Introduction:
Big-Bang Cosmology

Basic Assumptions

Principle of Relativity:

The laws of nature are the same everywhere and at all times

The Cosmological principle:

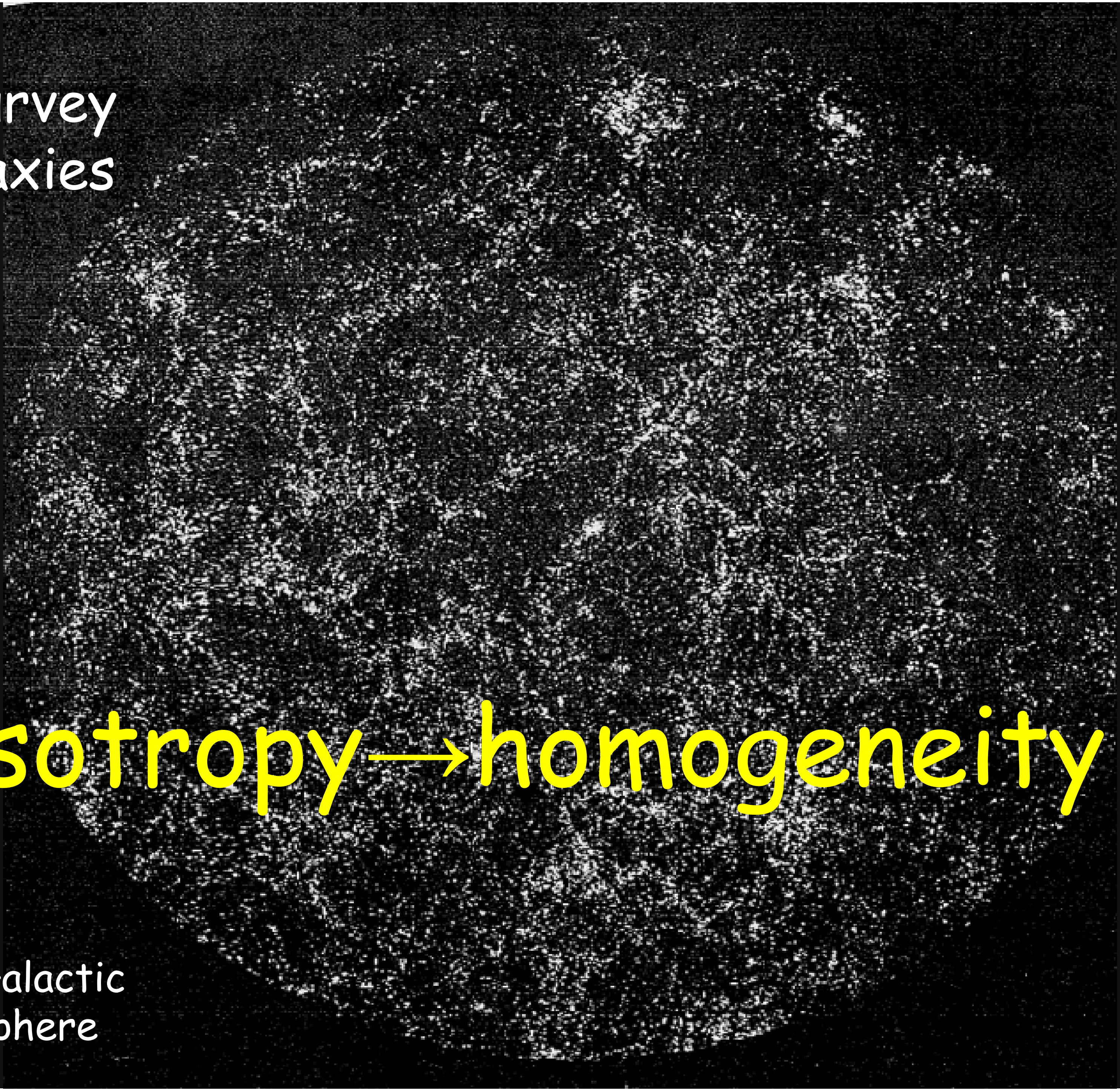
The universe is homogeneous and isotropic

Space time is simply connected, can be filled with comoving observers (CO).

Each CO performs local measurements of distance and time in it's own frame of reference, locally flat. No global inertial frame.

Cosmic time: synchronized clocks of COs in space at every given time.

Lick Survey
1M galaxies



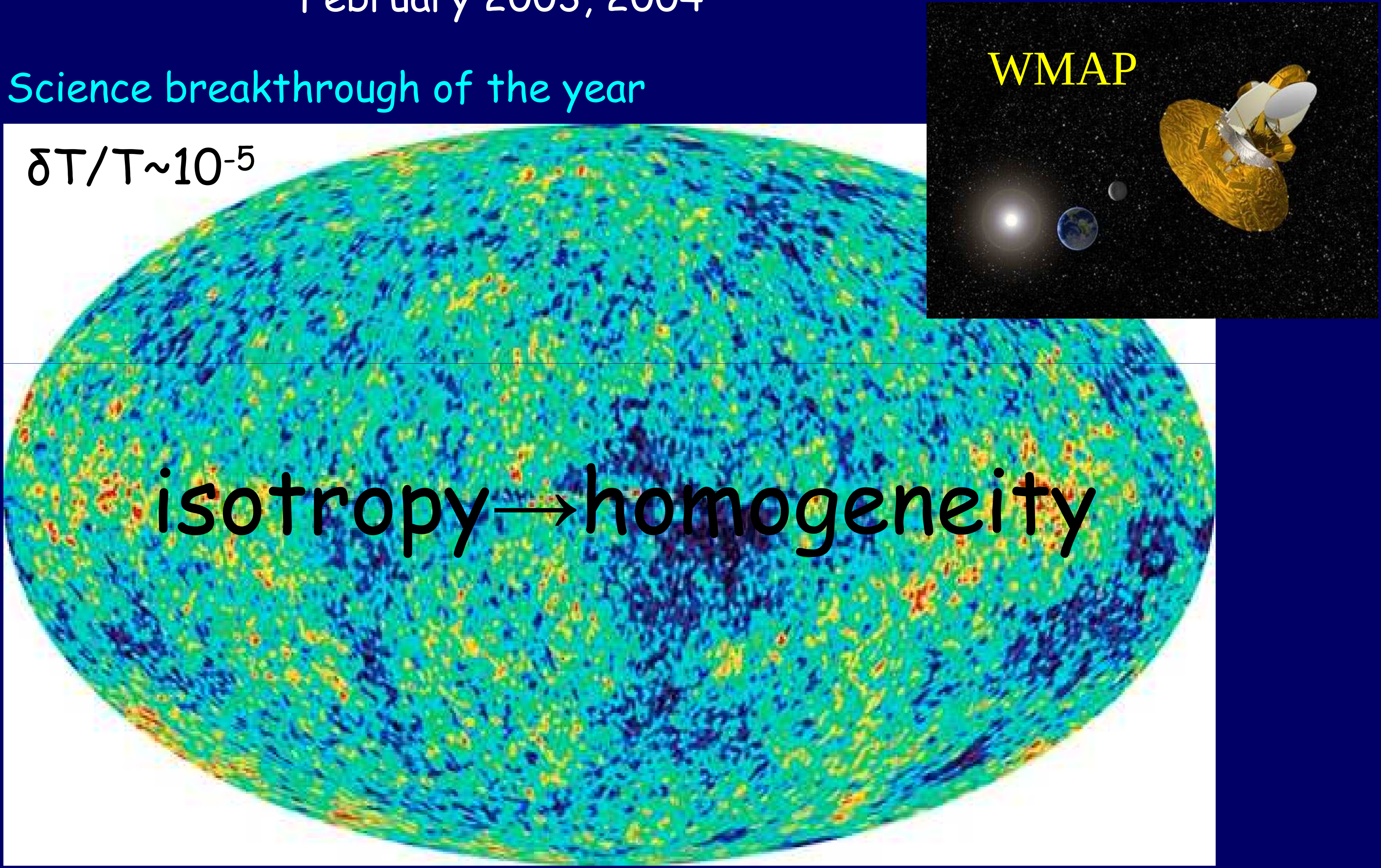
isotropy→homogeneity

North Galactic
Hemisphere

Microwave Anisotropy Probe

February 2003, 2004

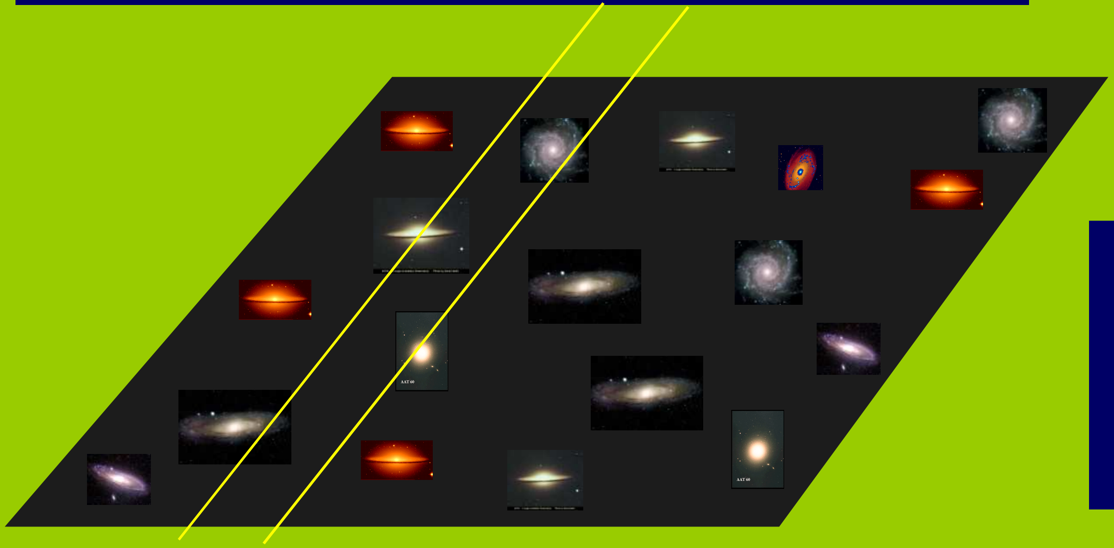
Science breakthrough of the year



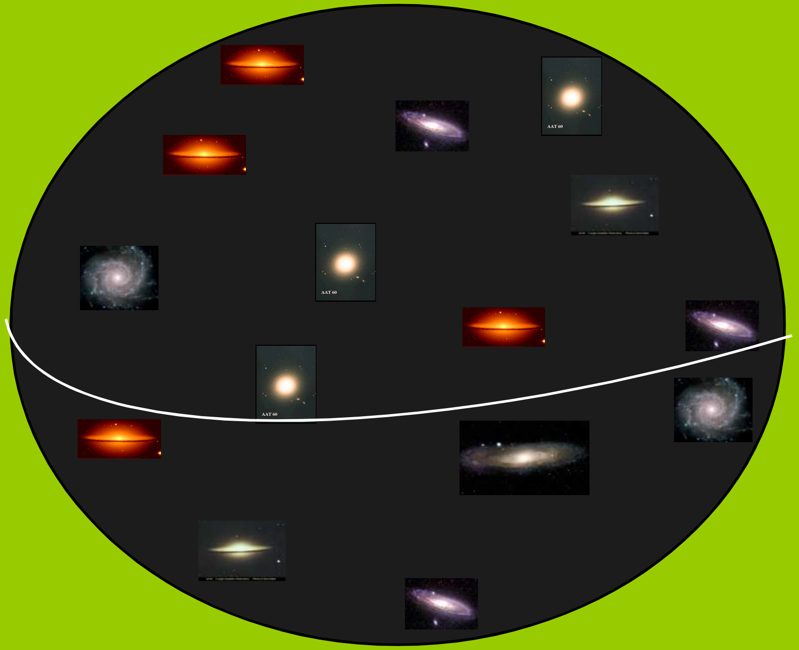
$\delta T/T \sim 10^{-5}$

isotropy→homogeneity

Homogeneous Universe:

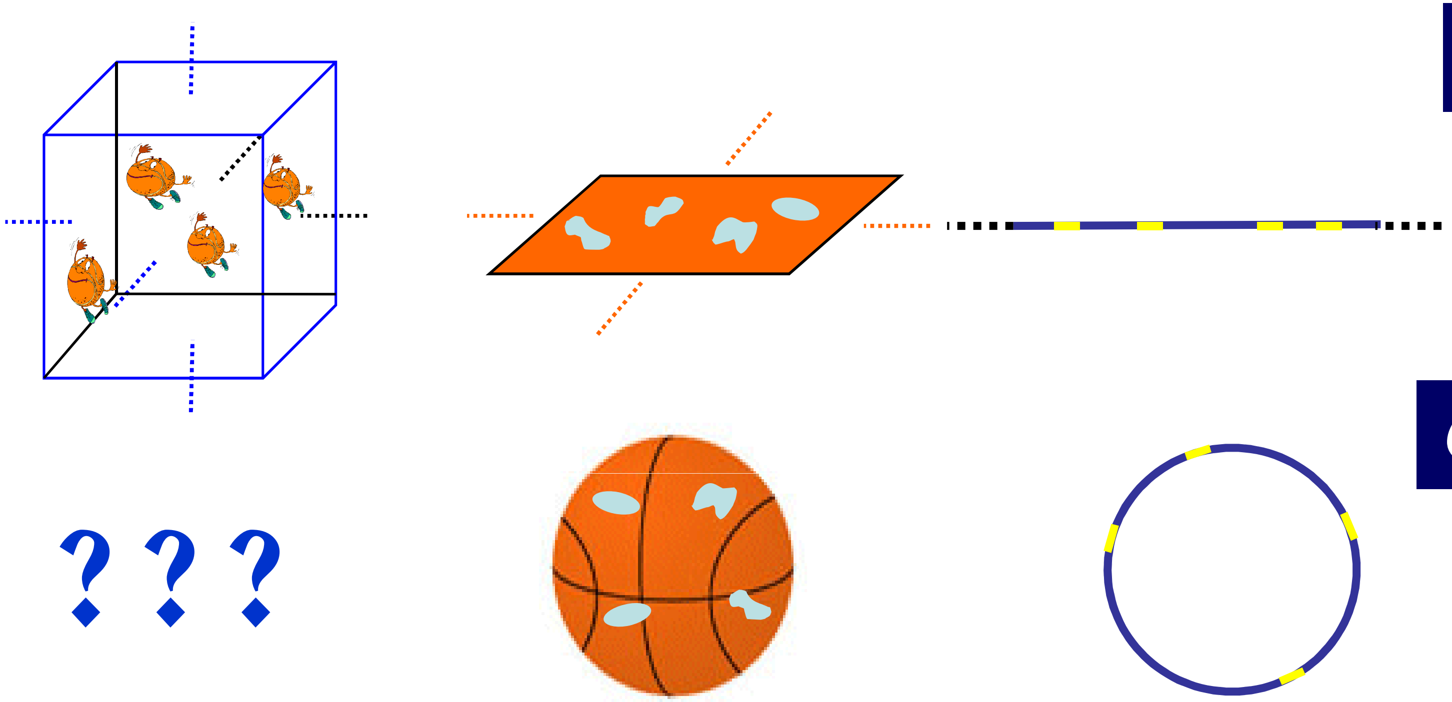


Flat (Euclidean), therefore open, infinite



Curved and closed, finite

Three-dimensional **Two dimensional** **One-dimensional**



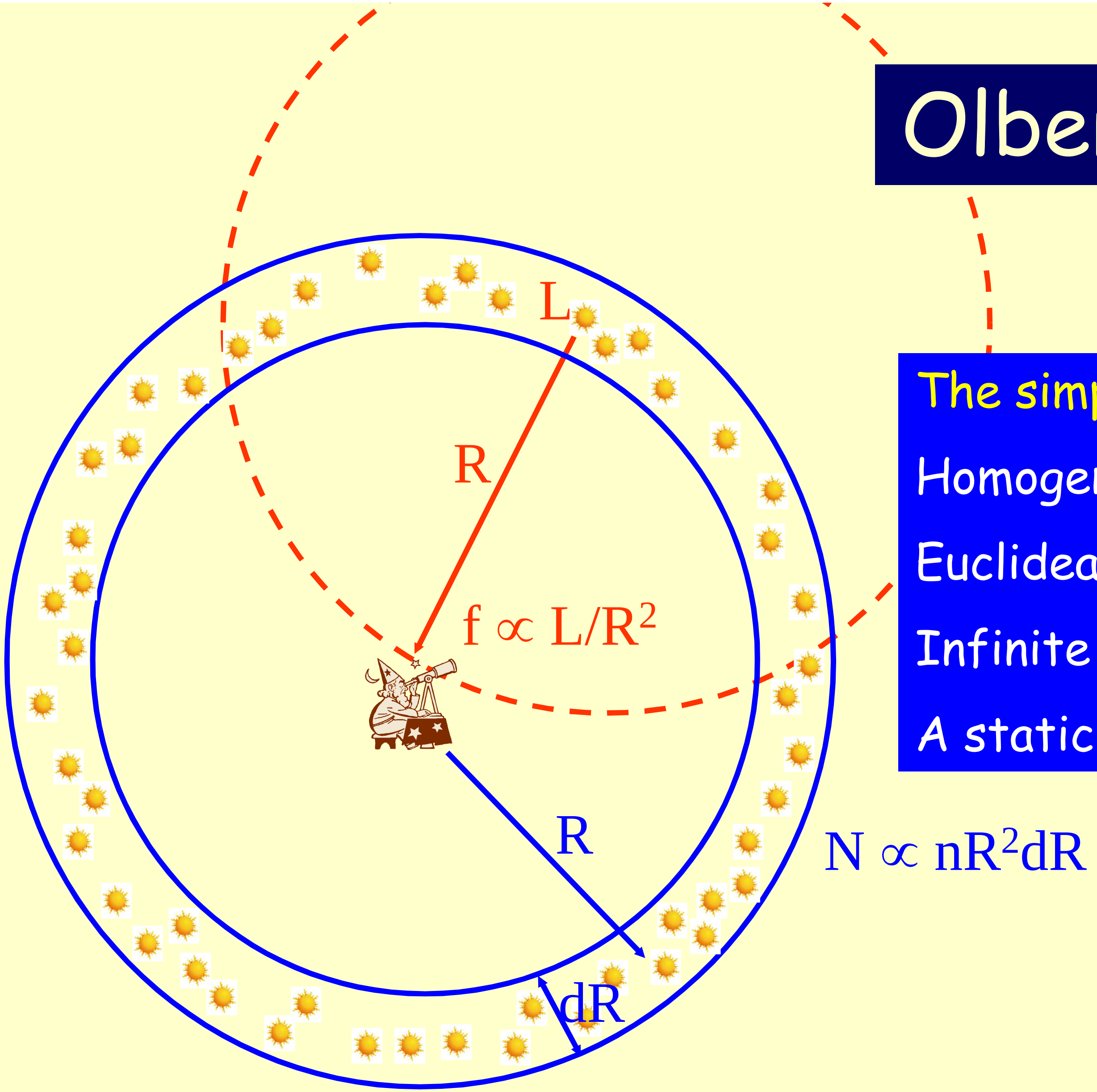
Open

Closed

? ? ?

