Long-Range Correlations in the Speckle Patterns of Directed Waves

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We develop a general method for calculating statistical properties of the speckle pattern of coherent waves propagating in disordered media. In some aspects this method is similar to the Boltzmann-Langevin approach for the calculation of classical fluctuations. We apply the method to the case where the incident wave experiences many small angle scattering events during propagation, but the total angle change remains small. In many aspects our results for this case are different from results previously known in the literature. The correlation function of the wave intensity at two points separated by a distance r, has a long-range character. It decays as a power of r and changes sign. We also consider sensitivities of the speckles to changes of external parameters, such as the wave frequency and the incidence angle.

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When a coherent wave propagates through an elastically scattering medium its intensity $I(\mathbf{r})$ exhibits sample specific random fluctuations known as speckles. Characterization of speckle statistics is an important problem relevant for a variety of physical systems. These range from propagation of electromagnetic waves through interstellar space or the atmosphere to ultrasound medical imaging and electron transport in disordered conductors.

The problem can be characterized by several length scales: The propagation distance of the wave, Z, the mean free path, ℓ, and the transport length, $\ell_{\rm tr}$, which is the typical distance for backscattering. In the limit of a single scattering event, $Z \ll \ell$, the problem was studied long ago [1]. In the diffusive regime, $Z \gg \ell_{\rm tr}$, the problem was investigated in Refs. [2–5]. It was shown, in particular, that the wave intensity correlations decay as a power law in space. If the single scattering cross section is a strongly anisotropic function of the scattering angle there is an intermediate regime of "directed waves", $\ell_{\rm tr} \gg Z \gg \ell$, where the wave experiences many small angle scattering events, but the total change of its propagation angle remains small. Investigation of this regime is especially important, for example, for laser communications through a turbulent atmosphere, or propagation of acoustic waves in the ocean [6–10], and for propagation of electron waves in ballistic contacts [11]. This regime was also studied in many papers, see for example [6–10] and references therein.

In this Letter we develop a general method for calculating speckle correlations, which enables us to treat both diffusive and directed wave regimes on equal footing. It is similar, in some of its aspects, to the Langevin scheme for the description of classical fluctuations [12–14]. We shall demonstrate the method by considering the directed wave regime, $\ell_{\rm tr} \gg Z \gg \ell$. In many aspects, our results differ from those obtained in the previous studies [7–10]. Among the differences are the slow power law decay of the intensity correlator as a function of coordinates and its change of sign, see Fig. 1. This affects interpretation of any measurements obtained with a finite aperture apparatus. We also consider the speckle pattern sensitivity to a change of external parameters, such as the scattering potential. The latter relates slow time-dependent fluctuations of the random medium to the spectrum of temporal intensity fluctuations.

We consider a scalar wave propagating through a disordered medium and described by the stationary wave equation,

$$[\nabla^2 + k^2 n^2(\mathbf{r})]\psi(\mathbf{r}) = 0, \tag{1}$$

where $n(\mathbf{r})$ is the index of refraction assumed to be a random Gaussian function. In this case the wave density is $I(\mathbf{r}) = |\psi(\mathbf{r})|^2$. The central object of our approach is the ray distribution function

$$f(\mathbf{r}, \mathbf{s}) = \int \frac{p^2 dp}{2\pi^2} \int d\mathbf{r}' \psi(\mathbf{r} - \mathbf{r}'/2)\psi^*(\mathbf{r} + \mathbf{r}'/2) e^{ip\mathbf{s}\cdot\mathbf{r}'}, \tag{2}$$

which is the probability of finding a ray at point $\mathbf{r}$ pointing in the direction specified by the unit vector $\mathbf{s}$, in particular $I(\mathbf{r}) = \int d\mathbf{s} f(\mathbf{r}, \mathbf{s})$. The average distribution function

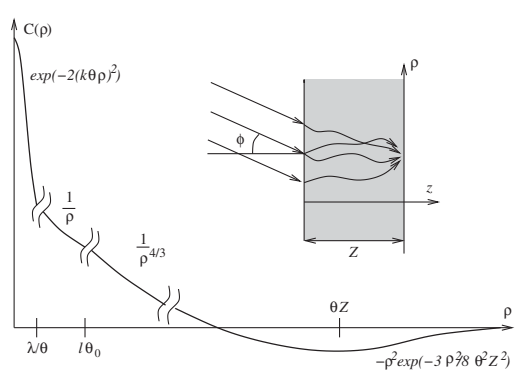

FIG. 1. The asymptotic behavior of the density correlation function $C(\rho) = \langle \delta I(\boldsymbol{\rho}) \delta I(0) \rangle$. λ is the light wavelength, ℓ is the elastic mean free path, Z is the slab width, and θ_0 and θ are the typical scattering angle of a ray traveling a distance ℓ and Z, respectively.

