

CO'CCO 77802

GO'ON AND FOR PRACTICE

1 AND

$$\frac{d}{dt} \sum_{N=0}^{\infty} P(N) = \sum_{N=0}^{\infty} \dot{P}(N) = \sum_{N=0}^{\infty} \left\{ F[P(N-1) - P(N)] + W[(N+1)P(N+1) - NP(N)] + K[(N+2)(N+1)P(N+2) - N(N-1)P(N)] \right\}$$

OPTICAL MASS X3D
~~N=0~~ ~~N=1~~ ~~N=2~~ ~~N=3~~ ~~N=4~~ ~~N=5~~ ~~N=6~~ ~~N=7~~ ~~N=8~~ ~~N=9~~ ~~N=10~~ ~~N=11~~ ~~N=12~~ ~~N=13~~ ~~N=14~~ ~~N=15~~ ~~N=16~~ ~~N=17~~ ~~N=18~~ ~~N=19~~ ~~N=20~~ ~~N=21~~ ~~N=22~~ ~~N=23~~ ~~N=24~~ ~~N=25~~ ~~N=26~~ ~~N=27~~ ~~N=28~~ ~~N=29~~ ~~N=30~~ ~~N=31~~ ~~N=32~~ ~~N=33~~ ~~N=34~~ ~~N=35~~ ~~N=36~~ ~~N=37~~ ~~N=38~~ ~~N=39~~ ~~N=40~~ ~~N=41~~ ~~N=42~~ ~~N=43~~ ~~N=44~~ ~~N=45~~ ~~N=46~~ ~~N=47~~ ~~N=48~~ ~~N=49~~ ~~N=50~~ ~~N=51~~ ~~N=52~~ ~~N=53~~ ~~N=54~~ ~~N=55~~ ~~N=56~~ ~~N=57~~ ~~N=58~~ ~~N=59~~ ~~N=60~~ ~~N=61~~ ~~N=62~~ ~~N=63~~ ~~N=64~~ ~~N=65~~ ~~N=66~~ ~~N=67~~ ~~N=68~~ ~~N=69~~ ~~N=70~~ ~~N=71~~ ~~N=72~~ ~~N=73~~ ~~N=74~~ ~~N=75~~ ~~N=76~~ ~~N=77~~ ~~N=78~~ ~~N=79~~ ~~N=80~~ ~~N=81~~ ~~N=82~~ ~~N=83~~ ~~N=84~~ ~~N=85~~ ~~N=86~~ ~~N=87~~ ~~N=88~~ ~~N=89~~ ~~N=90~~ ~~N=91~~ ~~N=92~~ ~~N=93~~ ~~N=94~~ ~~N=95~~ ~~N=96~~ ~~N=97~~ ~~N=98~~ ~~N=99~~ ~~N=100~~ ~~N=101~~ ~~N=102~~ ~~N=103~~ ~~N=104~~ ~~N=105~~ ~~N=106~~ ~~N=107~~ ~~N=108~~ ~~N=109~~ ~~N=110~~ 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$$\frac{d}{dt} \sum_{N=0}^{\infty} P(N) = \sum_{N=0}^{\infty} \dot{P}(N) = \sum_{N=0}^{\infty} \left\{ F[P(N-1) - P(N)] + W[(N+1)P(N+1) - NP(N)] + K[N(N-1)P(N) - N(N-2)P(N)] \right\}$$

$$\frac{d}{dt} \sum_{N=0}^{\infty} P(N) = \sum_{N=0}^{\infty} \dot{P}(N) = \sum_{N=0}^{\infty} \left\{ F[P(N-1) - P(N)] + W[N(N-1)P(N) - N(N-2)P(N)] + K[N(N-1)(N-2)P(N)] \right\}$$

$$\langle N \rangle = \sum_{N=0}^{\infty} N P(N) = \sum_{N=0}^{\infty} \left\{ FN[P(N-1) - P(N)] + WN[(N+1)P(N+1) - NP(N)] + K[N(N-1)(N-2)P(N)] + K[(N+2)(N+1)N P(N+2) - N^2(N-1)P(N)] \right\}$$

$$\langle N \rangle = F - W\langle N \rangle - 2K\langle N^2 \rangle + 2K\langle N \rangle$$

$$\langle N \rangle = F - W\langle N \rangle - 2K\langle N^2 \rangle$$

$$\langle N \rangle = F - W\langle N \rangle - 2K\langle N^2 \rangle$$

$$\langle N \rangle = F - W\langle N \rangle - 2K\langle N^2 \rangle$$

$$\therefore \text{for } \langle N \rangle = 0 \quad \gamma' \propto e^{-\lambda N}$$

$$2K\langle N \rangle^2 + W\langle N \rangle - F = 0$$

$$\langle N \rangle = \frac{-W + \sqrt{W^2 + 8KF}}{4K}$$

$\langle N \rangle \geq 0$ 5. 0.02 $\times 10^6 / \text{mole}$ (approx)
 (2) Re

: $\dot{P}(N) = 0 \Rightarrow \text{constant} \ L(N) + K \text{sgn } g \propto \text{const}$ (1)