$$x^2+y^2+z^2+w^2=R^2 \qquad x^2+y^2+z^2=R^2 \qquad x^2+y^2=R^2$$

Olbers Paradox



The simplest assumptions:

Homogeneity and isotropy

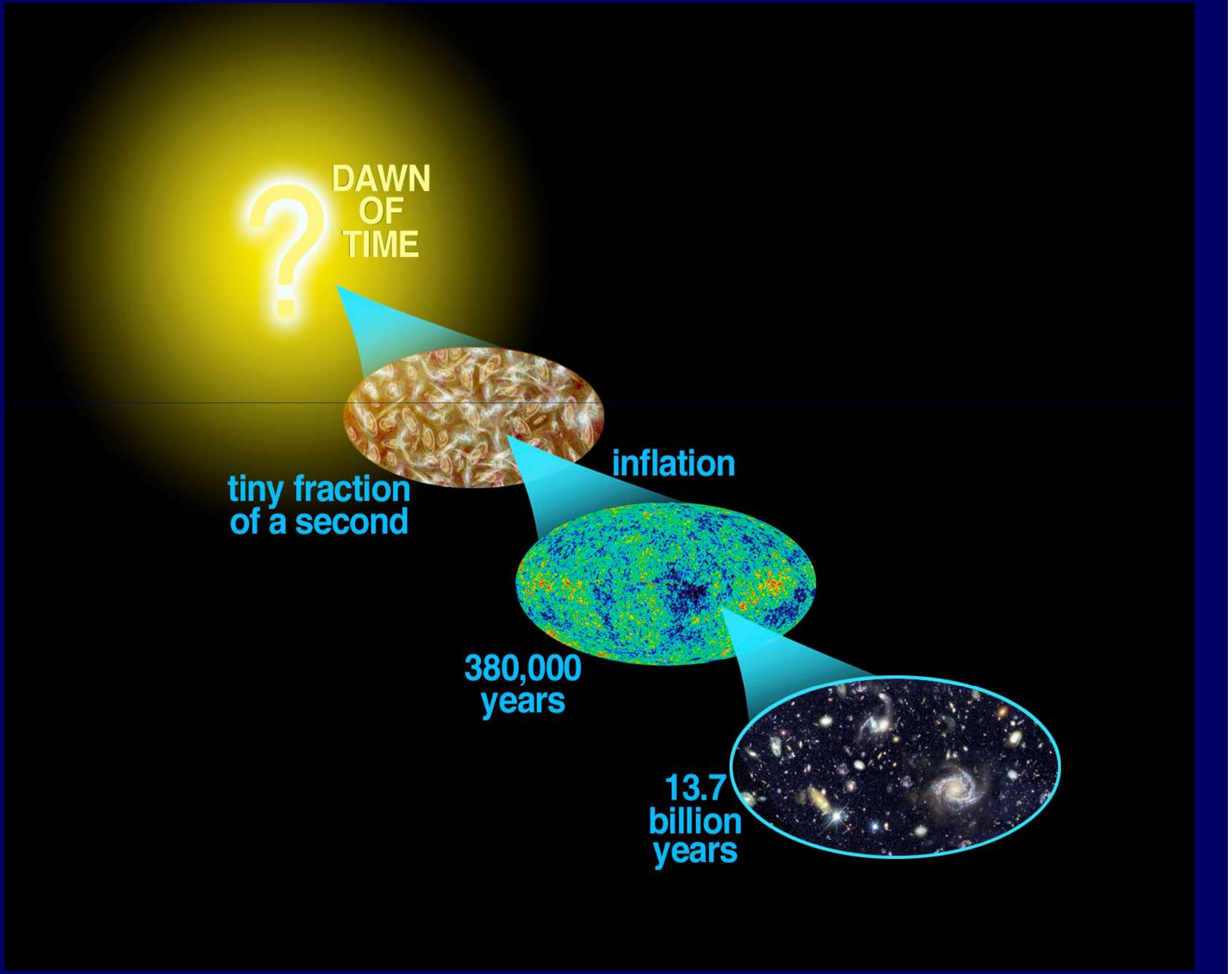
Euclidean geometry

Infinite space

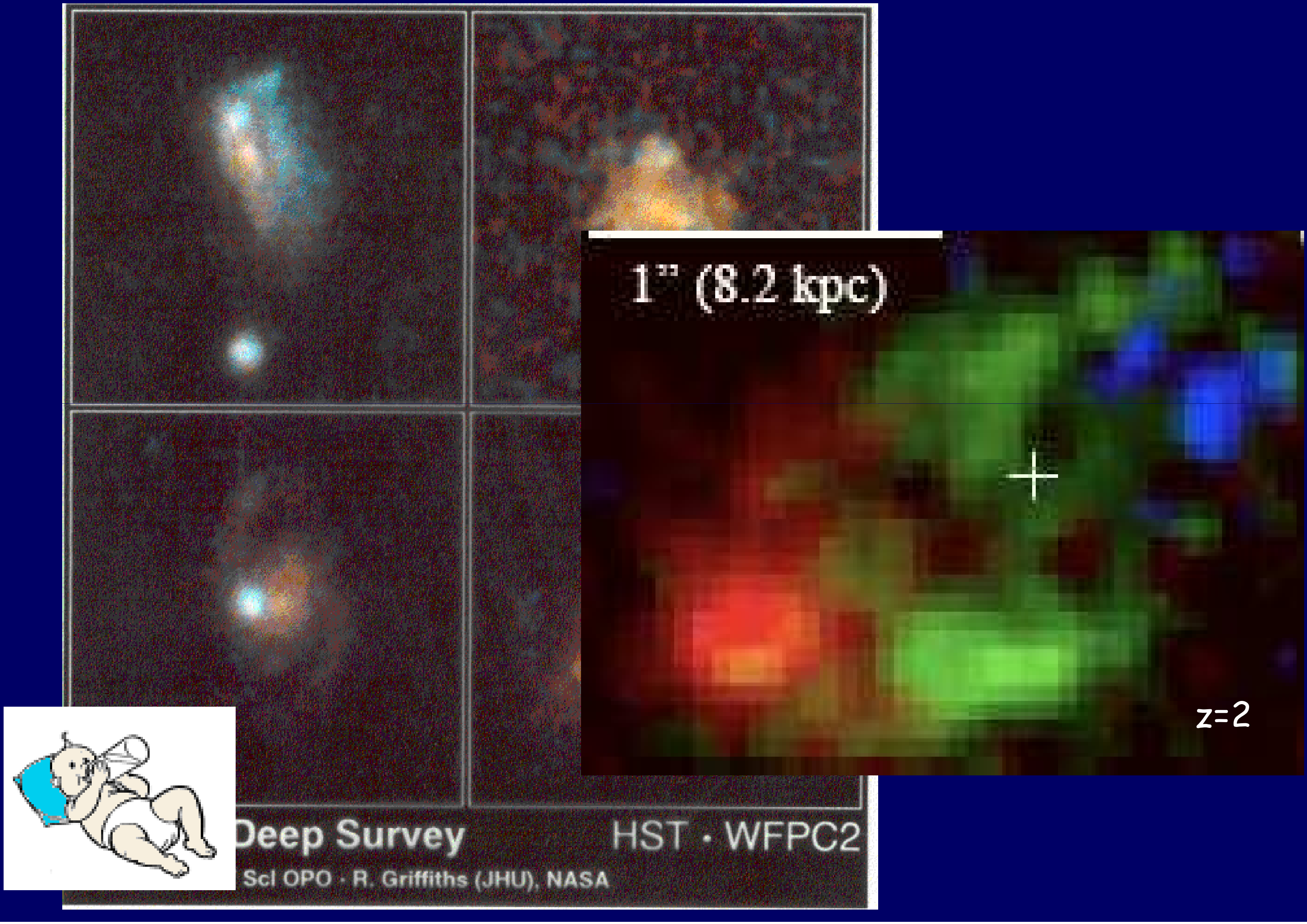
A static universe

$$\text{Flux} = \int_0^\infty \frac{L}{R^2} nR^2 dR = \int_0^\infty nL \, dR = nL + nL + nL + \ldots \to \infty$$

