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Partial Dynamical Symmetry in Odd-Mass Nuclei

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Abstract. Spectral features of the odd-mass nucleus ^{195}Pt are analyzed by means of an interacting boson-fermion Hamiltonian with SO(6) partial dynamical symmetry. For the latter, selected eigenstates are solvable and preserve the symmetry exactly, while other states are mixed. The analysis constitutes a first example of this novel symmetry construction in a mixed Bose-Fermi system.

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1 Introduction

The concept of dynamical symmetry (DS) has been widely used to interpret nuclear structure. A given DS admits an analytic solution for all states of the system, with characteristic degeneracies, quantum numbers and selection rules. Familiar examples are the U(5), SU(3) and O(6) DSs of the interacting boson model (IBM [1]) of even-even nuclei, which encode the dynamics of spherical, axially-deformed and γ-unstable nuclear shapes. The majority of nuclei, however, exhibit strong deviations from these solvable benchmarks. More often one finds that the assumed symmetry is not obeyed uniformly, *i.e.*, is fulfilled by some of the states but not by others. The need to break the DSs, but still preserve important symmetry remnants, has led to the introduction of partial dynamical symmetry (PDS) [2]. For the latter case, only selected eigenstates of the Hamiltonian retain solvability and good symmetry, while other states are mixed. Various types of PDSs were proposed and algorithms for constructing Hamiltonians with such property have been developed [2–4]. Bosonic Hamiltonians with PDS have been applied to nuclear spectroscopy, where extensive tests provide empirical evidence for their relevance to a broad range of nuclei [4–11]. Fermionic shell model Hamiltonians with PDS have been applied to light nuclei [12, 13] and seniority isomers [14–17]. These empirical manifestations and further applications to nuclear shape-phase transitions [18, 19], suggest a more pervasive role of PDSs in nuclei than heretofore realized.

All examples of PDS considered so far, were confined to systems of a given statistics. In the present contribution, we consider an extension of the PDS concept to mixed systems of bosons and fermions [20], of relevance to odd-mass nuclei. As an example of such novel symmetry construction, spectral features of ^{195}Pt are analyzed in the framework of the interacting boson fermion model.

2 SO$^{\rm BF}$(6) Dynamical Symmetry Limit of the IBFM

The interacting boson fermion model (IBFM [21]) describes properties of low-lying states in odd-mass nuclei, in terms of N bosons ($b^\dagger_{\ell,m}$) with $\ell = 0$ ($s^\dagger$) and $\ell = 2$ ($d^\dagger_m$), representing valence nucleon pairs, and a single fermion ($a^\dagger_{j,m}$) in a shell model orbit with angular momentum j. In the current study, $j = 1/2, 3/2, 5/2$, which can be divided into a pseudo-orbital angular momentum ($\tilde{\ell} = 0, 2$) coupled to a pseudo-spin ($\tilde{s} = 1/2$). The $\tilde{\ell}$-$\tilde{s}$ and jm bases are related by $c^\dagger_{\tilde{\ell}\tilde{m}_\ell;\tilde{s}\tilde{m}_s} = \sum_{j,m} (\tilde{\ell}, \tilde{m}_\ell; \tilde{s}, \tilde{m}_s | j, m)\, a^\dagger_{jm}$. The bilinear combinations $\{b^\dagger_{\ell,m} b_{\ell',m'}\}$ and $\{a^\dagger_{j,m} a_{j',m'}\}$ span, respectively, bosonic (B) and fermionic (F) algebras, forming a spectrum generating algebra $\mathrm{U^B}(6) \otimes \mathrm{U^F}(12)$. The IBFM Hamiltonian is expanded in terms of these generators and consists of Hermitian rotational-invariant interactions which conserve the total number of bosons, $\hat{N} = s^\dagger s + \sum_m d^\dagger_m d_m$, and of fermions $\hat{n} = \sum_{j,m} a^\dagger_{j,m} a_{j,m}$.

There exist several strategies to define DSs with $\mathrm{U^B}(6) \otimes \mathrm{U^F}(12)$ as a starting point [21]. They all define a chain of nested subalgebras, relying on the existence of isomorphisms between boson and fermion algebras and ending in the symmetry algebra. Here we focus on the $\mathrm{SO^{BF}}(6)$ DS limit of the model, corresponding to the classification:

$$
\begin{array}{cc}
\mathrm{U^B}(6) \otimes & \mathrm{U^F}(12) \\
\downarrow & \downarrow \\
[N] & [1]
\end{array}
$$

$$
\begin{array}{cccccccccccc}
\supset \Big(\mathrm{U^{BF}}(6) & \supset & \mathrm{SO^{BF}}(6) & \supset & \mathrm{SO^{BF}}(5) & \supset & \mathrm{SO^{BF}}(3) \Big) & \otimes & \mathrm{SU^F}(2) & \supset & \mathrm{Spin^{BF}}(3) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
[N_1, N_2] & & \langle \sigma_1, \sigma_2 \rangle & & (\tau_1, \tau_2) & & L & & \tilde{s} & & J
\end{array}
\tag{1}
$$

where underneath each algebra (G) the associated labels of the irreducible representations (irreps) are indicated. The indicated Bose-Fermi algebra $G^{\rm BF}$ is the direct sum of $G^{\rm B}$ and $G^{\rm F}$.

