Q, \hat{Q}\right] = \int \left[\frac{\delta S}{\delta \mathfrak{h}}\,\hat{\mathfrak{h}} + \frac{\delta S}{\delta Q}\,\hat{Q}\right]$$

$$= \int \hat{\mathfrak{h}}^*_\epsilon \left[\frac{N_{\ell,\hat{d}}}{G_d R^\epsilon_{\ell,\hat{d}}}\, r^{2\ell+\hat{d}+1}\left(\omega^2 + \partial_r^2 + \frac{2\ell+\hat{d}+1}{r}\partial_r\right)\mathfrak{h}^\epsilon - Q_\epsilon\right]$$

EOM found by varying w.r.t. $\hat{\mathfrak{h}}$

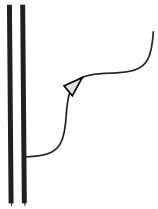
Feynman Rules



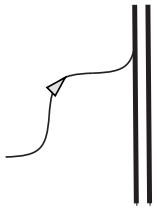
$$= \mathrm{G}^{\epsilon}_{\mathrm{ret}}(\mathrm{r}', \mathrm{r}) = -\mathrm{i}\,\mathrm{G}\,\omega^{2\ell+\hat{\mathrm{d}}}\,\mathrm{M}_{\ell,\hat{\mathrm{d}}}\,\mathrm{R}^{\epsilon}_{\ell,\hat{\mathrm{d}}}\,\tilde{\mathrm{j}}_{\alpha}(\omega \mathrm{r}_{<})\,\tilde{\mathrm{h}}^{+}_{\alpha}(\omega \mathrm{r}_{>})\delta_{\mathrm{LL}'}$$

$$\tilde{\mathrm{j}}_{\alpha} := \Gamma(\alpha+1)2^{\alpha}\frac{\mathrm{j}_{\alpha}(\mathrm{x})}{\mathrm{x}^{\alpha}} = \sum_{\mathrm{p}=0}^{\infty}\frac{(-)^{\mathrm{p}}\,(2\alpha)!!}{(2\mathrm{p})!!(2\mathrm{p}+2\alpha)!!}\mathrm{x}^{2\mathrm{p}} = 1 - \frac{\mathrm{x}^2}{2(2\alpha+2)} + \cdots$$

$$\tilde{\mathrm{h}}_{\alpha} := \Gamma(\alpha+1)2^{\alpha}\frac{\mathrm{h}_{\alpha}(\mathrm{x})}{\mathrm{x}^{\alpha}}$$

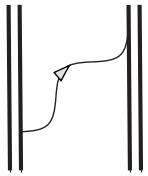


$$= -\mathrm{Q}^{\epsilon}_{\mathrm{L}\omega} \quad , \quad$$



$$= -\hat{\mathrm{Q}}^{\epsilon *}_{\mathrm{L}\omega}$$

$$\hat{\mathrm{S}} =$$



$$= \sum\left(\hat{\mathrm{Q}}\,\mathrm{G}\,\mathrm{Q}\right) \quad , \quad \mathrm{F} = \frac{\delta\hat{\mathrm{S}}}{\delta\hat{\mathrm{x}}}$$

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Quadrupole Moments (LO, NLO)

- Q_E^{ij} : Mass quadrupole

$$Q_S^{ij} \to \sum_{A=1}^{n} m_A \left(x^i x^j - \frac{1}{D} \delta^{ij} x^2 \right)_A$$

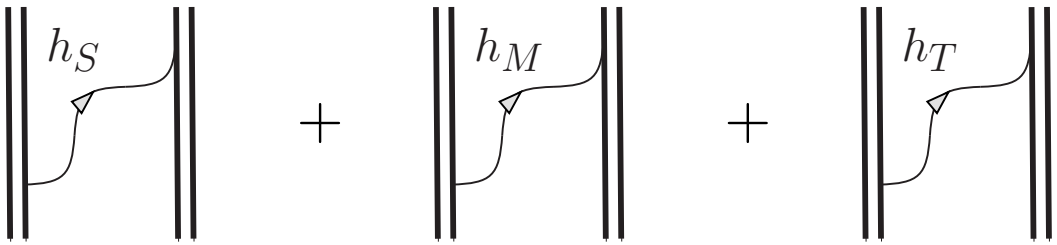
- Q_M^{ij} : Current quadrupole

$$Q_M^{L_2} = Q_M^{ij} = 2 \sum_{A=1}^{n} \left[m \left(\vec{r} \wedge \vec{J} \right)^{(i} x^{j)} \right]_A$$

