

# The Standard Model of Cosmology

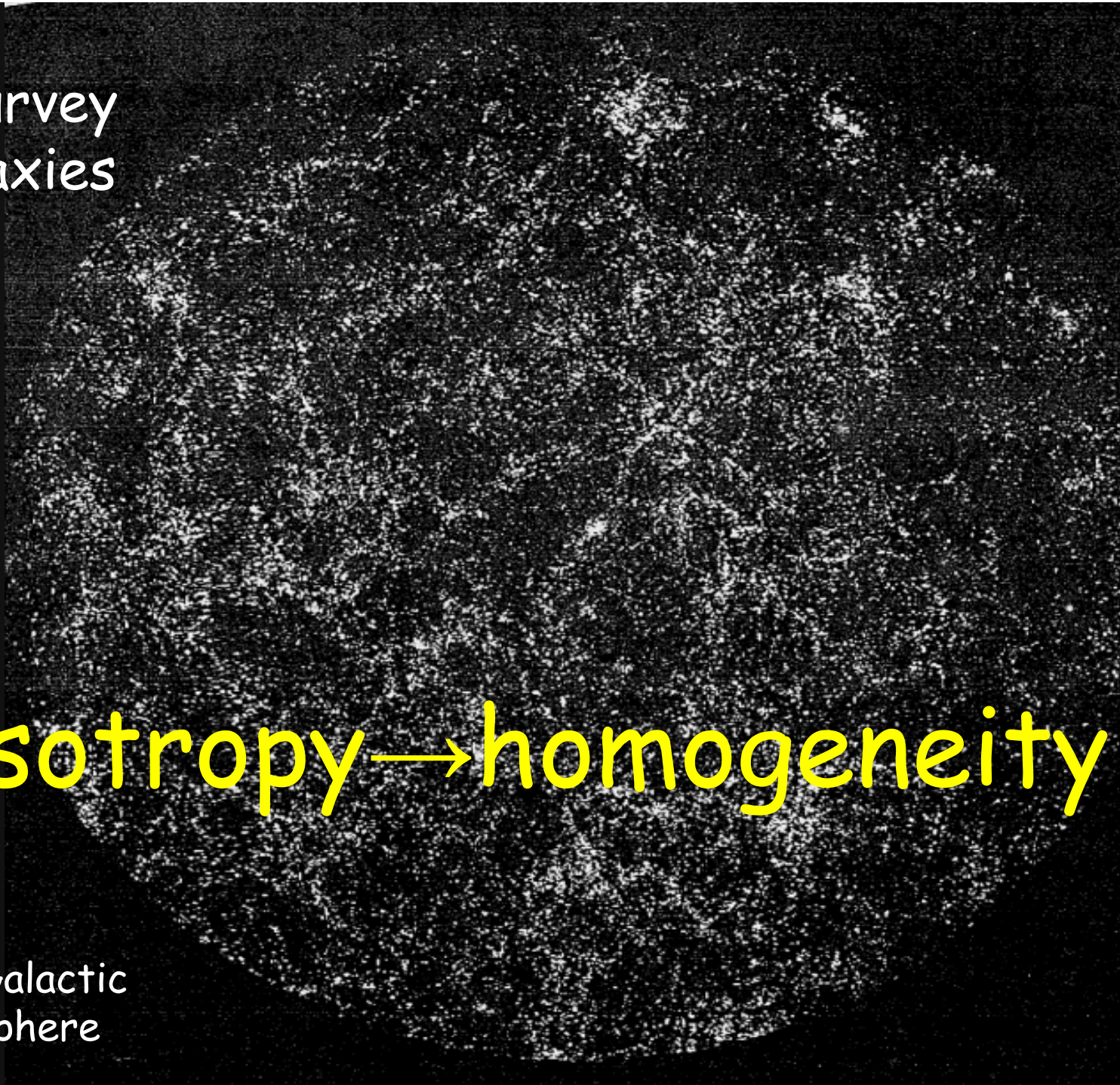
Avishai Dekel, HUJI, PANIC08



Lick Survey  
1M galaxies

isotropy → homogeneity

North Galactic  
Hemisphere





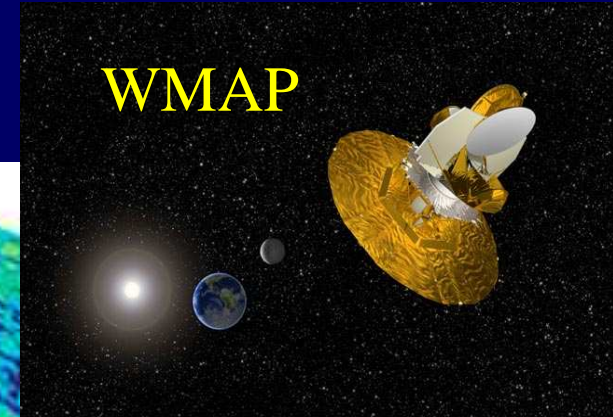
# Microwave Anisotropy Probe

February 2003, 2004

Science breakthrough of the year

$$\delta T/T \sim 10^{-5}$$

isotropy  $\rightarrow$  homogeneity



# Homogeneity & Isotropy - Robertson-Walker Metric

$$ds^2 = dt^2 - a^2(t) [du^2 + S_k(u) d\gamma^2]$$

expansion factor

comoving radius

$$r = a(t)u$$

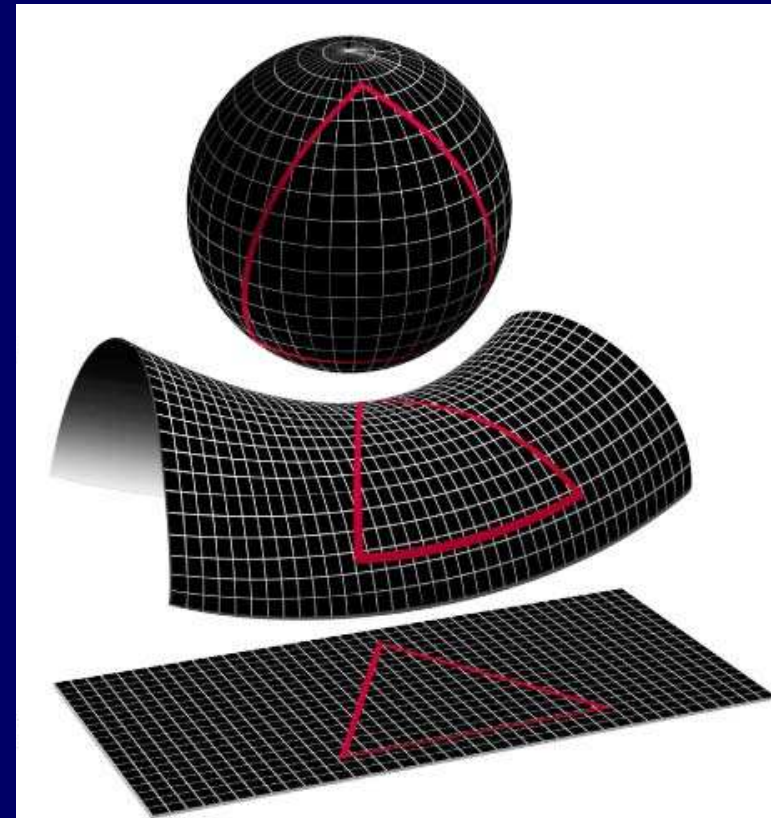
angular area

$$d\gamma^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$$

$$S_k(u) = \sin u \quad k = +1$$

$$S_k(u) = \sinh u \quad k = -1$$

$$S_k(u) = u \quad k = 0$$



# Friedman Equation

Homogeneity + Gravity ( $G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$ )  $\rightarrow$

$$H^2(t) \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G \rho(a)}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} = \frac{8\pi G}{3} \sum \rho_i(t)$$

kinetic potential curvature vacuum

$$\rho = \rho_m + \rho_r \quad \rho_m = \rho_{m0} a^{-3} \quad \rho_r = \rho_{r0} a^{-4}$$

$$1 = \Omega_m(t) + \Omega_k(t) + \Omega_\Lambda(t)$$

$$\Omega_i \equiv \frac{\rho_i}{3H^2/8\pi G}$$

$$\rho_{crit} \sim 10^{-29} \text{ g cm}^{-3}$$

Two basic free parameters

$$\Omega_{tot} \equiv \Omega_m + \Omega_\Lambda = 1 - \Omega_k$$

closed/open

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}\Omega_m - \Omega_\Lambda$$

decelerate/accelerate

# Solutions of Friedman eq.

$$\dot{a}^2 - \frac{2a^*}{a} = -k$$

matter era,  $\Lambda=0$

$$a^* \equiv \frac{4\pi G\rho_{m0}}{3} = \text{const.}$$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho_{m0}}{3a^3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

$$k=0: a \propto t^{2/3}$$

$$a \text{ small: } a \propto t^{2/3} \text{ any } k$$

$$a \text{ large, } k=-1: a \propto t \quad \Omega_m \ll 1$$

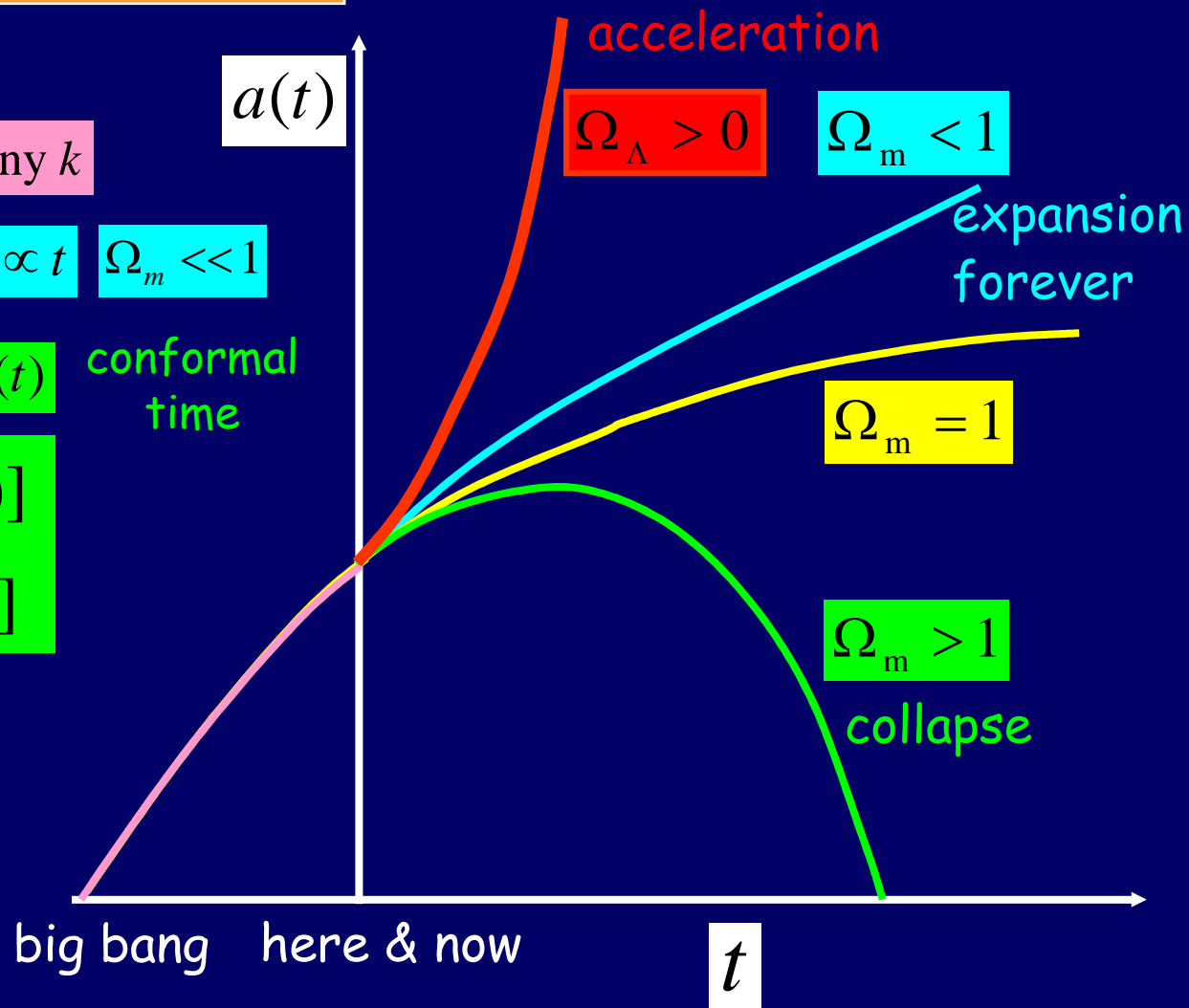
$$k=+1 \quad d\eta \equiv dt/a(t) \quad \text{conformal time}$$

$$a = a^* [1 - \cos(\eta)]$$

$$t = a^* [\eta - \sin(\eta)]$$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2}{3}$$