The eigenstates (1) are obtained with a Hamiltonian that is a combination of Casimir operators $\hat{C}_k[G]$ of order k of an algebra G appearing in the chain. Up to a constant energy, this Hamiltonian is of the form

$$\hat{H}_{\rm DS} = a\,\hat{C}_2[{\rm U}^{\rm BF}(6)] + b\,\hat{C}_2[{\rm SO}^{\rm BF}(6)] + c\,\hat{C}_2[{\rm SO}^{\rm BF}(5)] + d\,\hat{C}_2[{\rm SO}^{\rm BF}(3)] + d'\hat{C}_2[{\rm Spin}^{\rm BF}(3)]~. \tag{2}$$

Explicit expressions for the above Casimir operators are given in Table 1. The associated eigenvalue problem is analytically solvable, leading to the energy expression

$$E_{\rm DS} = a\,[N_1(N_1+5)+N_2(N_2+3)] + b\,[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)] + c\,[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + d\,L(L+1) + d'J(J+1)~. \tag{3}$$

The energy spectrum of the Hamiltonian (2) is then determined once the allowed values of $[N_1,N_2]$, $\langle\sigma_1,\sigma_2\rangle$, (τ_1,τ_2), L, and J for a given N are found. Such branching rules can be obtained with standard group-theoretical techniques [21]. The lowest-lying states in the spectrum of an odd-mass nucleus, described in terms of N bosons and one fermion, can be classified as $|[N+1]\langle N+1\rangle(\tau)LJM_J\rangle$ with $\tau = 0,1,\ldots N+1$. The next class of states belongs to $|[N,1]\langle N,1\rangle(\tau_1,\tau_2)LJM_J\rangle$ with $(\tau_1,\tau_2) = (\tau,0)$ or $(\tau,1)$ and $\tau = 1,2,,...N$. There is also some evidence from one-neutron transfer for $|[N,1]\langle N-1\rangle(\tau)LJM_J\rangle$ states [22], with $\tau = 0,1,\ldots N-1$. The L-values of these states are obtained from the known $SO(5)\supset SO(3)$ branching rules [21] and $J = L\pm 1/2$.

Table 1. Generators and Casimir operators, $\hat{C}_k[G]$, of order $k = 1,2$, for the Bose-Fermi algebras in Eq. (2). The generators are sums of bosonic: $(b^{\dagger}_{\ell}\tilde{b}_{\ell'})^{(L)}$, and fermionic operators: $K^{(\tilde{L})}_m(\tilde{\ell},\tilde{\ell}') = \sqrt{2}(c^{\dagger}_{\tilde{\ell},1/2}\,\tilde{c}_{\tilde{\ell}',1/2})^{(\tilde{L},0)}_{m,0}$, where $\tilde{b}_{\ell m_\ell} \equiv (-)^{\ell+m_\ell}b_{\ell,-m_\ell}$ and $\tilde{a}_{jm_j} \equiv (-)^{j+m_j}a_{j,-m_j}$. Here, $\hat{M}_0 = s^{\dagger}s + K^{(0)}_m(0,0)$, $\Pi^{(2)}_m = d^{\dagger}_m s + s^{\dagger}\tilde{d}_m + K^{(2)}_m(2,0) + K^{(2)}_m(0,2)$, $\bar{\Pi}^{(2)}_m = i[d^{\dagger}_m s - s^{\dagger}\tilde{d}_m + K^{(2)}_m(2,0) - K^{(2)}_m(0,2)]$, $U^{(\rho)}_m = (d^{\dagger}\,\tilde{d})^{(\rho)}_m + K^{(\rho)}_m(2,2)$, $\hat{L}_m = \sqrt{10}[U^{(1)}_m + K^{(1)}_m(2,2)]$ and $\hat{J}_m = \hat{L}_m + \hat{S}_m$, where $\hat{S}_m = \sum_{\tilde{\ell}=0,2}\sqrt{2\tilde{\ell}+1}(c^{\dagger}_{\tilde{\ell},1/2}\,\tilde{c}_{\tilde{\ell},1/2})^{(0,1)}_{0,m}$ are the pseudo-spin generators of SU^F(2). The boson- and fermion number operators are $\hat{N} = s^{\dagger}s + \sqrt{5}(d^{\dagger}\tilde{d})^{(0)}$ and $\hat{n} = K^{(0)}_m(0,0) + \sqrt{5}K^{(0)}_m(2,2)$, respectively.

