Semi-Analytic Models of Galaxy Formation

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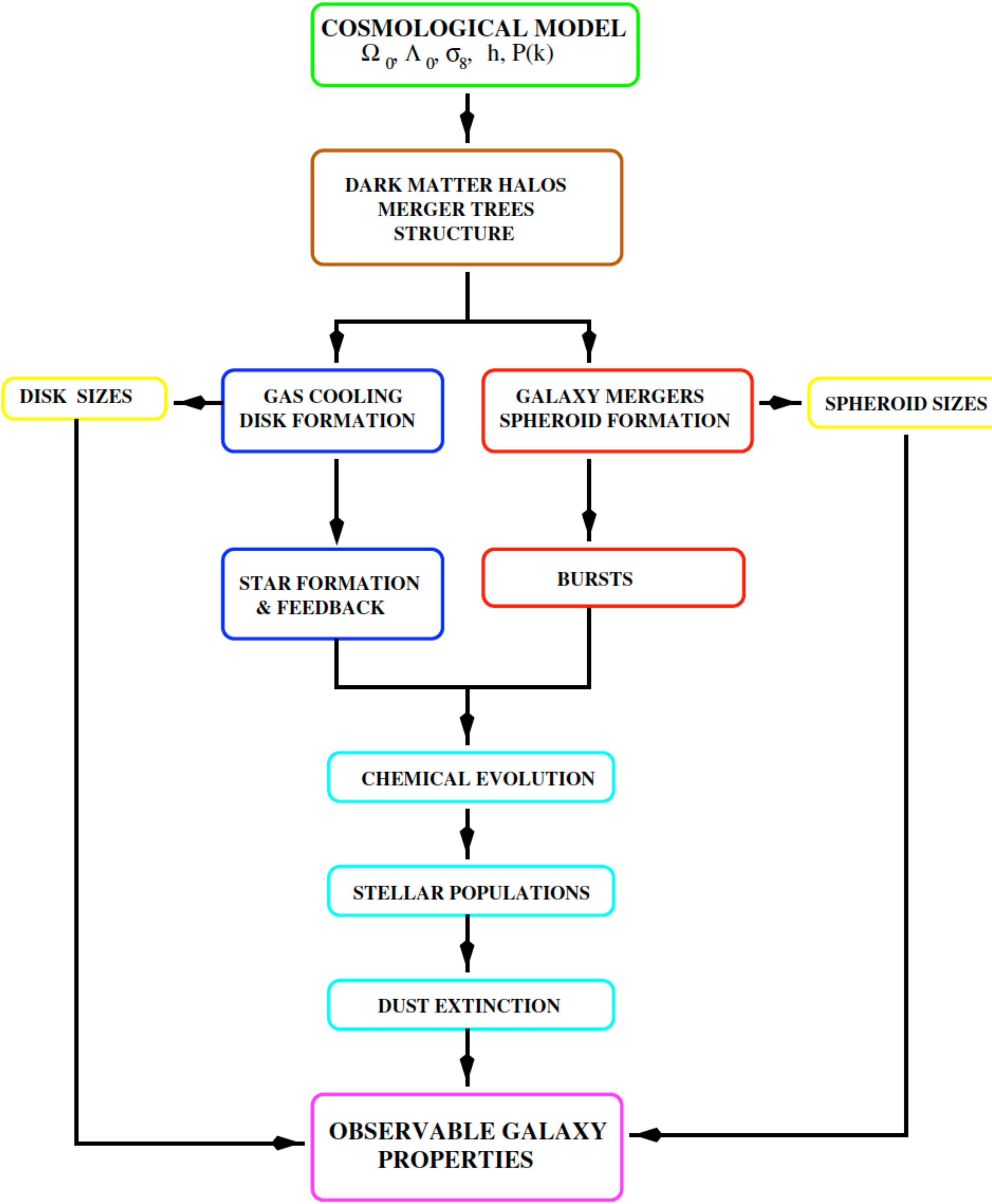
Lecture Notes for Advanced Cosmology 2020
Hebrew University of Jerusalem

Semi-Analytical Models (SAMs) for galaxy formation are phenomenological models that use approximate, analytical descriptions to describe the various processes relevant for galaxy formation in order to make predictions that can be compared to observations.



Ingredients of SAM

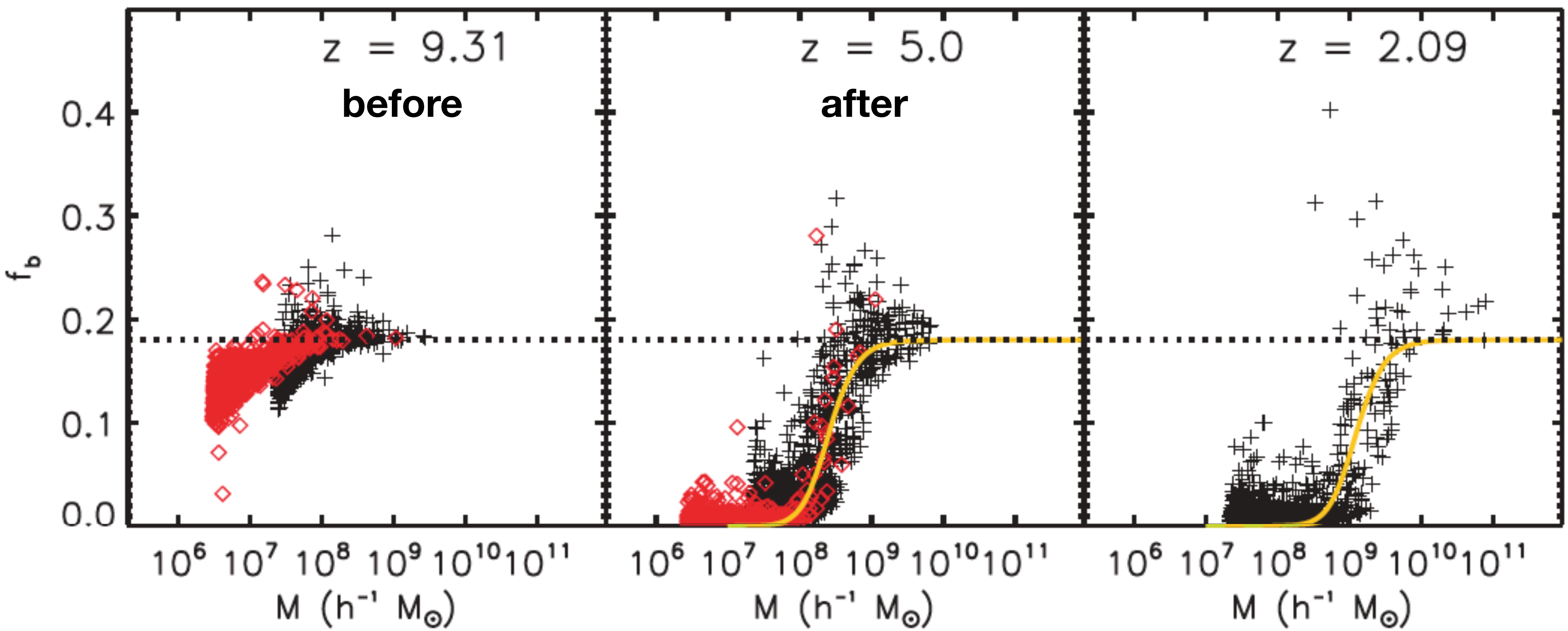
- Dark-matter halos and merger trees
- Reionization and gas infall
- Gas cooling
- Disk formation and angular momentum
- Star formation and supernova feedback
- Mergers, black-hole growth, and AGN feedback
- Satellite galaxies and environmental processes

We walk through the above ingredients of a recent “Munich” SAM by Henriques et al. (2015)

Reionization and gas infall

The fraction of baryonic matter that is bound to a dark matter (DM) halo is regulated by the potential depth and the pressure of the baryons. When the Universe reionizes, the temperature and thus the pressure of the baryons increases. This results in a strongly reduced baryon fraction in low mass halos.

Okamoto et al. (2008) used hydro simulations to study the impact of reinization imposing a uniform, ionizing background as computed by Haardt & Madau (2001).