$$\rightarrow a \propto e^{Ht}$$



# Solutions

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum \rho_i(t)$$

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

$$\Omega_\Lambda > 0$$

$$a \propto e^{Ht}$$

acceleration

$$\Omega_m < 1$$

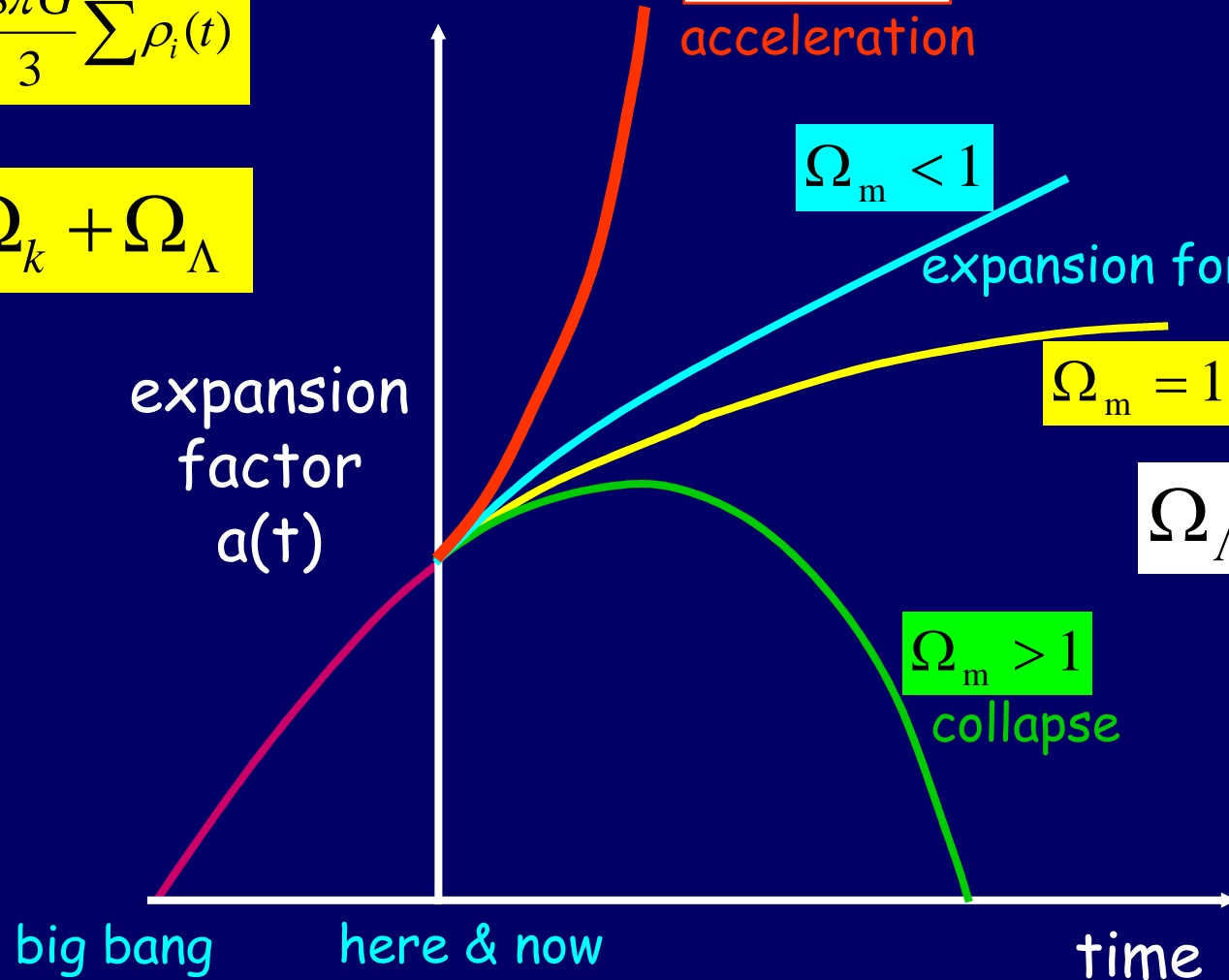
expansion forever

$$\Omega_m = 1$$

$$\Omega_\Lambda = 0$$

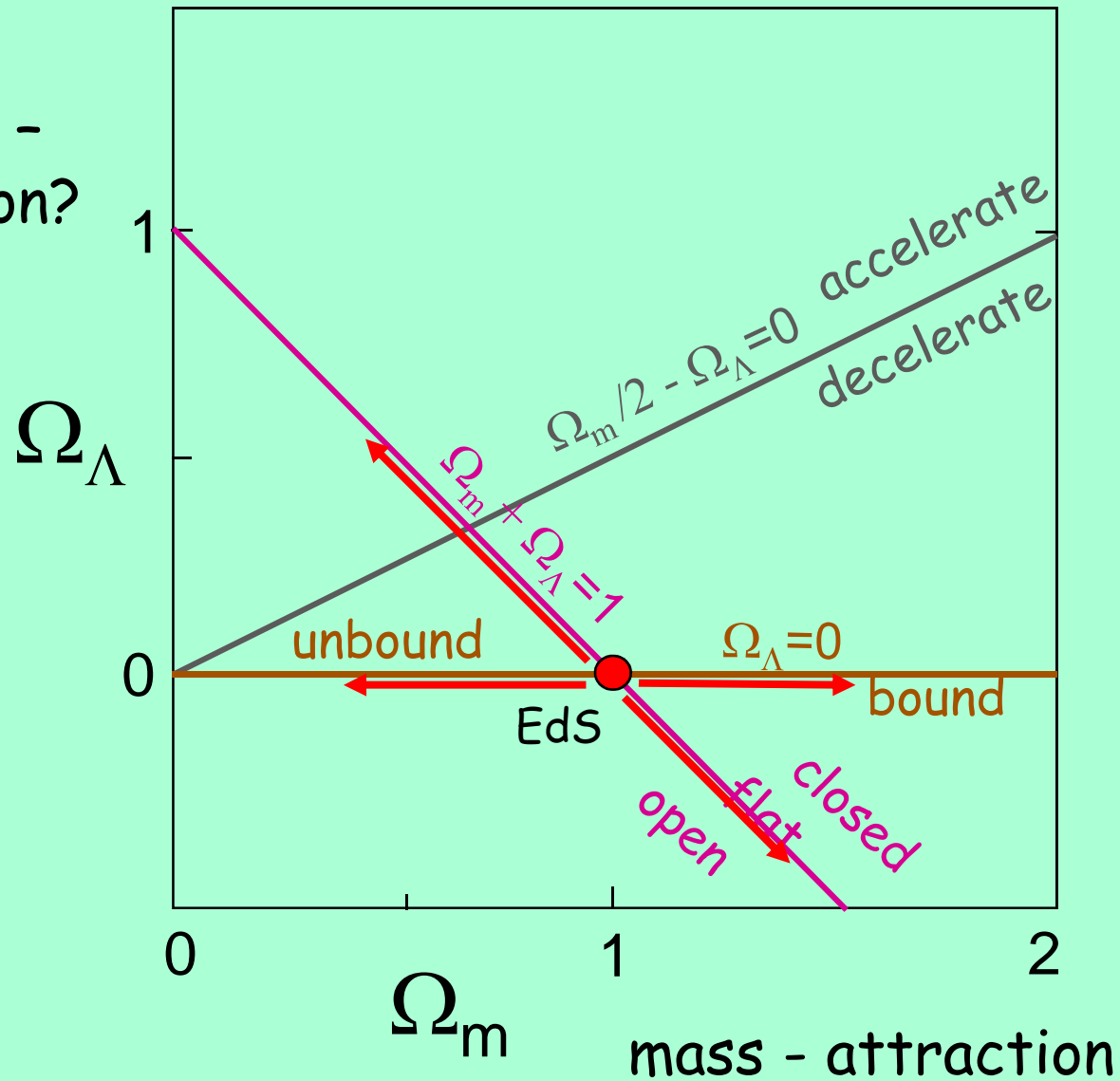
$$\Omega_m > 1$$

collapse



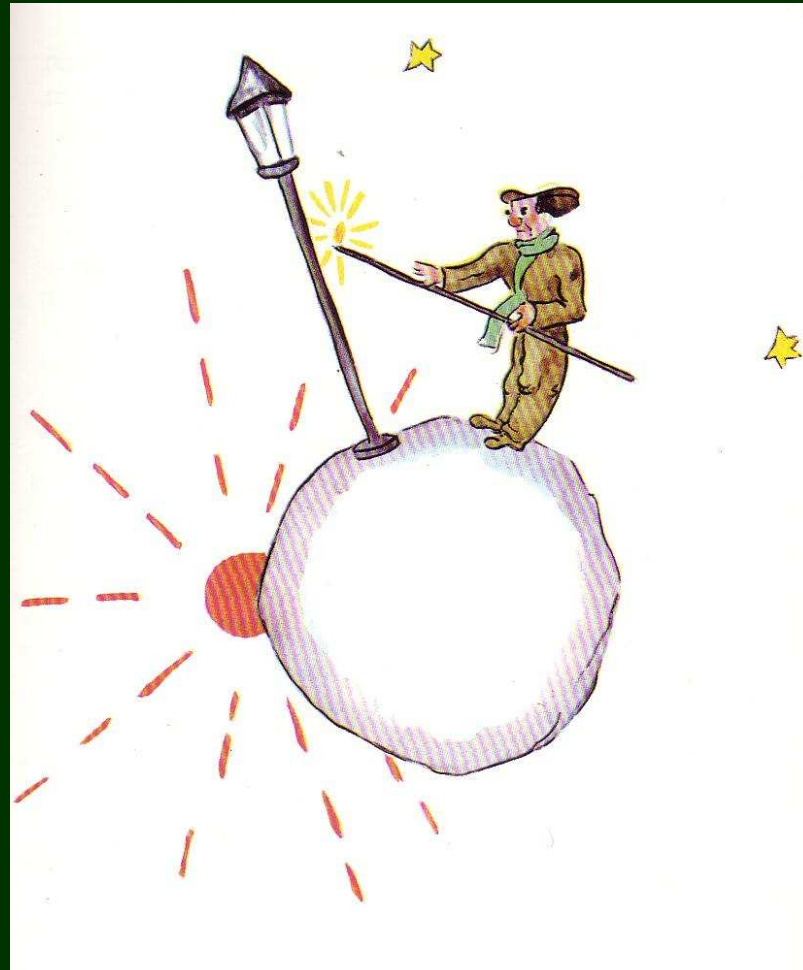
# Dark Matter and Dark Energy

vacuum -  
repulsion?