$\langle f(\mathbf{r},\mathbf{s})\rangle$ satisfies the Boltzmann kinetic equation,

$$\mathbf{s}\cdot\nabla\langle f(\mathbf{r},\mathbf{s})\rangle = I_{\rm st}\{\langle f(\mathbf{r},\mathbf{s})\rangle\}, \tag{3}$$

$$I_{\rm st}\{\langle f\rangle\} = \int d^2\tilde{s}W(\mathbf{s}-\tilde{s})[\langle f(\mathbf{r},\tilde{\mathbf{s}})\rangle - \langle f(\mathbf{r},\mathbf{s})\rangle]. \tag{4}$$

Here $\langle\ldots\rangle$ denotes averaging over the random realizations of $n(\mathbf{r})$, the integral over the directions is normalized to unity, $\int d^2s = 1$, and $W(\delta\mathbf{s}) = \frac{k^4}{\pi}\int d^3r g(\mathbf{r})e^{ik\delta\mathbf{s}\cdot\mathbf{r}}$ is the probability, per unit length of propagation, for changing the ray direction by $\delta\mathbf{s}$, and $g(\mathbf{r}-\mathbf{r}') = \langle n(\mathbf{r})n(\mathbf{r}')\rangle - \langle n(\mathbf{r})\rangle\langle n(\mathbf{r}')\rangle$ is the disorder correlation function.

Correlations of the fluctuations, $\delta f = f - \langle f\rangle$, may be evaluated using the Langevin-type equation

$$\mathbf{s}\cdot\nabla\delta f - I_{\rm st}\{\delta f\} = \mathcal{L}(\mathbf{r},\mathbf{s}), \tag{5}$$

where $\mathcal{L}$ is a random Langevin source, with a zero mean and the variance given by

$$\langle\mathcal{L}(\mathbf{r},\mathbf{s})\mathcal{L}(\mathbf{r}',\mathbf{s}')\rangle = \frac{2\pi}{k^2}\delta(\mathbf{r}-\mathbf{r}')\Big[\delta(\mathbf{s}-\mathbf{s}')\langle f(\mathbf{r},\mathbf{s})\rangle \times\int d^2s_1 W(\mathbf{s}-\mathbf{s}_1)\langle f(\mathbf{r},\mathbf{s}_1)\rangle - \langle f(\mathbf{r},\mathbf{s})\rangle W(\mathbf{s}-\mathbf{s}')\langle f(\mathbf{r},\mathbf{s}')\rangle\Big]. \tag{6}$$

One can prove Eqs. (3)–(6) using the standard impurity diagram technique [15]. The Boltzmann Eq. (3) and (4) is obtained by summing the ladder diagrams shown in Fig. 2(a), which describe the classical beam propagation. Equations (5) and (6) follow from diagrams 2(b)–2(d). They describe two beams meeting at a Hikami box [16] which acts as a beam splitter and gives rise to "partition noise". The latter is described by the Langevin sources.

Equations (3)–(6) are valid when the mean free path is sufficiently large, $\ell = [\int ds^2 W(\mathbf{s})]^{-1} \gg \xi^2/\lambda$, and $|\mathbf{r}-\mathbf{r}'| \gg \lambda$. Here $\mathbf{r}$ and $\mathbf{r}'$ are observation points, $\xi = [\int d^3r r^2 g(r)/3\int d^3r g(r)]^{1/2} > \lambda$ is the disorder correlation length, and $\lambda = 2\pi/k$ is the wave length. For $|\mathbf{r}-\mathbf{r}'| \le \lambda$ Equations (3)–(6) are not valid and to evaluate the correlation function one has to calculate the diagram shown in Fig. 2(e).