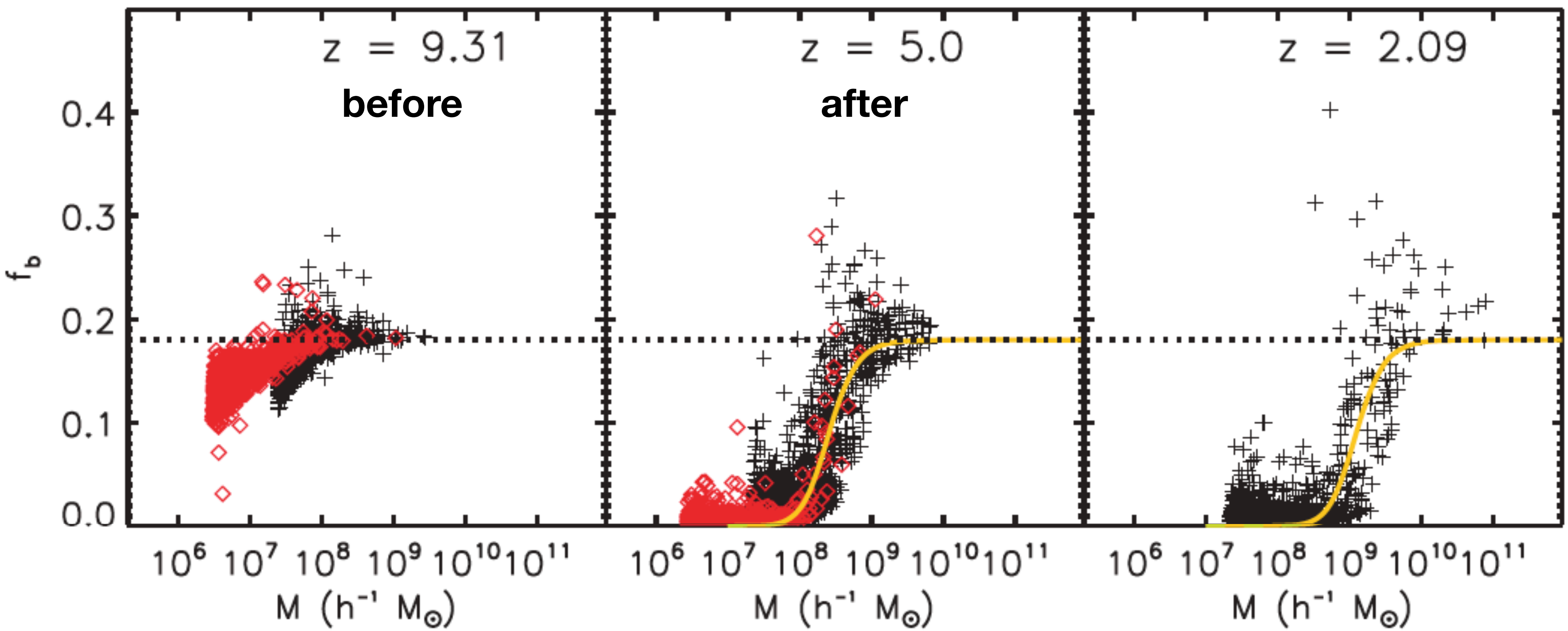


Reionization and gas infall

The fraction of baryons that remains bound to their halos is well fit by (Gnedin 2000):

$$f_b(z, M_{200c}) = f_b^{\cos}\left(1 + (2^{\alpha/3} - 1)\left[\frac{M_{200c}}{M_F(z)}\right]^{-\alpha}\right)^{-3/\alpha}$$

where M_F is the z-dependent characteristic mass at which half the baryons are photo-evaporated from their host halo, which increases from ~10^8 M⊙ shortly after reionization (z~9), to ~7×10^9 M⊙ at z=0 (Okamoto et al. 2008). M ~7×10^9 M⊙ corresponds to V_{max} ~ 25 km/s.



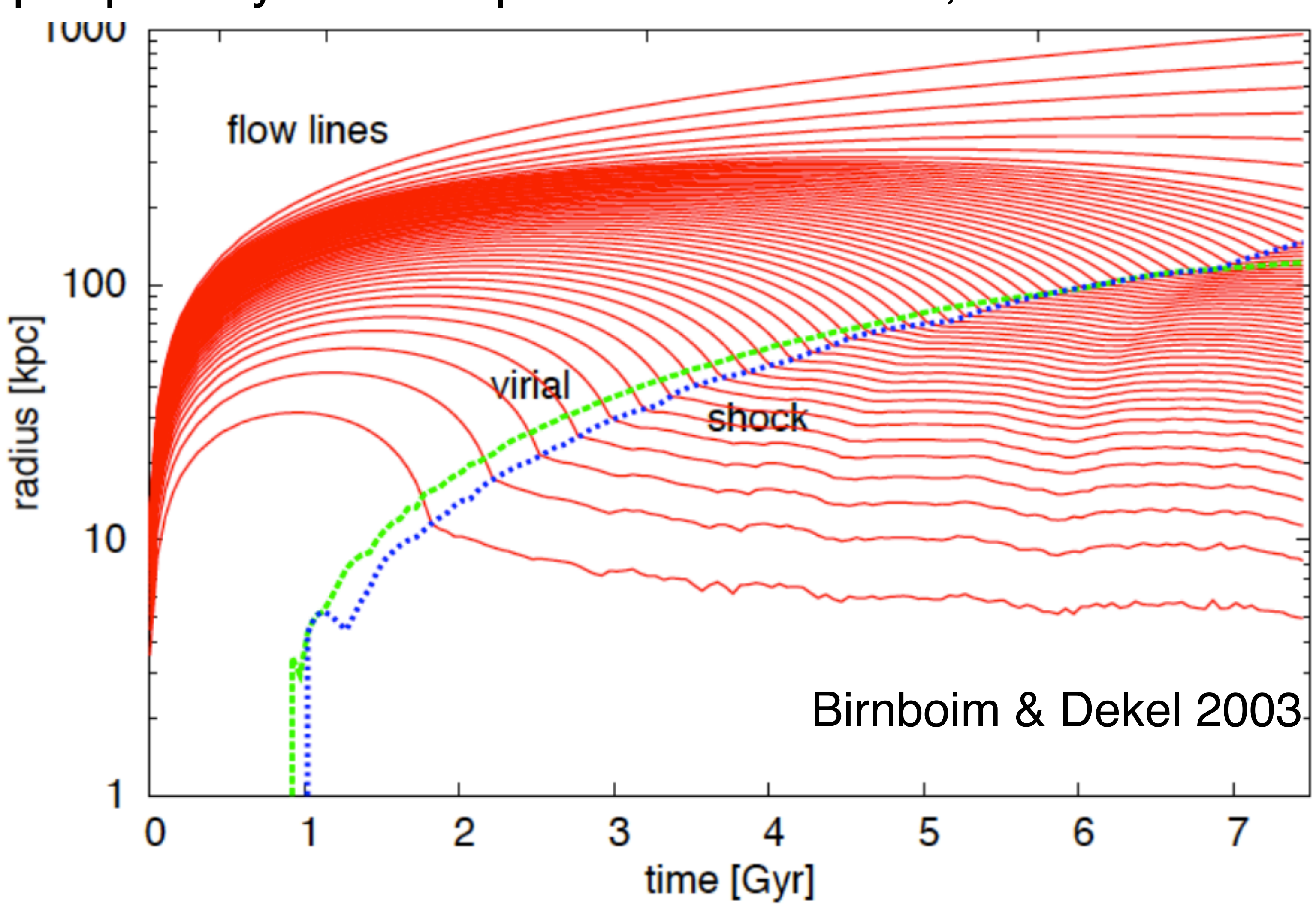
Cooling — accretion shocks & viral temperature

When infalling gas passes the halo's accretion shock, its kinetic energy is thermalized, and therefore heated to of order the **virial temperature**:

$$T_{\rm vir} = \frac{\mu m_{\rm p}}{2k_{\rm B}} V_{\rm c}^2 \simeq 3.6 \times 10^5\,{\rm K} \left(\frac{V_{\rm c}}{100\,{\rm km\,s^{-1}}} \right)^2$$

where V_c is the circular velocity at the viral radius (assuming halos are singular isothermal spheres).

In general, gas inside halo has a T-profile, and cannot be described by a single T. But the concept of T_{vir} is useful for order of magnitude estimates — when you hear people say "the temperature of a halo", it refers to this T_{vir}.



The build-up of a virial shock (discontinuity in velocity) at around the virial radius in a collapsing structure (halo during formation). Based on 1D calculations in an expanding Universe.

Cooling — radiative cooling & cooling time

The primary cooling processes relevant for galaxy formation are two-body radiative processes in which gas loses energy through the emission of photons as a consequence of two-body interactions.

The **cooling time**, the time it takes the gas to radiate away its internal energy, is given by (White & Frenk 1991; Springel et al. 2001):

$$t_{\rm cool}(r) = \frac{3\mu m_{\rm H} k T_{\rm 200c}}{2\rho_{\rm hot}(r)\Lambda(T_{\rm hot}, Z_{\rm hot})}$$

Denser gas cools faster.

where μm_H is the mean particle mass, k is the Boltzmann constant, $\rho_{hot}(r)$ is the hot gas density and Z_{hot} is the hot gas metallicity. $\Lambda(T_{hot},Z_{hot})$ is the equilibrium **cooling function** for collisional processes which depends both on the metallicity and temperature of the gas but ignores radiative ionization effects (Sutherland & Dopita 1993).

Cooling — radiative cooling & cooling time

The cooling time, when compared with a dynamical timescale of a halo, tells us if a system is efficient in cooling.

$$t_{\rm cool}(r) = \frac{3\mu m_{\rm H} k T_{200c}}{2\rho_{\rm hot}(r)\Lambda(T_{\rm hot}, Z_{\rm hot})}$$

$$t_{\rm dyn,h} = R_{200c}/V_{200c} = 0.1 H(z)^{-1}$$

$$t_{\rm cool} \propto (1+z)^{-3}$$

$$t_{\rm dyn,h} \propto (1+z)^{-3/2}$$

Cooling is more efficient at higher redshifts.