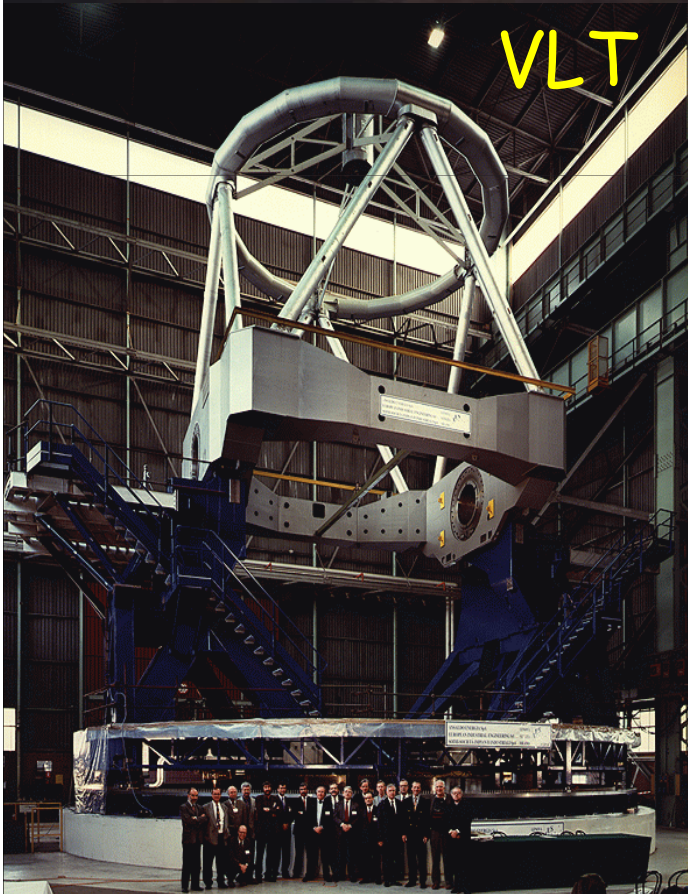


# Luminous Matter





Keck Telescope 10  
meter





# Hubble Space Telescope

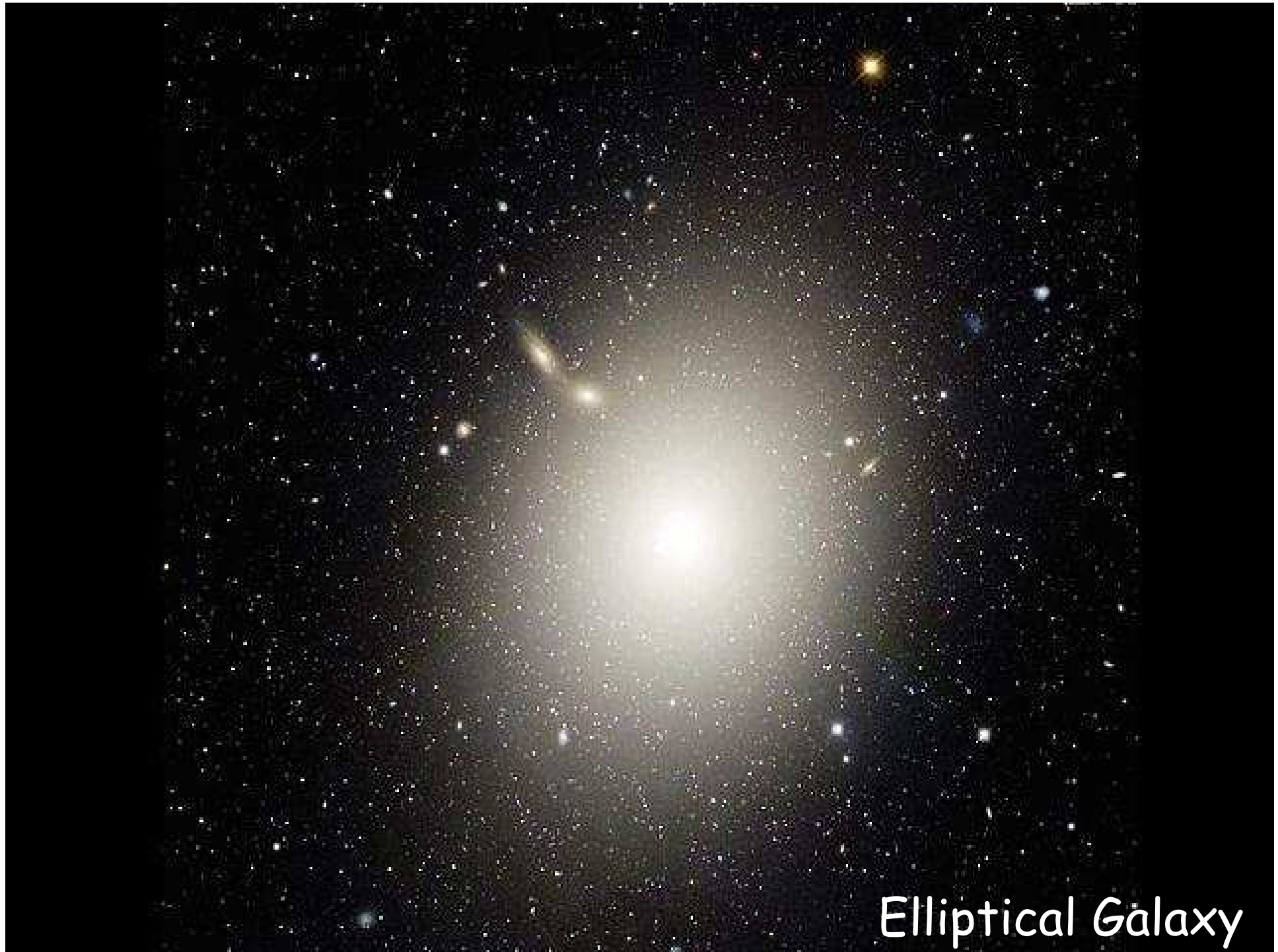




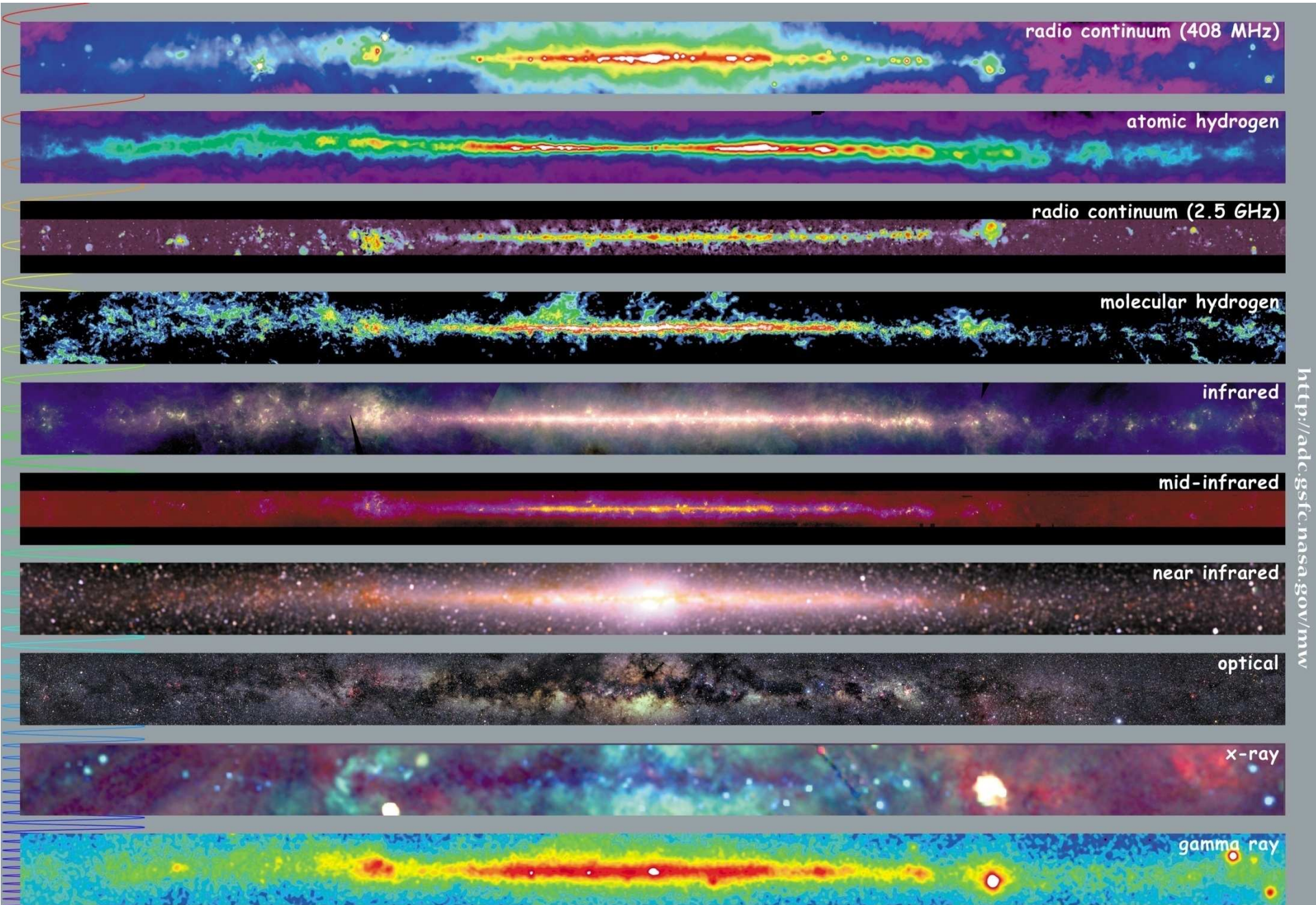


Spiral galaxy M74





Elliptical Galaxy



<http://adc.gsfc.nasa.gov/mw>



# Multiwavelength Milky Way

# Luminous mass

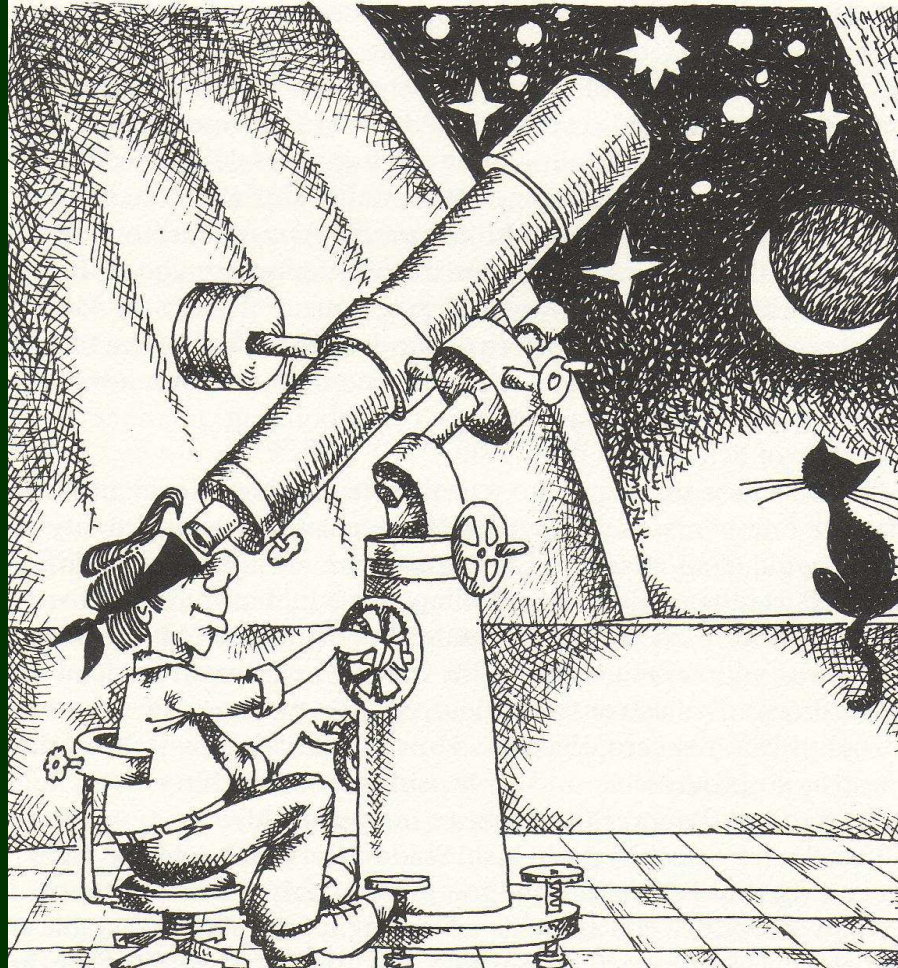
all sources, all wavelengths

$$\Omega_{\text{luminous}} \approx 0.01$$

With  $\Lambda=0$ , Universe unbound and infinite?