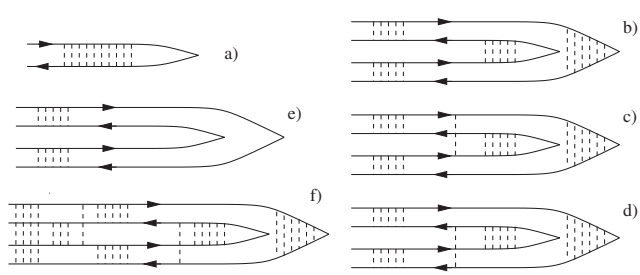

FIG. 2. Diagrams describing wave density correlations. The solid lines denote the disorder-averaged Green functions of Eq. (1) and the dashed lines denote the disorder correlator, $4k^4\langle\delta n(\mathbf{r})\delta n(\mathbf{r}')\rangle$. The ladder diagrams describe the classical intensity propagation, Eqs. (3) and (4). The Langevin source fluctuations (6) are determined by the mixing of two beams arriving to the Hikami box as shown in panels b, c, and d.

In the regime of small angle scattering, $\theta_0 \ll 1$, and when $|\mathbf{r}-\mathbf{r}'| \gg l$ the rays undergo diffusion in the space of directions, $\mathbf{s}$. In this case Eqs. (3)–(6) reduce to a set of angular diffusion equations,

$$\mathbf{s}\cdot\nabla\langle f(\mathbf{r},\mathbf{s})\rangle = D_\theta\nabla_s^2\langle f(\mathbf{r},\mathbf{s})\rangle, \tag{7}$$

$$\mathbf{s}\cdot\nabla\delta f(\mathbf{r},\mathbf{s}) = \nabla_s[D_\theta\nabla_s\delta f(\mathbf{r},\mathbf{s}) - \mathbf{j}_L(\mathbf{r},\mathbf{s})], \tag{8}$$

$$\langle j_\alpha^L(\mathbf{r},\mathbf{s})j_\beta^L(\tilde{\mathbf{r}},\tilde{\mathbf{s}})\rangle = 2\pi D_\theta\langle f\rangle^2 k^{-2}\delta_{\alpha\beta}\delta(\mathbf{s}-\tilde{\mathbf{s}})\delta(\mathbf{r}-\tilde{\mathbf{r}}). \tag{9}$$

Here $\mathbf{j}^L(\mathbf{r},\mathbf{s})$ are the Langevin current sources, $D_\theta = \frac{1}{2}\ell_{\rm tr}^{-1} = \frac{1}{2}\int d^2s'(1-\mathbf{s}\cdot\mathbf{s}')W(\mathbf{s}-\mathbf{s}')$ is the diffusion constant in the space of angles $\mathbf{s}$, and $\nabla_s = \hat{\theta}\frac{\partial}{\partial\theta} + \frac{\hat{\phi}}{\sin(\theta)}\frac{\partial}{\partial\phi}$ is the gradient operator, with $\hat{\phi} = (-\sin\phi, \cos\phi, 0)$, and $\hat{\theta} = (\cos\phi\cos\theta, -\sin\phi\cos\theta, -\sin\phi)$.

Further simplification emerges at larger spatial scales, $|\mathbf{r}-\mathbf{r}'| \gg \ell_{\rm tr}$. In this case Eqs. (3)–(6) can be reduced to diffusion equations [2,5], $\nabla^2\langle I\rangle = 0$, and $\nabla(-D\nabla\delta I + \mathbf{J}) = 0$. Here $D = \ell_{\rm tr}/3$ is the (real space) diffusion constant, and the correlation function for Langevin currents has the form: $\langle J_\alpha(\mathbf{r})J_\beta(\mathbf{r}')\rangle = \frac{\lambda^2\ell_{\rm tr}}{6\pi}\langle I(\mathbf{r})\rangle^2\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')$. In this case, the correlation function $\langle\delta I(\mathbf{r})\delta I(\mathbf{r}')\rangle$ decays as $1/r$ and as $1/r^2$ for $r \gg \ell_{\rm tr}$ and $\lambda \ll r \ll \ell_{\rm tr}$, respectively.

The scheme presented above can be generalized to treat speckle sensitivities to changes of external parameters, such as the wave number, Δk, or smooth changes of the refractive index, $\Delta n(\mathbf{r})$. We characterize the sensitivities by the correlation function $\langle\delta f(0)\delta f(\gamma)\rangle$, where $\gamma = \Delta k + k\Delta n(\mathbf{r})$. In this case the Langevin source correlator is given by

