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Gravitational Radiation from Colliding Black Holes

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It is shown that there is an upper bound to the energy of the gravitational radiation emitted when one collapsed object captures another. In the case of two objects with equal masses m and zero intrinsic angular momenta, this upper bound is $(2-\sqrt{2})m$.

Weber[1-3] has recently reported coinciding measurements of short bursts of gravitational radiation at a frequency of 1660 Hz. These occur at a rate of about one per day and the bursts appear to be coming from the center of the galaxy. It seems likely[3,4] that the probability of a burst causing a coincidence between Weber's detectors is less than $\frac{1}{10}$. If one allows for this and assumes that the radiation is broadband, one finds that the energy flux in gravitational radiation must be at least 10^{10} erg/cm^2 day.[4] This would imply a mass loss from the center of the galaxy of about $20\,000 M_\odot$/yr. It is therefore possible that the mass of the galaxy might have been considerably higher in the past than it is now.[5] This makes it important to estimate the efficiency with which rest-mass energy can be converted into gravitational radiation. Clearly nuclear reactions are insufficient since they release only about 1% of the rest mass. The efficiency might be higher in either the nonspherical gravitational collapse of a star or the collision and coalescence of two collapsed objects. Up to now no limits on the efficiency of the processes have been known. The object of this Letter is to show that there is a limit for the second process. For the case of two colliding collapsed objects, each of mass m and zero angular momentum, the amount of energy that can be carried away by gravitational or any other form of radiation is less than $(2-\sqrt{2})m$.

I assume the validity of the Carter-Israel conjucture[6,7] that the metric outside a collapsed object settles down to that of one of the Kerr family of solutions[8] with positive mass m and angular momentum a per unit mass less than or equal to m. (I am using units in which $G=c=1$.) Each of these solutions contains a nonsingular *event horizon*, two-dimensional sections of which are topographically spheres with area[9]

$$8\pi m[m+(m^2-a^2)^{1/2}]. \qquad (1)$$

The event horizon is the boundary of the region of space-time from which particles or photons can escape to infinity. I shall consider only space-times which are asymptotically flat in the sense of being weakly asymptotically simple.[10] The metric on such a space M can be extended conformally to a manifold with boundary $\bar{M}$ which consists of M and two null hypersurfaces $\mathscr{I}^+$ and $\mathscr{I}^-$. These represent future and past null infinity, respectively. The event horizon can then be defined as $\dot{J}^-(\mathscr{I}^+)$, where a dot indicates the boundary, and for any set $\mathcal{S}$, $J^-(\mathcal{S})$ is the causal past of $\mathcal{S}$, i.e., the set of all points that can be reached from $\mathcal{S}$ by past-directed nonspacelike curves.