$$0 = \frac{dg}{dt} = \sum_{N=0}^{\infty} \dot{P}(N) S^N = \sum_{N=0}^{\infty} F P(N-1) S^N - F \sum_{N=0}^{\infty} S^N P(N) + W \sum_{N=0}^{\infty} (N+1) P(N+1) S^N$$

then $\int dN$ from 0 to ∞

$$-W \sum_{N=0}^{\infty} N P(N) S^N + K \sum_{N=0}^{\infty} (N+2)(N+1) P(N+2) S^N - K \sum_{N=0}^{\infty} N(N-1) P(N) S^N$$

$$= F S g(s) - F g(s) + W \frac{d}{ds} \sum_{N=1}^{\infty} P(N) S^N - W S \frac{d}{ds} \sum_{N=0}^{\infty} P(N) S^N + K \frac{d^2}{ds^2} \sum_{N=0}^{\infty} P(N+2) S^{N+2}$$

$$- K S^2 \frac{d^2}{ds^2} \sum_{N=0}^{\infty} P(N) S^N$$

: $\text{const} \rightarrow \text{const} \propto N! \propto N^N$

$$0 = -F(1-s)g(s) + W(1-s) \frac{dg}{ds} + K(1-s^2) \frac{d^2g}{ds^2}$$

$$g = r^{-\delta} f(r)$$

$$g' = -\delta r^{\delta-1} f + r^{\delta} f'$$

$$g'' = +\delta(\delta+1) r^{\delta-2} f - 2\delta r^{\delta-1} f' + r^{\delta} f''$$

$$ds = dr$$

$$0 = -F r^{\delta} f + W \left(\delta r^{\delta} f' - \delta r^{\delta-1} f \right) + K r \left(\delta(\delta+1) r^{\delta-2} f - 2\delta r^{\delta-1} f' + r^{\delta} f'' \right)$$

$$0 = \left(-\frac{E}{Kr} - \frac{W\delta}{Kr^2} + \frac{\delta(\delta+1)}{r^2} \right) f + \left(\frac{W}{Kr} - \frac{2\delta}{r} \right) f' + f''$$

$$\alpha = \frac{F}{2K} \quad \rightarrow \quad \delta = \frac{W}{2K}$$

$$O = \left(\frac{2\alpha}{r} + \frac{\delta(1-\delta)}{r^2} \right) f + f''$$

ר' ב' י' ו' מ' נ' מ' נ' ב' מ' נ' ב'

שאלה 3

נתונה מערכת חד-מימנית של N כדורים קשיחים בעלי אורך צלע, a , על רצועה באורך L , $Na < L$.

$$u(x_i - x_j) = \begin{cases} \infty & |x_i - x_j| < a \\ 0 & 0 \end{cases}$$

א. חשבו במדוקיק את פונקציית החלוקה ואת משווהת המצב.
פונקציית החלוקה ניתנת ע"י המכפלת $Z_K Z_V$. $Z = Z_K Z_V$ היא תרומה הקיינטית:

$$Z_K = \frac{1}{N!} \left(\frac{1}{2\pi\hbar} \int dp e^{-\beta \frac{p^2}{2m}} \right)^N = \frac{1}{N!} \left(\frac{2\pi n k_B T}{h^2} \right)^{N/2}$$

ז' היא תרומה הפטונצייאלית-חלוקה:

$$Z_V = \int dx_1 \dots dx_N \exp \left[-\frac{1}{k_B T} \sum_{i < j} v(x_i - x_j) \right]$$

$$\exp \left[-\frac{1}{k_B T} v(x_i - x_j) \right] = \begin{cases} 0 & |x_i - x_j| < a \\ 1 & |x_i - x_j| \geq a \end{cases}$$

את החלוקים נ"ז שיתנו פקטור $N!$ לאינטגרל (ו庵):

$$Z_V = N! \int_{\frac{a}{2}}^{L-(2N-1)\frac{a}{2}} dx_1 \int_{x_1+a}^{L-(2N-3)\frac{a}{2}} dx_2 \dots \int_{x_{N-1}+a}^{L-\frac{a}{2}} dx_N$$

$$N=1 \Rightarrow Z_V = 1!(L-a) = L-a$$

$$N=2 \Rightarrow Z_V = 2! \frac{(L-2a)^2}{2} = (L-2a)^2$$

באנדרוקצייה נ. $Z_V = (L-Na)^N$

$$\begin{aligned} Z_V &= N! \int_{\frac{a}{2}}^{L-(2N-1)\frac{a}{2}} dx_1 \int_{x_1+a}^{L-(2N-3)\frac{a}{2}} dx_2 \dots \int_{x_{N-1}+a}^{L-\frac{a}{2}} dx_N = \\ &= N \int_{\frac{a}{2}}^{L-(2N-1)\frac{a}{2}} dx_1 (N-1)! \left[\int_{x_1+a}^{L-(2N-3)\frac{a}{2}} \dots \int_{x_{N-1}+a}^{L-\frac{a}{2}} dx_N \right]_{\tilde{x}_i = x_i - x_1 - \frac{a}{2}} = \\ &= N \int_{\frac{a}{2}}^{L-(2N-1)\frac{a}{2}} dx_1 (N-1)! \left[\int_{x_1+a}^{L-(2N-3)\frac{a}{2}} \dots \int_{x_{N-1}+a}^{L-\frac{a}{2}} dx_N \right]_{\tilde{x}_i = x_i - x_1 - \frac{a}{2}} = \end{aligned}$$