Algebra	Generators and Casimir operators $\hat{C}_k[G]$
${\rm U}^{\rm BF}(6)$	$\hat{M}_0$, $\Pi^{(2)}$, $\bar{\Pi}^{(2)}$, ${\rm U}^{(\rho)}$ $\rho = 0{-}4$; $\hat{C}_1[{\rm U}^{\rm BF}(6)] = \hat{N} + \hat{n}$
	$\hat{C}_2[{\rm U}^{\rm BF}(6)] = \hat{M}_0^2 + \frac{1}{2}\Pi^{(2)}\cdot\Pi^{(2)} + \frac{1}{2}\bar{\Pi}^{(2)}\cdot\bar{\Pi}^{(2)} + \sum_{\rho=0\text{–}4}U^{(\rho)}\cdot U^{(\rho)}$
${\rm SO}^{\rm BF}(6)$	$\Pi^{(2)}$, ${\rm U}^{(1)}$, ${\rm U}^{(3)}$; $\hat{C}_2[{\rm SO}^{\rm BF}(6)] = 2\sum_{\rho=1,3}U^{(\rho)}\cdot U^{(\rho)} + \Pi^{(2)}\cdot\Pi^{(2)}$
${\rm SO}^{\rm BF}(5)$	${\rm U}^{(1)}$, ${\rm U}^{(3)}$; $\hat{C}_2[{\rm SO}^{\rm BF}(5)] = 2\sum_{\rho=1,3}U^{(\rho)}\cdot U^{(\rho)}$
${\rm SO}^{\rm BF}(3)$	$\hat{L}_m$; $\hat{C}_2[{\rm SO}^{\rm BF}(3)] = \hat{L}\cdot\hat{L}$
${\rm Spin}^{\rm BF}(3)$	$\hat{J}_m$; $\hat{C}_2[{\rm Spin}^{\rm BF}(3)] = \hat{J}\cdot\hat{J}$

3 SO$^{\rm BF}$(6) Partial Dynamical Symmetry in the IBFM

While $\hat{H}_{\rm DS}$ (2) is *completely* solvable, the question arises whether terms can be added that preserve solvability for *part* of its spectrum. This can be achieved by the construction of a PDS.

The algorithm to construct an Hamiltonian with a PDS is based on its expansion, $\hat{H}' = \sum_{\alpha,\beta} u_{\alpha\beta}\, \hat{B}^\dagger_\alpha \hat{B}_\beta$, in terms of tensors which annihilate prescribed set of states [3, 4]. The tensors of interest in the present study, are listed in Table 2. They are composed of two-particle operators (either two bosons or a boson-fermion pair), and have definite character under the chain (1), $B^\dagger_{[N_1,N_2]\langle\sigma_1,\sigma_2\rangle(\tau_1,\tau_2)\mathcal{L}(\mathcal{J})} \equiv \mathcal{T}^{\mathcal{L}(\mathcal{J})}_{+,M_\mathcal{J}}$. The corresponding annihilation operators with the correct tensor properties follow from $\tilde{\mathcal{T}}^{\mathcal{L}(\mathcal{J})}_{-,M_\mathcal{J}} = (-)^{\mathcal{J}+M_\mathcal{J}} \left(\mathcal{T}^{\mathcal{L}(\mathcal{J})}_{+,-M_\mathcal{J}}\right)^\dagger$, where $\mathcal{T} = \mathcal{U}$ or $\mathcal{V}$. All these operators annihilate particular states, hence lead to a PDS of some kind. For example, the operators with U$^{\rm BF}$(6) labels $[N_1, N_2] = [1, 1]$ satisfy

$$\tilde{\mathcal{U}}^{\mathcal{L}(\mathcal{J})}_{-,M_\mathcal{J}} |[N+1]\langle\sigma\rangle(\tau)LJM_J\rangle = 0\,, \tag{4}$$

for all permissible $(\sigma\tau LJM_J)$. This is so because a state with $N-1$ bosons and no fermion has the U$^{\rm BF}$(6) label $[N-1]$. Given the multiplication rule $[N-1]\otimes[1,1] = [N,1]\oplus[N-1,1,1]$, the action of a $\mathcal{U}^{\mathcal{L}(\mathcal{J})}_{+,-M_\mathcal{J}}$ operator on an $(N-1)$-boson state can never yield a boson-fermion state with the U$^{\rm BF}$(6) labels

Table 2. Two-particle tensor operators in the SO$^{\rm BF}$(6) limit. For the Bose-Fermi pairs, the superscript $\mathcal{L}(\mathcal{J})$ stands for the coupling $\mathcal{J} = \mathcal{L} \pm 1/2$.