The Universe is evolving in time



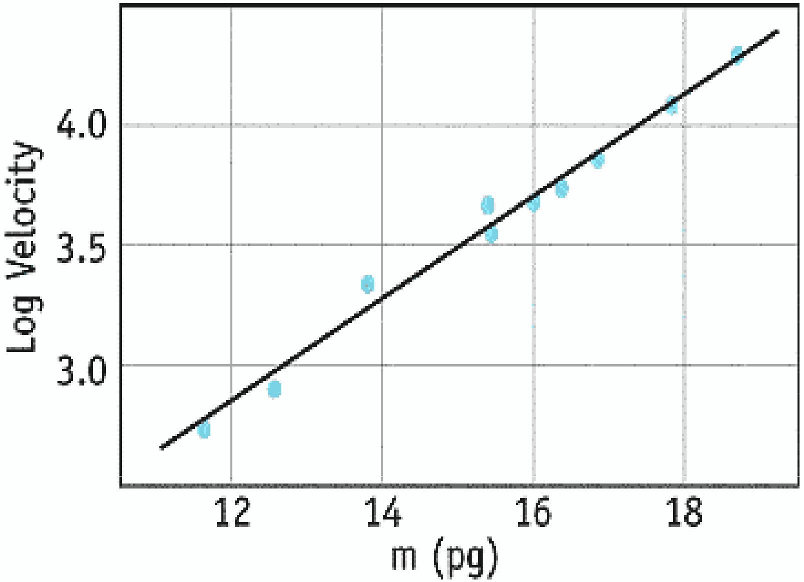
Baby galaxies in early universe

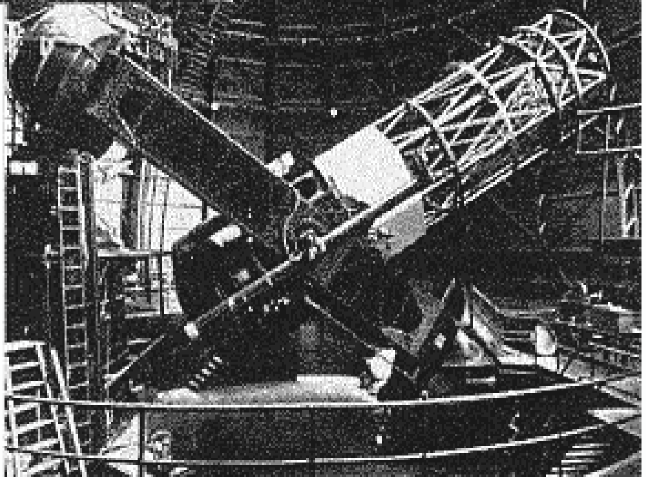


the universe is expanding

DISCOVERY OF EXPANDING UNIVERSE 1929

Edwin Hubble

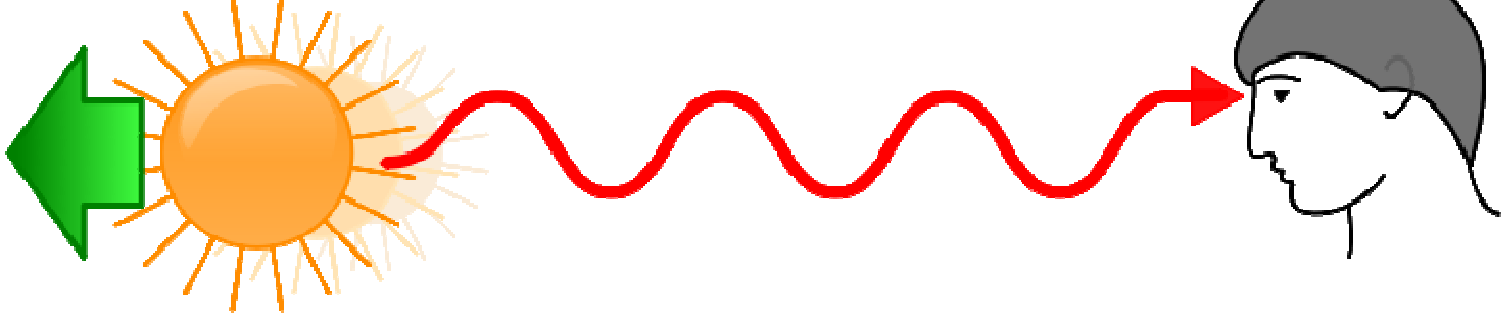




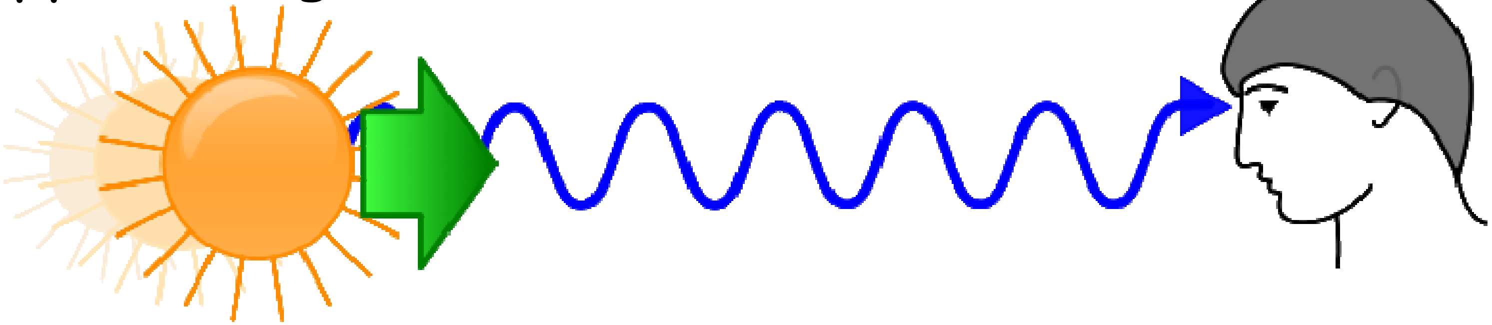
Mt. Wilson
100 Inch
Telescope

Doppler Shift

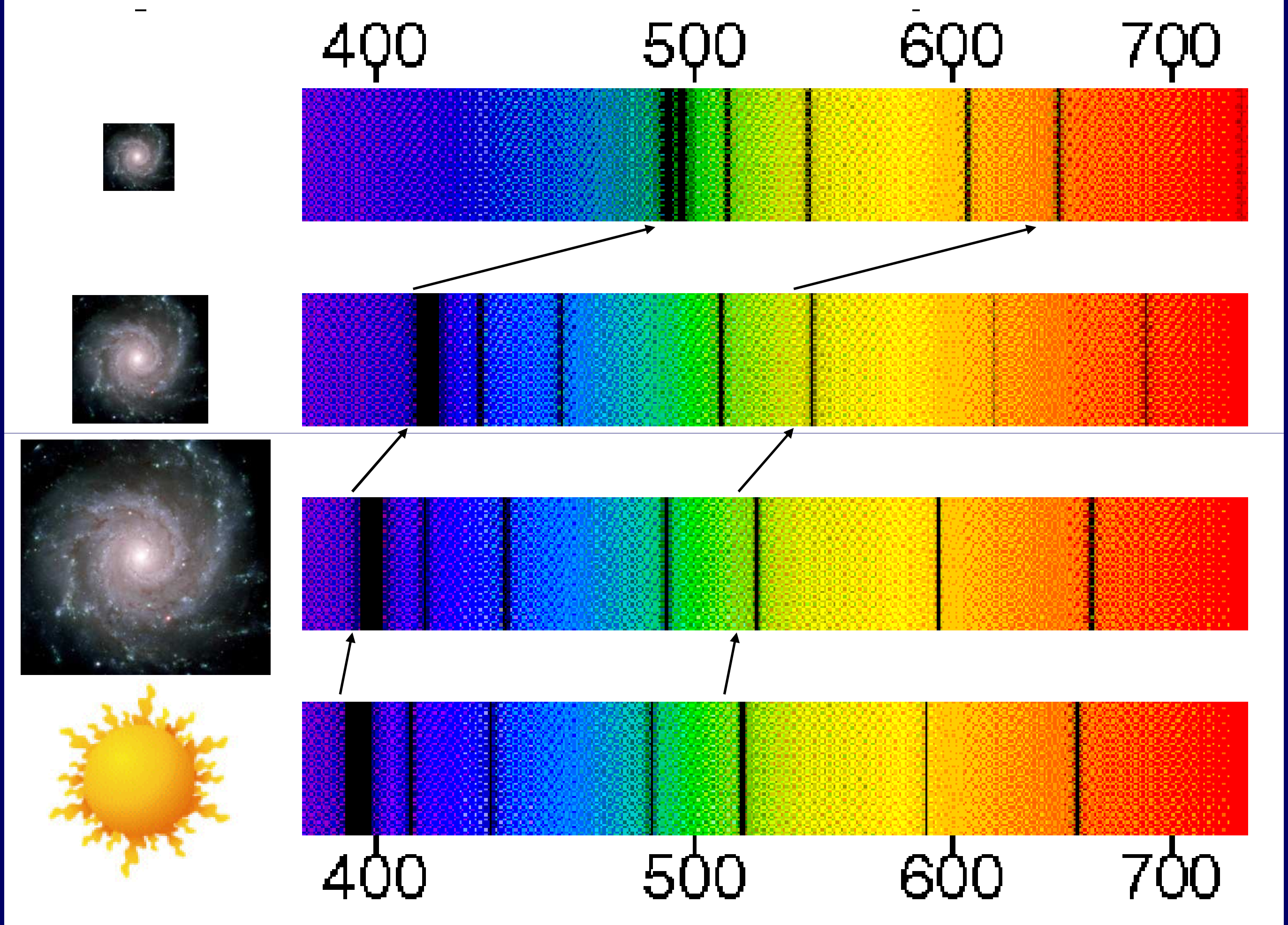
receding source



approaching source



Red-shift



distance

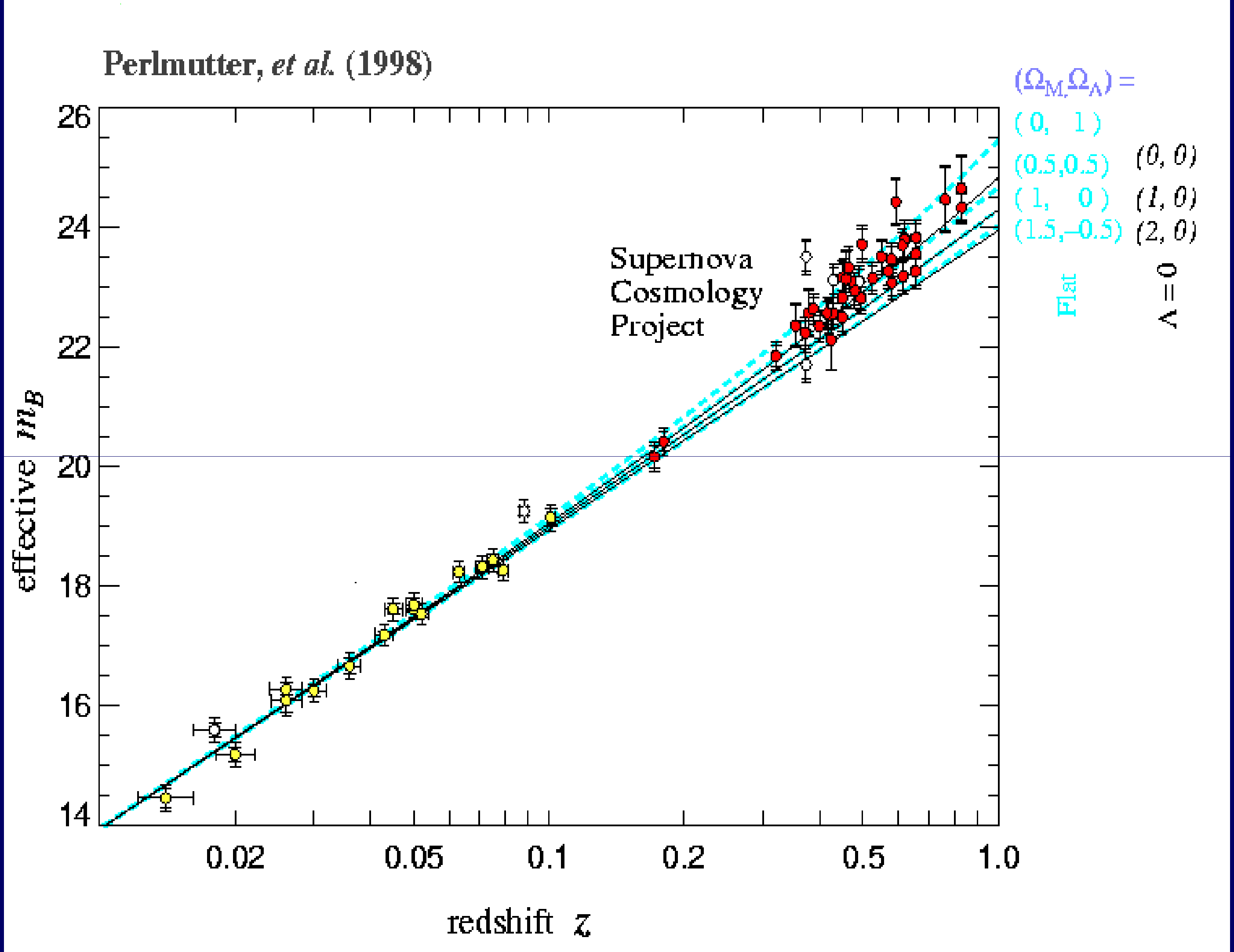
wavelength

Hubble Expansion: $V = H R$

↑

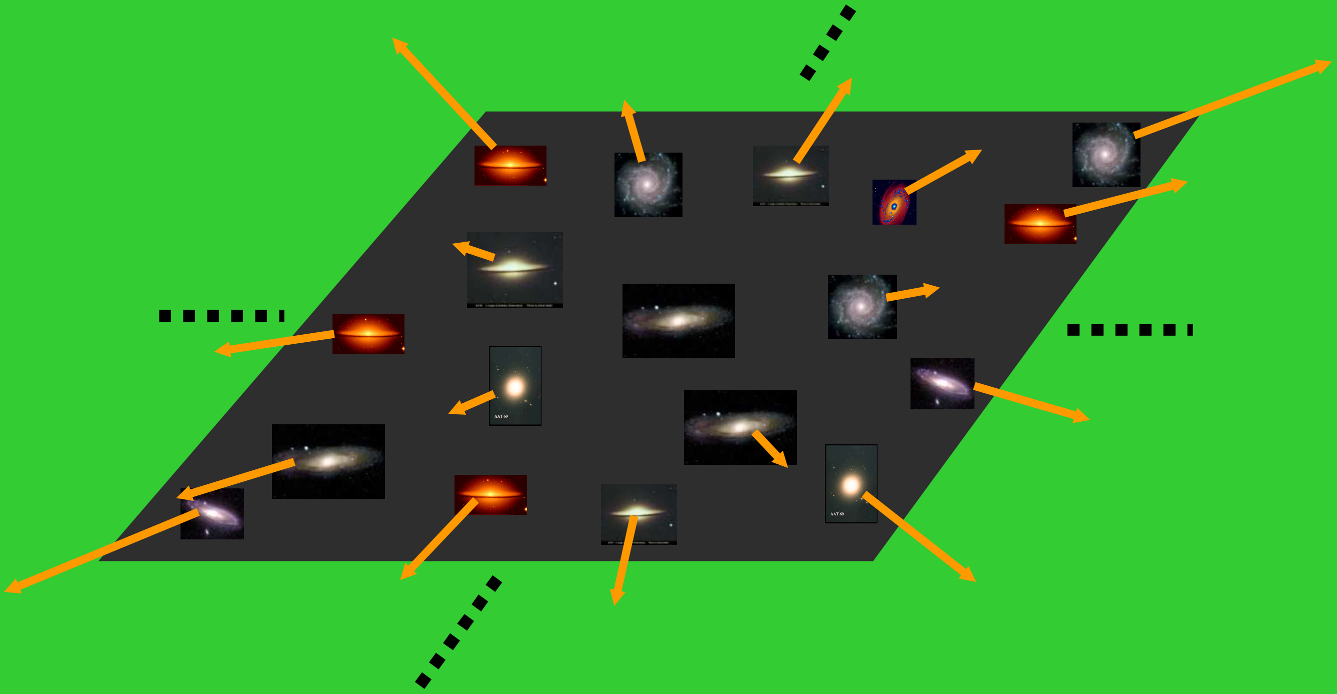
Distance

R



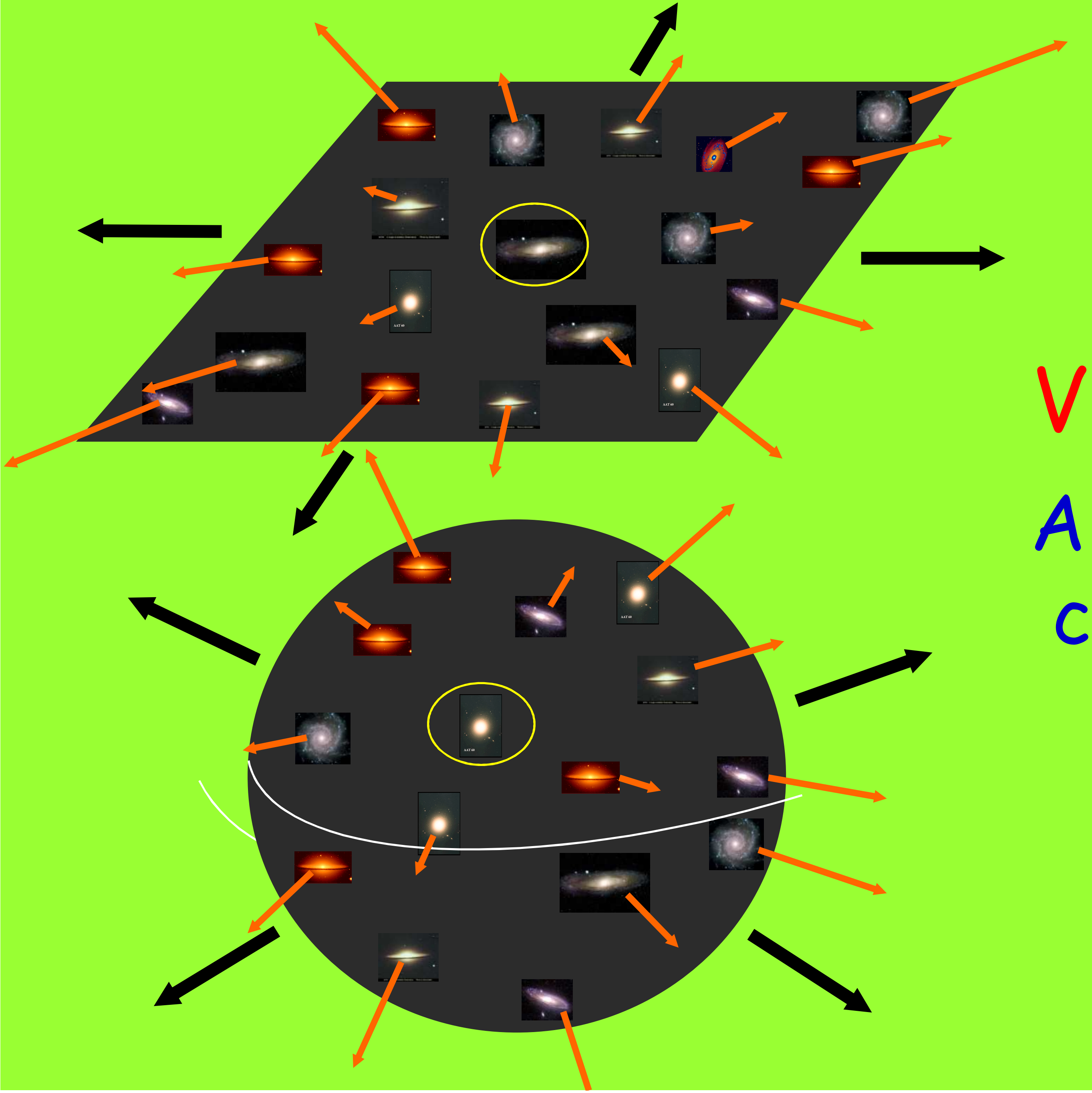
Velocity V ⟶

Hubble Expansion



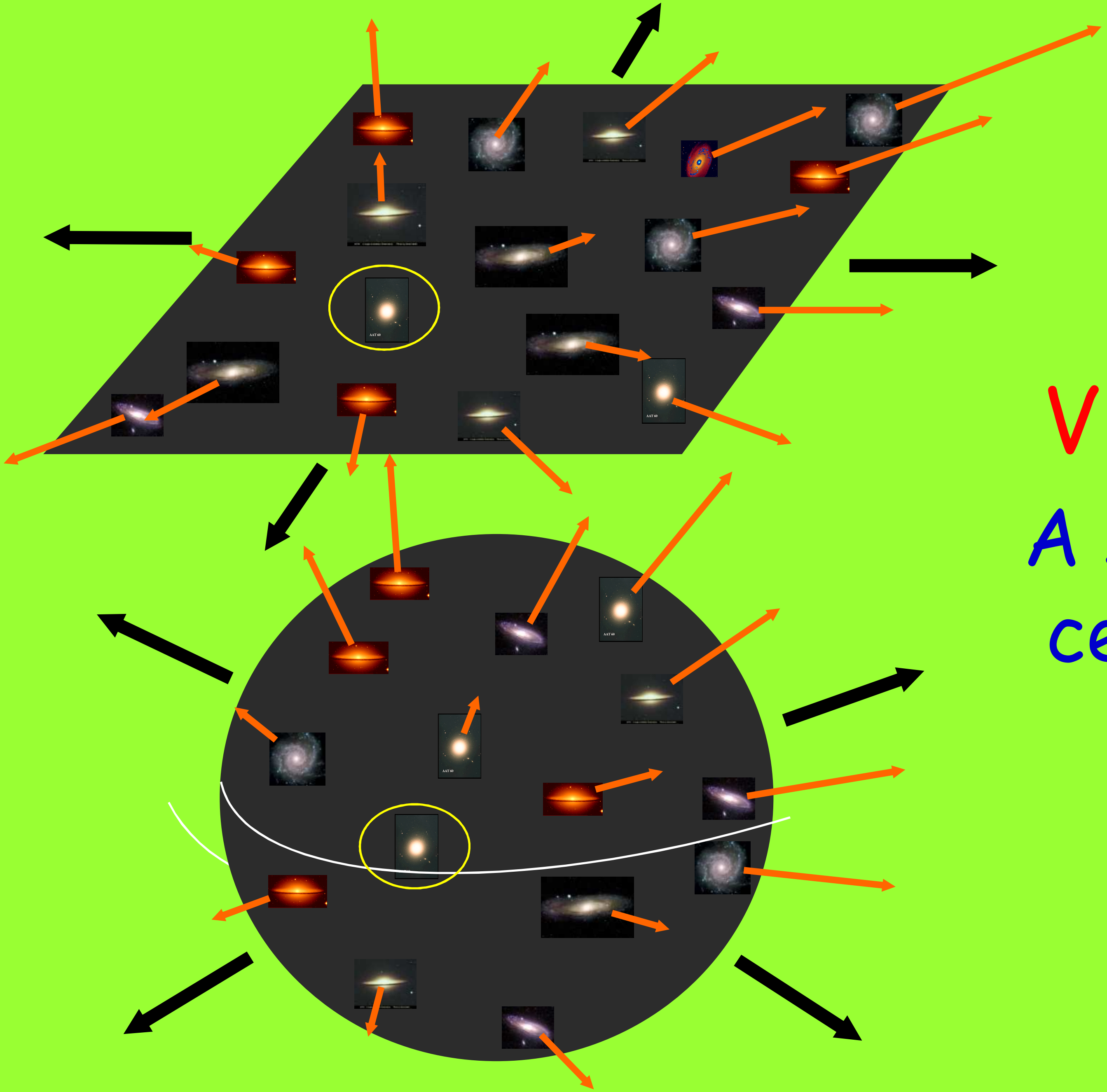
velocity — $V = HR$ — distance

Hubble constant



$V = H R$

A special center?