Hot mode: $t_{cool} > t_{dyn}$

System is in quasi-hydrostatic equilibrium. It evolves on cooling time scale. Gas contracts slowly as it cools, but system has sufficient time to continue to re-establish hydrostatic equilibrium

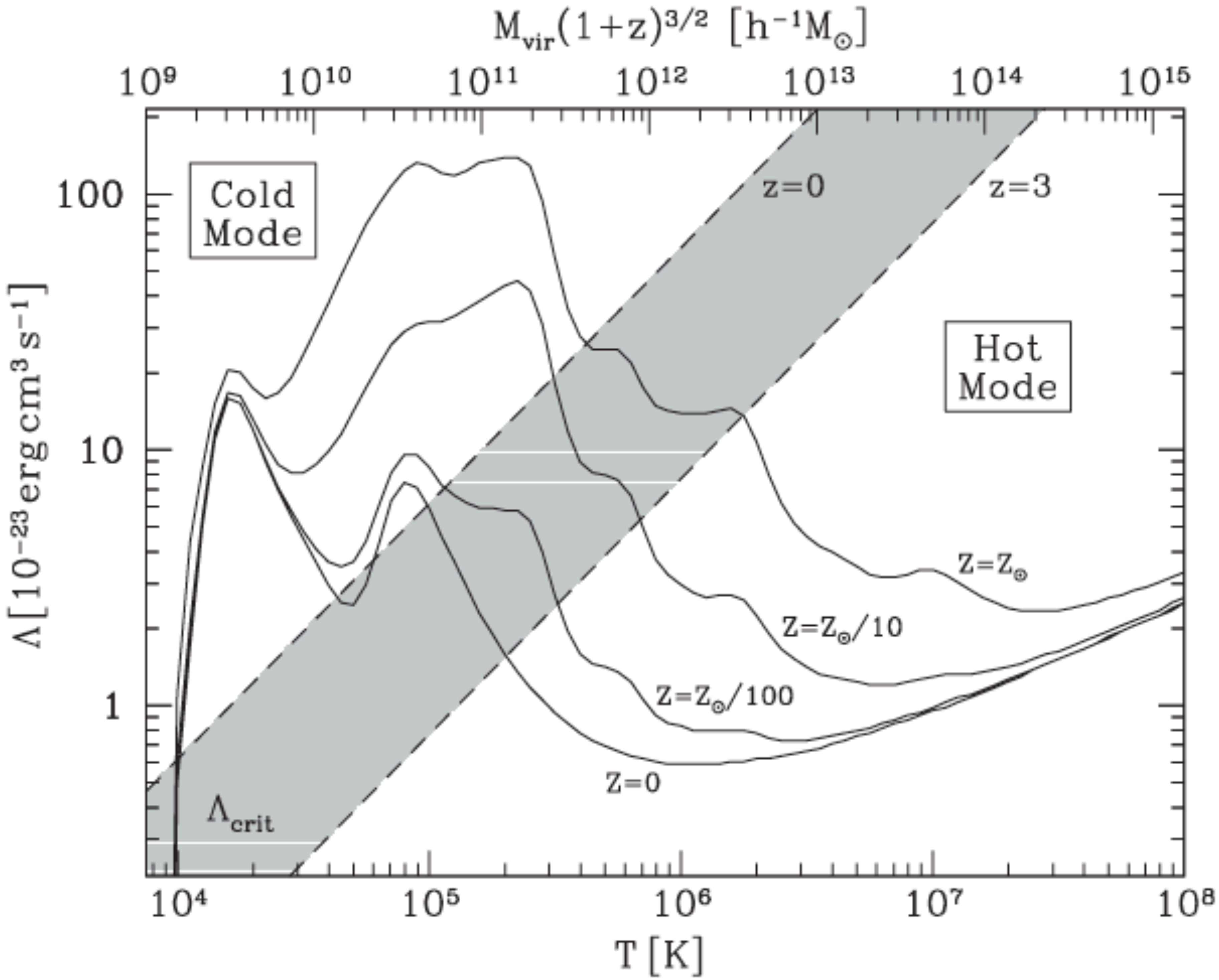
Cold mode: $t_{cool} < t_{dyn}$

Cooling is catastropic. Gas cannot respond fast enough to loss of pressure. Since cooling time decreases with increasing density, cooling proceeds faster and faster (=catastrophic). Gas falls to center of dynamic system on free-fall time.

See Birnboim & Dekel (2003) for more details.

Cooling — radiative cooling & cooling time

Assuming collisional ionization equilibrium (CIE), the cooling function $\Lambda(T_{hot},Z_{hot})$ is given by



Cooling function/ rate depends on metallicity.

Different peaks correspond to different metal species — H, He, C, O, Ne, Fe.

Cooling — cooling radius & cooling rate

Almost all SAMs treat cooling by defining a cooling radius, r_{cool}, as the radius where $t_{cool}(r) = t_{dyn}$

$$r_{\rm cool} = \left[\frac{t_{\rm dyn,h} M_{\rm hot} \Lambda(T_{\rm hot}, Z_{\rm hot})}{6\pi \mu m_{\rm H} k T_{\rm 200c} R_{\rm 200c}} \right]^{\frac{1}{2}}$$

Once the cooling radius has been computed, the cooling rate is computed as

$$\dot{M}_{\rm cool} = M_{\rm hot} \frac{r_{\rm cool}}{R_{\rm 200c}} \frac{1}{t_{\rm dyn,h}} \quad \text{if} \quad r_{\rm cool} < R_{\rm 200c}$$

i.e., the halo is in the cooling flow regime with gas cooling from the quasi-static hot atmosphere.

$$\dot{M}_{\rm cool} = \frac{M_{\rm hot}}{t_{\rm dyn,h}} \quad \text{if} \quad r_{\rm cool} > R_{\rm 200c}$$

i.e., the halo is in the rapid infall regime and material accretes onto the central object in free fall.

Angular momentum and disk size

As dark matter and primordial gas accretes onto a halo, its dark matter and baryonic components are expected to have similar specific angular momenta (but see also Danovich et al. 2015, Jiang et al. 2019). Some of this gas is subsequently added to the central galaxy, and its remaining angular momentum (AM) then determines the radius at which it settles within the galactic disk.

The AM of the cold gas disk changes as a result of star formation and of gas accretion through cooling and minor merger events (Guo et al. 2011):

$$\Delta \vec{J}_{\rm gas} = \delta \vec{J}_{\rm gas,cooling} + \delta \vec{J}_{\rm gas,SF} + \delta \vec{J}_{\rm gas,acc}$$

$$= \frac{\vec{J}_{\rm DM}}{M_{\rm DM}} \dot{M}_{\rm cool}\delta t - \frac{\vec{J}_{\rm gas}}{M_{\rm gas}}((\underline{1 - \mathrm{R_{ret}})\dot{\mathrm{M}}_\star}\delta \mathrm{t} + \underline{\Delta \mathrm{M_{reheat}}}) + \frac{\vec{J}_{\rm DM}}{M_{\rm DM}} \underline{M_{\rm sat,gas}}$$

formation rate of long-lived stars

cold gas reheated into the hot atmosphere as a result of star formation activity

cold gas mass of any merging satellite(s)

The stellar disk gains the AM which is removed from the gas disk by star formation:

$$\delta \vec{J}_\star = (\vec{J}_{\rm gas}/M_{\rm gas})(1 - \mathrm{R_{ret}})\dot{\mathrm{M}}_\star \delta \mathrm{t}$$

Angular momentum and disk size

AM is used to predict disk size:

Assume that both the stellar and gaseous disks are thin, and follow exponential surface-density profiles:

$$\Sigma_\star(R) = \Sigma_{\star,0}\,\exp(-R/R_\star)$$
$$\Sigma_{\rm gas}(R) = \Sigma_{\rm gas,0}\,\exp(-R/R_{\rm gas})$$

where, taking Guo et al. (2011) as an example, the scale lengths are given by:

$$R_\star = \frac{J_\star/M_{\star,\rm d}}{2V_{\rm max}} \qquad R_{\rm gas} = \frac{J_{\rm gas}/M_{\rm gas}}{2V_{\rm max}}$$

halo maximum circular velocity, ~ V_{rot} ~ V_{200c} ~ V_{vir}

More generally (Mo, Mao, & White 1998), the scale radius (R_e) of the stellar or the gaseous components of an isolated galaxy that has not experienced major merger is given by:

$$R_{\rm e} \simeq \frac{j_{\rm gal}}{j_{\rm halo}}\frac{j_{\rm halo}}{R_{\rm vir}V_{\rm vir}}\frac{V_{\rm vir}}{V_{\rm rot}}R_{\rm vir} \simeq f_j\lambda_{\rm halo}R_{\rm vir}$$

AM "retention" factor

halo spin

That is, if AM is conserved (f_j=1), galaxy size is determined by the spin and the viral radius of the host halo (but see also Jiang et al. 2019).

Star formation

Stars are assumed to form from cold gas within the disk of each galaxy. The star formation rate (SFR) is taken to be:

$$\dot{M}_\star = \underline{\alpha_{\rm SF}} \frac{(M_{\rm gas} - M_{\rm crit})}{\underline{t_{\rm dyn,disk}}}$$

SF efficiency ~0.01

dynamical time of the disk, = $R_\star$ / V_{max}

where M_{crit} is a threshold mass for star formation motivated by observations (Kauffmann et al. 1999):

$$M_{\rm crit} = 3.8 \times 10^9\,{\rm M_\odot} \left(\frac{V_{\rm 200c}}{200\,{\rm km\,s^{-1}}}\right) \left(\frac{R_{\rm gas}}{10\,{\rm kpc}}\right)$$

Obviously, the SFR follows the Kennicutt-Schmidt law (an almost linear relation observed b/w SFR surface density and cold gas surface density), and takes account of a critical surface density for star formation, in agreement with empirical findings (e.g., Kennicutt 1998).

Among the newly formed stars, a fraction R_{ret} (~0.4, depending on the initial mass function) is in massive, short-lived stars, and is returned to the cold gas reservoir immediately.