$B^\dagger_{[N_1,N_2]\langle\sigma_1,\sigma_2\rangle(\tau_1,\tau_2)\mathcal{L}(\mathcal{J})} \equiv \mathcal{T}^{\mathcal{L}(\mathcal{J})}_{+,M_\mathcal{J}}$		
$B^\dagger_{[2,0]\langle0,0\rangle(0,0)0(0)}$	$\equiv \mathcal{V}^{0(0)}_{+}$	$= \sqrt{\frac{5}{12}}(d^\dagger d^\dagger)^{(0)}_0 - \sqrt{\frac{1}{12}}(s^\dagger s^\dagger)^{(0)}_0$
$B^\dagger_{[2,0]\langle0,0\rangle(0,0)0(1/2)}$	$\equiv \mathcal{V}^{0(1/2)}_{+,\mu}$	$= -\sqrt{\frac{1}{6}}(s^\dagger a^\dagger_{1/2})^{(1/2)}_\mu - \sqrt{\frac{1}{3}}(d^\dagger a^\dagger_{3/2})^{(1/2)}_\mu + \sqrt{\frac{1}{2}}(d^\dagger a^\dagger_{5/2})^{(1/2)}_\mu$
$B^\dagger_{[1,1]\langle1,1\rangle(1,1)1(1/2)}$	$\equiv \mathcal{U}^{1(1/2)}_{+,\mu}$	$= \sqrt{\frac{3}{5}}(d^\dagger a^\dagger_{3/2})^{(1/2)}_\mu + \sqrt{\frac{2}{5}}(d^\dagger a^\dagger_{5/2})^{(1/2)}_\mu$
$B^\dagger_{[1,1]\langle1,1\rangle(1,1)1(3/2)}$	$\equiv \mathcal{U}^{1(3/2)}_{+,\mu}$	$= -\sqrt{\frac{3}{10}}(d^\dagger a^\dagger_{3/2})^{(3/2)}_\mu + \sqrt{\frac{7}{10}}(d^\dagger a^\dagger_{5/2})^{(3/2)}_\mu$
$B^\dagger_{[1,1]\langle1,1\rangle(1,1)2(3/2)}$	$\equiv \mathcal{U}^{2(3/2)}_{+,\mu}$	$= \sqrt{\frac{1}{2}}(s^\dagger a^\dagger_{3/2})^{(3/2)}_\mu - \sqrt{\frac{1}{2}}(d^\dagger a^\dagger_{1/2})^{(3/2)}_\mu$
$B^\dagger_{[1,1]\langle1,1\rangle(1,1)2(5/2)}$	$\equiv \mathcal{U}^{2(5/2)}_{+,\mu}$	$= \sqrt{\frac{1}{2}}(s^\dagger a^\dagger_{5/2})^{(5/2)}_\mu - \sqrt{\frac{1}{2}}(d^\dagger a^\dagger_{1/2})^{(5/2)}_\mu$
$B^\dagger_{[1,1]\langle1,1\rangle(1,1)3(5/2)}$	$\equiv \mathcal{U}^{3(5/2)}_{+,\mu}$	$= \sqrt{\frac{4}{5}}(d^\dagger a^\dagger_{3/2})^{(5/2)}_\mu + \sqrt{\frac{1}{5}}(d^\dagger a^\dagger_{5/2})^{(5/2)}_\mu$
$B^\dagger_{[1,1]\langle1,1\rangle(1,1)2(7/2)}$	$\equiv \mathcal{U}^{3(7/2)}_{+,\mu}$	$= -\sqrt{\frac{1}{10}}(d^\dagger a^\dagger_{3/2})^{(7/2)}_\mu + \sqrt{\frac{9}{10}}(d^\dagger a^\dagger_{5/2})^{(7/2)}_\mu$

$[N+1]$. Similar arguments involving SO(6) multiplication lead to the following properties for the $\mathcal{V}$ operators which have SO(6) tensor character $\langle 0,0\rangle$:

$$\tilde{\mathcal{V}}^{0(\mathcal{J})}_{-,M_{\mathcal{J}}}|[N+1]\langle N+1\rangle(\tau)LJM_J\rangle = 0\ , \tag{5a}$$

$$\tilde{\mathcal{V}}^{0(\mathcal{J})}_{-,M_{\mathcal{J}}}|[N,1]\langle N,1\rangle(\tau_1,\tau_2)LJM_J\rangle = 0\ . \tag{5b}$$

Normal-ordered interactions with PDS can now be constructed out of the $\mathcal{T}$-operators in Table 2, as

$$\begin{aligned}\hat{H}' = {} & x^{0}_{00}\,(\mathcal{V}^{0(0)}_{+}\tilde{\mathcal{V}}^{0(0)}_{-})^{(0)} + x^{1/2}_{00}\,\sqrt{2}\,(\mathcal{V}^{0(1/2)}_{+}\tilde{\mathcal{V}}^{0(1/2)}_{-})^{(0)}\\ & + \sum_{\mathcal{L}\mathcal{L}'\mathcal{J}} x^{\mathcal{J}}_{\mathcal{L}\mathcal{L}'}\,\sqrt{2\mathcal{J}+1}\,[(\mathcal{U}^{\mathcal{L}(\mathcal{J})}\tilde{\mathcal{U}}^{\mathcal{L}'(\mathcal{J})}_{-})^{(0)} + \text{H.c.}]\\ & + x^{1/2}_{10}\,\sqrt{2}\,[(\mathcal{U}^{1(1/2)}_{+}\tilde{\mathcal{V}}^{0(1/2)}_{-})^{(0)} + \text{H.c.}]\ ,\end{aligned} \tag{6}$$