The situation that I consider is one in which there are initially two collapsed objects or "black holes" a considerable distance apart in asymptotically flat space. The black holes are assumed to have formed at some earlier time as a result of either gravitational collapses or the amalgamation of smaller black holes. This situation can be described in terms of a spacelike hypersurface Σ_i which is a Cauchy surface for the asymptotically flat region of space-time. On Σ_i there will be two separate regions, B_1 and B_2, which contain closed, trapped surfaces.[11] In fact the surface Σ_i need not exist within the regions B_1 and B_2 since what happens there cannot affect what happens outside. Just outside B_1 and B_2 will be two two-spheres which are the intersection of $\dot{J}^-(\mathscr{I}^+)$ with Σ_i. By the Carter-Israel conjecture the areas of these two spheres will be to a good approximation given by formula (1) for the values m_1, a_1 and m_2, a_2 of the masses and specific angular momenta of the two black holes. Suppose now that the two black holes capture each other to form a single black hole which settles down to a Kerr solution with parameters m_3 and a_3. There will then be a later spacelike hypersurface Σ_f on which there will be a single region B_3 containing closed trapped surfaces. Just outside B_3 will be a two-sphere $\Sigma_f \cap \dot{J}^-(\mathscr{I}^+)$ whose area is given by formula (1) with m_3 and a_3. Now $\dot{J}^-(\mathscr{I}^+)$ is generated by null geodesic segments which may have past end points but which can have no future end points.[10,12] I shall take it that there are no singularities in the exterior region $\bar{J}^-(\mathscr{I}^+) \cap J^+(\Sigma_i)$. Whether such "naked" singularities can occur or whether the singularities are always hidden behind the event horizon is an unresolved question.[13] One would not expect to obtain a limit on the energy that could be carried to infinity by gravitational radiation if there were a naked singularity since what happens at a singularity is not governed by any known laws. I shall therefore consider only those situations where what happens in $\bar{J}^-(\mathscr{I}^+) \cap J^+(\Sigma_i)$ is determined by data on Σ_i, i.e., where Σ_i is a Cauchy surface for $\bar{J}^-(\mathscr{I}^+) \cap J^+(\Sigma_i)$. One can choose Σ_f so that it is a Cauchy surface for $\bar{J}^-(\mathscr{I}^+) \cap J^+(\Sigma_f)$ and show that every null geodesic generating segment of $\mathring{J}^-(\mathscr{I}^+)$ in $J^+(\Sigma_i) \cap J^-(\Sigma_f)$ must intersect Σ_f. By the Carter-Israel conjecture the solution in $\bar{J}^-(\mathscr{I}^+) \cap J^+(\Sigma_f)$ resembles the exterior Kerr solution and so contains no singularities. Thus each null geodesic generating segment of $\dot{J}^-(\mathscr{I}^+)$ in $J^+(\Sigma_f)$ will be geodesically complete in the future direction. I assume also that the weak energy condition holds[11]: $T_{ab}K^aK^b \geq 0$ for every null vector K^a. It then follows that the convergence ρ of the null geodesic generators of $\dot{J}^-(\mathscr{I}^+)$ cannot be positive anywhere since if it were there would be a caustic and hence a future end point of some of the generators.[10] This shows that the area of $\Sigma_f \cap \dot{J}^-(\mathscr{I}^+)$ must be greater than or equal to the area of $\Sigma_i \cap \dot{J}^-(\mathscr{I}^+)$ (in fact, it must be strictly greater). Therefore,

$$m_3[m_3 + (m_3^2 - a_3^2)^{1/2}] > m_1[m_1^2 + (m_1^2 - a_1^2)^{1/2}] + m_2[m_2 + (m_2^2 - a_2^2)^{1/2}]. \tag{2}$$

By the conservation law for asymptotically flat space,[14,15] the energy emitted in gravitational or other forms of radiation is $m_1 + m_2 - m_3$. The efficiency $\epsilon = (m_1 + m_2)^{-1}(m_1 + m_2 - m_3)$ is limited by Eq. (2). The highest limit on ϵ is $\frac{1}{2}$ which occurs if $m_1 = m_2 = a_1 = a_2$, $a_3 = 0$. If $a_1 = a_2 = 0$, then $\epsilon < (1 - 2^{-1/2})$. It should be stressed that these are upper limits. The actual efficiency might be much less.

Imagine a situation in which there were initially a large number of black holes. First these could combine in pairs, and then the resulting holes could combine, and so on. On dimensional grounds one could expect the efficiency to be the same at each stage. Thus one would extract a very large fraction of the original mass (this argument was suggested by C. W. Misner). At each stage the emitted bursts would be longer and stronger. This might explain Weber's observations.

One can also apply this result that the area on the event horizon increases to the case of a single black hole on which various particles and fields impinge. One then has that the final mass m_2 and specific angular momentum a_2 must satisfy the inequality

$$m_2[m_2 + (m_2^2 - a_2^2)^{1/2}] > m_1[m_1 + (m_1^2 - a_1^2)^{1/2}],$$

where m_1 and a_1 are the initial values. In particular, if a_2 is equal to zero, then $2m_2^2 > m_1[m_1 + (m_1^2 - a_1^2)^{1/2}]$. Using an idea of Penrose,[13] Christodolou[16] has shown that one can get arbitrarily near this limit. One can interpret this as extracting the rotational energy of the black hole.