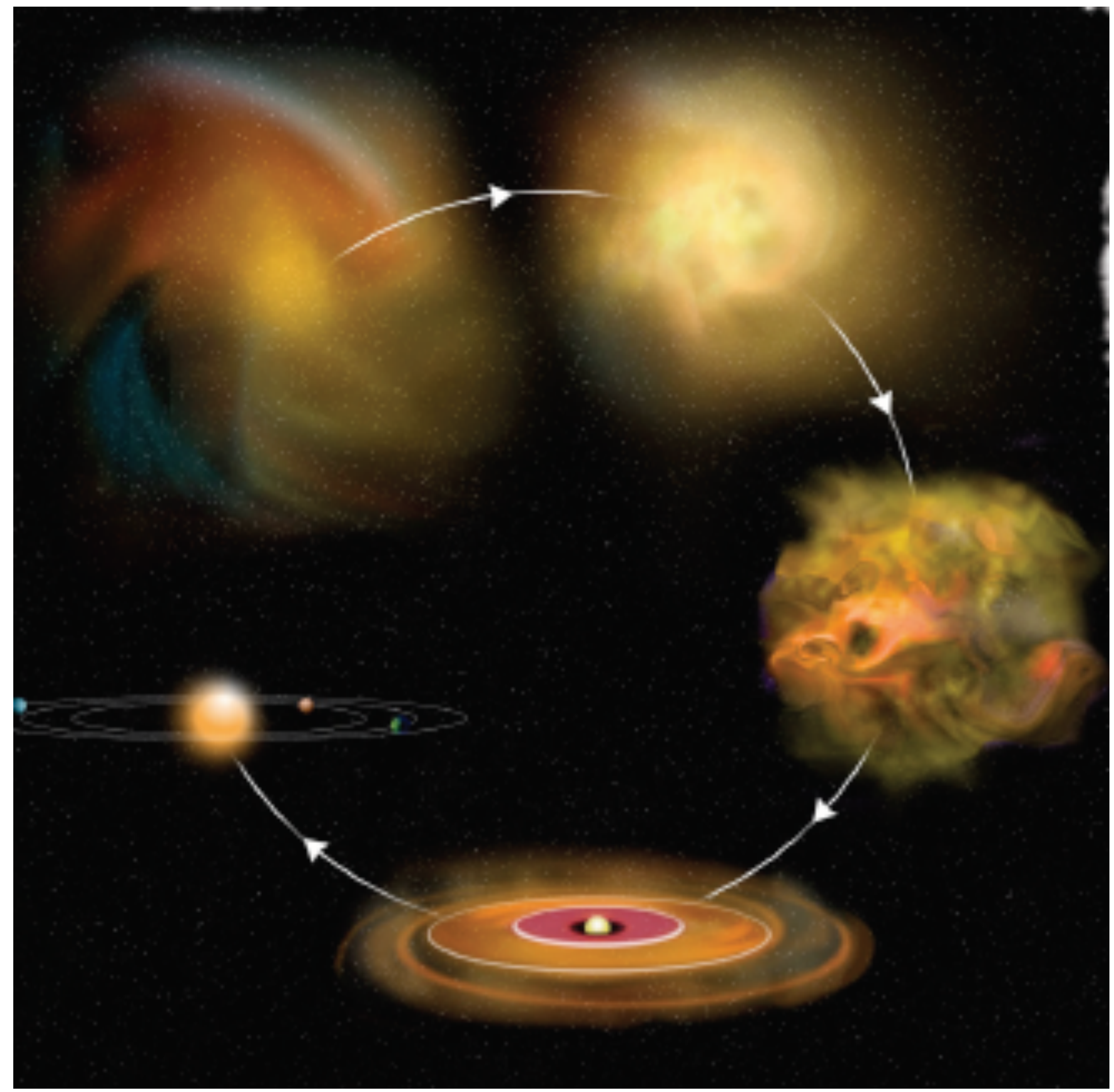
SN feedback

Supernovae (SN) feed energy back to the surrounding gas. From the amount of newly formed stars $\Delta M_\star$, the energy available to the surrounding gas is taken to be (Henriques et al. 2015):

$$\Delta E_{\rm SN} = \epsilon_{\rm halo} \times \frac{1}{2} \Delta M_\star V_{\rm SN}^2$$

where $0.5V_{SN}^2$ is the mean energy per unit stellar mass formed ($V_{SN} \sim 630$km/s depending on IMF), and the efficiency is taken to be:

$$\epsilon_{\rm halo} = \eta \times \left[0.5 + \left(\frac{V_{\rm max}}{V_{\rm eject}}\right)^{-\beta_2}\right]$$



The mass of cold gas reheated by star formation and added to the hot atmosphere is assumed to be directly proportional to the amount of stars formed:

$$\Delta M_{\rm reheat} = \epsilon_{\rm disk} \Delta M_\star$$

where

$$\epsilon_{\rm disk} = \epsilon \times \left[0.5 + \left(\frac{V_{\rm max}}{V_{\rm reheat}}\right)^{-\beta_1}\right]$$

This reheating is assumed to require energy

$$\Delta E_{\rm reheat} = \frac{1}{2} \Delta M_{\rm reheat} V_{\rm 200c}^2$$

SN feedback

$$\Delta E_{\rm SN} = \epsilon_{\rm halo} \times \frac{1}{2}\Delta M_\star V_{\rm SN}^2$$

$$\Delta E_{\rm reheat} = \tfrac{1}{2}\Delta M_{\rm reheat} V_{\rm 200c}^2$$

If $\Delta E_{\rm reheat} > \Delta E_{\rm SN}$, the reheated mass is assumed to saturate at

$$\Delta M_{\rm reheat} = \Delta E_{\rm SN} / \left(\tfrac{1}{2} V_{\rm 200c}^2\right)$$

Otherwise, the remaining SN energy is used to eject a mass ΔM_{eject} of hot gas into an external reservoir, which is given by

$$\frac{1}{2}\Delta M_{\rm eject} V_{\rm 200c}^2 = \Delta E_{\rm SN} - \Delta E_{\rm reheat}$$

Gas ejected in "galactic wind" can be re-incorporated into the galaxy's hot gas component, at a rate (Henriques et al. 2013)

$$\dot{M}_{\rm ejec} = -\frac{M_{\rm ejec}}{t_{\rm reinc}}$$

$$t_{\rm reinc} = \gamma' \frac{10^{10}\,{\rm M}_\odot}{M_{\rm 200c}}$$

I will skip metal enrichment and dust, see e.g., De Lucia et al. (2014) for details.

Satellite galaxies

The growth of structure in a CDM universe affects galaxies as they and their halos fall into larger systems and are influenced by tides, by hydrodynamical forces from the hot gas through which they move, and by encounters with other galaxies. Such environmental effects remove material and modify the structure and evolution of the galaxies, in some cases leading to their complete disruption.

As soon as a halo falls into a larger system, its mass growth stops as tidal forces begin to remove dark matter. No new baryonic material is added to the system and its hot gas atmosphere is stripped away in proportion to its dark matter mass:

$$\frac{M_{\rm hot}(R_{\rm tidal})}{M_{\rm hot,infall}} = \frac{M_{\rm DM}}{M_{\rm DM,infall}}$$

$M_{\rm DM}$: remaining dark-matter mass of the subhalo (after being tidally stripped)

$M_{\rm DM,infall}$: mass of the subhalo when entering the host halo

For an isothermal halo and ignoring centrifugal force, the “tidal radius” is simply

$$R_{\rm tidal} = \left(\frac{M_{\rm DM}}{M_{\rm DM,infall}}\right) \underline{R_{\rm DM,infall}}$$