where H.c. stands for Hermitian conjugate. Particular linear combinations of terms in Eq. (6) yield the Casimir operators in $\hat{H}_{\rm DS}$, Eq. (2). Specifically, the quadratic Casimir operator of $\mathrm{U^{BF}}(6)$ is obtained for

$$\begin{aligned}\hat{\mathcal{M}}_6 &\equiv (\hat{N}+\hat{n})(\hat{N}+\hat{n}+5) - \hat{C}_2[\mathrm{U^{BF}}(6)]\\ &= 2\left[\hat{U}^{1/2}_{1} + \hat{U}^{3/2}_{1} + \hat{U}^{3/2}_{2} + \hat{U}^{5/2}_{2} + \hat{U}^{5/2}_{3} + \hat{U}^{7/2}_{3}\right]\ ,\end{aligned} \tag{7}$$

where $\hat{U}^{\mathcal{J}}_{\mathcal{L}} \equiv \sqrt{2\mathcal{J}+1}(\mathcal{U}^{\mathcal{L}(\mathcal{J})}_{+}\tilde{\mathcal{U}}^{\mathcal{L}(\mathcal{J})}_{-})^{(0)}_{0}$, and the quadratic Casimir of $\mathrm{SO^{BF}}(6)$ is obtained for

$$(\hat{N}+\hat{n})(\hat{N}+\hat{n}+4) - \hat{C}_2[\mathrm{SO^{BF}}(6)] = \hat{\mathcal{M}}_6 + 12\left[\hat{V}^{0}_{0} + \hat{V}^{1/2}_{0}\right]\ , \tag{8}$$

where $\hat{V}^{\mathcal{J}}_{\mathcal{L}} \equiv \sqrt{2\mathcal{J}+1}(\mathcal{V}^{\mathcal{L}(\mathcal{J})}_{+}\tilde{\mathcal{V}}^{\mathcal{L}(\mathcal{J})}_{-})^{(0)}_{0}$. In general, $\hat{H}'$ of Eq. (6) is not invariant under $\mathrm{U^{BF}}(6)$ nor $\mathrm{SO^{BF}}(6)$, yet the relations in Eqs. (4)-(5) ensure that a specific band of states will remain solvable with good $\mathrm{U^{BF}}(6)$ and $\mathrm{SO^{BF}}(6)$ quantum numbers $[N+1]\langle N+1\rangle$. The combined effect of adding $\hat{H}'$ to the DS Hamiltonian (2), $\hat{H}_{\rm PDS} = \hat{H}_{\rm DS} + \hat{H}'$, gives rise to a rich variety of Hamiltonians with PDS, for which only selected states are solvable with good symmetry, while other states are mixed.

4 A Case Study: $\mathrm{SO^{BF}}$(6) PDS in ^{195}Pt

The SO(6) limit of the interacting boson model [23] is known to be of relevance for the even-even platinum isotopes [24]. Accordingly, the classification (1) is proposed in the context of the IBFM to describe odd-mass isotopes of platinum with the odd neutron in the orbits $3p_{1/2}$, $3p_{3/2}$, and $2f_{5/2}$, which are dominant

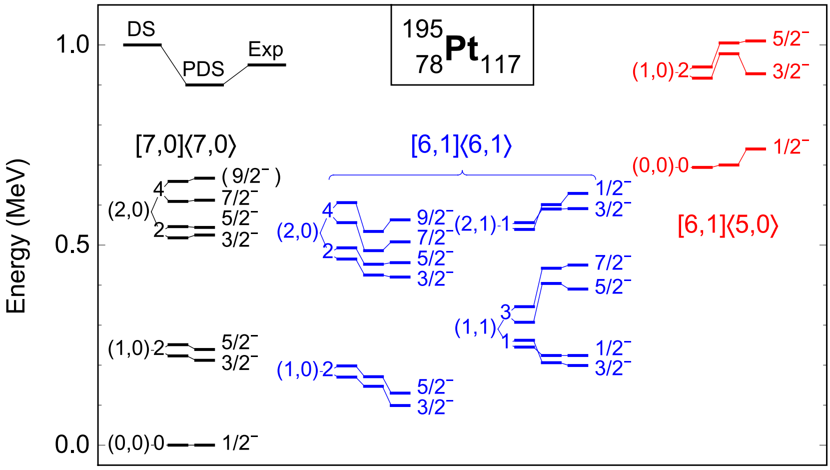

Figure 1. (Color online). Observed and calculated energy spectrum of ^{195}Pt. The levels in black are the solvable $[7,0]\langle 7,0\rangle$ eigenstates of $\hat{H}_{\rm DS}$ (2), whose structure and energy remain unaffected by the added PDS interactions in Eq. (9). The levels in blue (red) are the $[6,1]\langle 6,1\rangle$ ($[6,1]\langle 5,0\rangle$) eigenstates of $\hat{H}_{\rm DS}$ (2) and are subsequently perturbed by the PDS interactions in Eq. (9). Adapted from [20].