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Heavy Mesons and Neutral Axial-Vector Currents*

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It is suggested that the eighth and ninth axial-vector currents are dominated by the $J^P = 0^-$ mesons $\eta(549)$, $\eta'(958)$, and the (presumed) $J^P = 1^+$ mesons $D(1285)$ and $E(1422)$. Predictions are made concerning the decay modes of D and E as well as for the DNN and ENN couplings.

The existence of an octet of axial-vector currents may be regarded as well established. However the status of a possible ninth current $\mathfrak{F}_{0\mu}^5$, an "axial baryon current," has remained obscure, with some authors doubting its existence and others ascribing special virtues to it. In the present note we suggest that the $\eta'(960)$ and $E(1422)$ are manifestations of this current and suggest experimental means of illuminating this question. At the same time we present a parallel interpretation of the eighth axial current $\mathfrak{F}_{8\mu}^5$ in terms of the properties of the $\eta(550)$ and $D(1285)$ mesons.

The decay modes of D and E are rather similar, with $\eta\pi\pi$ and $K\bar{K}\pi$ [with $K\bar{K}$ coming from the 0^+ state $\pi_N(1016)$] observed for both mesons.[1,2] (The E meson also decays through $\bar{K}K^*$, which is kinematically forbidden for D.) Apart from the $K^*\bar{K}$ mode, we interpret the $\eta\pi\pi$ and $K\bar{K}\pi$ final states as due to the sequence $D(E) \to \epsilon\eta$ or $D(E) \to \pi_N\pi$ with subsequent decays $\epsilon \to 2\pi$, $\pi_N \to \pi\eta$, and $\pi_N \to K\bar{K}$. As in an earlier paper[3] we consider ϵ and π_N to be members of a scalar nonet.[4] In the following we shall relate the axial-scalar-pseudoscalar couplings to the scalar-pseudoscalar-pseudoscalar couplings by a "universality" argument based on partial conservation of axial-vector current (PCAC) for all nine axial-vector currents.

We consider the nine axial-vector currents $\mathfrak{F}_{i\mu}^5$ ($i = 0, \cdots, 8$). The matrix elements of the isospin current $\mathfrak{F}_\mu^5$ may be regarded as dominated by the π (0^-) and the A_1 (1^+). Since there is a nonet of pseudoscalars [the ninth being $\eta'(958)$], it is reasonable to ask about a companion nonet of axial-vector mesons and further to speculate that the ($0^-, 1^+$) nonets dominate the corresponding currents. The experimental situation is unclear, but it appears that the $A_1(1070)$, $D(1285)$, $K_A(1240)$, and $E(1422)$ are good candidates for a nonet.[5,6] Comparison with the pseudoscalar octet suggests that D is mostly octet and E the singlet. With this assignment the assumed A_1 and K_A masses imply that the mixing is negligible. Since $\eta\eta'$ mixing is small ($\lesssim 10°$), we shall ignore mixing here and associate the pairs (η, D) and (η', E) with the unmixed currents $\mathfrak{F}_{8\mu}^5$ and $\mathfrak{F}_{0\mu}^5$, respectively.

The coupling constants $g_{\epsilon\eta D}$, $g_{\pi_N\pi D}$, etc., may be related to $g_{\epsilon\eta\eta}$, $g_{\pi_N\pi\eta}$ by a generalization of a universality argument proposed in Ref. 3 for the (π, A_1) complex. The matrix element $\langle\pi|\mathfrak{F}_\mu^5|\epsilon\rangle$ is given in terms of the form factors F_1 and F_2