- Q_E^{ijk} : Mass octupole

$$Q_S^{L_3} = Q_S^{ijk} = \sum_{A=1}^{n} m_A \left[x^i x^j x^k - \frac{1}{D+2} \left(\delta^{ij} x^k + \delta^{ik} x^j + \delta^{jk} x^i \right) x^2 \right]_A$$

General Dimension - RR Effective Action

$\hat{\mathrm{S}}_{\mathrm{linear}} =$ 

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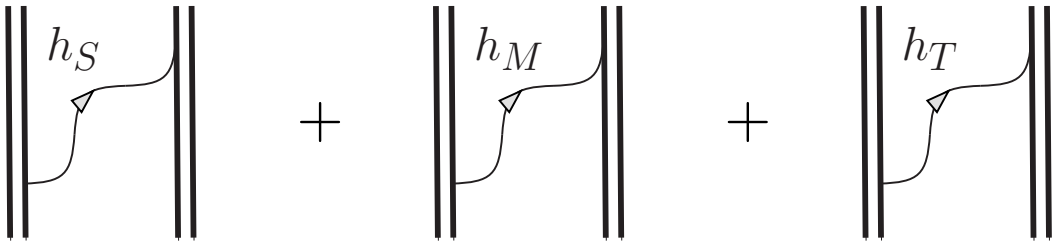
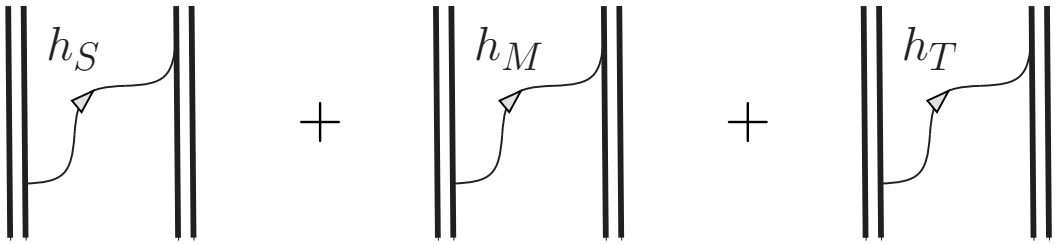
$\hat{\mathrm{S}}_{\mathrm{linear}} =$ 

$$= \int \mathrm{dt} \sum_{\ell} \frac{\mathrm{G_d}\,(-)^{\ell+\hat{\mathrm{d}}}\left(\ell+\hat{\mathrm{d}}+1\right)}{\hat{\mathrm{d}}!!(2\ell+\hat{\mathrm{d}})!!\,(\ell-1)} \left[\frac{\hat{\mathrm{d}}\left(\ell+\hat{\mathrm{d}}\right)}{\ell} \hat{\mathrm{Q}}^{\mathrm{L}}_{(\mathrm{E})} \partial_{\mathrm{t}}^{2\ell+\hat{\mathrm{d}}} \mathrm{Q}^{(\mathrm{E})}_{\mathrm{L}} + \frac{\left(\hat{\mathrm{d}}+1\right)\ell}{2\left(\ell+\hat{\mathrm{d}}\right)} \hat{\mathrm{Q}}^{\mathrm{L}}_{(\mathrm{M})} \partial_{\mathrm{t}}^{2\ell+\hat{\mathrm{d}}} \mathrm{Q}^{(\mathrm{M})}_{\mathrm{L}} + \#_{(\mathrm{T})} \right]$$

General Dimension - RR Effective Action

$\hat{\mathrm{S}}_{\mathrm{linear}} =$ 

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$$\hat{\mathrm{S}}^{(\mathrm{d}=4)}_{\mathrm{LO+NLO}} = \mathrm{G_d} \int \mathrm{dt} \left[-\frac{1}{5} \hat{\mathrm{Q}}^{\mathrm{ij}}_{\mathrm{E}}\, \partial_{\mathrm{t}}^{5} \mathrm{Q}^{\mathrm{ij}}_{\mathrm{E}} - \frac{4}{45} \hat{\mathrm{Q}}^{\mathrm{ij}}_{\mathrm{M}}\, \partial_{\mathrm{t}}^{5} \mathrm{Q}^{\mathrm{ij}}_{\mathrm{M}} + \frac{1}{189} \hat{\mathrm{Q}}^{\mathrm{ijk}}_{\mathrm{E}}\, \partial_{\mathrm{t}}^{7} \mathrm{Q}^{\mathrm{ijk}}_{\mathrm{E}} \right]$$