for these isotopes [25, 26]. In the current application of PDS to ^{195}Pt, we take a restricted Hamiltonian which, in the notation of Eqs. (7)-(8), has the form

$$\begin{aligned}\hat{H}_{\rm PDS} &= \hat{H}_{\rm DS} + a_0 \hat{V}_0^{1/2} + a_1'(2\hat{U}_1^{1/2} - \hat{U}_1^{3/2}) \\ &\quad + a_2(\hat{U}_2^{3/2} + \hat{U}_2^{5/2}) + a_3(\hat{U}_3^{5/2} + \hat{U}_3^{7/2})\ , \end{aligned} \tag{9}$$

and $N = 6$. These interactions can be transcribed as tensors with total pseudo-orbital $\tilde{L}$ and pseudo-spin $\tilde{S}$ coupled to zero total angular momentum. In particular, the a_1' term in Eq. (9) has $\tilde{L} = \tilde{S} = 1$, while the a_0, a_2 and a_3 terms have $\tilde{L} = \tilde{S} = 0$.

The experimental spectrum of ^{195}Pt is shown in Figure 1, compared with the DS and PDS calculations. The coefficients c, d, and d' in $\hat{H}_{\rm DS}$ (2) are adjusted to the excitation energies of the $[7,0]\langle 7,0\rangle$ levels which are reproduced with a root-mean-square (rms) deviation of 12 keV. The remaining two coefficients a and b are obtained from an overall fit. The resulting (DS) values are (in keV): $a = 45.3$, $b = -41.5$, $c = 49.1$, $d = 1.7$, and $d' = 5.6$. The fit for the PDS calculation proceeds in stages. First, the parameters c, d, and d' in Eq. (2) are taken at their DS values. This ensures the same spectrum for the $[7,0]\langle 7,0\rangle$ levels (drawn in black in Figure 1) which remain eigenstates of $\hat{H}_{\rm PDS}$ (9). Next, one considers the $[6,1]\langle 6,1\rangle$ levels and improves their description by adding the three PDS U interactions. The resulting coefficients are (in keV): $a_1' = 10$, $a_2 = -97$, and $a_3 = 112$. Eq. (4) ensures that the energies of the $[7,0]\langle 7,0\rangle$ levels do

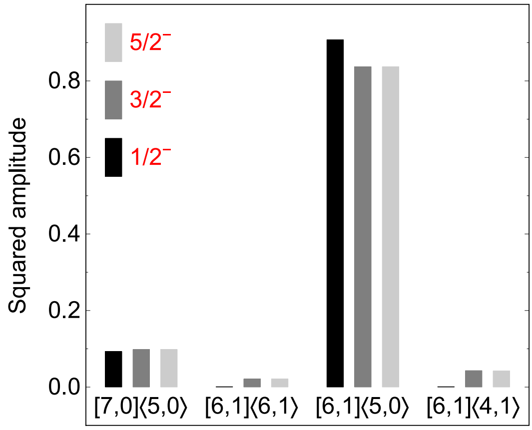

Figure 2. (Color online). $\mathrm{SO}^{\mathrm{BF}}(6)$ decomposition of the eigenstates of $\hat{H}_{\mathrm{PDS}}$ (9), shown in red in Figure 1. Adapted from [20].

not change while the agreement for the $[6,1]\langle 6,1\rangle$ levels is improved (blue levels in Figure 1). The rms deviation for both classes of levels is 20 keV. In particular, unlike in the DS calculation, it is possible to reproduce the observed inversion of the $1/2^{-}$-$3/2^{-}$ doublets without changing the order of other doublets. The additional PDS terms necessitate a readjustment of the a coefficient in Eq. (2), for which the final (PDS) value is $a = 37.7$ keV, while the coefficient b is kept unchanged. Finally, the position of the $[6,1]\langle 5,0\rangle$ levels (red levels in Figure 1) is corrected by considering the PDS V interaction with $a_0 = 306$ keV which, due to Eq. (5), has a marginal effect on lower bands. As seen in Figure 1, the agreement is very good for yrast and non-yrast levels. As shown in Figure 2, while the states $[7,0]\langle 7,0\rangle$ of the ground band are pure, other eigenstates of $\hat{H}_{\mathrm{PDS}}$ in excited bands can have substantial $\mathrm{SO}^{\mathrm{BF}}(6)$ mixing.