# Dark Matter





# Measuring Dark Matter

Disk galaxies: rotation curves

Clusters and Elliptical galaxies: virial theorem

All scales: gravitational Lensing

Clusters: X-ray

Clusters: scattering of CMB by gas

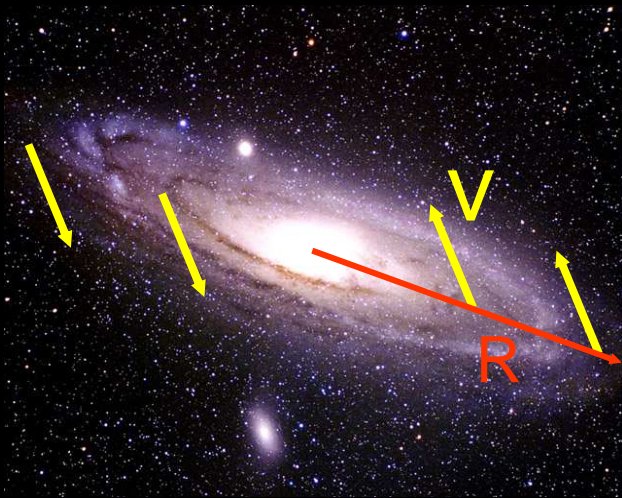
Large-Scale: cosmic flows

Large-Scale: power-spectrum of density fluctuations

Cluster abundance at early times

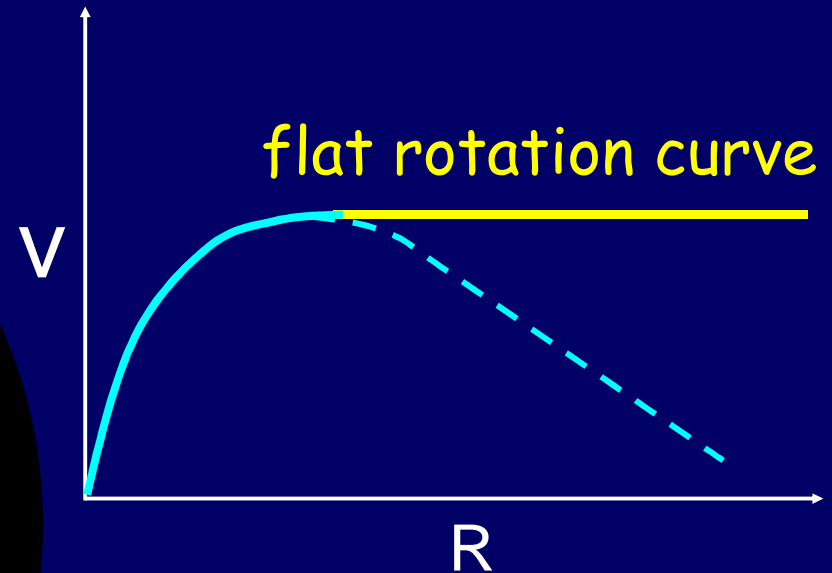
# Dark-Matter Halos in Galaxies

dark halo



3,000 light years

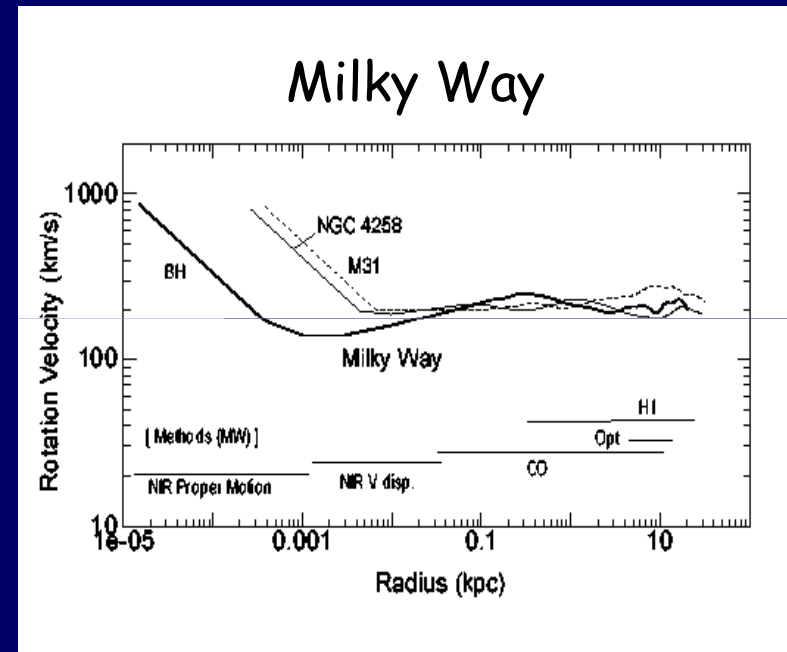
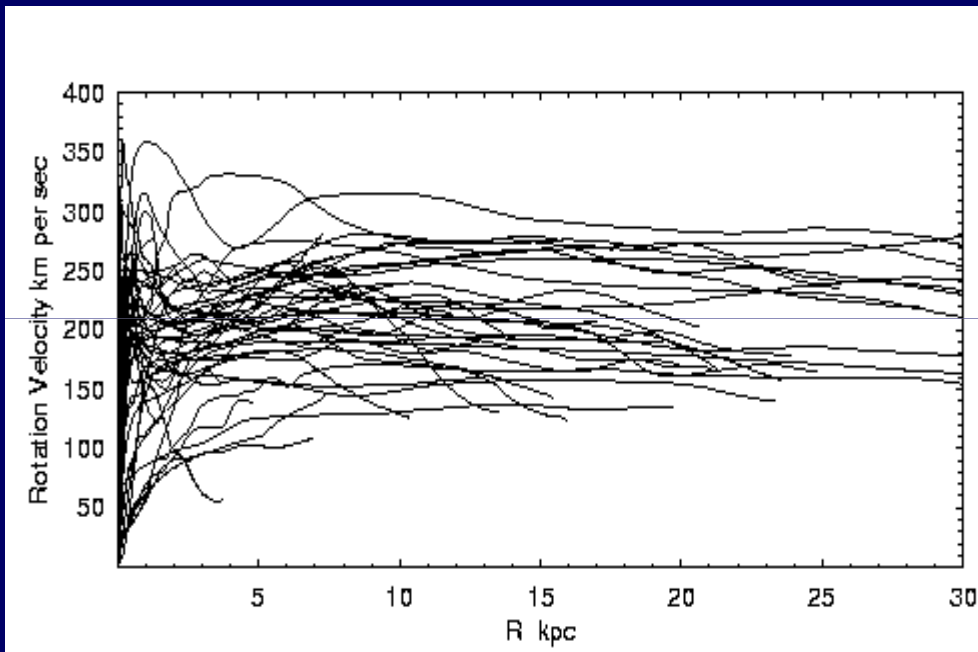
30,000 ly



$$V^2 = \frac{GM(R)}{R}$$

$$\rightarrow M(R) \propto R$$

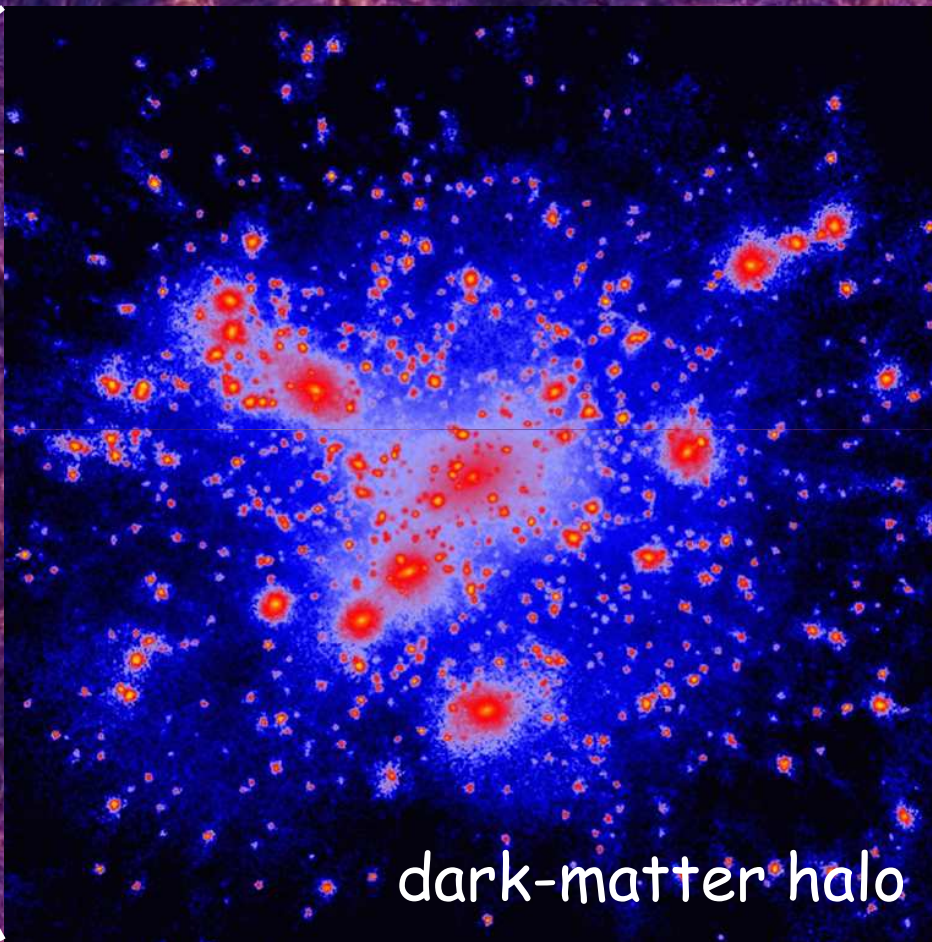
# Flat Rotation Curves: Extended Massive Dark-Matter Halos in Disk Galaxies



Sofue & Rubin 2001



# The Cosmic Web of Dark Matter



dark-matter halo

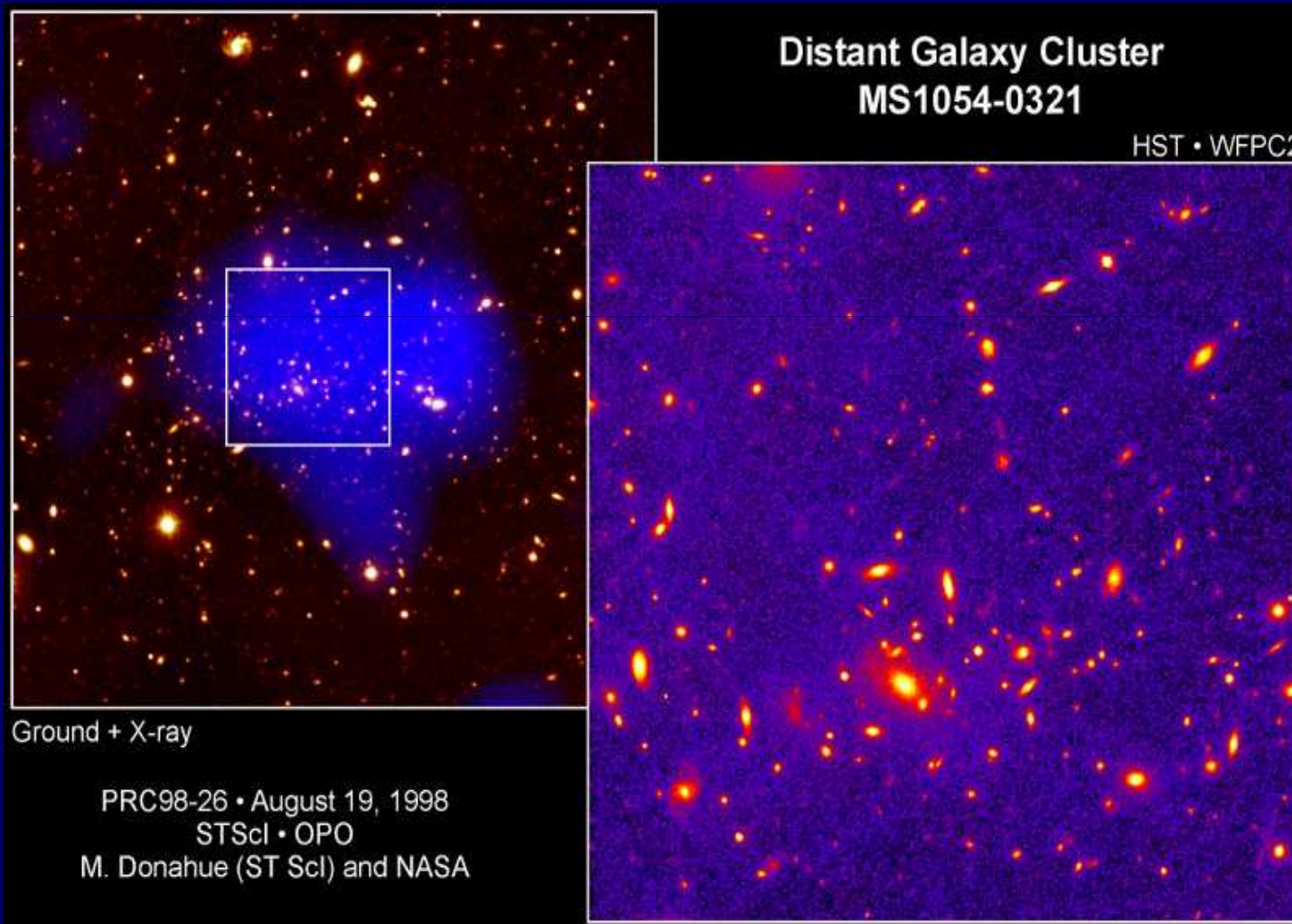
the millenium cosmological simulation



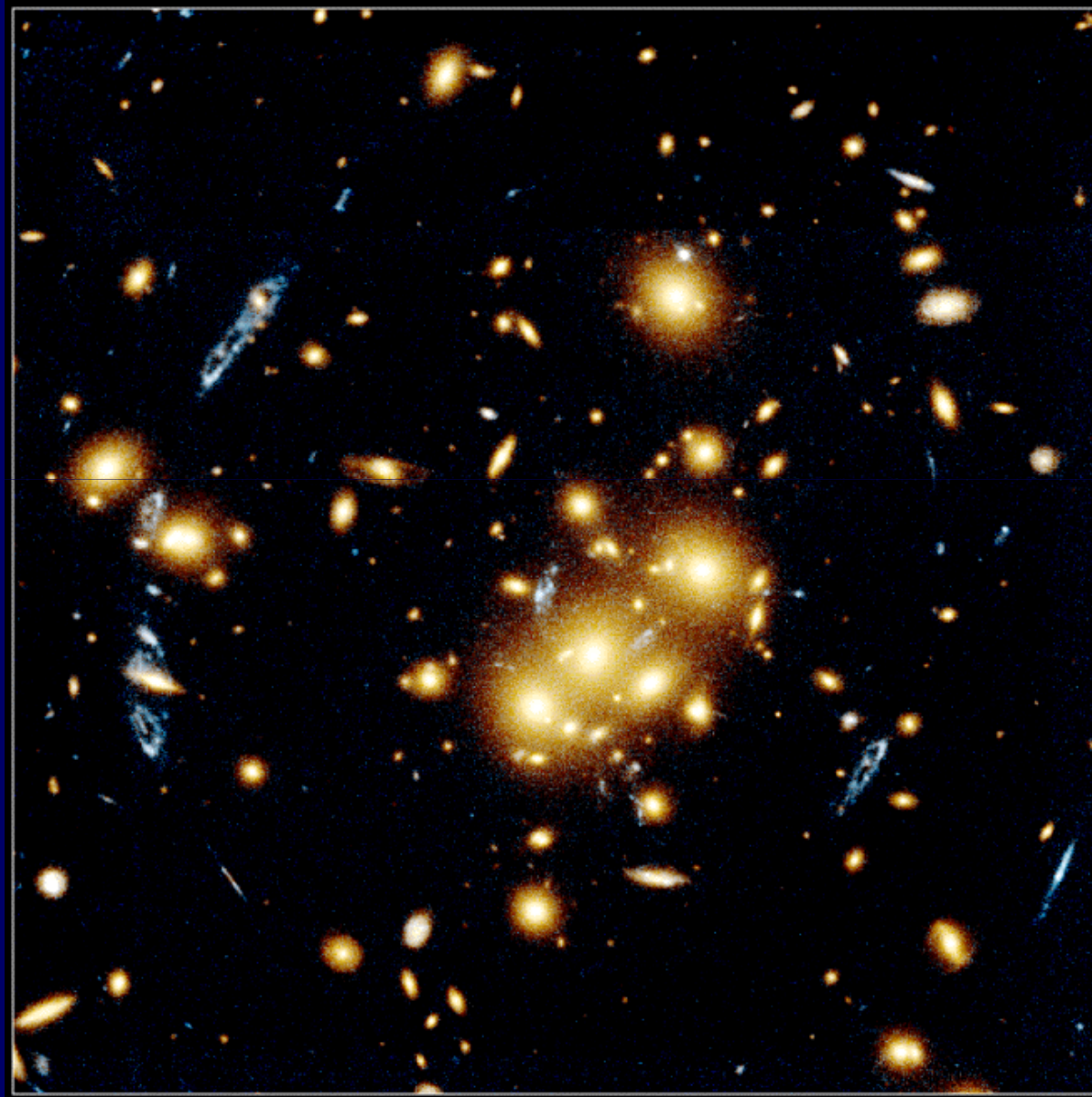
# Virial Equilibrium in Clusters of Galaxies

$$V^2 = \frac{GM}{R}$$

$V \sim 1500 \text{ km/s}$     $R \sim 1.5 \text{ Mpc}$     $\rightarrow M \sim 7 \times 10^{14} M_{\odot}$