$$\langle\mathcal{L}(\mathbf{r},\mathbf{s};0)\mathcal{L}(\mathbf{r}',\mathbf{s}';\gamma)\rangle = \frac{\pi}{k^2}\delta(\mathbf{r}-\mathbf{r}') \times\sum_{\nu=\pm}\Big[\delta(\mathbf{s}-\mathbf{s}')f_\nu(\mathbf{r},\mathbf{s}) \times\int d^2\mathbf{s}_1 W(\mathbf{s}-\mathbf{s}_1)f_{-\nu}(\mathbf{r},\mathbf{s}_1) - f_\nu(\mathbf{r},\mathbf{s})W(\mathbf{s}-\mathbf{s}')f_{-\nu}(\mathbf{r},\mathbf{s}')\Big], \tag{10}$$

where $f_\pm(\mathbf{r},\mathbf{s})$ satisfies the equation

$$\mathbf{s}\cdot\nabla f_\pm(\mathbf{r},\mathbf{s}) - I_{\rm st}\{f_\pm(\mathbf{r},\mathbf{s})\} = \pm i\gamma f_\pm(\mathbf{r},\mathbf{s}). \tag{11}$$

The method based on Eqs. (3)–(11) is similar to the Langevin approach describing classical time and space fluctuations of a single particle distribution function [12–14]. The fundamental difference between our problem and that of classical fluctuations manifests itself in the form of the correlation function of the Langevin sources. In the

classical problem the Langevin sources are δ-correlated both in time and space and their variance is proportional to the average distribution function $\langle f\rangle$. In contrast, in Eqs. (6) and (10) the Langevin source variance is quadratic in $\langle f\rangle$ and δ-correlated only in space.

To illustrate the use of Eqs. (3)–(11) we consider the case when a plane wave of intensity I_0 is incident on a disordered slab of thickness Z, such that $\ell \ll Z \ll \ell_{\rm tr}$, as shown in the inset of Fig. 1. The results presented below are calculated to leading order in the small total scattering angle, $\theta^2 \equiv D_\theta Z$. In this case the correlation function $\mathcal{C}(\mathbf{r}-\mathbf{r}') = \langle \delta I(\mathbf{r})\delta I(\mathbf{r}')\rangle$ is strongly anisotropic [here $\delta I(\mathbf{r}) = I(\mathbf{r}) - \langle I(\mathbf{r})\rangle$]. Therefore below we shall use the notation: $\mathbf{r} = (z, \boldsymbol{\rho})$, where $\boldsymbol{\rho}$ denotes a two-component vector in the plane perpendicular to the z axis and z denotes the distance between observation points along the z axis. When $\rho = 0$, i.e., the observation points are located along the z axis, the correlation function for $z \ll Z$ is

$$\mathcal{C}(z) = I_0^2/(4k^2\theta^4 z^2). \tag{12}$$

When θ is of order unity, Eq. (12) matches the results for the case $Z \gg \ell_{\rm tr}$ and $z \ll \ell_{\rm tr}$ [2,5].

When $z < \rho/\theta$, i.e., the observation points are located essentially on a plane perpendicular to the z axis, a general formula for $\mathcal{C}(\boldsymbol{\rho})$, can be derived from Eqs. (7)–(9)

$$\mathcal{C}(\rho) = \frac{I_0^2}{4D_\theta k^2}\int_0^{Z-\ell}\frac{d\zeta}{\zeta - Z}\int_0^\infty dq q J_0(q\rho)\frac{d}{d\zeta} \times \exp\left[-\frac{2}{\ell}\int_0^\zeta d\eta\left\{1 - \tilde{g}\left(\frac{q}{k}\eta\right)\right\}\right], \tag{13}$$

where $\tilde{g}(\rho) = \int dz g(\sqrt{\rho^2 + z^2})/\int dz g(z)$. The integral in Eq. (13) contains a term proportional to a δ-function, $\frac{\pi I_0^2}{2D_\theta k^2 Z}\delta(\boldsymbol{\rho})$. This term represents the rapidly decaying (at $\rho \sim \lambda/\theta$) part of the correlator and corresponds to the part of diagram 2(b) without the impurity ladders after the Hikami box, see diagram 2(e). The δ-function term results from the semiclassical approximation employed in the derivation of Eqs. (3)–(11), which limits the spacial resolution to $\delta\rho \gg \lambda$. In order to resolve the spacial structure of the short distance part of the correlator diagram 2(e) needs to be evaluated more accurately. This gives the following asymptotic behavior;