$R_{\rm DM,infall}$: virial radius of the subhalo when entering the host halo

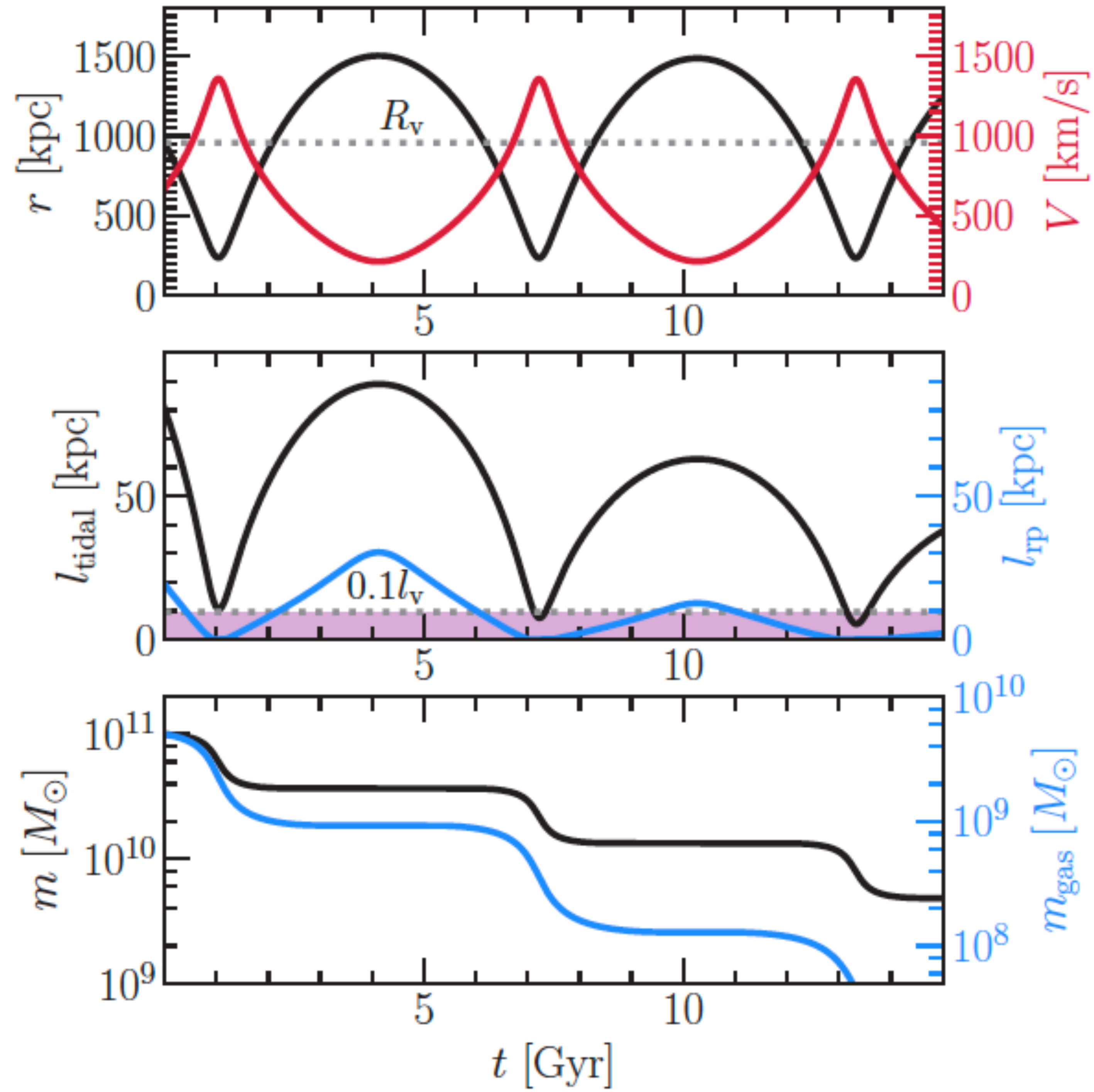
Satellite galaxies

Hot gas can also be stripped by ram-pressure, which is followed starting when the satellite first falls within the virial radius of its host. One can define a ram-pressure-stripping radius $R_{r.p.}$, beyond which the hot gas of the satellite is deposited to the central galaxy, by equating the ram pressure from the hot gas of the host halo to the self-gravity of the satellite:

$$\underline{\rho_{\rm sat}\left(R_{\rm r.p.}\right)V_{\rm sat}^2} = \underline{\rho_{\rm par}\left(R\right)V_{\rm orbit}^2}$$

self-gravity of the satellite halo: $\rho_{sat}(R_{r.p.})$ is the hot gas profile of the satellite; V_{sat} is the virial velocity of the satellite

ram pressure from the hot gas of the host: $\rho_{par}(R)$ is the hot gas density of the parent (host) halo at host-centric distance R; V_{orbit} is the orbital velocity of the satellite



The satellite is completely disrupted (Guo et al. 2011) if

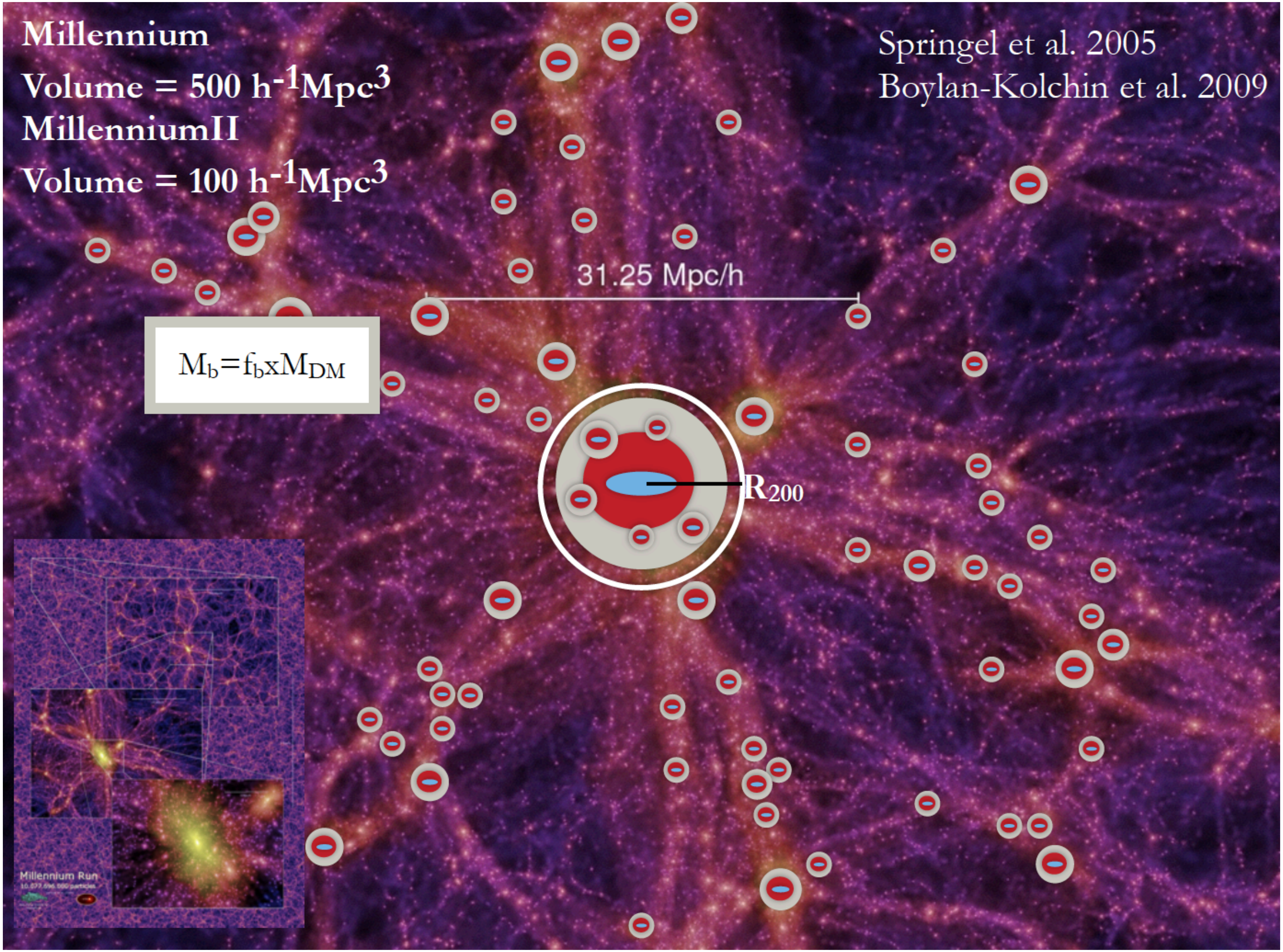
$$\frac{M_{\rm DM,halo}(R_{\rm peri})}{R_{\rm peri}^3} \equiv \rho_{\rm DM,halo} > \rho_{\rm sat} \equiv \frac{M_{\rm sat}}{R_{\rm sat,half}^3}$$

and its stars are added to the intracluster light (ICL) and its cold gas is added to the hot gas atmosphere of the central galaxy.

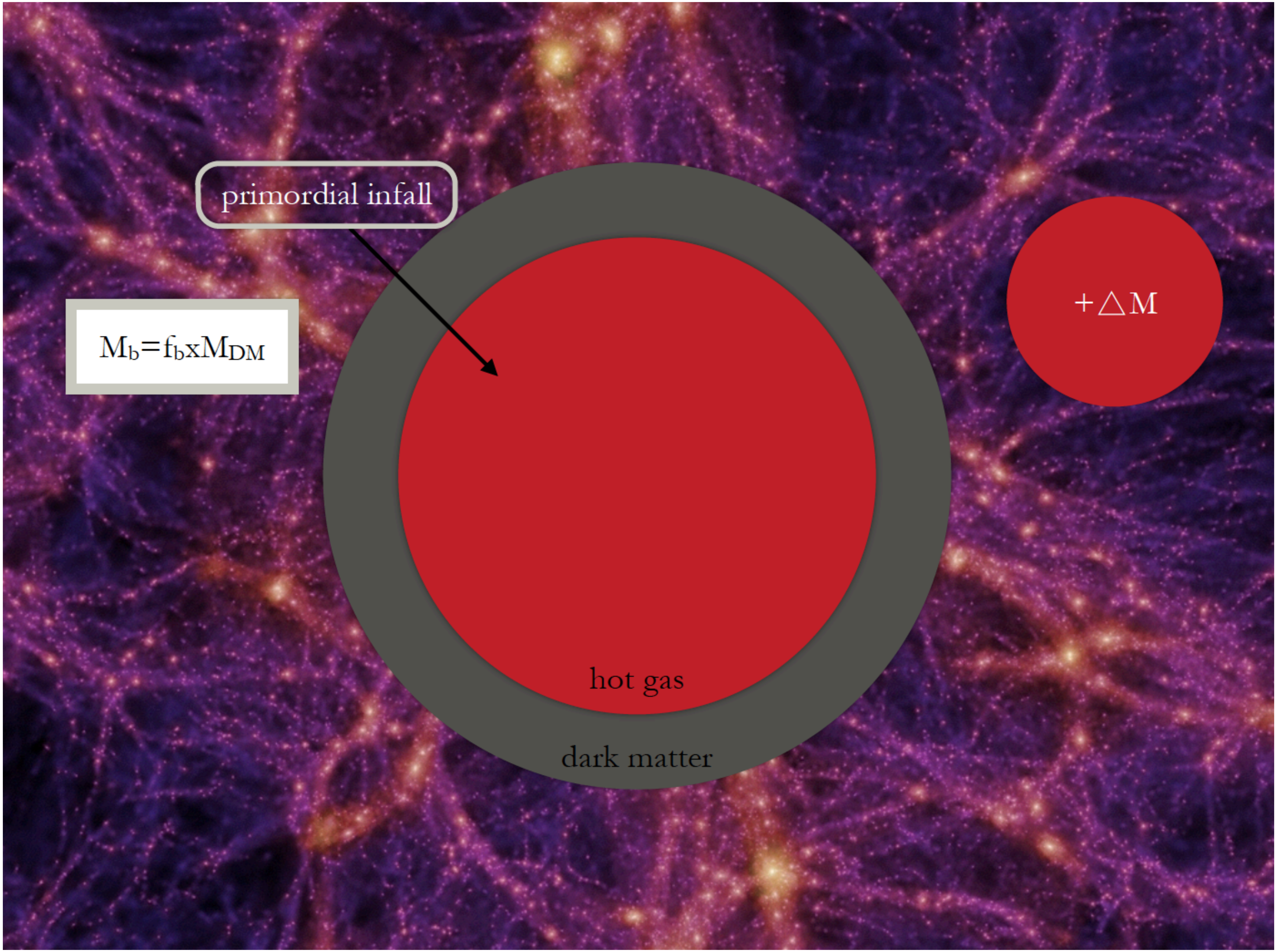
Mergers, black-hole growth, and AGN feedback

See my lecture next time.