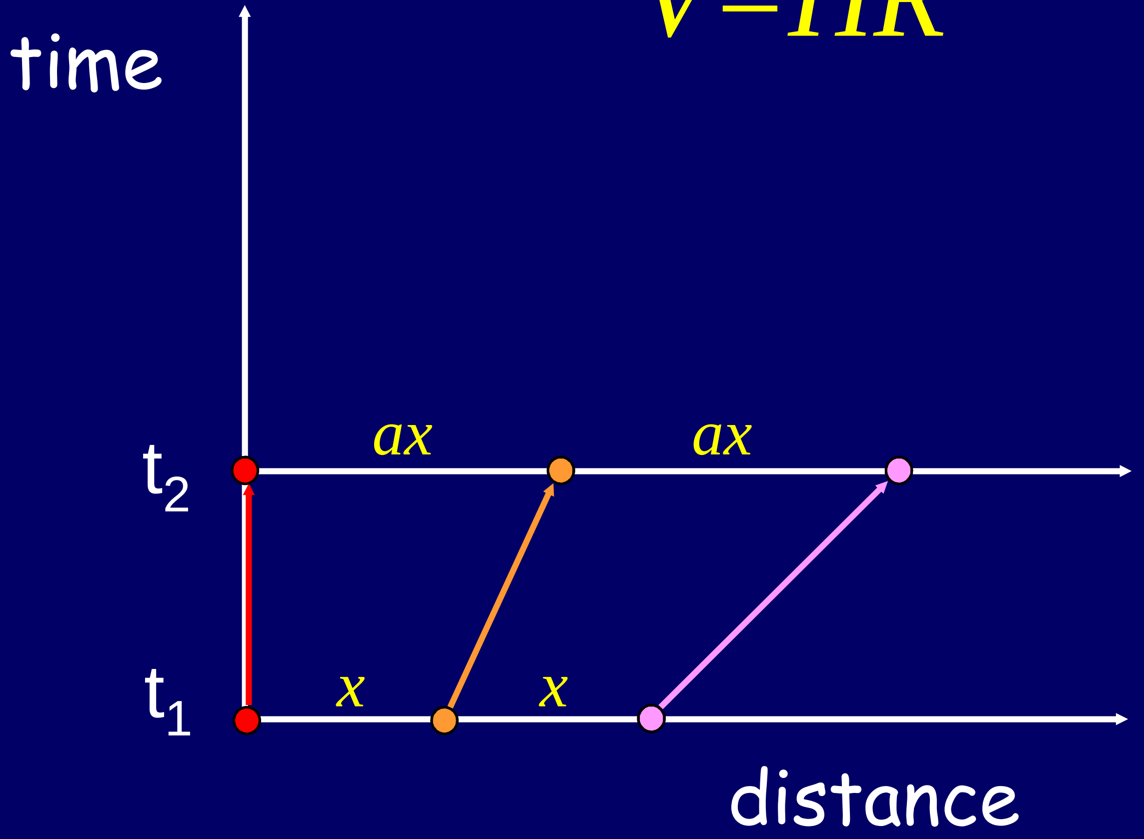


$V = H R$

A special center?

Prediction from homogeneity: The Hubble Law

$V=HR$

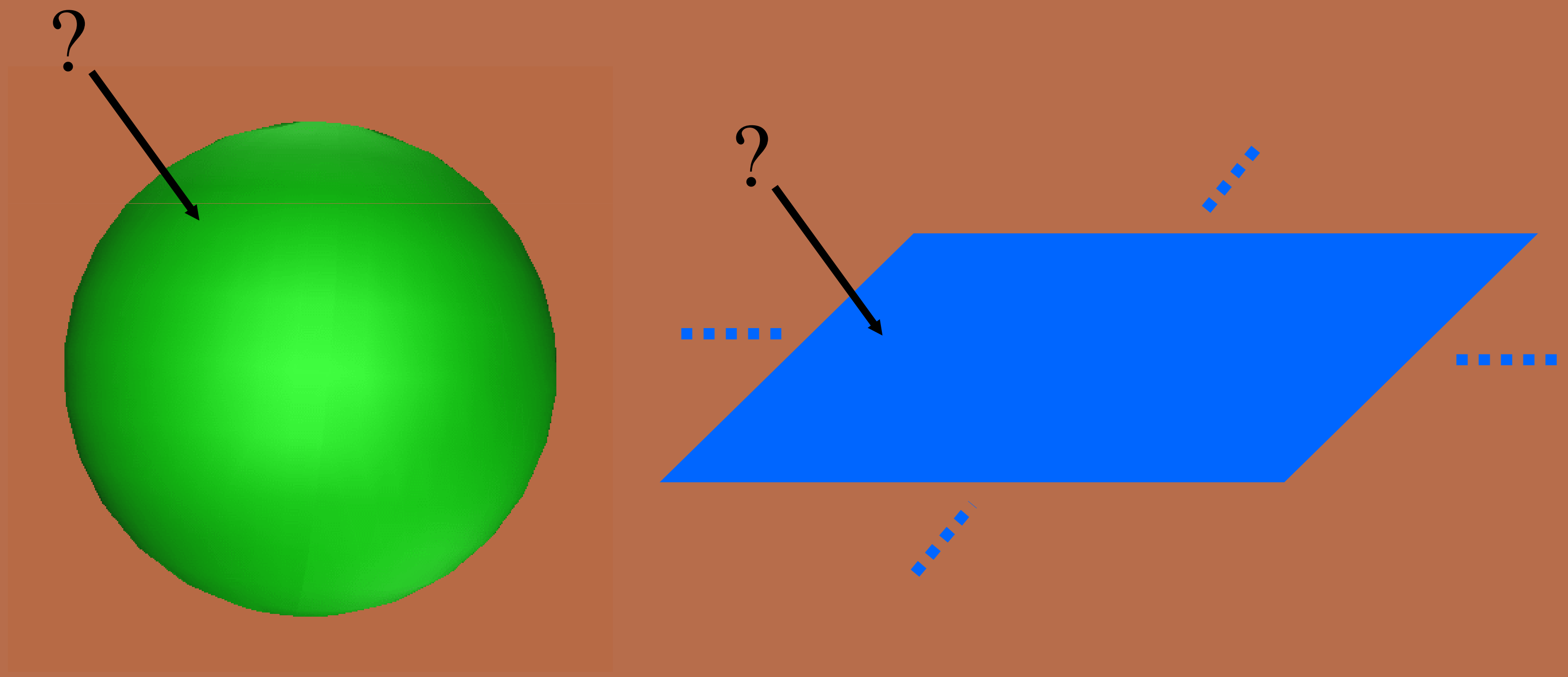


$$r(x,t)=a(t)x \quad \rightarrow \quad \dot{r}=\dot{a}x=\frac{\dot{a}}{a}r \quad H=\frac{\dot{a}}{a}$$

The Big Bang $t_0 \approx 13.7$ Gyr

היכן היה המפץ הגדול?

בנקודה אחת? בהרבה נקודות?



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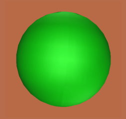
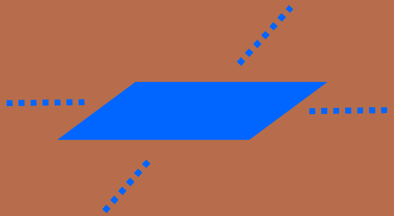
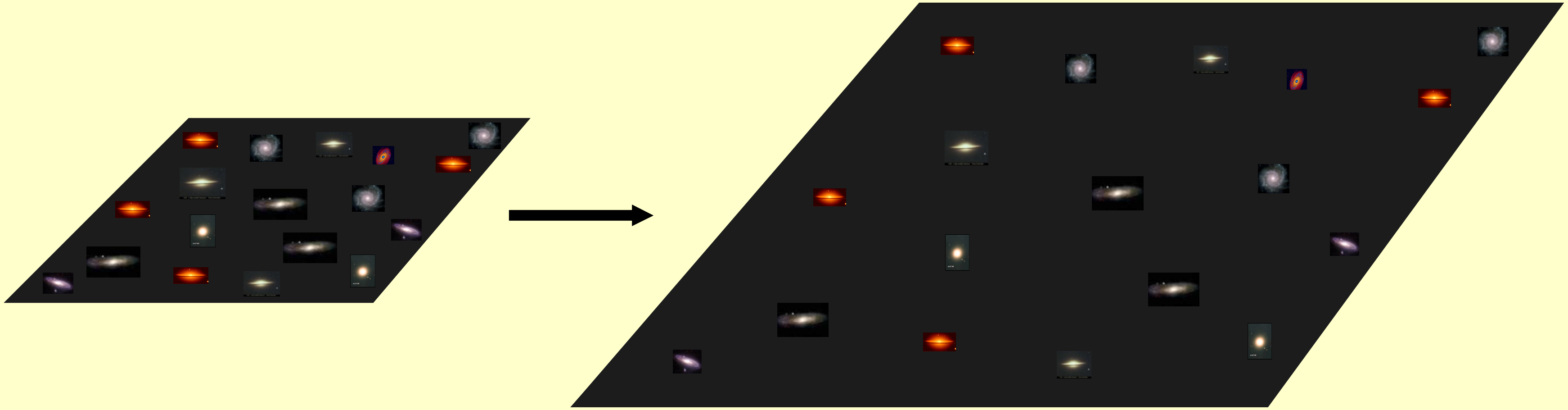


כל הנקודות מתלכדות לנקודה אחת

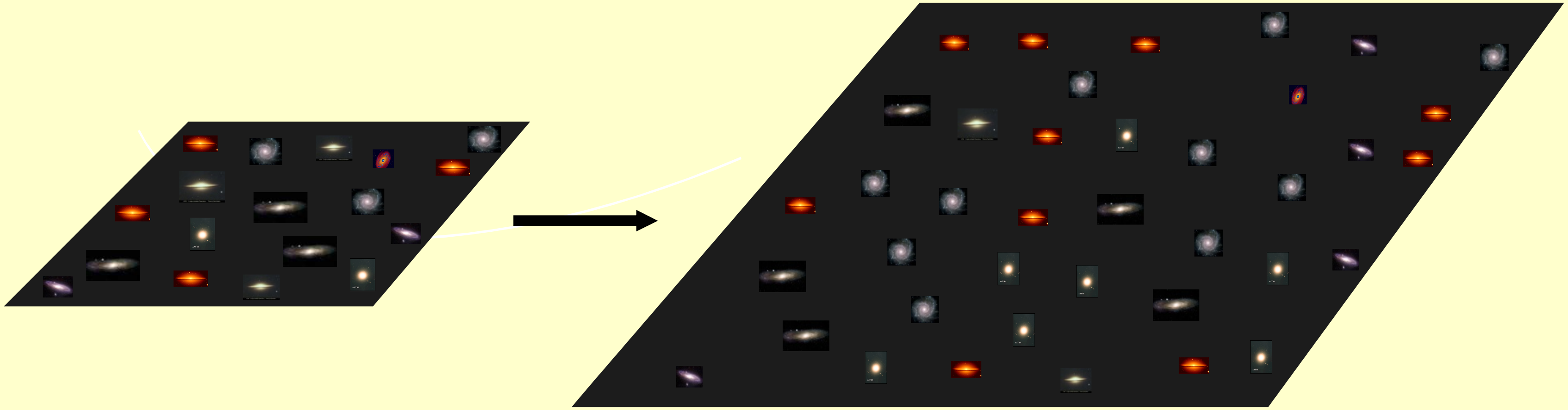


מפץ באינסוף נקודות

The Big Bang model



The Steady State model

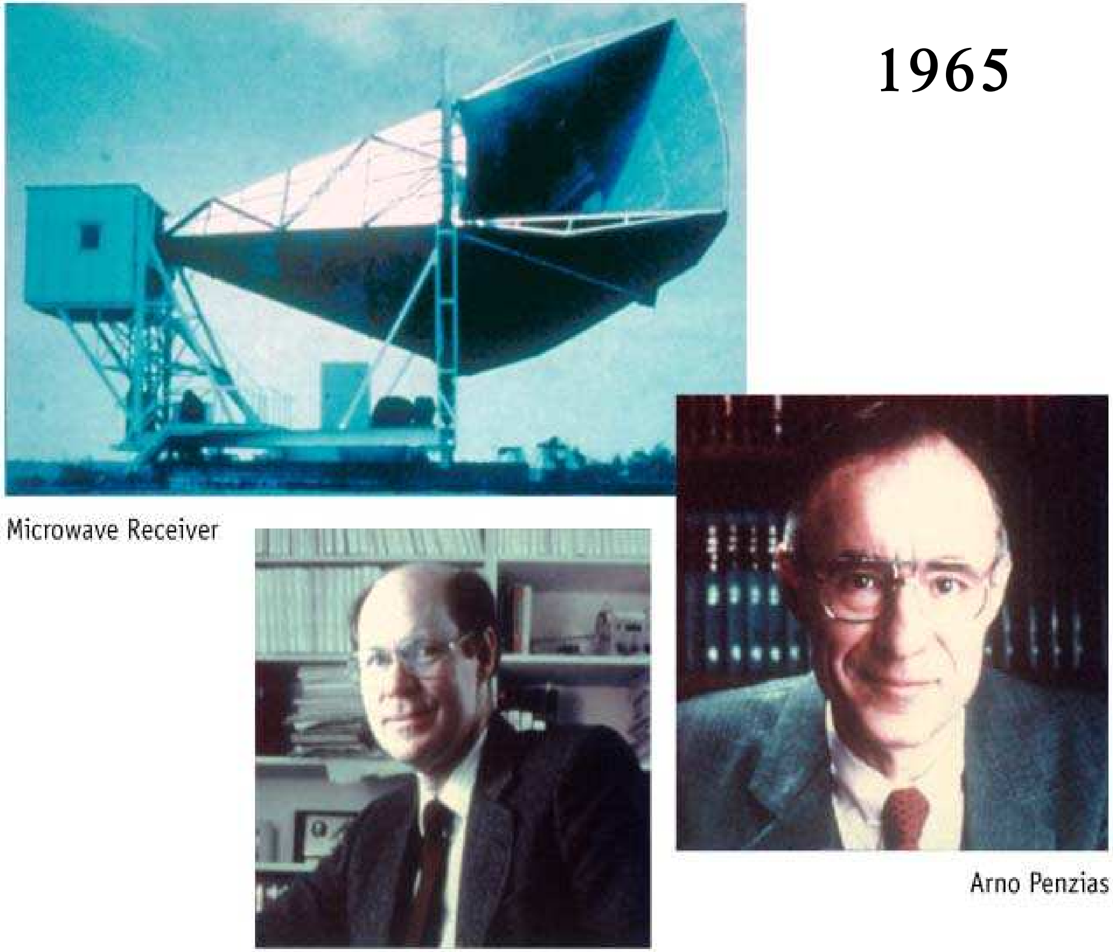


The Steady
State model

Cosmic Microwave Background Radiation

DISCOVERY OF COSMIC BACKGROUND

1965

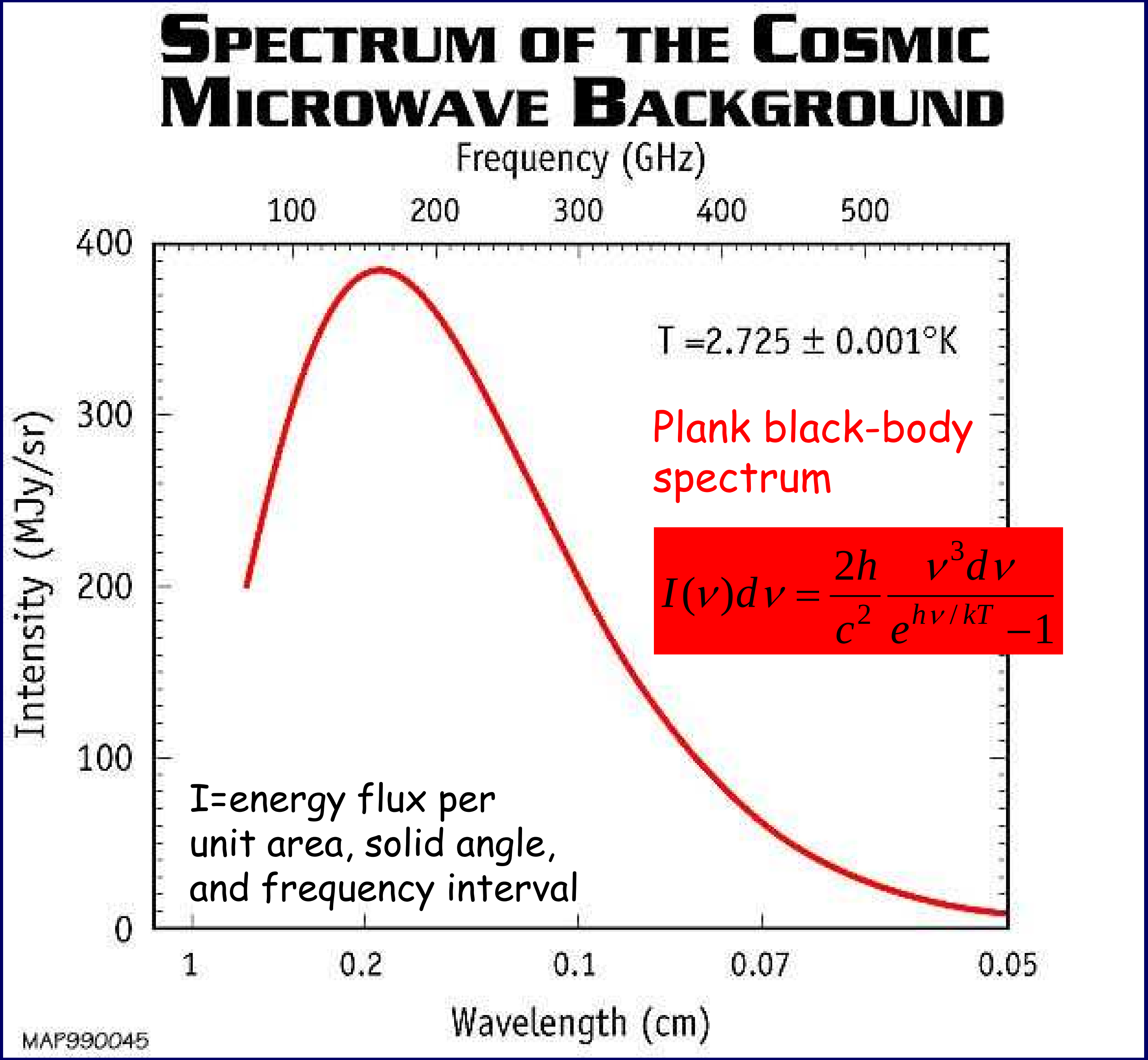


Microwave Receiver

Arno Penzias

Robert Wilson

MAP990045



Plank black-body spectrum

$$I(\nu)d\nu = \frac{2h}{c^2}\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}$$