$$\frac{\mathcal{C}(\rho)}{I_0^2} \approx \begin{cases} e^{-2(k\theta\rho)^2} & \text{if } \rho \sim \alpha\lambda/\theta, \\ \frac{b_1}{k^2\theta^2\theta_0\rho} & \text{if } \alpha\lambda/\theta \ll \rho \ll \ell\theta_0, \\ \frac{b_2 D_\theta^{2/3}}{k^2\theta^4\rho^{4/3}} & \text{if } \ell\theta_0 \ll \rho \ll \theta Z, \\ \frac{-b_3\rho^2}{k^2\theta^6 Z^4}e^{-(3\rho^2/8\theta^2 Z^2)} & \text{if } \theta Z \ll \rho \ll \frac{Z\theta^2}{\theta_0}, \end{cases} \tag{14}$$

where $\alpha^2 = \log(k\ell\theta^3/\theta_0)$, $b_1 = \int_0^\infty dx\tilde{g}(x)$ is a constant of order unity, $b_2 = 3^{1/3}\Gamma(5/3)/8 \approx 0.163$, and $b_3 = 27/128 \approx 0.21$. The tail of the correlation function (the regime $\rho > Z\theta^2/\theta_0$) is also described by Eq. (13) and depends on the precise form of the disorder correlator $g(r)$, since this limit is dominated by rare scattering events. The qualitative form of the function $\mathcal{C}(\rho)$ is shown in Fig. 1. Its power low decay in the regime $\ell\theta_0 \ll \rho \ll \theta Z$, follows from the classical (superdiffusive) ray spreading, $\rho^2 \sim D_\theta z^3$. Thus the density correlation between two points, separated by ρ, is generated by Langevin sources located at a distance $\Delta z \sim (\rho^2/D_\theta)^{1/3}$, from the screen. The flux emitted by these sources decays as $1/\Delta z^2$. This produces the power low correlations at the screen, $\mathcal{C}(\rho) \propto 1/\rho^{4/3}$.

Let us consider now the statistics of density, integrated over a disk of radius R, $P = \int_{\rho\le R} d^2\rho I(\boldsymbol{\rho})$. Using Eqs. (13) and (14) we get that the fluctuations of this quantity are characterized by

$$\frac{\langle \delta P^2\rangle}{I_0^2\pi R^2} \approx \begin{cases} \frac{\pi}{2k^2\theta^2} + \frac{b_1'(D_\theta R)}{k^2\theta^4\theta_0}, & \frac{\alpha\lambda}{\theta} \ll R \ll \ell\theta_0, \\ \frac{\pi}{2k^2\theta^2} + \frac{b_2'(D_\theta R)^{2/3}}{k^2\theta^4}, & \ell\theta_0 \ll R \ll \theta Z, \\ b_3'\frac{Z}{k^2 R\theta}, & \theta Z \ll R \ll \frac{Z\theta^2}{\theta_0}, \end{cases} \tag{15}$$

where $b_1' = 2b_1\pi/3$, $b_2' = 3^{4/3}\Gamma(5/6)\pi/2^{11/3}\Gamma(7/6)$, and $b_3' = \sqrt{3\pi/2}$.

Consider now the sensitivity of the integrated density $P(\omega)$ to a change in the wave frequency $\Delta\omega = c\Delta k$, where c is the speed of the wave. It can be characterized by the experimentally accessible quantity

$$\langle (P(\omega + \Delta\omega) - P(\omega))^2\rangle/\langle(\delta P)^2\rangle \approx (\Delta\omega)^2/(\omega^*)^2, \tag{16}$$

where $\omega^* = \sqrt{15/2}c/(\theta^2 Z)$.

A qualitative explanation of the scale ω^* is similar to that given for the sensitivity of the conductance fluctuations [17,18]. Let us estimate the characteristic change in the phase of a typical orbit due to the frequency change $\Delta\omega$: The typical length spread of the orbits is of order $\theta^2 Z$. Therefore the phase difference is $\Delta k Z\theta^2$ where $\Delta k = \Delta\omega/c$ is the change in the wave number. Thus a complete change of the speckle pattern occurs when the phase, $\Delta\omega Z\theta^2/c$ is of order one, namely $\Delta\omega \sim c/Z\theta^2$, in agreement with Ref. [10].