# Gravitational Lensing: Dark Matter in Galaxy Clusters

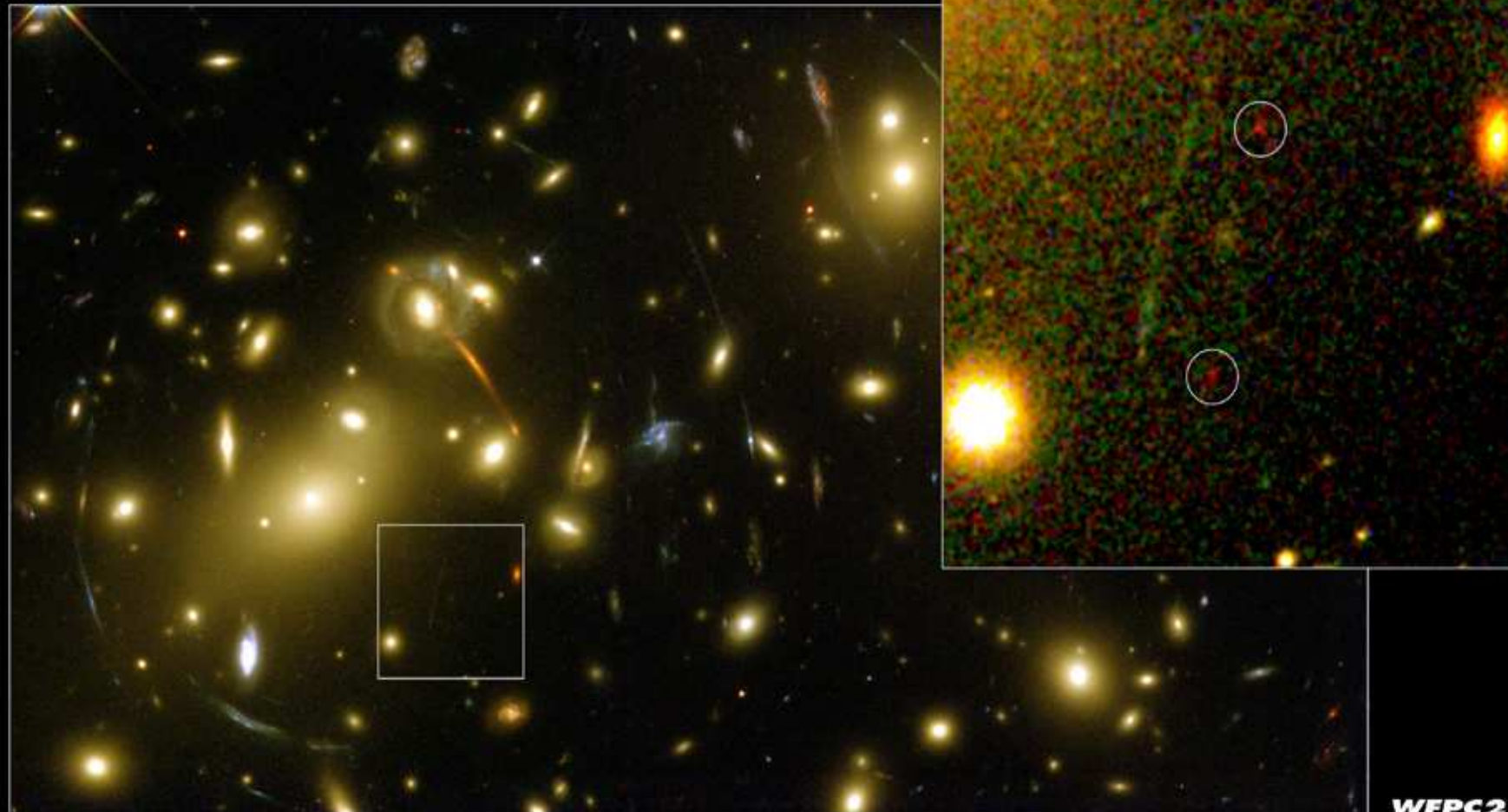


HST



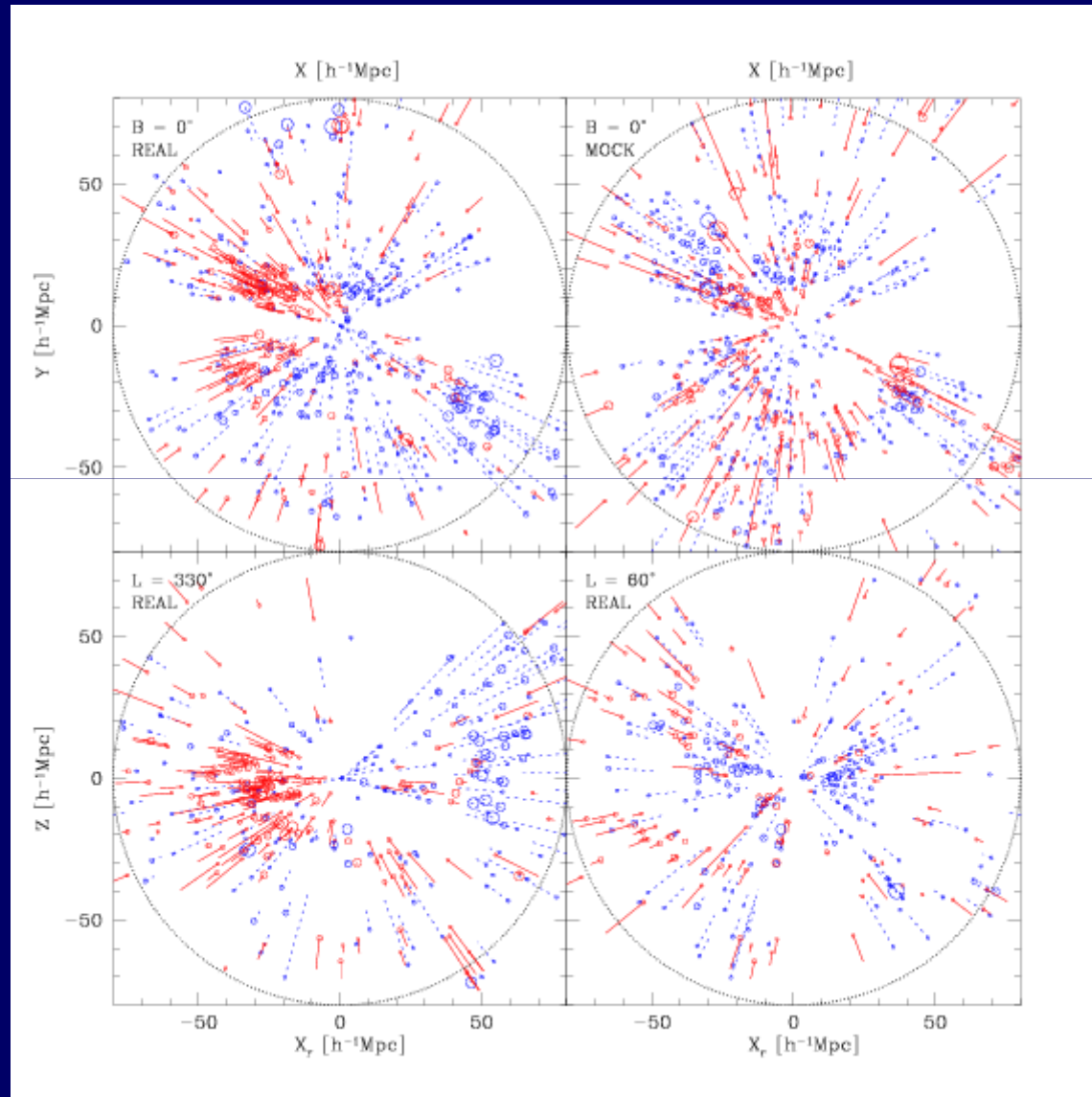
# Gravitational Lensing

HEIC0113



WFPC2

# Observed Radial Peculiar Velocities



Mark III



# POTENT: Cosmic Flows

Observe radial peculiar velocities:  $cz = H_0 r + v_r$

Potential flow:  $\vec{v}(\vec{r}) = -\vec{\nabla} \phi(\vec{r}) \quad (\vec{\nabla} \times \vec{v} = 0)$

Smooth the radial velocity field.

Integrate from the origin along radial trajectories to obtain the potential at any point in space.

$$\phi(\vec{r}) = -\int_0^{\vec{r}} v_r dr$$

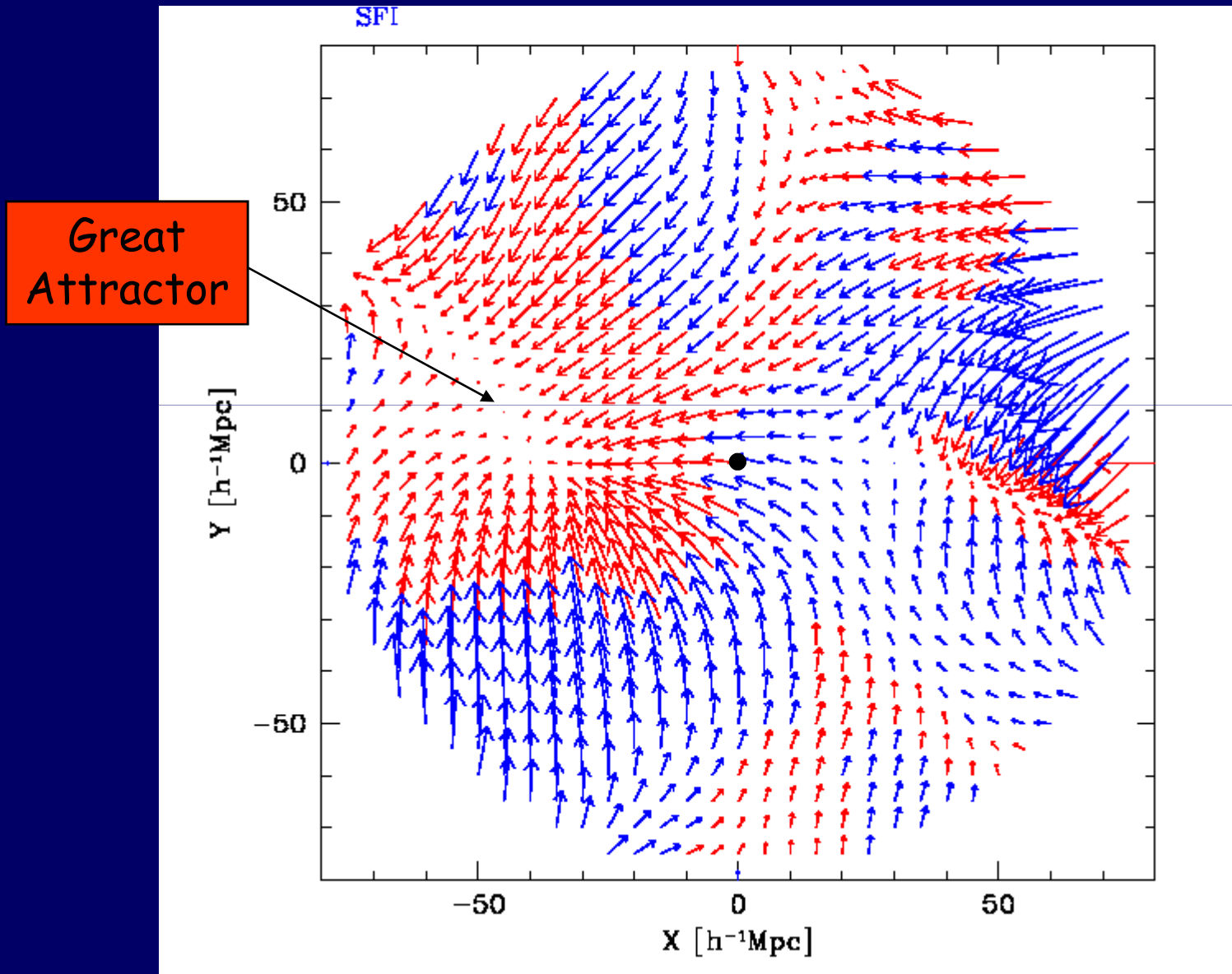
Differentiate to obtain the 3-dimensional velocity field.