I=energy flux per
unit area, solid angle,
and frequency interval

COBE
1992

Nobel
Prize
2006 to
Smoot
and
Mather

Homogeneity and Isotropy: Robertson-Walker Metric

Metric Distance

Metric distance $\ell(A,B;t) \equiv \sum \ell_i$

Hubble expansion in curved space:

$$\frac{d\ell}{dt} = \sum \frac{d\ell_i}{dt} = \sum v_i = \sum H(t)\ell_i = H(t)\sum \ell_i = H(t)\ell$$

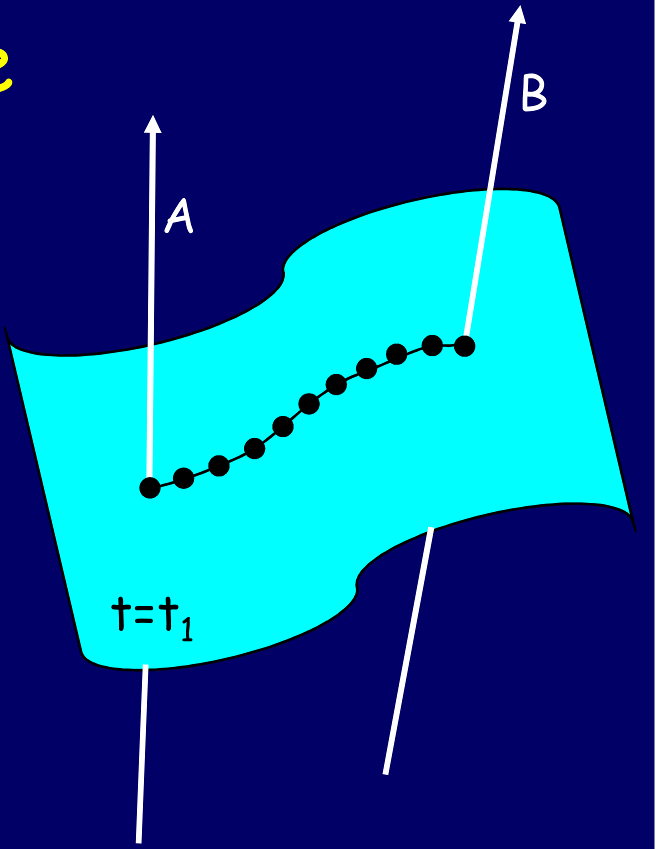
local Hubble law

$$\frac{d\ln\ell}{dt} = H(t) \longrightarrow \ell(A,B;t) = w(A,B)\, a(t)$$

comoving distance

universal expansion factor

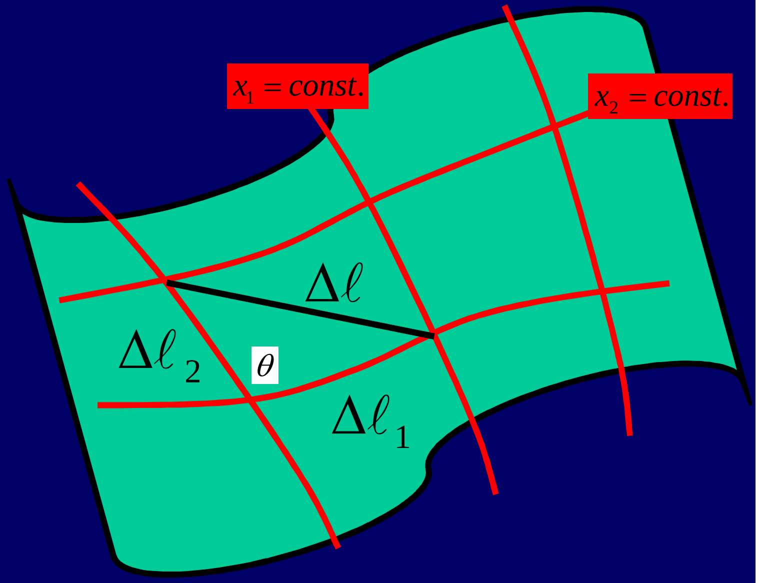
$$H(t) = \frac{\dot{a}}{a}$$



Metric

Coordinate system: x_1, x_2 (2d example)

In a small neighborhood (locally flat)



$$\Delta\ell^2 = \Delta\ell_1^2 + \Delta\ell_2^2 - 2\Delta\ell_1\Delta\ell_2\cos\theta$$

$$= \left(\frac{\partial\ell}{\partial x_1}\right)^2 \Delta x_1{}^2 + \left(\frac{\partial\ell}{\partial x_2}\right)^2 \Delta x_2{}^2 - 2\left(\frac{\partial\ell}{\partial x_1}\right)\left(\frac{\partial\ell}{\partial x_2}\right)\cos\theta\,\Delta x_1\Delta x_2$$

$$d\ell^2 = g_{11}dx_1^2 + g_{22}dx_2^2 + 2g_{12}dx_1dx_2$$

Line element: $$d\ell^2 = g_{ij}dx^i dx^j$$

The metric: $$g_{ij}(\vec{x}) = \frac{1}{2}\frac{\partial^2\ell^2}{\partial x^i\partial x^j} = \left(\frac{\partial\ell}{\partial x_1}\right)\left(\frac{\partial\ell}{\partial x_2}\right) \quad \ell \to 0$$

Specifies the geometry uniquely.
Exact form depends on the choice of coordinates

Orthogonal coordinates: $g_{ij}=0$ for $i\neq j$

Example: E_3

Coordinates: cartezian spherical

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

interval $$d\ell^2 = dx^2 + dy^2 + dz^2 = dr^2 + r^2\underbrace{(d\theta^2 + \sin^2\theta\, d\phi^2)}_{\equiv d\gamma^2}$$

angular distance

cartezian spherical

$$g = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

$$d\ell_r = dr$$
$$d\ell_\theta = r\, d\theta$$
$$d\ell_\phi = r\sin\theta\, d\phi$$

Example: a 2D sphere embedded in E_3 : r=const. $d\ell^2 = d\theta^2 + \sin^2\theta\, d\phi^2$

Area: $dA = d\ell_\theta d\ell_\phi = r^2\sin\theta\, d\theta\, d\phi$ $\quad A = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} dA = 4\pi r^2$

The Metric of a Homogeneous and Isotropic Universe (Robertson-Walker)

$d\ell^2 = a^2(t)\, dw^2$ t=const.

In comoving spherical coordinates $u = r/a, \quad \theta, \quad \phi$

Isotropy: $dw^2 = du^2 + \sigma^2(u)\, d\gamma^2$ $d\gamma^2 \equiv d\theta^2 + \sin^2\theta\, d\phi^2$

Three solutions: $\sigma(u) =$ $\boxed{S_k(u) = \begin{cases} \sin(u) & k=1 \\ u & k=0 \\ \sinh(u) & k=-1 \end{cases}}$ $\sinh(x) = (e^x - e^{-x})/2$

$$\boxed{d\ell^2 = a^2(t)[du^2 + S_k^2(u)\, d\gamma^2]}$$

Space-time interval $ds^2 = dt^2 - a^2(t)[du^2 + S_k^2(u)\, d\gamma^2]$

k=0 flat space (E_3)

$$d\ell^2 = a^2(t)\,[du^2 + u^2(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

$0 \le u \le \infty \quad 0 \le \theta \le \pi \quad 0 \le \phi \le 2\pi$ infinite volume

k=+1 a closed space

$$d\ell^2 = a^2(t)\,[du^2 + \sin^2(u)\,(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

$$0 \le u \le \pi \quad 0 \le \theta \le \pi \quad 0 \le \phi \le 2\pi$$

For visualization: a 3D sphere embedded in E_4: (w, x, y, z)

a 3D sphere $$w^2 + x^2 + y^2 + z^2 = a^2$$

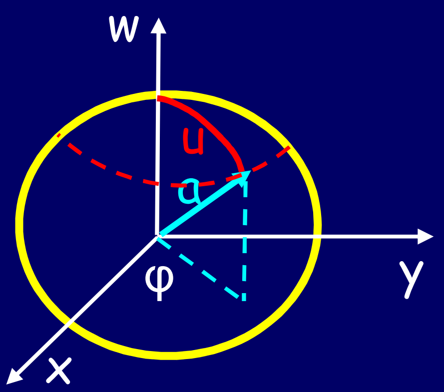
the embedding is defined by the transformation:

$$\begin{aligned} w &= a\cos u \\ z &= a\sin u\cos\theta \\ x &= a\sin u\sin\theta\cos\phi \\ y &= a\sin u\sin\theta\sin\phi \end{aligned}$$

consistent with $$d\ell^2 = dw^2 + dx^2 + dy^2 + dz^2$$

To visualize plot subspace $\theta = \pi/2 \quad (z=0)$

2D sphere in w,x,y,z=0 $$d\ell^2 = a^2(du^2 + \sin^2 u\, d\phi^2)$$



u=const. is a sphere of comoving radius u

$$d\ell^2 = a^2\sin^2 u\,(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$A = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} a\sin u\, d\theta \;\; a\sin u \;\; \sin\theta\, d\phi \;= 4\pi a^2\sin^2 u$$

A grows for 0 < u < π/2 and decreases for π/2 < u < π

$$V = \int_{u=0}^{\pi} A\, a\, du = 4\pi a^3 \int_0^{\pi} \sin^2 u\, du = 2\pi^2 a^3(t)$$

k=-1 an open space

$$d\ell^2 = a^2(t)\,[du^2 + \sinh^2(u)\,(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

Homogeneity and Isotropy → Robertson-Walker Metric

$$ds^2 = dt^2 - a^2(t)[du^2 + S_k^2(u)\,d\gamma^2]$$

expansion factor

comoving radius

$r = a(t)u$

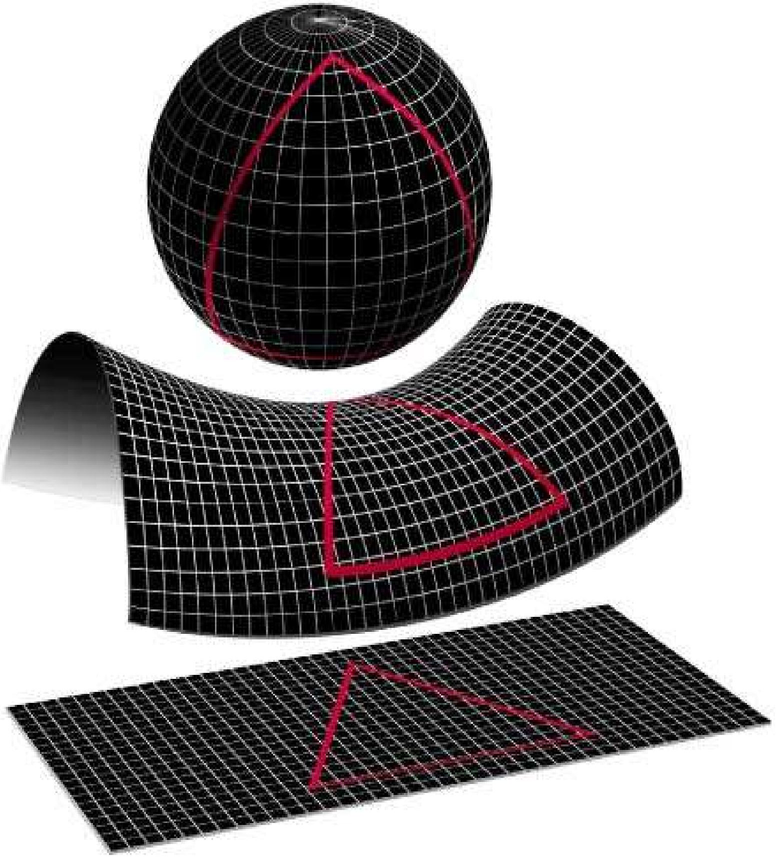
angular area

$d\gamma^2 \equiv d\theta^2 + \sin^2\theta\, d\varphi^2$

$$S_k(u) = \sin u \qquad k = +1$$

$$= \sinh u \qquad k = -1$$

$$= u \qquad k = 0$$



Redshift

$$z \equiv \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

$$ds^2 = dt^2 - a^2(t)[du^2 + S_k^2(u)d\gamma^2]$$

Radial ray $$d\gamma = 0 \quad ds^2 = 0 \quad \rightarrow \quad dt = \pm a(t)du \quad \rightarrow \quad u = \pm\int_{t_e}^{t_o} \frac{dt}{a(t)}$$

nearby observers along a light path, separated by δr:

$$\frac{\delta\nu}{\nu} = -\delta V = -H\delta r = -Ha\delta u = -H\delta t = -\frac{\dot{a}}{a}\delta t = -\frac{\delta a}{a}$$

local Hubble local flat

$$\boxed{a \propto \nu^{-1} \propto \lambda \propto (1+z)^{-1} \quad \propto T^{-1}}$$

For Black-Body radiation, Planck's spectrum: $$dN = \frac{8\pi\nu^2}{c^3}\frac{Vd\nu}{\exp(h\nu/kT)-1}$$

Also for free massive particles: $p \propto a^{-1}$ like photons: $p = h\nu/c$

de Broglie wavelength (particles or photons): $\lambda = h/p \propto a$

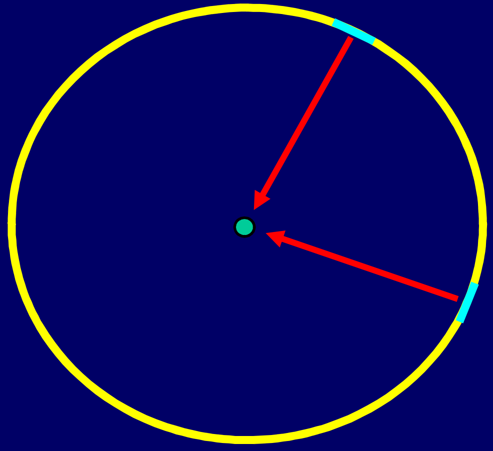
useful:
conformal time

$$d\eta = \frac{dt}{a} \quad \rightarrow \quad u = \eta_o - \eta_e \quad ds^2 = a^2(\eta)[d\eta^2 - du^2 - S_k^2(u)d\gamma^2]$$

Horizon

$$u = \int_{t_e}^{t_o} \frac{dt}{a(t)} < \lim_{t_e \to 0} u(t_o, t_e) \equiv u_{\text{horizon}}(t_o)$$

limit exists



$$r_H = a(t) u_H(t))$$

$$V_H = 4\pi a^3 \int_0^{u_H} S_k^2(u) du$$

Example: EdS (k=0, Λ=0)

$$a \propto t^{2/3} \quad \rightarrow \quad u_H \propto \int_0^t \frac{dt}{t^{2/3}} \propto t^{1/3} \quad r_H \propto t$$

$$M_H \propto u_H^3 \propto t \propto a^{3/2}$$

Causality problem:

$$\frac{M_H(t_{rec})}{M_H(t_0)} \approx (10^{-3})^{3/2} \sim 10^{-4}$$

Our Horizon is not causally connected: what is the origin of the isotropy?