$$\begin{aligned}
Z_V &= N \int_{\frac{a}{2}}^{L-(2N-1)\frac{a}{2}} dx_1 (N-1)! \left[\int_{\frac{a}{2}}^{\left(L-x_1+\frac{a}{2}\right)-(2N-3)\frac{a}{2}} dx_2 \dots \int_{x_{N-1}+\frac{a}{2}}^{\left(L-x_1+\frac{a}{2}\right)-\frac{a}{2}} dx_N \right] = \\
&= N \int_{\frac{a}{2}}^{L-(2N-1)\frac{a}{2}} dx_1 \left(L - x_1 + \frac{a}{2} - (N-1)a \right)^{N-1} = \\
&= - \left(L - x_1 + \frac{a}{2} - (N-1)a \right)^N \Big|_{\frac{a}{2}}^{L-(2N-1)\frac{a}{2}} = (L-Na)^N
\end{aligned}$$

ב)

$$P = kT \left(\frac{\partial}{\partial L} \ln Z \right)_{T,N} = kT \left(\frac{\partial}{\partial L} \ln Z_V \right)_{T,N} = \frac{NkT}{L} \left(1 - \frac{Na}{L} \right)^{-1} \approx \frac{NkT}{L} \left(1 + a \frac{N}{L} \right)$$

ב. חשבו את B_2

$$B_2 = \frac{1}{2L} \int_0^L dx_1 \int_0^L dx_2 \exp \left(-\frac{1}{k_B T} v(x_1 - x_2) \right) \Big|_{\substack{x=x_1-x_2 \\ X=\frac{x_1+x_2}{2}}} = \frac{1}{2L} \int_0^L dX \int_L^L dx \exp \left(-\frac{1}{k_B T} v(x) \right) = a$$

ג. הראו כי התוצאה שקיבלתם קונסיסטנטיות עם סעיף א'.

$$\text{בסעיף ב קיבלנו} , \frac{PL}{NkT} = 1 + a \frac{N}{L} \text{, כמו בסעיף א'}$$

שאלה 4

נתונה מערכת בת N חלקיקים במרחב \mathbb{V} וטמפרטורה T . בין כל שני חלקיקים קיימת אינטראקציה התלויה רק במרחק בין החלקרים ונתונה ע"י:

$$u(r) = \begin{cases} \infty & 0 < r < b \\ -\epsilon & b \leq r \leq R \\ 0 & R < r \end{cases}$$

א. חשבו את מקדם הפיתוח הייריאלי $B_2(T)$ של המערכת.

מתוך הנוסחה שקיבלנו בכיתה:

$$\begin{aligned}
B_2(T) &= 2\pi \int_0^\infty r^2 (1 - \exp(-\beta u(r))) = \\
&= 2\pi \left[\int_0^b r^2 dr - \int_b^R r^2 e^{\beta \epsilon} dr \right] = \\
&= 2\pi \left[\frac{R^3}{3} - e^{\beta \epsilon} \left(\frac{R^3}{3} - \frac{b^3}{3} \right) \right] = \\
&= \frac{2\pi b^3}{3} \left[1 - \left(\frac{R^3}{b^3} - 1 \right) (e^{\beta \epsilon} - 1) \right]
\end{aligned}$$

ב. הראו כי בקרוב מסוים נותן הפיתוח עד סדר שני את משוואת המצב של גז ואנ-דר-וילס.
מהו קרוב זה?
ונאן דר וילס:

$$P = \frac{NkT}{V - Nb_0} - \left(\frac{N}{V}\right)^2 a_0 \approx \frac{NkT}{V} \left(1 + \frac{N}{V} b_0\right) - \left(\frac{N}{V}\right)^2 a_0 = \frac{NkT}{V} \left(1 + \frac{N}{V} \left(b_0 - \frac{a_0}{kT}\right)\right)$$

כלומר, מקדם הפיתוח הויריאלי השני בן ונאן דר וילס הוא

נתoor לבעיה שלנו, ונשים לב כי הקירוב בו הטמפרטורה גבוהה ביחס לאנרגיה האופיינית של הפוטנציאל הדו נפי מתקיים:

$$B_2(T) = \frac{2\pi b^3}{3} \left[1 - \left(\frac{R^3}{b^3} - 1 \right) (e^{\beta\varepsilon} - 1) \right] \underset{\frac{\varepsilon}{kT} \ll 1}{\approx} \frac{2\pi b^3}{3} \left[1 - \frac{\varepsilon}{kT} \left(\frac{R^3}{b^3} - 1 \right) \right]$$

$$\text{ואמנם } a_0 = \frac{2\pi\varepsilon}{3} (R^3 - b^3), b_0 = \frac{2\pi b^3}{3}$$