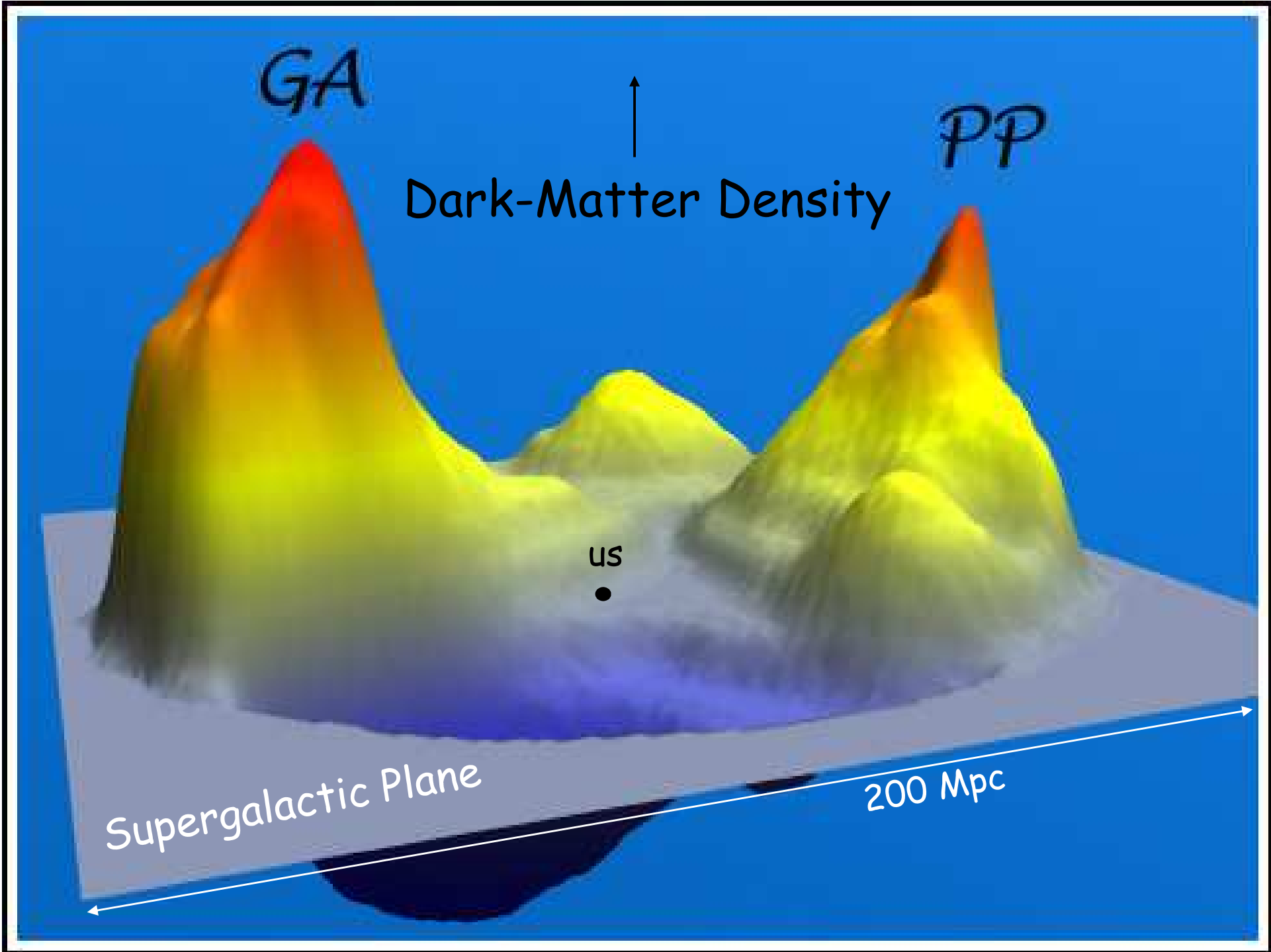
Compute density-fluctuation field by another differentiation:

$$\frac{\delta\rho}{\rho} \approx -\frac{1}{H_0 f(\Omega_m)} \vec{\nabla} \cdot \vec{v}$$

$$\delta = \left\| I - \frac{1}{Hf} \frac{\partial \vec{v}}{\partial \vec{x}} \right\| - 1$$

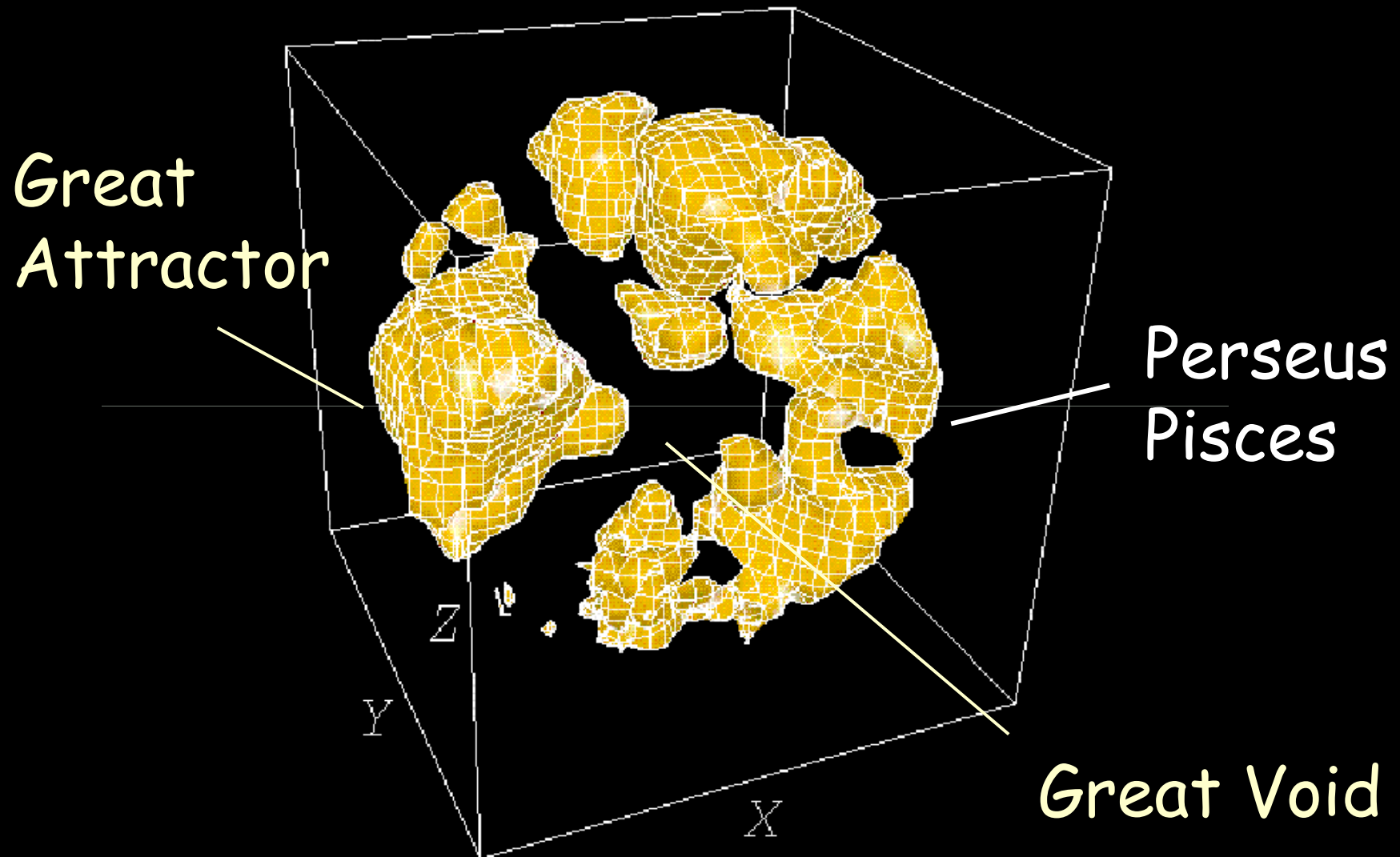
# Large-Scale Cosmic Flows - POTENT





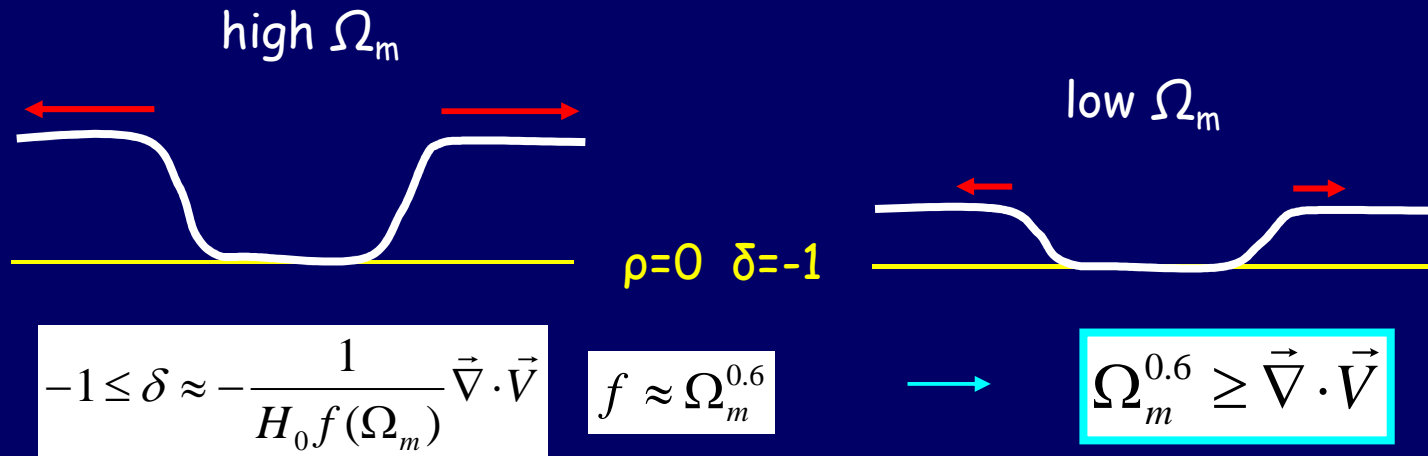


# Mass Density in 3D



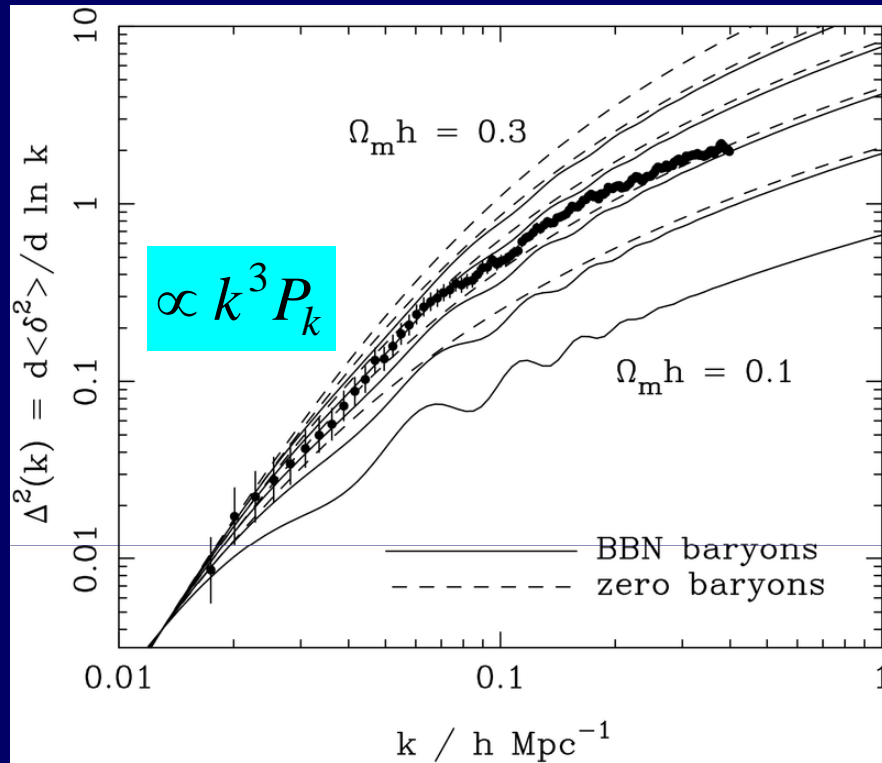
# Matter Density from Void Outflows

Dekel & Rees 1993



rees/void21.gif

# power spectrum



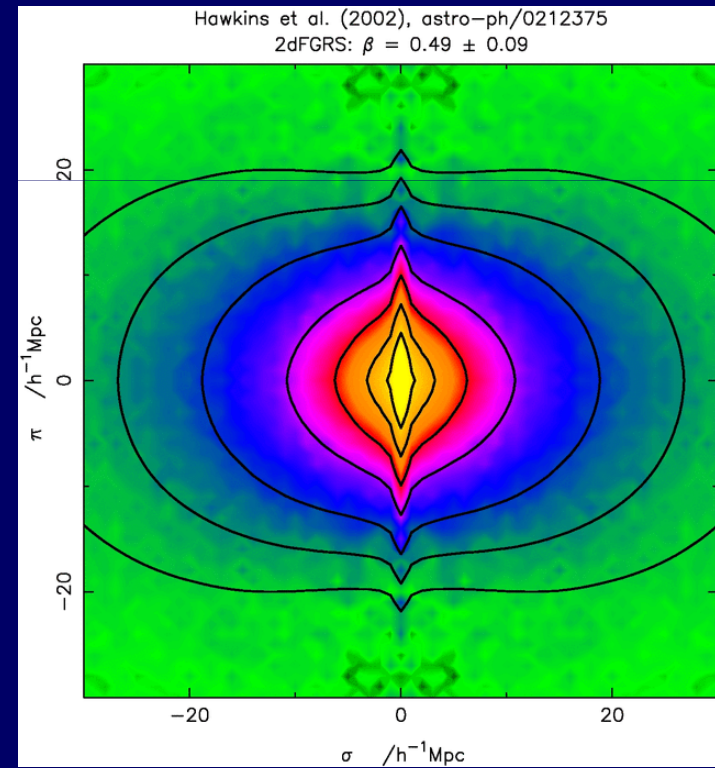
$$\Omega_m h \approx 0.2 \rightarrow \Omega_m \approx 0.3$$

$$\Omega_m^{0.6} \approx 0.5 \pm 0.1 \rightarrow \Omega_m \approx 0.3$$

redshift

# Measuring $\Omega_m$ from the 2dF Survey

## 2D correlation function anisotropy in z-space



angular separation



# Total Mass

exerting gravitational attraction

$$\Omega_m = 0.28 \pm 0.02$$

With  $\Lambda=0$ , Universe still unbound and infinite?