Putting everything together



Putting everything together



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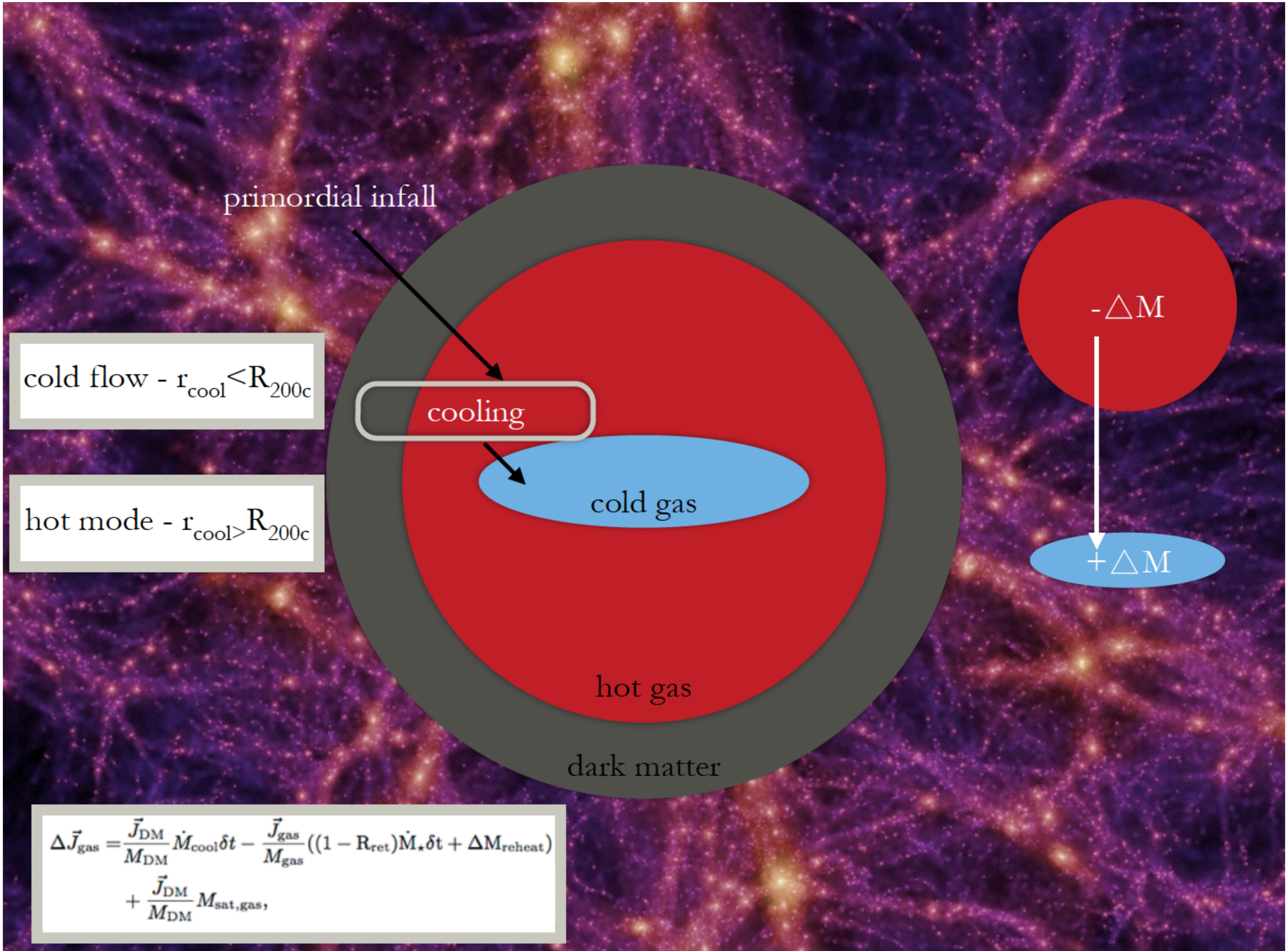
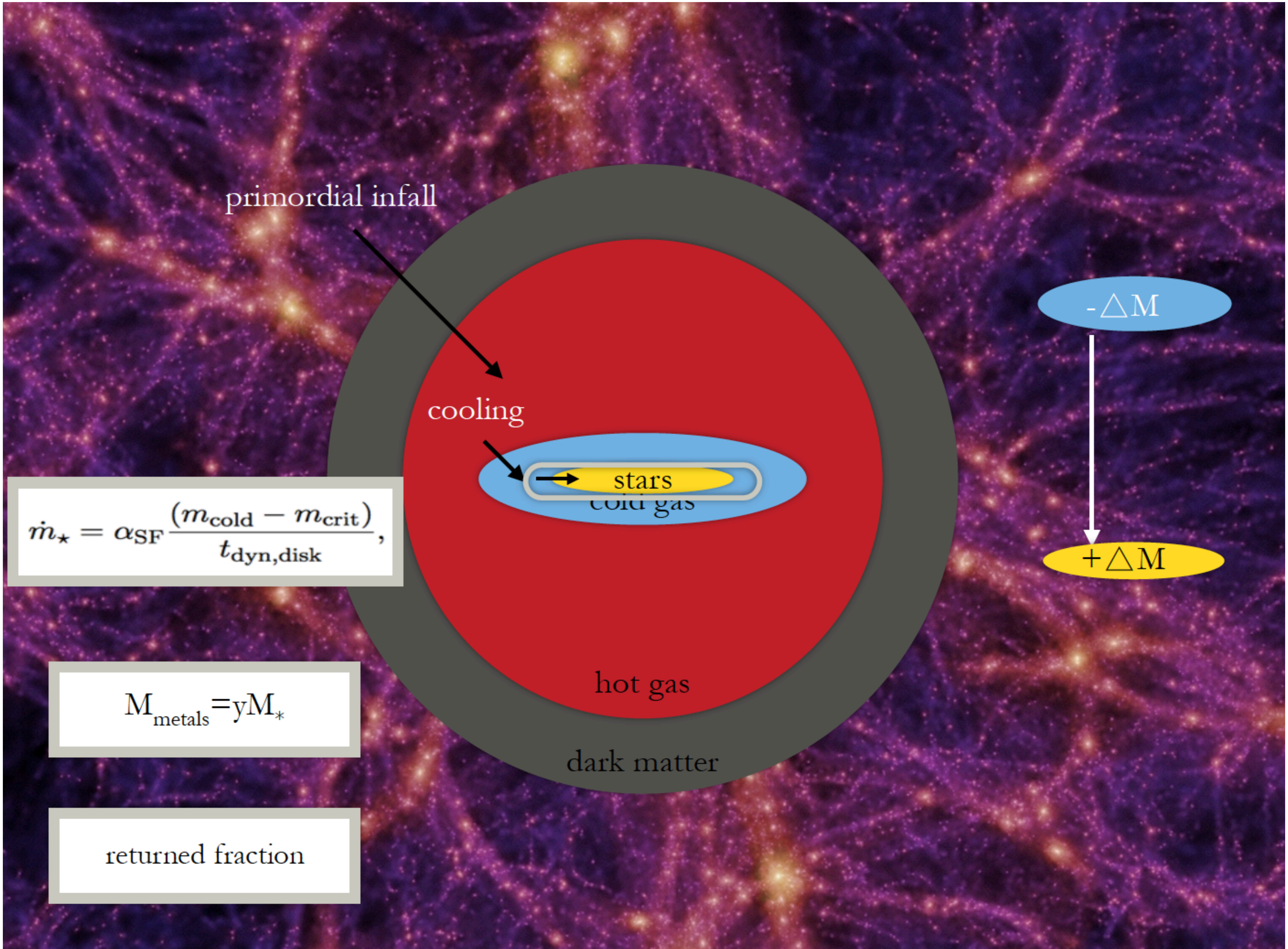


$$\Delta \vec{J}_{\rm gas} = \frac{\vec{J}_{\rm DM}}{M_{\rm DM}} \dot{M}_{\rm cool}\delta t - \frac{\vec{J}_{\rm gas}}{M_{\rm gas}}((1 - \mathrm{R_{ret}})\dot{\mathrm{M}}_{\star}\delta \mathrm{t} + \Delta \mathrm{M_{reheat}}) + \frac{\vec{J}_{\rm DM}}{M_{\rm DM}} M_{\rm sat,gas},$$