As another application of our scheme, let us consider the sensitivity of speckles to a change in the incidence angle, ϕ, of the wave; see the inset in Fig. 1 (thus the incident wave function now has the form $\psi = \sqrt{I_0}\exp[ikz\cos\phi + ik\rho\sin\phi]$). One may characterize this sensitivity by the correlation function

$$\frac{\langle \delta P(\phi)\delta P(0)\rangle}{\langle(\delta P)^2\rangle} \approx e^{-2/3(\phi^2/\phi^{*2})} + \frac{3(e^{-1/8(\phi^2/\theta^2)} - e^{-2/3(\phi^2/\phi^{*2})})}{8k^2\ell^4 D_\theta^2}, \tag{17}$$

where $\phi^* = (\theta k Z)^{-1}$ and it is assumed that $R \ll \theta Z$. The first term of this equation follows from formula (5) and the correlator (10), where $f_\pm$ satisfies Eq. (11) with initial conditions

$$f_{\pm}(\boldsymbol{\rho}, z = 0) = I_0 e^{\pm ik\mathbf{s}_\perp \boldsymbol{\rho}} \delta(\mathbf{s} - \mathbf{s}_0), \tag{18}$$

where $\mathbf{s}_0 = (\cos\phi, \mathbf{s}_\perp) \approx (1, \mathbf{s}_\perp)$, with $|\mathbf{s}_\perp| = \sin\phi \approx \phi$, assuming $\phi \ll 1$. The second term in the right-hand side of Eq. (17) is computed from the diagrams of the type shown in Fig. 2(f), containing two Hikami boxes. It represents a small correction in the parameter $\xi/\ell\theta_0$; however, this term becomes the dominant contribution when $\phi \gg \phi^*$.

Our results can be extended to cases with light polarization, optically active media, Faraday effect, and coherent short wave pulses as long as their duration is longer than $\tau = \ell/c$. These issues are left for future studies.

The problem considered here is similar to the problem of universal conductance fluctuations in metallic samples [17,19], which is also of interference nature. Therefore we would like to discuss the relation between the two problems. In the single particle approximation the conductance of a metallic sample, $G \sim \int d\phi T(\phi)$ can be expressed in terms of the electron transmission probability through the sample, $T(\phi) \propto \int d\boldsymbol{\rho} d^2 s (\mathbf{z} \cdot \mathbf{s}) f(\boldsymbol{\rho}, \mathbf{s})$, integrated over the incidence angle of the incoming wave, ϕ. Thus the variance of the conductance fluctuations, δG is proportional to a double integral of the correlation function $\langle \delta T(\phi) \delta T(\phi') \rangle$. In principle, the latter can be calculated using Eqs. (3)–(11), or, equivalently, by calculating diagrams shown in Figs. 2(b)–2(d). However, as we explain below, this does not account for the conductance fluctuations which arise from diagrams of the from shown in Fig. 2(f).

In the limit of directed waves, $Z \ll \ell_{\rm tr}$, there is no backscattering. Therefore the transmission probability does not fluctuate, $\delta T(\phi) \sim \delta G = 0$. Thus to compare the two problems we have to consider the diffusive case, $Z \gg l_{\rm tr}$, where $\delta G \sim e^2/\hbar$. The correlation function $\langle \delta T(\phi) \delta T(\phi') \rangle$, in the diffusive regime, still has a structure similar to the correlation function given by Eq. (17) [5]. Namely, it contains two contributions. The first contribution comes from diagrams 2(b)–2(d), and describes relatively strong fluctuations of the transmission coefficient. However, it is very sensitive to the change of ϕ, and after the integration over ϕ gives a small contribution to δG^2. The second contribution originates from diagrams of the type of Fig. 2(f), and is analogous to the second term in Eq. (17). Although its amplitude is smaller than that of the first term, it is insensitive to the change of ϕ, and after the integration over ϕ yields the dominant contribution to δG^2. Thus, conductance fluctuations are not described by Eqs. (3)–(11), and should be calculated from the diagrams of the type shown in Fig. 2(f) (see the corresponding discussion in Ref. [5]).

The results presented above substantially differ from those known in the literature [7–10]. First, the correlation function (13) exhibits a universal long-range power law behavior in a wide range of values of ρ. The only nonuniversal regimes are at the tail, $\rho \gg Z\theta^2/\theta_0$, and the short distance region, $\rho \sim \xi$. In contrast, in the results presented in Refs. [7–10] $\mathcal{C}(\rho)$ depends on the detailed form of $g(r)$, and usually decays exponentially at $\rho > \xi$. Second, in contrast with previous results, $\mathcal{C}(\rho)$ changes its sign as a function of ρ, which is a consequence of the current conservation. This conservation law also implies, that the fluctuations of the integrated intensity over disks of radius $R > Z\theta$ are proportional to R, see Eq. (15), rather than R^2, as would follow from Refs. [7–10]. The reason for these differences is that previous studies did not take into account the Hikami box diagrams shown in Fig. 2.

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