What is the dark matter made of?

# Baryonic Mass

Baryonic dark matter:  
planets, black holes, ...

Big-Bang Nucleosynthesis:

$$\Omega_{\text{baryons}} = 0.044 \pm 0.004$$

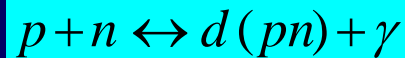


# Big Bang Nucleosynthesis

$$m_n > m_p \Rightarrow n + \nu \rightarrow p + e^-$$

only 12.5% n left after decaying to p → 75% H + 25% He (in mass)

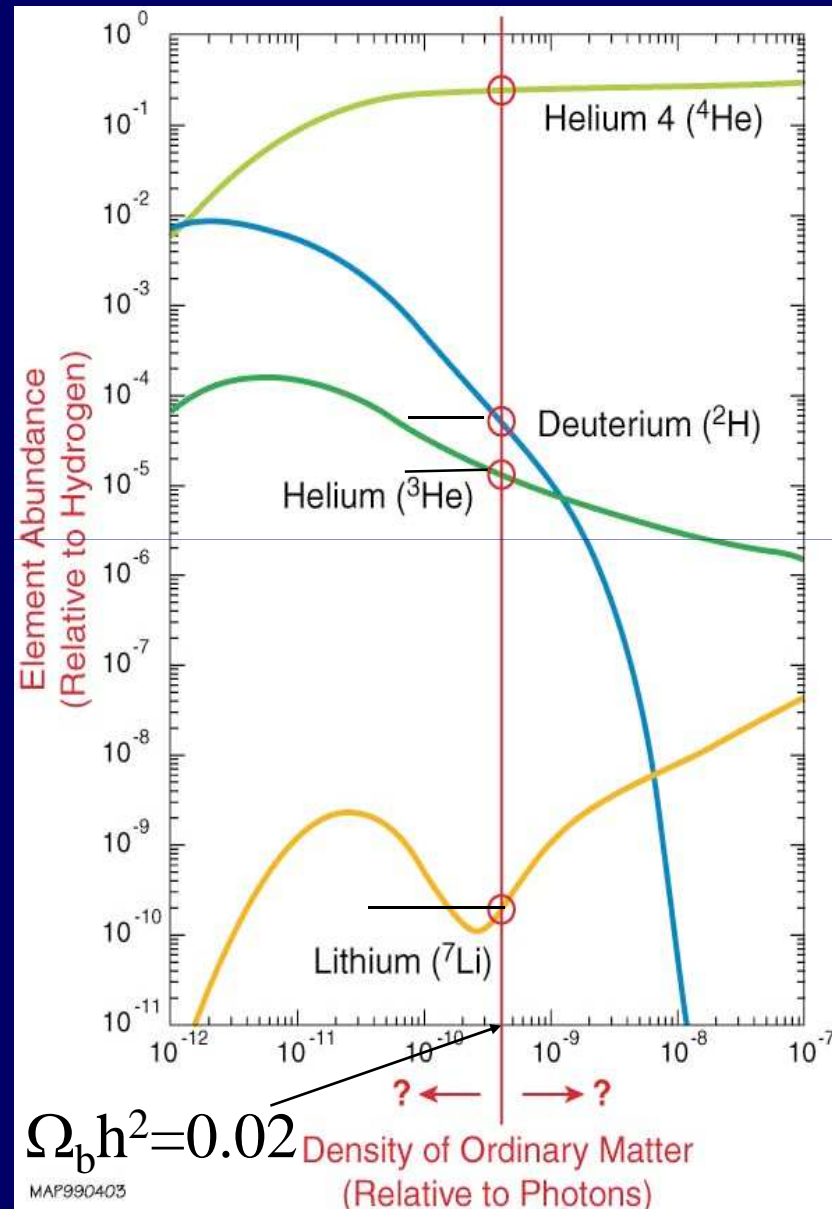
At  $T \sim 10^9$  K deuterium becomes stable and nucleosynthesis starts:



A minute later p becomes too cold to penetrate the Coulomb barrier by p in d and the process stops.

Rate  $\propto n_p^2 \rightarrow$  abundances of d and  ${}^3\text{He}$  decrease with  $\Omega_b$

$$\Omega_b = 0.04 \pm 0.01$$



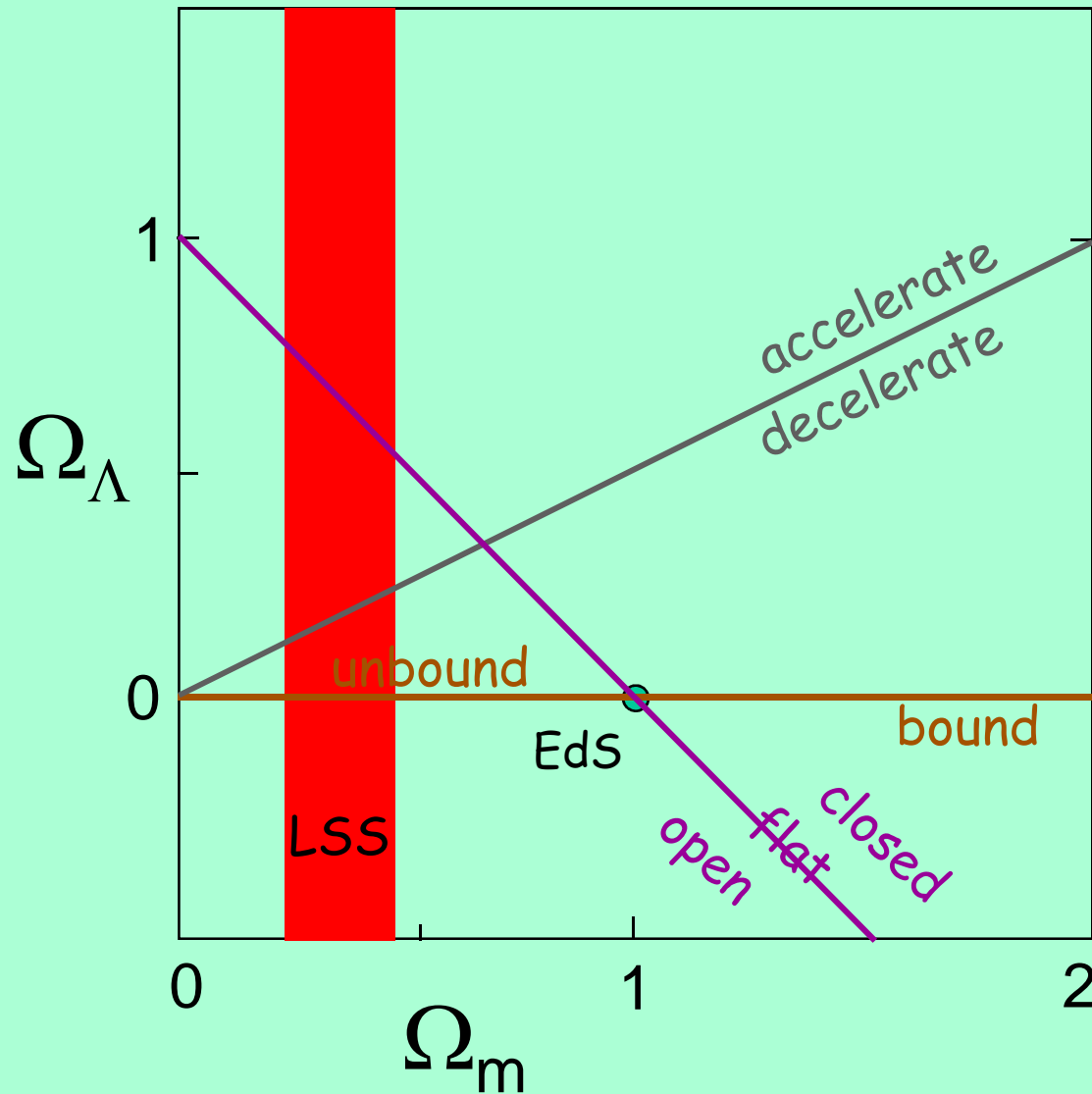


# Non-Baryonic Dark-Matter Particles

Neutralinos, photinos, axions, ... all those damn super-symmetric particles you can't see... that's what drove me to drink... but **now** I can see them!