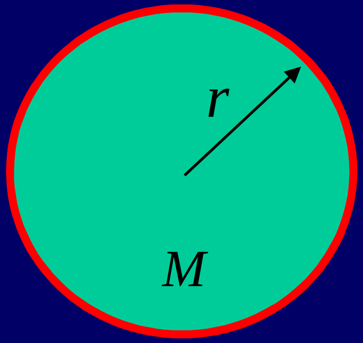
Friedman's Equation and its solutions

Newtonian Gravity

shell

$$\frac{E}{m} = \frac{1}{2}v^2 - \frac{GM}{r} \underset{\uparrow}{=} \left(\frac{1}{2}H^2 - \frac{4\pi G}{3}\rho\right) r^2$$

$$v = Hr \quad M = \frac{4\pi}{3}\rho r^3$$



3 types of solutions, depending on the sign of E

H=0 at maximum expansion, possible only if E<0

$$\rho_{crit} = \frac{3H^2}{8\pi G} \quad \Omega \equiv \frac{\rho}{\rho_{crit}}$$

$$r = au$$

$$\frac{\dot{a}^2}{a^2} - \frac{8\pi G}{3}\rho(t) = \frac{2E}{ma^2u^2} \equiv \varepsilon$$

$$\varepsilon = 0, \pm 1$$

independent of
r because lhs is

The Friedman Equation

Newton's gravity: space fixed, external force determining motions $\nabla^2\phi = 4\pi G\,\rho$

Einstein's equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$ $G = c = 1$

Gravity is an intrinsic property of space-time. geometry <-> energy density.
Particles move on geodesics (local straight lines) determined by the local curvature.

left side of E's eq. is the most general function of g and its 1st and 2nd time derivatives that reduces to Newton's equation

For the isotropic RW metric $ds^2 = dt^2 - a^2(t)[du^2 + S_k(u)\,d\gamma^2]$

Einstein's tensor $G_{tt} = 3\frac{\dot a^2}{a^2} + \frac{3k}{a^2} \qquad G_{uu} = G_{\vartheta\vartheta} = G_{\varphi\varphi} = -2\frac{\ddot a}{a} - \frac{\dot a^2}{a^2} - \frac{k}{a^2}$

Stress-energy tensor $T_{tt} = \rho \qquad T_{uu} = T_{\vartheta\vartheta} = T_{\varphi\varphi} = P$

Add Λ

$$\frac{\dot a^2}{a^2} = \frac{8\pi\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

energy conservation

$$\frac{2\ddot a}{a} = -\frac{\dot a^2}{a^2} - \frac{k}{a^2} - 8\pi P + \Lambda$$

eq. of motion

mass conservation

$$\rho_m V = const. \quad \rightarrow \quad \rho_m \propto a^{-3}$$

conservation of number of photons

$$N = \frac{\rho_r V}{h\nu} \propto \frac{\rho_r a^3}{a^{-1}} = const. \quad \rightarrow \quad \rho_r \propto a^{-4}$$

A differential equation for a(t)

Solutions of Friedman eq. (matter era)

$$\frac{\dot{a}^2}{a^2} = \frac{2a^*}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$a^* \equiv \frac{4\pi G \rho_{m0}}{3} = const.$$

$$a \to 0, \text{ any } k: \quad \dot{a}^2 = \frac{2a^*}{a} \quad \to \quad a \propto t^{2/3}$$

radiation era

$$\dot{a}^2 \propto a^{-2} \quad \to \quad a \propto t^{1/2}$$

$$\Lambda = 0 \quad \dot{a}^2 = \frac{2a^*}{a} - k \quad \to \quad \dot{a}^2 \downarrow$$

$$k = 0 \quad \dot{a}^2 = \frac{2a^*}{a} \quad \to \quad a \propto t^{2/3}$$

$$k = -1 \quad \dot{a}^2 = 1 + \frac{2a^*}{a} \quad a \propto t \;\; (a \to \infty)$$

$$k = +1 \quad \dot{a}^2 = -1 + \frac{2a^*}{a} \quad \to \quad \text{turnaround}$$

conformal time

$$d\eta \equiv dt / a(t)$$

$$a = a^*[1 - \cos(\eta)]$$

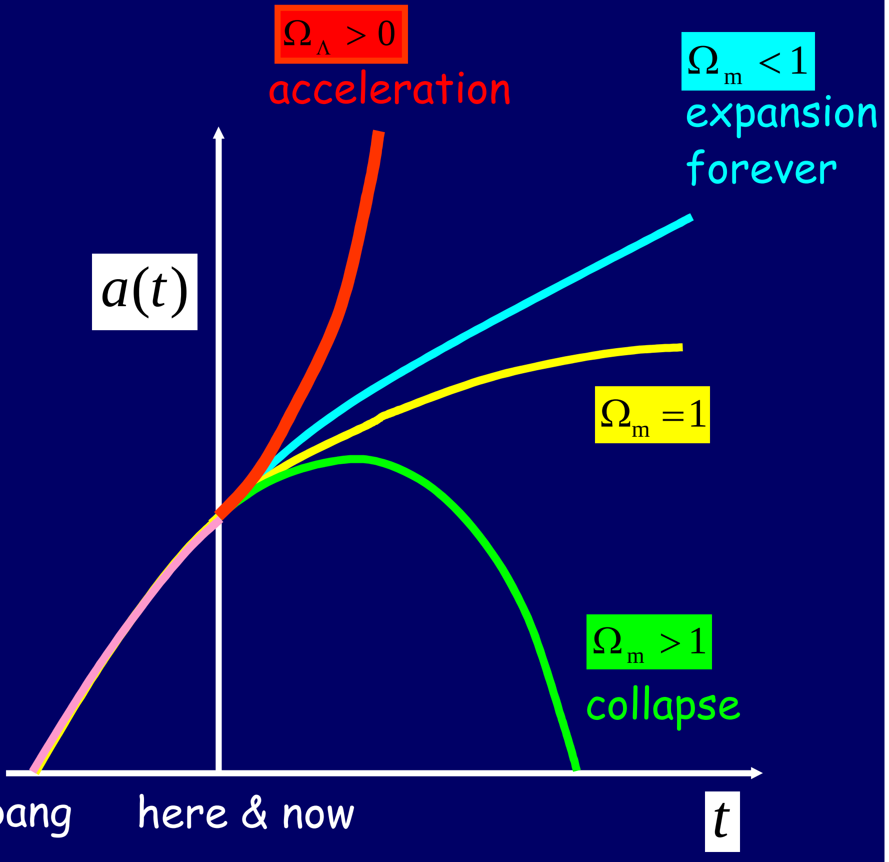
$$t = a^*[\eta - \sin(\eta)]$$

critical density

$$\rho = \frac{3H^2}{8\pi G} + \frac{3k}{8\pi G a^2}$$

$$\Omega \equiv \frac{\rho}{3H^2 / 8\pi G}$$

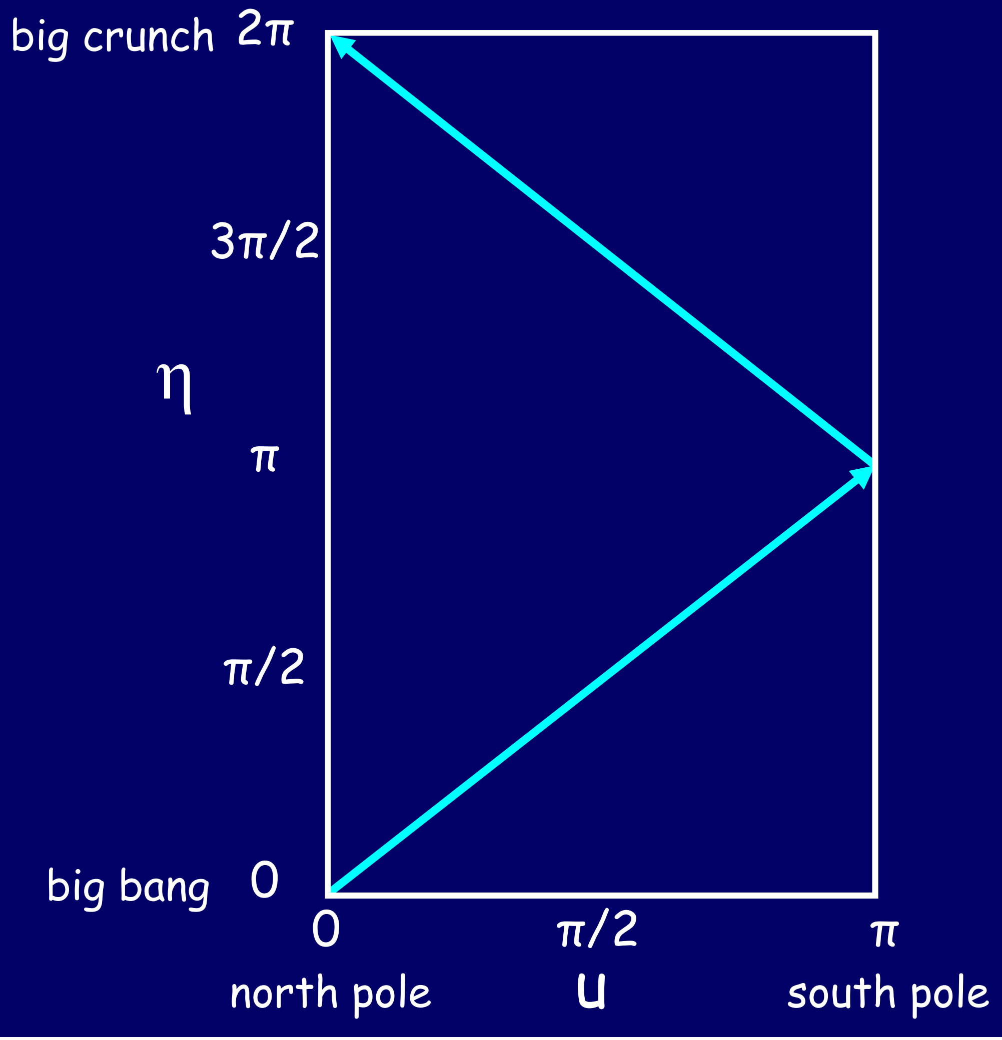
$$\Lambda > 0 \quad H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2}{3} \quad (a \to \infty) \quad \to \quad a \propto e^{Ht}$$

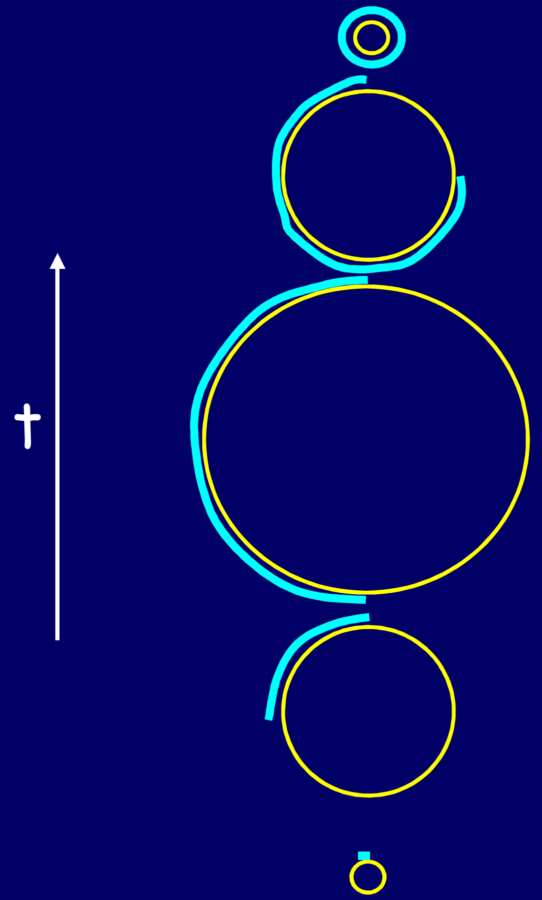


Light travel in a closed universe

A photon is emitted at the origin (u_e=0) right after the big-bang (t_e=0)

conformal time $d\eta \equiv \frac{dt}{a(t)}$ photon: $ds = 0 \quad \rightarrow \quad d\eta = du$





H and t

$\Lambda = 0$

$$\begin{array}{llll} \Omega_m << 1 & \rightarrow & a \propto t & \rightarrow & Ht \approx 1 \\ \Omega_m = 1 & \rightarrow & a \propto t^{2/3} & \rightarrow & Ht = 2/3 \\ \Omega_m > 1 & \rightarrow & \text{collapse} & \rightarrow & Ht < 2/3 \end{array}$$

$k = 0$

$$Ht = \frac{2}{3} \sinh^{-1}\left[\left(\frac{1-\Omega_m}{\Omega_m}\right)^{1/2}\right] / (1-\Omega_m)^{1/2}$$

General:

$$Ht \approx \frac{2}{3} S^{-1}\left[\left(\frac{|1-\Omega_a|}{\Omega_a}\right)^{1/2}\right] / (|1-\Omega_a|)^{1/2}$$

$$\Omega_a \equiv 0.7\Omega_m - 0.3\Omega_\Lambda + 0.3$$

$$S \equiv \sin\ (\Omega_a > 1), \quad \sinh\ (\Omega_a \leq 1)$$

Carrol, Press, Turner 1992,
Ann Rev A&A 30, 499

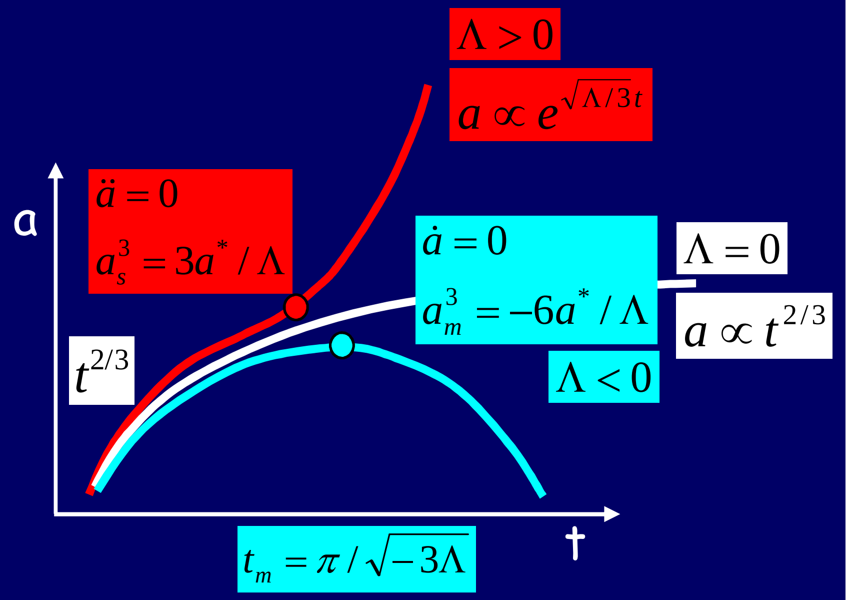
Solutions with a Cosmological Constant

$$\dot a^2 = -k + \frac{\Lambda}{3}a^2 + \frac{2a^*}{a} \qquad (\text{matter})$$

k=0

$\Lambda > 0$ $\quad a^3 = \frac{3a^*}{\Lambda}[\cosh(\sqrt{3\Lambda}\,t)-1]$

$\Lambda < 0$ $\quad a^3 = \frac{3a^*}{-\Lambda}[1-\cosh(\sqrt{-3\Lambda}\,t)]$



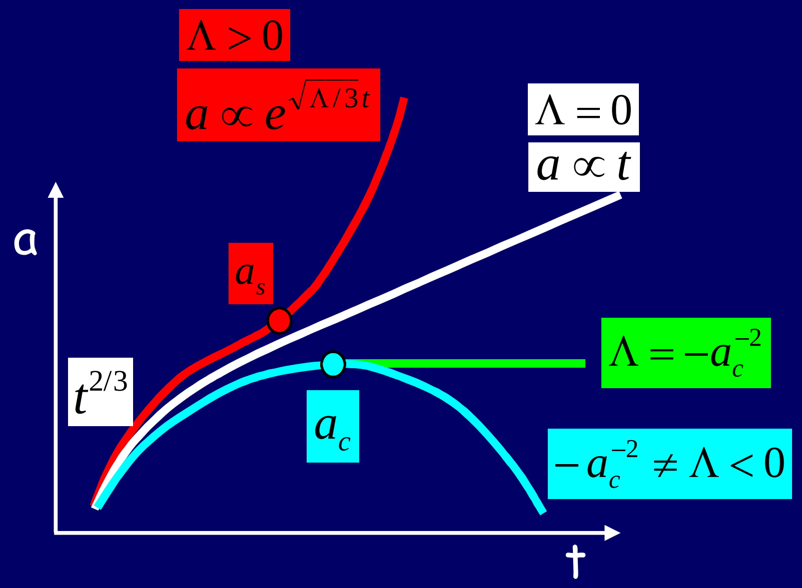
k=-1 $\quad \Lambda > 0$ same asymptotic behavior as k=0