Putting everything together



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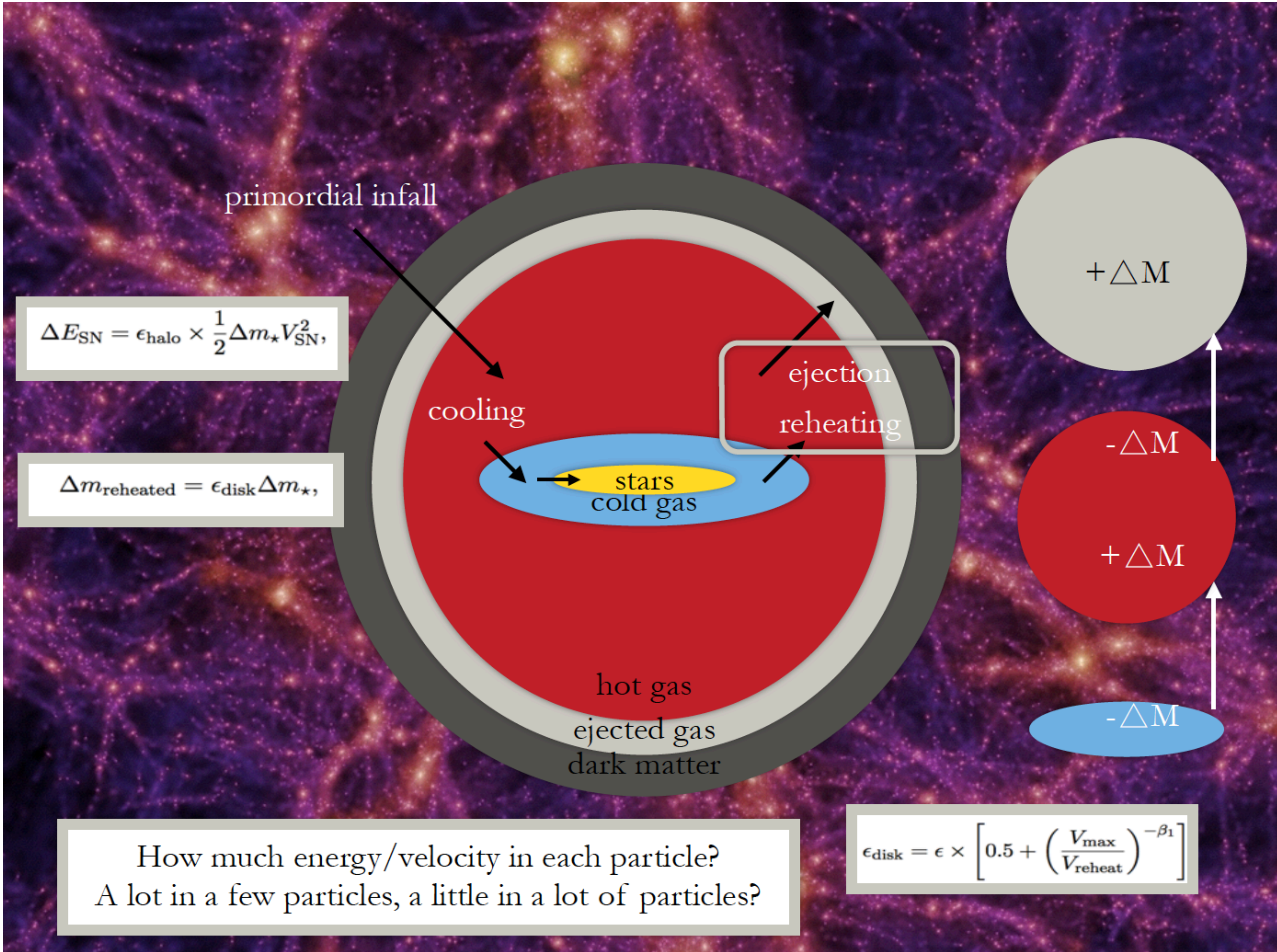


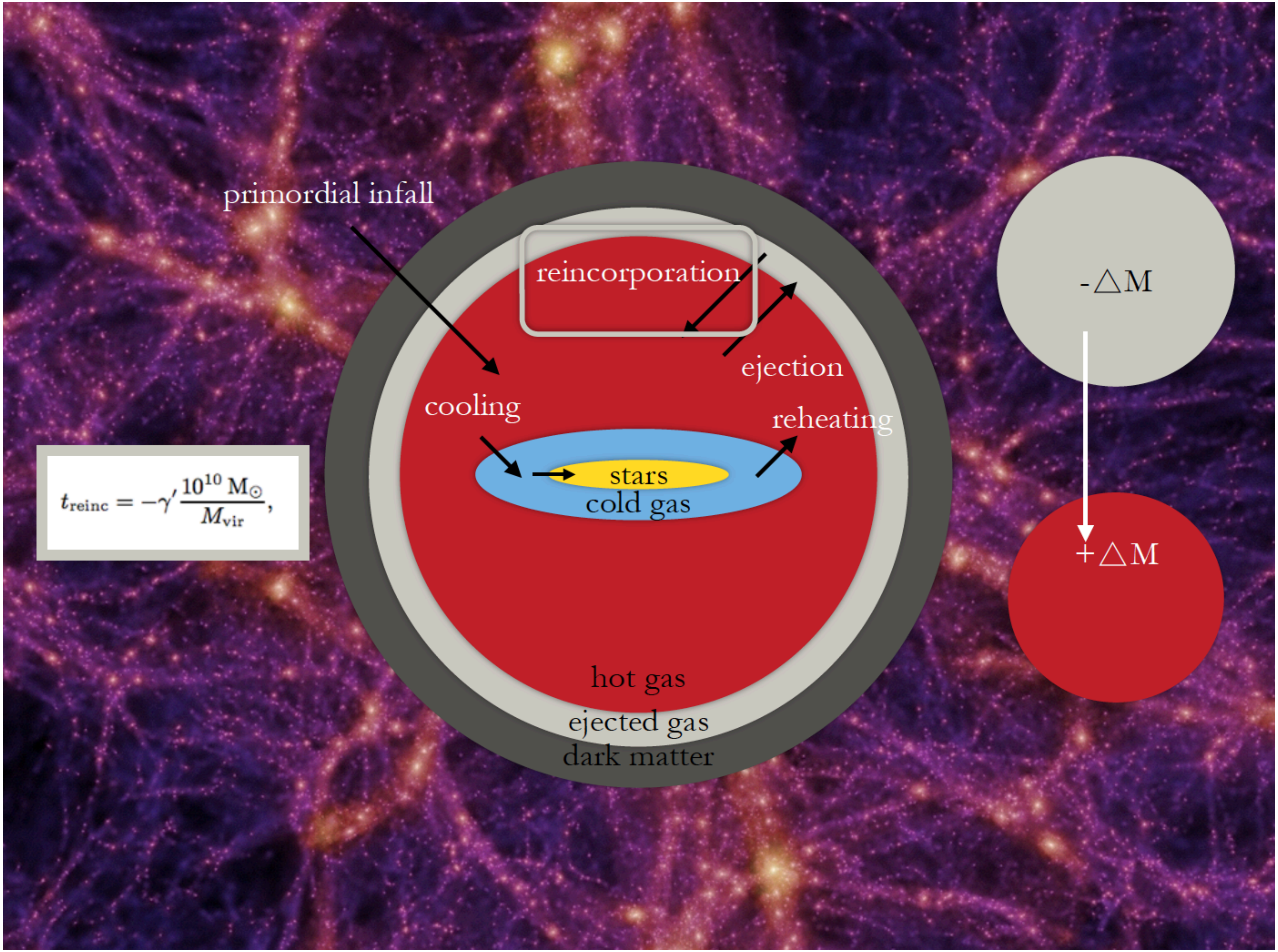
$$\Delta E_{\rm SN} = \epsilon_{\rm halo} \times \frac{1}{2}\Delta m_\star V_{\rm SN}^2,$$

$$\Delta m_{\rm reheated} = \epsilon_{\rm disk}\Delta m_\star,$$

$$\epsilon_{\rm disk} = \epsilon \times \left[0.5 + \left(\frac{V_{\rm max}}{V_{\rm reheat}}\right)^{-\beta_1}\right]$$

How much energy/velocity in each particle?
A lot in a few particles, a little in a lot of particles?