A large amount of information also exists on electromagnetic transition rates and spectroscopic strengths. In Table 3, 25 measured B(E2) values in ^{195}Pt are compared with the DS and PDS predictions. The same E2 operator is used as in previous studies [27, 28] of the $\mathrm{SO}^{\mathrm{BF}}(6)$ limit, $\hat{T}_{\mu}(\mathrm{E2}) = e_{\mathrm{b}}\hat{Q}^{\mathrm{B}}_{\mu} + e_{\mathrm{f}}\hat{Q}^{\mathrm{F}}_{\mu}$, where $\hat{Q}^{\mathrm{B}}_{\mu} = s^{\dagger}\tilde{d}_{\mu} + d^{\dagger}_{\mu}s$ is the boson quadrupole operator, $\hat{Q}^{\mathrm{F}}_{\mu} = K^{(2)}(2,0) + K^{(2)}(0,2)$ is its fermion analogue (see Table 1), and e_{b} and e_{f} are effective boson and fermion charges, with the values $e_{\mathrm{b}} = -e_{\mathrm{f}} = 0.151$ eb. Table 3 is subdivided in four parts according to whether the initial and/or final state in the transition has a DS structure (as in Refs. [27, 28]) or whether it is mixed by the PDS interaction. It is seen that when both have a DS structure the B(E2) value does not change, only slight differences occur when either the initial or the final state is mixed, and the biggest changes arise when both are mixed.

Table 3. Observed $B(\mathrm{E2}; J_\mathrm{i} \to J_\mathrm{f})$ values between negative-parity states in ^{195}Pt compared with the DS and PDS predictions of the $\mathrm{SO^{BF}}(6)$ limit. The solvable (mixed) states are members of the ground (excited) bands shown in Figure 1. Adapted from [20].

E_i (keV)	J_i	E_f (keV)	J_f	$B(\mathrm{E2}; J_\mathrm{i} \to J_\mathrm{f})$ $(10^{-3}\ e^2\mathrm{b}^2)$		
				Exp	DS	PDS
			Solvable $\to$ solvable			
212	$3/2$	0	$1/2$	$190(10)$	179	179
239	$5/2$	0	$1/2$	$170(10)$	179	179
525	$3/2$	0	$1/2$	$17(1)$	0	0
525	$3/2$	239	$5/2$	≤ 19	72	72
544	$5/2$	0	$1/2$	$8(4)$	0	0
612	$7/2$	212	$3/2$	$170(70)$	215	215
667	$9/2$	239	$5/2$	$200(40)$	239	239
			Solvable $\to$ mixed			
239	$5/2$	99	$3/2$	$60(20)$	0	0
525	$3/2$	99	$3/2$	≤ 33	7	3
525	$3/2$	130	$5/2$	$9(5)$	3	2
612	$7/2$	99	$3/2$	$5(3)$	9	11
667	$9/2$	130	$5/2$	$12(3)$	10	12
			Mixed $\to$ solvable			
99	$3/2$	0	$1/2$	$38(6)$	35	34
130	$5/2$	0	$1/2$	$66(4)$	35	33
420	$3/2$	0	$1/2$	$15(1)$	0	0
456	$5/2$	0	$1/2$	≤ 0.04	0	0
508	$7/2$	212	$3/2$	$55(17)$	20	18
563	$9/2$	239	$5/2$	$91(22)$	22	22
199	$3/2$	0	$1/2$	$25(2)$	0	0
390	$5/2$	0	$1/2$	$7(1)$	0	0
			Mixed $\to$ mixed			
420	$3/2$	99	$3/2$	$5(4)$	177	165
508	$7/2$	99	$3/2$	$240(50)$	228	263
563	$9/2$	130	$5/2$	$240(40)$	253	284
390	$5/2$	99	$3/2$	$200(70)$	219	179
390	$5/2$	130	$5/2$	≤ 14	55	35

5 PDS and Intrinsic States

An alternative way of constructing Hamiltonians with PDS for an algebra G, is to identify n-particle operators which annihilate a lowest-weight state of a prescribed G-irrep [3]. In the IBFM, such a state, which transforms as $[N+1]$ and $\tilde{s} = 1/2$ under $\mathrm{U^{BF}}(6) \otimes \mathrm{SU^{F}}(2)$, is given by

$$|\Psi_\mathrm{g}\rangle \propto [b_c^\dagger(\beta)]^N f_{\tilde{m}_s}^\dagger(\beta)|0\rangle \ , \tag{10}$$

where $b_c^\dagger(\beta) \propto (\beta\, d_0^\dagger + s^\dagger)$ and $f_{\tilde{m}_s}^\dagger(\beta) \propto (\beta\, c_{2,0;1/2,\tilde{m}_s}^\dagger + c_{0,0;1/2,\tilde{m}_s}^\dagger)$ in the $\tilde{\ell}$-$\tilde{s}$ basis defined above. $|\Psi_{\rm g}\rangle$ is a condensate of N bosons and a single fermion, and represents an intrinsic state for the ground band with deformation β. The Hermitian conjugate of the following two-particle operators