$\Lambda < 0$ $\quad \dot a = 0$ at a_c

3rd-order polynomial
– one real root exists

$$0 = +1 + \frac{\Lambda}{3}a^2 + \frac{2a^*}{a}$$

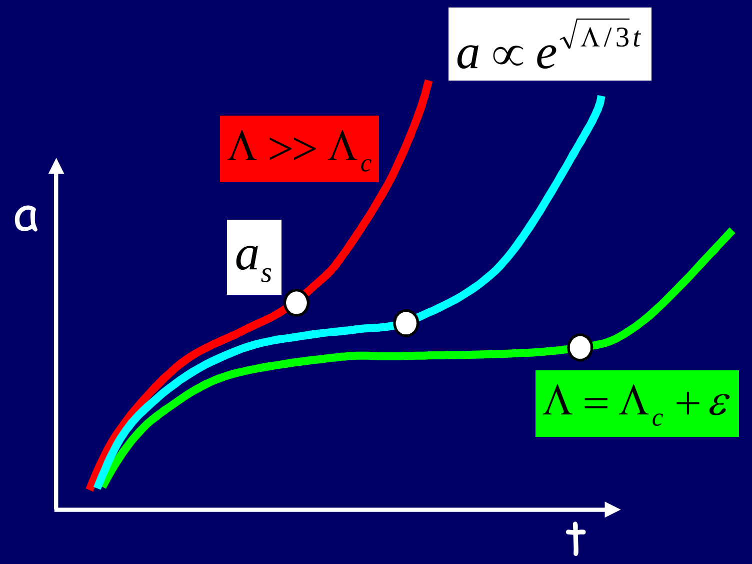
$$\ddot a(a = a_c) = 0 \quad \text{if} \quad \Lambda = -a_c^{-2}$$



Solutions with a Cosmological Constant (cont.)

$$\dot{a}^2 = -k + \frac{\Lambda}{3} a^2 + \frac{2a^*}{a} \quad (\text{matter})$$

k=+1 (closed) a critical value: $\Lambda_c \equiv \frac{1}{9a^{*2}}$



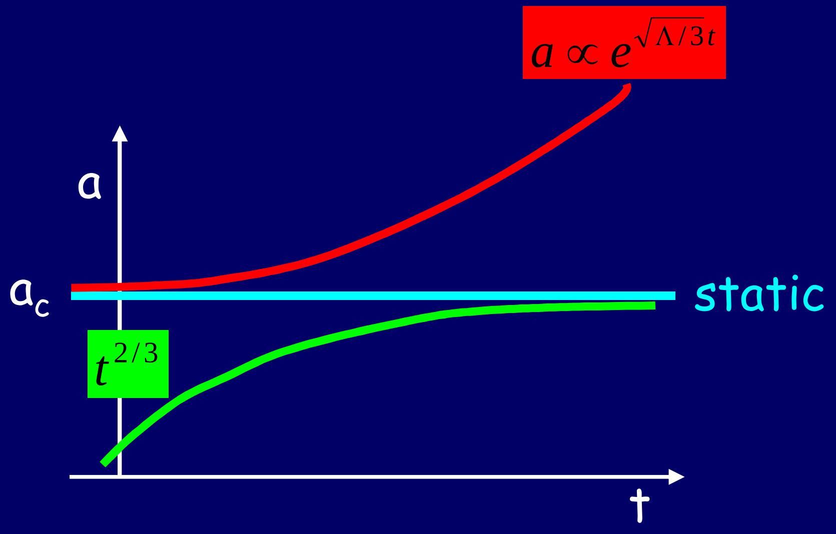
Lemaitre

$\Lambda > \Lambda_c$ $\rightarrow \dot{a}^2 > 0$ a solution for every a

the rhs at a_s to be >0

$\Lambda = \Lambda_c$ $\rightarrow \dot{a}^2 \geq 0$

a double root at $a_c = 3a^* \rightarrow \dot{a}_c = \ddot{a}_c = 0$



Einstein's static universe (unstable)

static expanding to inflation
Eddington-Lematre

big-bang expanding to static

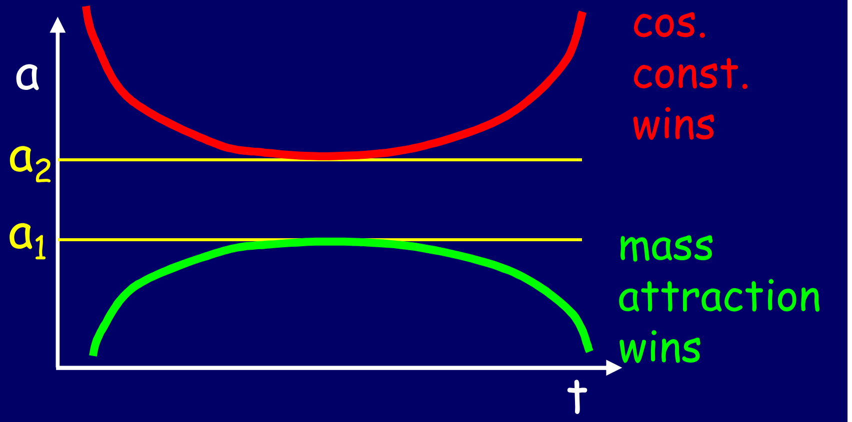
Solutions with a Cosmological Constant (cont.)

$$\dot{a}^2 = -k + \frac{\Lambda}{3} a^2 + \frac{2a^*}{a} \quad (\text{matter})$$

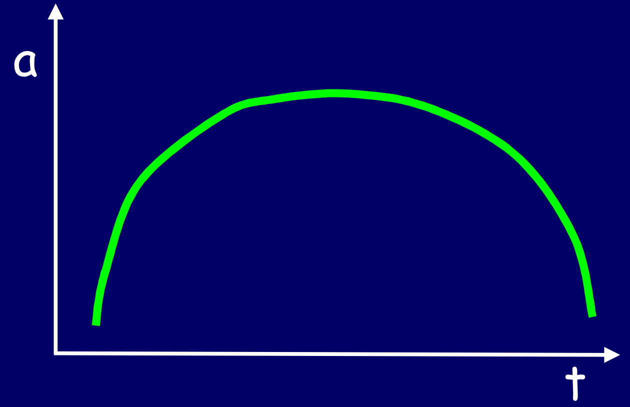
k=+1 (closed)   a critical value: $\Lambda_c \equiv \frac{1}{9a^{*2}}$

$0 < \Lambda < \Lambda_c$ $\rightarrow \dot{a}^2 > 0$ for a small and large

$\rightarrow \dot{a}^2 < 0$ for $a_1<a<a_2$ – no solution



$\Lambda \leq 0$ $\rightarrow \dot{a}^2 \downarrow$ only attraction



Friedman Equation Ω_m, Ω_Λ

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

kinetic potential curvature vacuum

$$\rho = \rho_m + \rho_r$$

$$\rho_m = \rho_{m0} a^{-3} \qquad \rho_r = \rho_{r0} a^{-4}$$

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

Carrol, Press, Turner 1992,
Ann Rev A&A 30, 499

$$\Omega_m \equiv \frac{\rho_m}{3H^2/8\pi G} \qquad \Omega_k \equiv -\frac{kc^2}{a^2H^2} \qquad \Omega_\Lambda \equiv \frac{\Lambda c^2}{3H^2}$$

two free parameters

$$\Omega_{tot} \equiv \Omega_m + \Omega_\Lambda = 1 - \Omega_k$$

closed/open

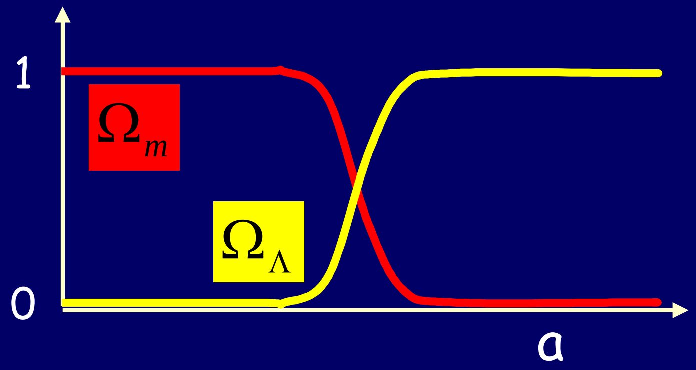
$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}\Omega_m - \Omega_\Lambda$$

decelerate/accelerate

Flat: k=0 $\Omega_{tot} = 1 \quad \Omega_\Lambda = 1 - \Omega_m$

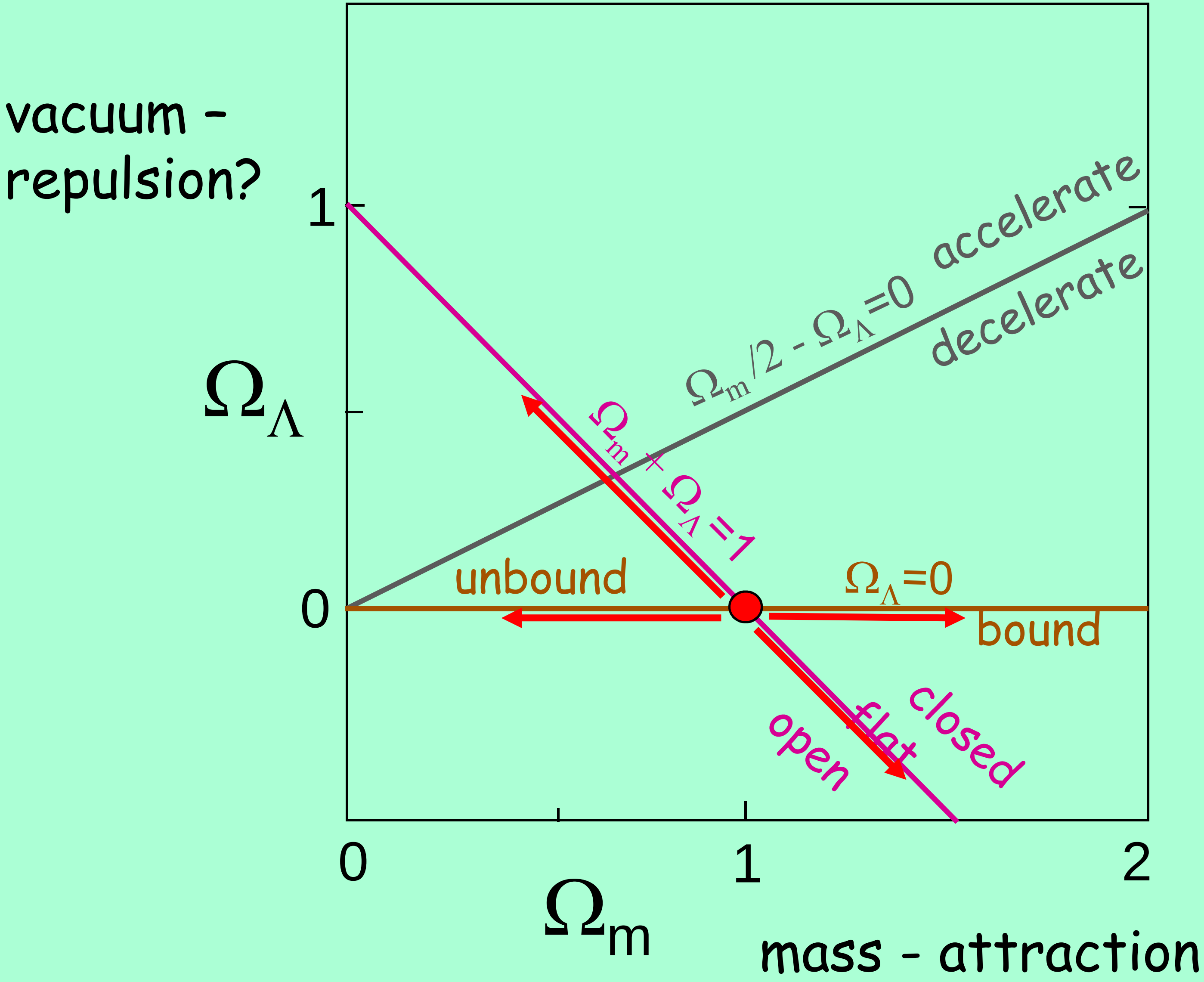
$$\Omega_m = \frac{\Omega_{m0}}{a^3 + (1 - a^3)\Omega_{m0}}$$

$$H^2 / H_0^2 = 1 + (a^{-3} - 1)\Omega_{m0}$$



$$\rho_m = \Omega_{m0}\frac{3H_0^2}{8\pi G}\frac{1}{a^3} \approx 1.88\times10^{-29}\Omega_{m0}h_0^2 a^{-3} \approx 2.76\times10^{-30}\left(\frac{\Omega_{m0}}{0.3}\right)\left(\frac{h_0}{0.7}\right)^2 a^{-3} g\, cm^{-3}$$

Dark Matter and Dark Energy



Cosmological Constant: Newtonian Analog

vacuum energy density

$$\Lambda \equiv -4\pi\rho_\Lambda$$

$$M_\Lambda = \frac{4\pi}{3}\rho_\Lambda r^3 = -\frac{1}{3}\Lambda r^3$$

force per mass on shell

$$F = -\frac{M_\Lambda}{r^2}\hat{r} = \frac{1}{3}\Lambda\, r\,\hat{r}$$

vs.

$$-\frac{M}{r^2}\hat{r}$$

potential

$$\phi = -\int_0^r F dr = -\frac{1}{6}\Lambda r^2$$

vs.

$$-\frac{M}{r}$$

in Einstein's eqs.

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\frac{\rho_{m0}}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

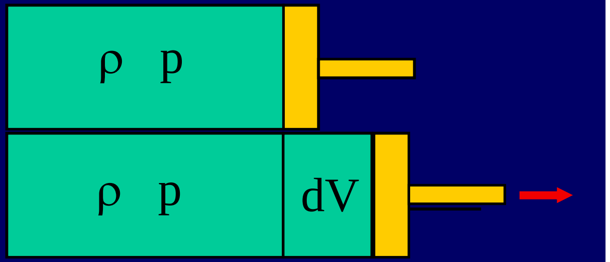
$$\ddot{a} = -\frac{4\pi}{3}\frac{\rho_{m0}}{a^2} + \frac{\Lambda}{3}a$$

Acceleration, pressure, energy density

FRW: $\dot{a}^2 = \frac{8\pi}{3}\rho a^2 \quad (k=0)$ $\rho = \rho_\Lambda + \rho_{m0}a^{-3} + \rho_{r0}a^{-4}$ $G=1$

Energy change by work $d(\rho c^2 a^3) = -p\,d(a^3)$ (2)

$$\Longrightarrow \ddot{a} = -\frac{4\pi}{3}a\left(\rho + \frac{3p}{c^2}\right)$$



if ρ_m dominates $\rho_m \propto a^{-3} \rightarrow^{(2)} p_m \approx 0 \quad \ddot{a} = -\frac{4\pi}{3}\frac{\rho_m a^3}{a^2}$

if ρ_r dominates $\rho_r \propto a^{-4} \rightarrow^{(2)} p_r = \frac{1}{3}\rho_r c^2 \quad \ddot{a} = -\frac{8\pi}{3}\frac{\rho_r a^4}{a^3}$

if ρ_Λ dominates $\rho_\Lambda = \text{const.} \rightarrow^{(2)} p_\Lambda = -\rho_\Lambda c^2 \quad \ddot{a} = +\frac{8\pi}{3}\rho_\Lambda a$

Construct a static model:

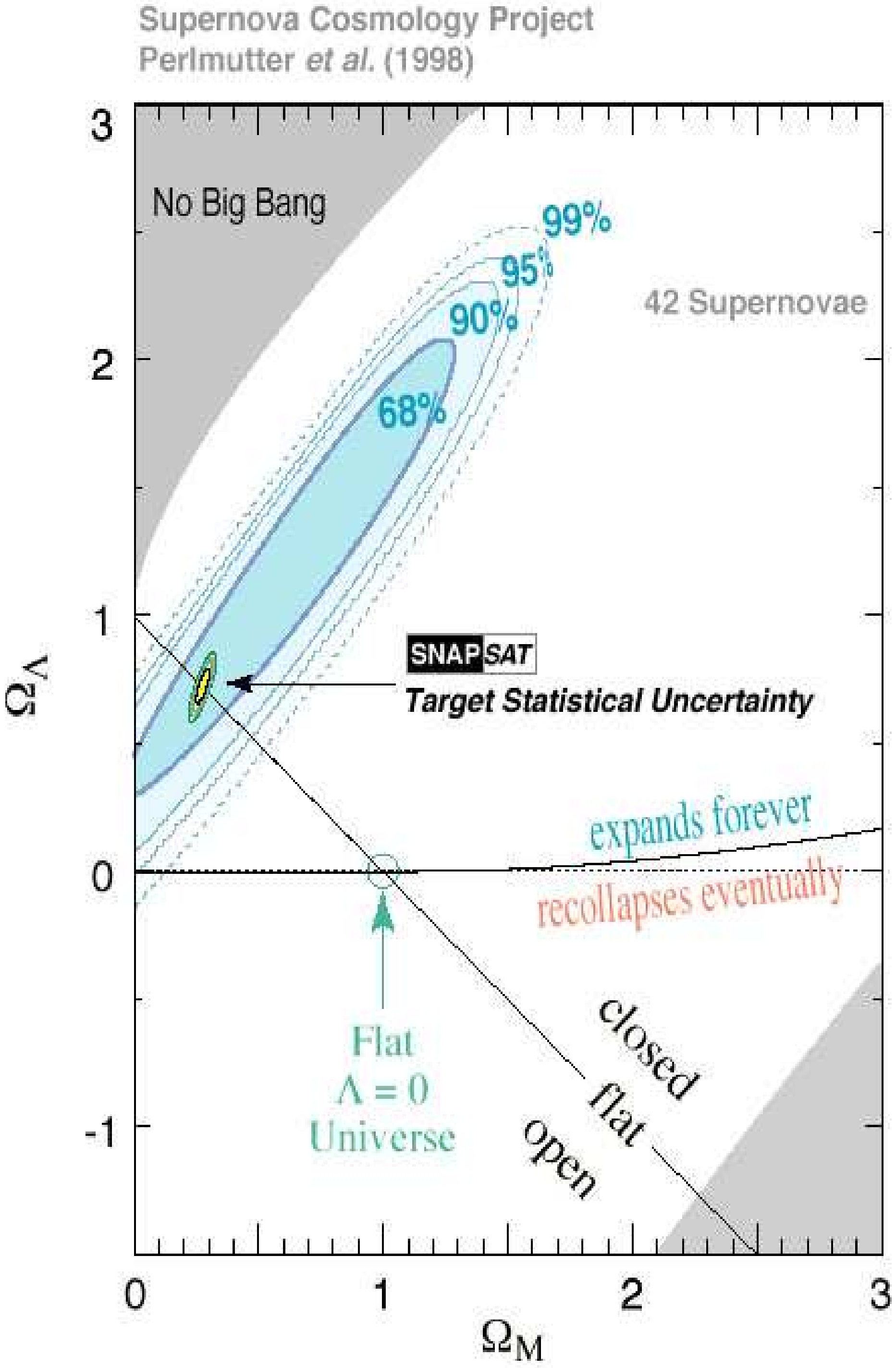
$\dot{a} = 0 \rightarrow$ in FRW: $\frac{8\pi}{3}\rho a^2 = kc^2$

$\ddot{a} = 0 \rightarrow^{(2)} \rho = -\frac{3p}{c^2} = 3\rho_\Lambda \rightarrow \rho_m = 2\rho_\Lambda \rightarrow \rho > 0 \rightarrow k = +1$

de Sitter: $H^2 = \dot{a}^2/a^2 = \Lambda c^2/3 \rightarrow a \propto e^{Ht}$

Quintessence $p/c^2 \equiv \omega\rho$ $\Lambda \leftrightarrow \omega = -1$

for inflation need $\ddot{a} > 0$ to exceed $r_h \propto ct \rightarrow \ddot{r}_h = 0$ ⟶ $\omega < -1/3$

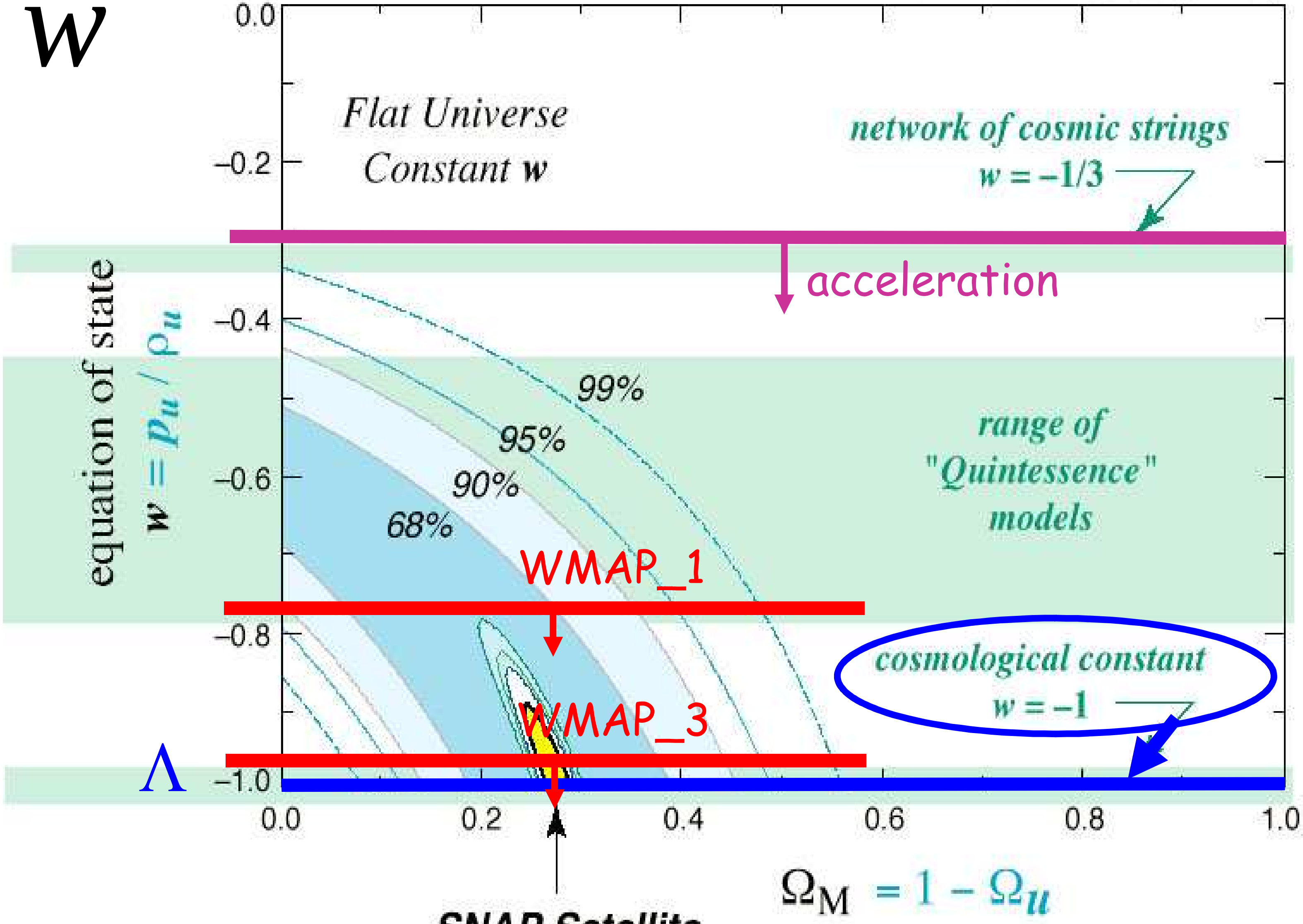


Future SN Cosmology Project

Dark Energy

w

Supernova Cosmology Project
Perlmutter *et al.* (1998)



SNAP Satellite
Target Statistical Uncertainty

Friedman Equation

Homogeneity + Gravity $(G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}) \rightarrow$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

kinetic potential curvature vacuum

$$\rho = \rho_m + \rho_r$$

$$\rho_m = \rho_{m0} a^{-3} \qquad \rho_r = \rho_{r0} a^{-4}$$

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

$$\Omega_m \equiv \frac{\rho_m}{\boxed{3H^2/8\pi G}} \qquad \Omega_k \equiv -\frac{kc^2}{a^2 H^2} \qquad \Omega_\Lambda \equiv \frac{\Lambda c^2}{3H^2}$$

two free parameters

$\rho_{crit} \sim 10^{-29}\,\mathrm{g\ cm}^{-3}$

$$\Omega_{tot} \equiv \Omega_m + \Omega_\Lambda = 1 - \Omega_k$$

closed/open

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}\Omega_m - \Omega_\Lambda$$

decelerate/accelerate

Solutions of Friedman eq.

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho_{m0}}{3a^3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

$$\dot{a}^2 - \frac{2a^*}{a} = -k$$

matter era, Λ=0

$$a^* \equiv \frac{4\pi G\rho_{m0}}{3} = const.$$

$k = 0: \quad a \propto t^{2/3}$

a small: $\quad a \propto t^{2/3}$ any k

a large, $k = -1: \quad a \propto t$ $\quad \Omega_m << 1$

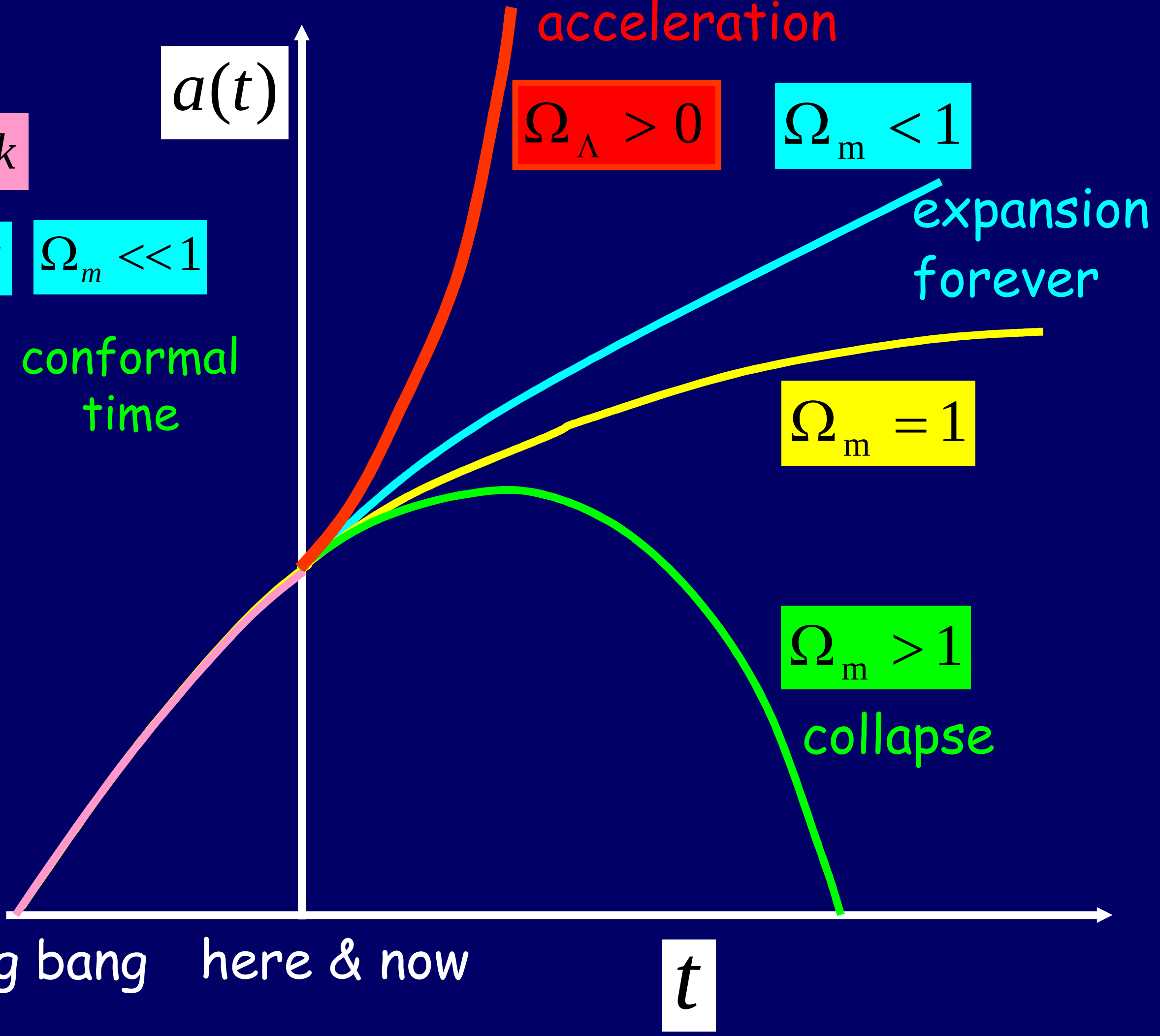
$k = +1$ $\quad d\eta \equiv dt / a(t)$ conformal time

$$a = a^*[1 - \cos(\eta)]$$

$$t = a^*[\eta - \sin(\eta)]$$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2}{3}$$

$$\rightarrow a \propto e^{Ht}$$



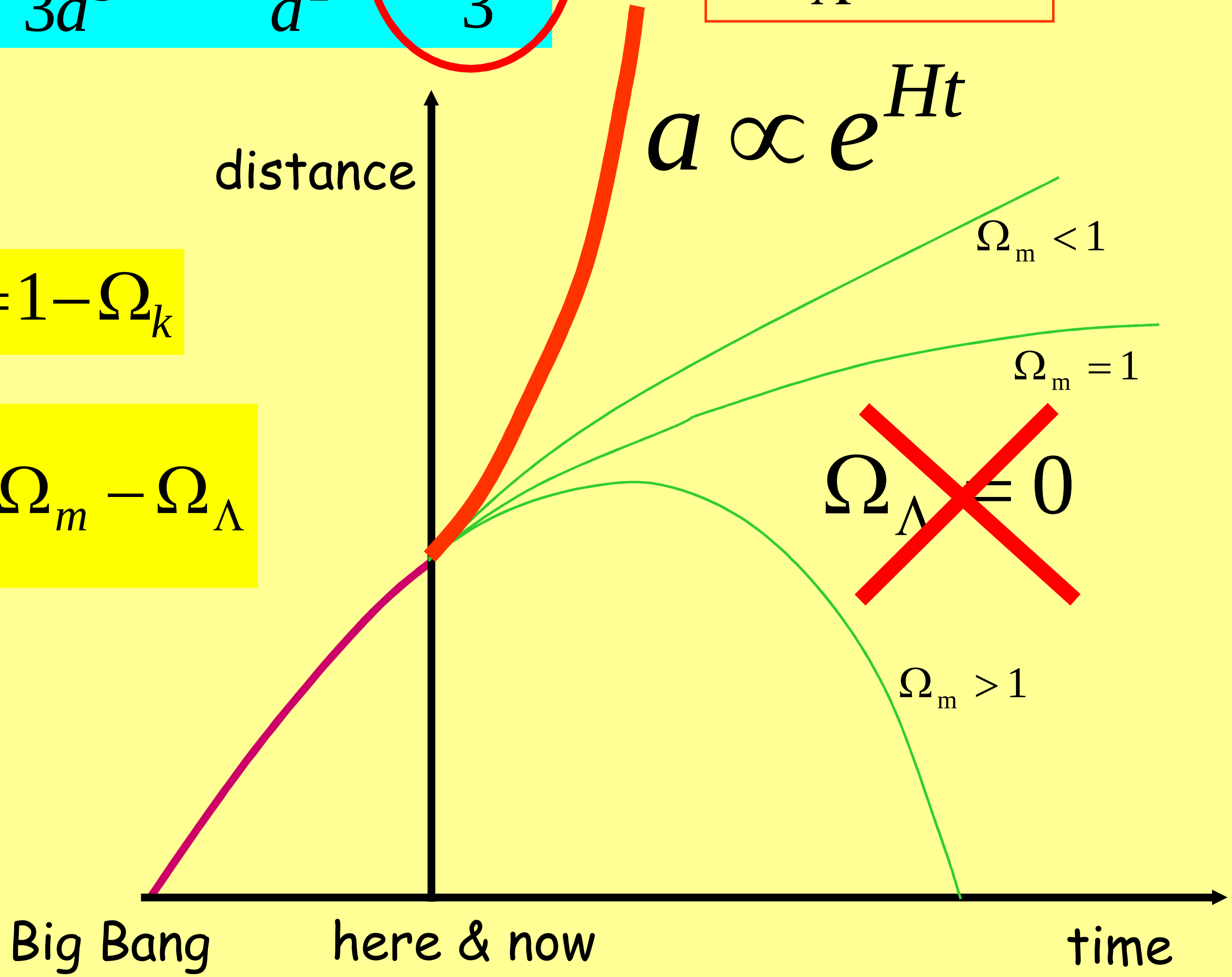
Acceleration by a cosmological constant:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho_{m0}}{3a^3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\boxed{\Omega_\Lambda > 0}$$

$$\Omega_m + \Omega_\Lambda = 1 - \Omega_k$$

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}\Omega_m - \Omega_\Lambda$$



Generalized Dark Energy

Energy conservation during expansion:

$$d(\rho_{tot} c^2 a^3) = -p\, d(a^3)$$

Cosmological constant:

$$\rho_{tot} = \rho_\Lambda = const.$$

Equation of state:

$$\rightarrow p = -\rho c^2$$

negative pressure

General eq. of state:

$$p \equiv w \rho c^2$$

e.g. Quintessence

$$w(x,t)?$$

$$\Lambda \quad \leftrightarrow \quad w = -1$$

$$\ddot{a} > 0 \quad \leftrightarrow \quad w < -1/3$$

$$\text{FRW } (k=0) \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho_{tot}$$

$$\rightarrow \ddot{a} = -\frac{4\pi G}{3} a \left(\rho + \frac{3p}{c^2} \right)$$