General d Gravitation RR Effective Action

$$S_{\rm eff}^{\rm LO} = (-)^{\frac{\hat{d}+1}{2}} G_d \int dt \frac{\hat{d}(\hat{d}+2)(\hat{d}+3)}{2\,\hat{d}!!\,(\hat{d}+4)!!} \hat{Q}_S^{L_2} \partial_t^{d+1} Q_S^{L_2}$$

$$\begin{aligned} S_{\rm eff}^{\rm NLO} \;=\; (-)^{\frac{\hat{d}+1}{2}} G_d \int dt \Bigg[& \frac{\hat{d}(\hat{d}+2)(\hat{d}+3)}{2\hat{d}!!(\hat{d}+4)!!} \left(\hat{Q}_S^{L_2} \partial_t^{d+1} \delta^1 Q_S^{L_2} + \delta^1 \hat{Q}_S^{L_2} \partial_t^{d+1} Q_S^{L_2} \right) \\ & - \frac{\hat{d}(\hat{d}+4)(\hat{d}+3)}{6\,\hat{d}!!(\hat{d}+6)!!} \hat{Q}_S^{L_3} \partial_t^{d+3} Q_S^{L_3} \\ & + \frac{2\,\hat{d}\,(\hat{d}+3)}{3\,\hat{d}!!\,(\hat{d}+4)!!} \hat{Q}_M^{L_2} \partial_t^{d+1} Q_M^{L_2} \Bigg] . \end{aligned}$$

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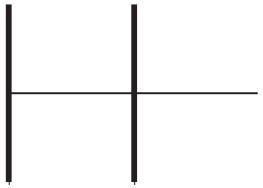
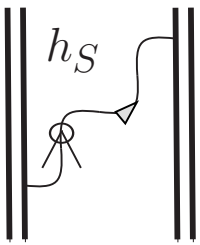
In System Zone: Quadrupole Moments (LO, NLO)

- $\delta^1 Q_E^{ij}$: Mass quadrupole (+1PN corrected including first system zone nonlinear effect: gravitating potential energy, $\sim -\frac{G m_A m_B}{r^{\hat{d}}}$)

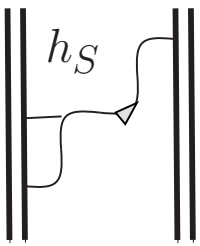


$$\delta^1 Q_S^{L_2} = \sum_{A=1}^{n} m_A \left[\left(\frac{\hat{d}+2}{2\,\hat{d}} v_A^2 - \sum_{B\neq A} \frac{G_d\, m_B}{\|\vec{x}_A - \vec{x}_B\|^{\hat{d}}} \right) x_A^{L_2} - \frac{2(\hat{d}+1)}{\hat{d}(\hat{d}+2)} \partial_t \left(\vec{x}_A \cdot \vec{v}_A\, x_A^{L_2} \right) + \frac{\left(\hat{d}^2+6\hat{d}+4\right)}{2\hat{d}(\hat{d}+2)(\hat{d}+6)} \partial_t^2 \left(r_A^2\, x_A^{L_2} \right) \right]$$

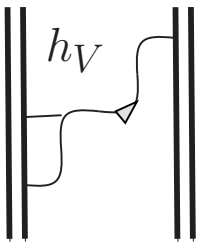
In Radiation Zone: Background Interactions



Scalar / Cosmological Constant Λ



Scalar / Total Mass Curvature $(+(1+\frac{\hat{d}}{2})\mathrm{PN})$



Vector / Total Mass Curvature $(+(2+\frac{\hat{d}}{2})\mathrm{PN})$

Spin Effects

	co-rotation	maximal-spin
Spin-Orbit	$1 + 2/\hat{\mathrm{d}}$	$1/2 + 1/\hat{\mathrm{d}}$
Spin-Spin	$4/\hat{\mathrm{d}}$	$2/\hat{\mathrm{d}}$

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GR radiation-reaction in general dimension

- Leading order and Next-to-leading order in any dimension
- Joint analytical Action formulation of radiation & reaction
- Economization of traditional computations
- Nonlinear effects - seperately for Sys. and Rad. zones
- Beginnings of +1.5PN, +2PN in any dimension
- Ready platform for other effects

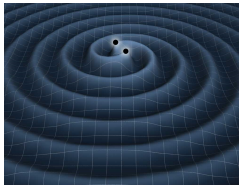
Plans for the future

- The 2-body problem for systems of compact objects:
  - Complete 4PN, 4.5PN, 5PN radiative effects
  - Use EFT to manage Spin and tidal effects
  - Study scattering trajectories in 4d and higher
  - Numerical recipes from EFT
  - Parameter determination from GW signals
- Other high-D systems
  - Explore hyper-relativistic EM problem (Cerenkov)
  - Explore ultra-relativistic systems
- Develope Analytical basis for Effective One Body

Thank you for your attention

Questions?

Image credits


K. Thorne (Caltech) & T. Carnahan (NASA GSFC)

J. Tenniel in L. Carroll, “Through the Looking-Glass, and What Alice Found There" (1871)

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