$$\mathcal{V}_{+}^{0(0)} \propto \sqrt{5}(d^\dagger d^\dagger)_0^{(0)} - \beta^2 (s^\dagger s^\dagger)_0^{(0)}, \tag{11a}$$

$$\mathcal{V}_{+,\mu}^{0(1/2)} \propto \sqrt{5}(d^\dagger c_{2;1/2}^\dagger)_\mu^{0(1/2)} - \beta^2 (s^\dagger c_{0;1/2}^\dagger)_\mu^{0(1/2)}, \tag{11b}$$

$$\mathcal{U}_{+,\mu}^{\mathcal{L}(\mathcal{J})} \propto (d^\dagger c_{2;1/2}^\dagger)_\mu^{\mathcal{L}(\mathcal{J})}, \quad \mathcal{L} = 1, 3, \tag{11c}$$

$$\mathcal{U}_{+,\mu}^{2(\mathcal{J})} \propto (s^\dagger c_{2;1/2}^\dagger)_\mu^{2(\mathcal{J})} - (d^\dagger c_{0;1/2}^\dagger)_\mu^{2(\mathcal{J})}, \tag{11d}$$

satisfy $\tilde{\mathcal{T}}_{-,\mu}^{\mathcal{L}(\mathcal{J})}|\Psi_{\rm g}\rangle = 0$. The $\mathcal{V}$ operators of Eqs. (11a)-(11b) satisfy also $\tilde{\mathcal{V}}_{-,\mu}^{0(\mathcal{J})}|\Psi_{\rm e}\rangle = 0$, where

$$|\Psi_{\rm e}\rangle \propto [b_c^\dagger(\beta)\, c_{2,1;1/2,\tilde{m}_s}^\dagger - d_1^\dagger\, f_{\tilde{m}_s}^\dagger(\beta)][b_c^\dagger(\beta)]^{N-1}|0\rangle \tag{12}$$

is an intrinsic state, with $\mathrm{U^{BF}}(6)$ label $[N,1]$, representing an excited band in the odd-mass nucleus. For $\beta = 1$, $|\Psi_{\rm g}\rangle$ and $|\Psi_{\rm e}\rangle$ become the lowest-weight states in the $\mathrm{SO^{BF}}(6)$ irreps $\langle N+1\rangle$ and $\langle N,1\rangle$, respectively, from which the $|(\tau_1,\tau_2)LJM_J\rangle$ states of Eq. (5) can be obtained by $\mathrm{SO^{BF}}(5)$ projection, and the operators (11) coincide with those listed in Table 2.

In case of axially-symmetric shapes, $\mathrm{SO^{BF}}(5)$ is no longer a conserved symmetry and the following additional operators can contribute to Hamiltonians with other types of PDS,

$$\mathcal{W}_{+,\mu}^{2(2)} \propto \sqrt{2}\beta s^\dagger d_\mu^\dagger + \sqrt{7}(d^\dagger d^\dagger)_\mu^{(2)}, \tag{13a}$$

$$\mathcal{W}_{+,\mu}^{2(\mathcal{J})} \propto \beta(s^\dagger c_{2;1/2}^\dagger)_\mu^{2(\mathcal{J})} + \beta(d^\dagger c_{0;1/2}^\dagger)_\mu^{2(\mathcal{J})} + \sqrt{14}(d^\dagger c_{2;1/2}^\dagger)_\mu^{2(\mathcal{J})}. \tag{13b}$$

The above operators contain a mixture of components with different $\mathrm{SO^{BF}}(5)$ character ($\tau = 1,2$), and annihilate the intrinsic states of Eqs. (10) and (12). The solvable states are now obtained by angular momentum $\mathrm{Spin^{BF}}(3)$ projection. The operators in Eqs. (11) and (13) are the Bose-Fermi analog of the proton-neutron boson-pair operators comprising the intrinsic part of the IBM-2 Hamiltonian [29], and used in the study of F-spin PDS [30].

6 Summary and Outlook

We have considered an extension of the PDS notion to Bose-Fermi systems and exemplified it in ^{195}Pt. The analysis highlights the ability of a PDS to select and add to the Hamiltonian, in a controlled fashion, required symmetry-breaking terms, yet retain a good DS for a segment of the spectrum. These virtues greatly enhance the scope of applications of algebraic modeling of complex systems. The operators of Eqs. (11) and (13) can be used to explore additional PDSs in odd-mass nuclei, *e.g.*, $\mathrm{SU^{BF}}(3)$ PDS for $\beta = \sqrt{2}$. Partial supersymmetry, of relevance to nuclei [31], can be studied by embedding $\mathrm{U^B}(6) \otimes \mathrm{U^F}(12)$ in a graded Lie algebra. Work in these directions is in progress.

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