Putting everything together



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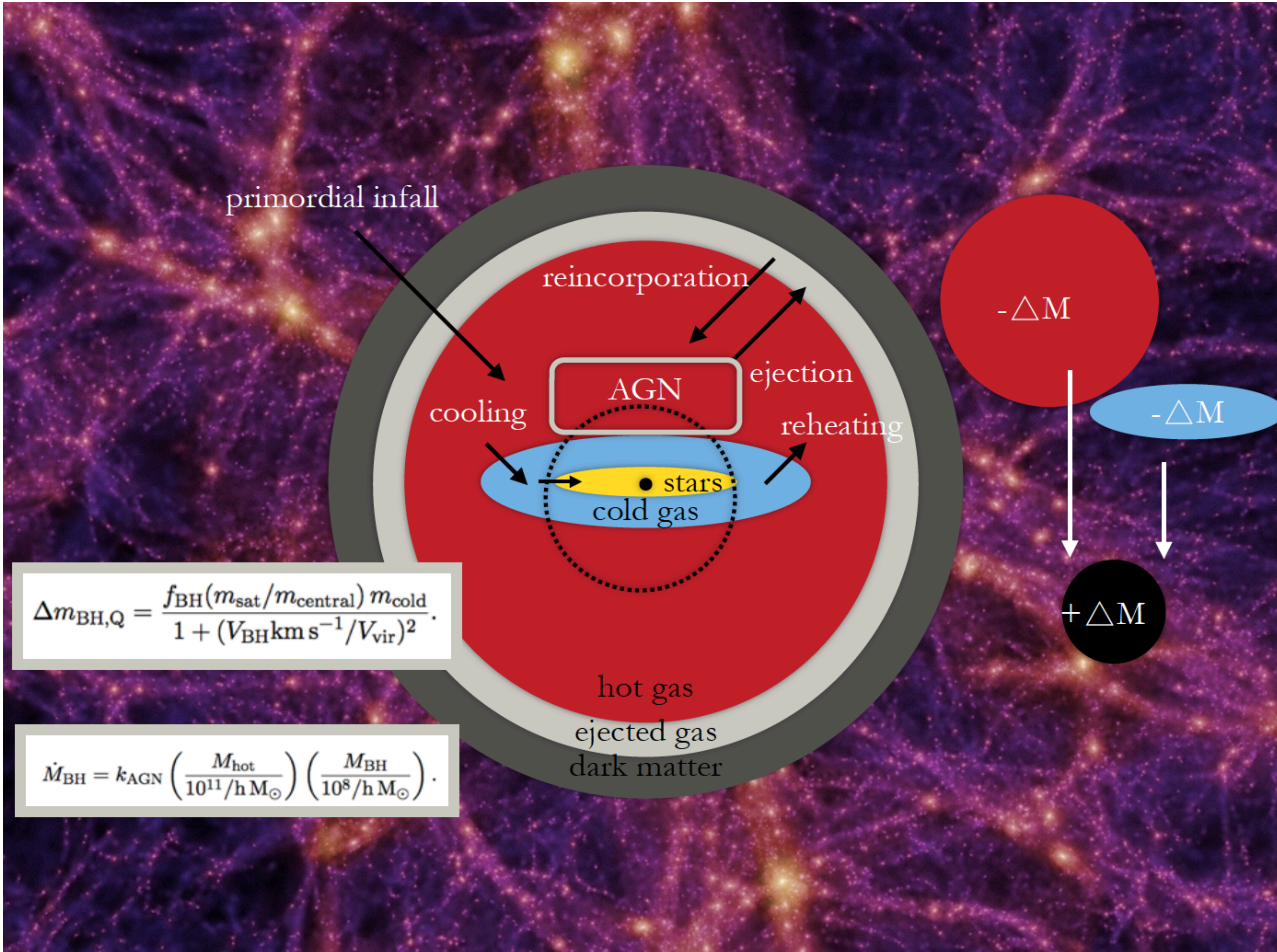
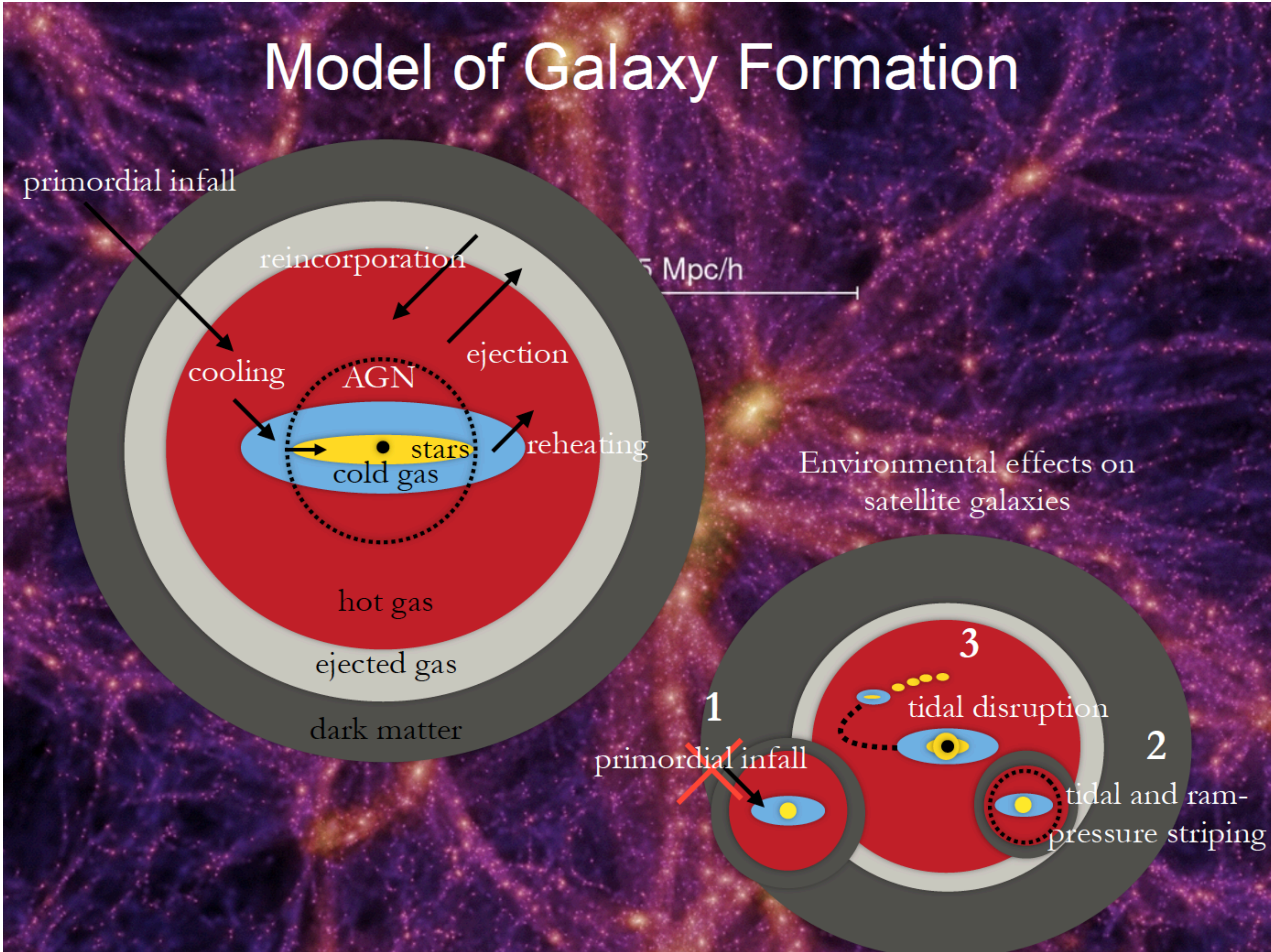


$$\Delta m_{\rm BH,Q} = \frac{f_{\rm BH}(m_{\rm sat}/m_{\rm central})\, m_{\rm cold}}{1 + (V_{\rm BH}{\rm km\,s^{-1}}/V_{\rm vir})^2}.$$

$$\dot{M}_{\rm BH} = k_{\rm AGN} \left(\frac{M_{\rm hot}}{10^{11}/{\rm h\,M_\odot}}\right)\left(\frac{M_{\rm BH}}{10^{8}/{\rm h\,M_\odot}}\right).$$

Putting everything together



SAMs in the literature

Currently there are many different SAMs available in the literature, all with different treatments of different physical processes.

Feature	Model				
	Durham[1]	Munich[2]	Santa Cruz[3]	Morgana[4]	Galics[5]
Merger trees					
→ Analytic	Modified ePS[6]	ePS[7]	ePS	Pinocchio[8]	×
→ N-body	✓[9]	✓	✓	×	✓
Halo Profiles	Einasto[10]	Isothermal	NFW	NFW	Empirical[11]
Cooling Model					
→ Metal dependent	✓	✓	✓	✓	✓
Star Formation	✓	✓	✓	✓	✓
Feedbacks					
→ SNe	✓	✓	✓	✓	✓
→ AGN	✓[12]	✓	✓	✓	✓[13]
→ Reionization	✓[10]	×	✓	✓[14]	✓[15]
Merging					
→ Substructure[16]	N-body[17]	N-body[17]	DF[18]	DF[18]	N-body[17]
→ Substructure–Substructure[19]	✓[20,10]	×	✓[21,22]	×	✓[21]
Environments					
→ Ram pressure stripping	✓[23]	✓[24]	×	×	✓[25]
→ Tidal Stripping	✓[10]	×	✓	✓	✓
→ Harassment	×	×	×	×	×
Disks					
→ Disk stability	✓	✓	✓[26]	✓	✓
→ Dynamical friction[27]	✓[28]	×	×	×	×
→ Thickness	✓[28]	×	×	×	×
Sizes					
→ Adiabatic contraction	✓	×	✓	✓	×
Chemical enrichment	✓[delayed[10]]	✓[instant[29]]	✓[delayed[30]]	✓[instant]	✓[delayed[31]]
Dust	Grasil[32]	Screen[33]	Slab[34]	Grasil[32,35]	Slab[34]

Reading List

- Somerville & Primack, 1999, MNRAS, 310, 1087 (Santa Cruz SAM)
- Baugh, 2006, Rep. Prog. Phys., 69, 3101 (SAM review)
- Benson, 2010, Physics Reports, 495, 33 (SAM review)
- Henriques et al., 2015, MNRAS, 451, 2663 (Munich SAM)
- Benson 2012, New Astronomy, 17, 175 (Durham SAM)

Predictions of SAM

See my lecture next time.

- galaxy luminosity function
- galaxy-halo connection (Tully-Fisher relation)
- star-forming main sequence
- satellite quenching
- … … (many other applications beyond this lecture)