# Dark Matter and Dark Energy



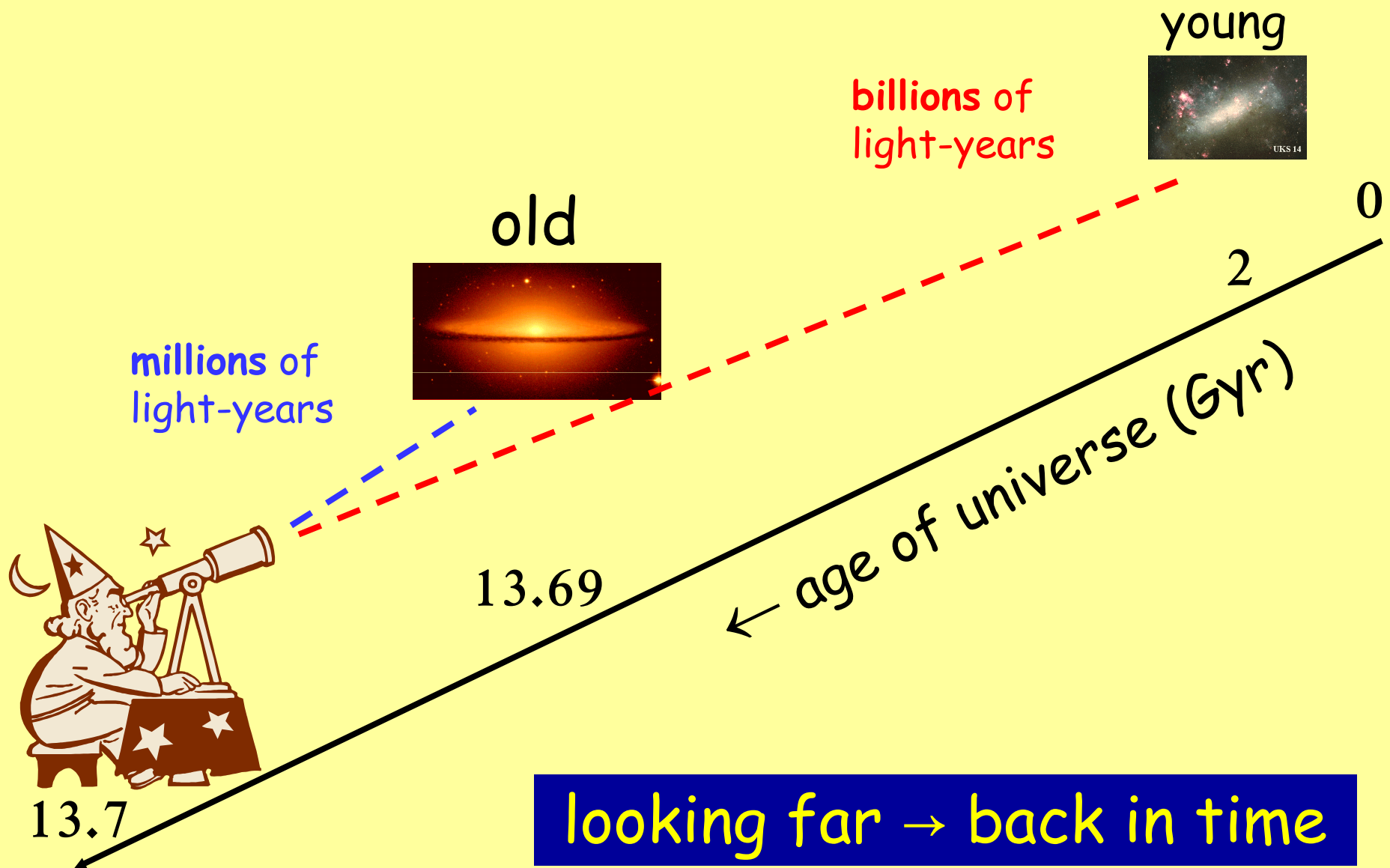
# Dark Energy



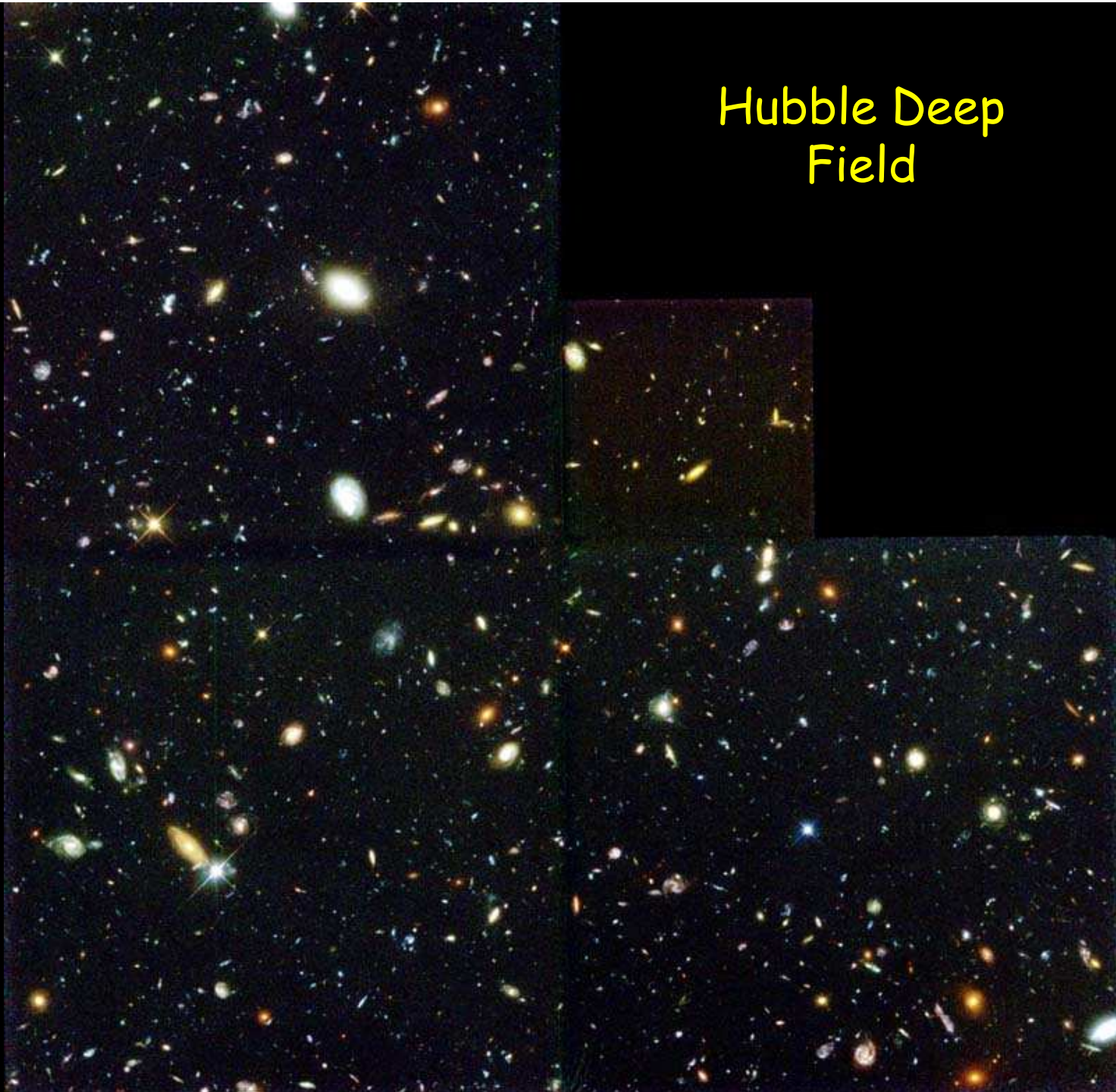
measure the expected deceleration  
under gravitational attraction



# The Telescope as a Time Machine

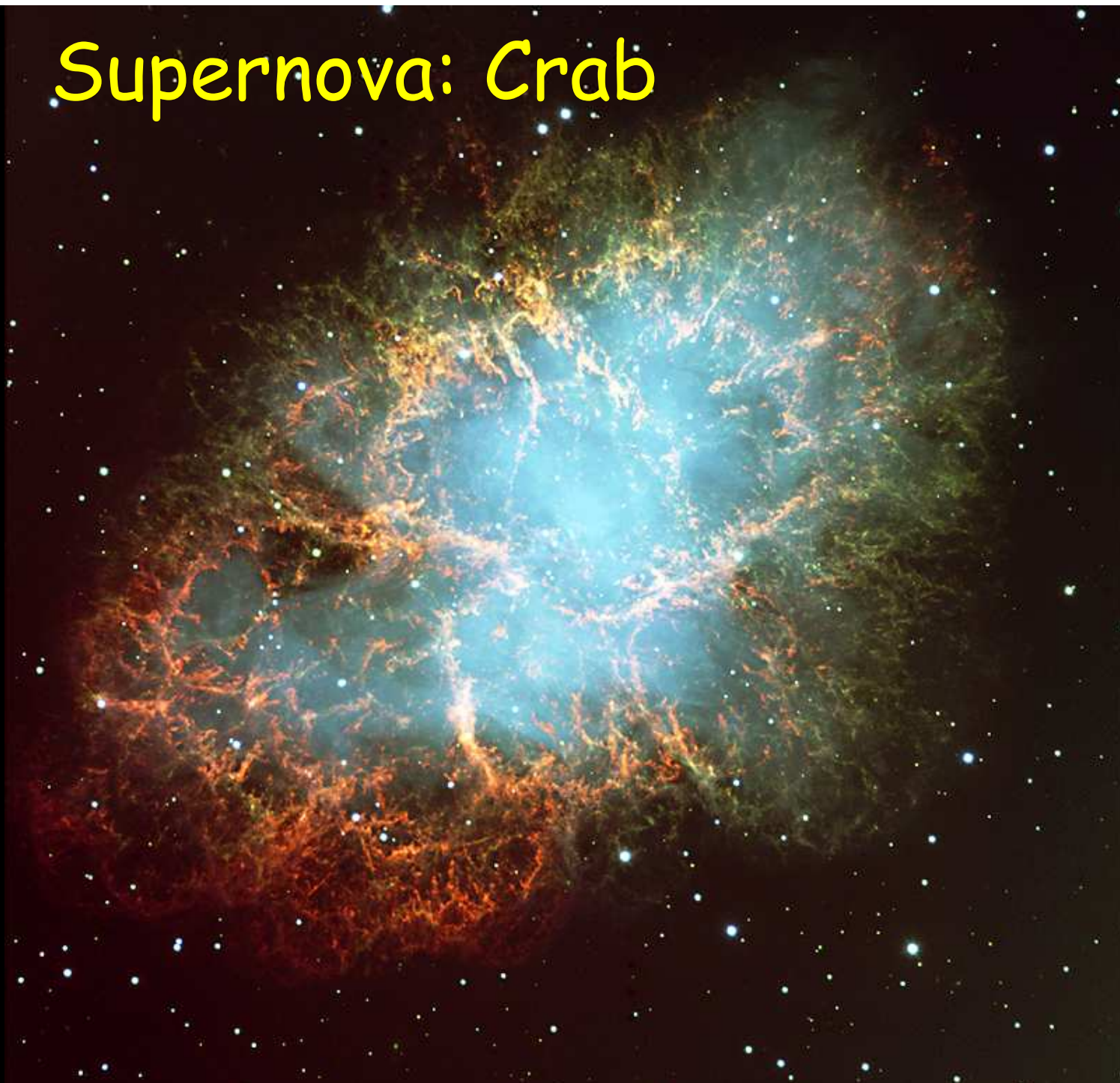


Hubble Deep  
Field

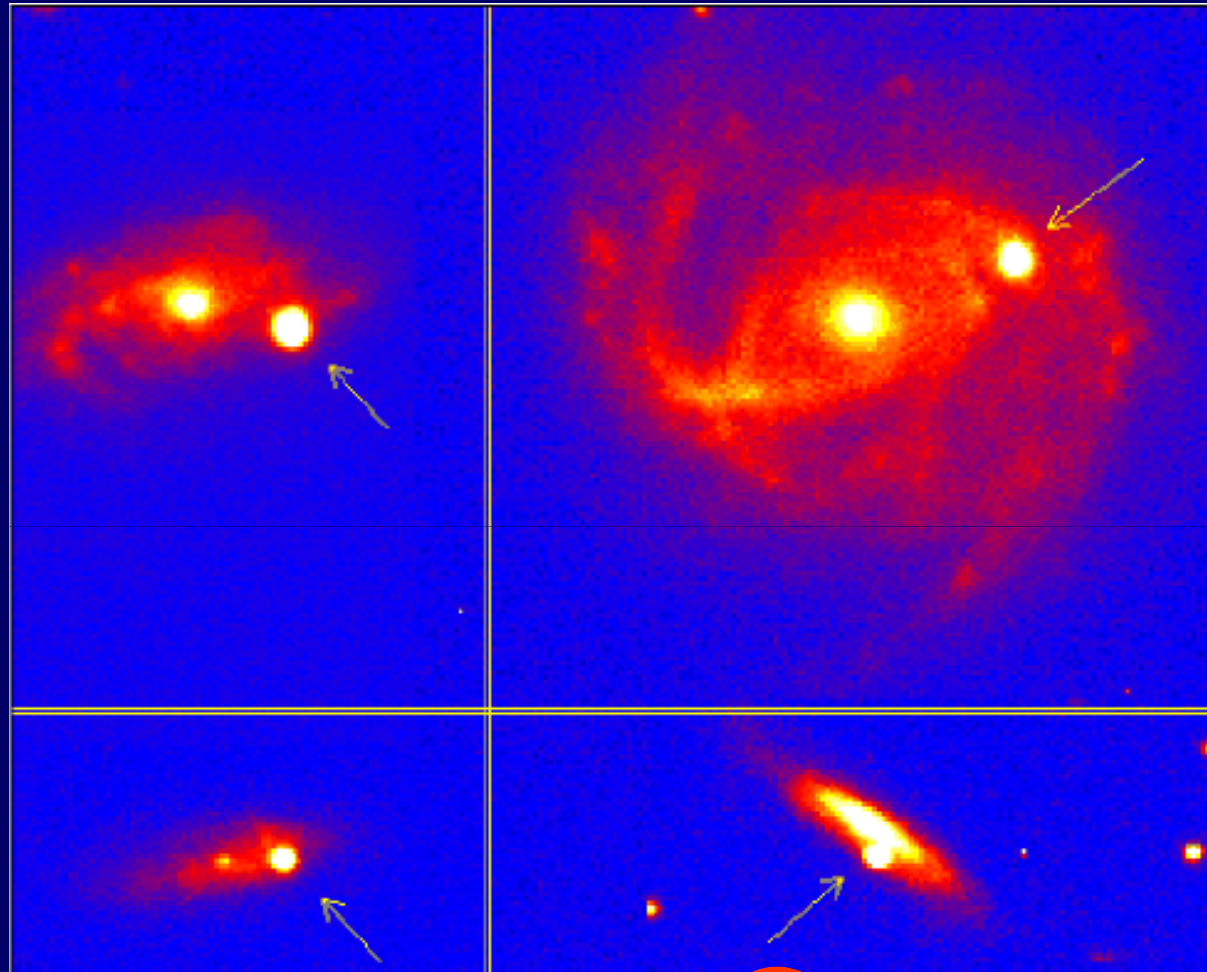




# Supernova: Crab



# Bright Standard Candle: Supernovae Type Ia



$$\frac{\text{observed luminosity}}{\text{intrinsic luminosity}} = \frac{\ell}{L} = 4\pi d_L^{-2}(z; H_0, \Omega_m, \Omega_\Lambda)$$



# Luminosity-distance to a standard candle

$$l \sim L/d^2 \quad \text{magnitude} = -2.5 \log(\text{luminosity}) + \text{const.}$$

$$m(z) = M + 5 \log D_L(z; \Omega_m, \Omega_\Lambda) - 5 \log H_0 + 25$$

Luminosity distance ( $D_L = d_L H_0$ )

$$D_L(z; \Omega_m, \Omega_\Lambda) = c(1+z) |\Omega_k|^{-1/2} S_k \left( |\Omega_k|^{1/2} \int_0^z [(1+z')^2 (1 + \Omega_m z') - z'(2+z') \Omega_\Lambda]^{-1/2} dz' \right)$$

$$\xrightarrow{z \ll 1} c z$$

Observe a sample of z-m

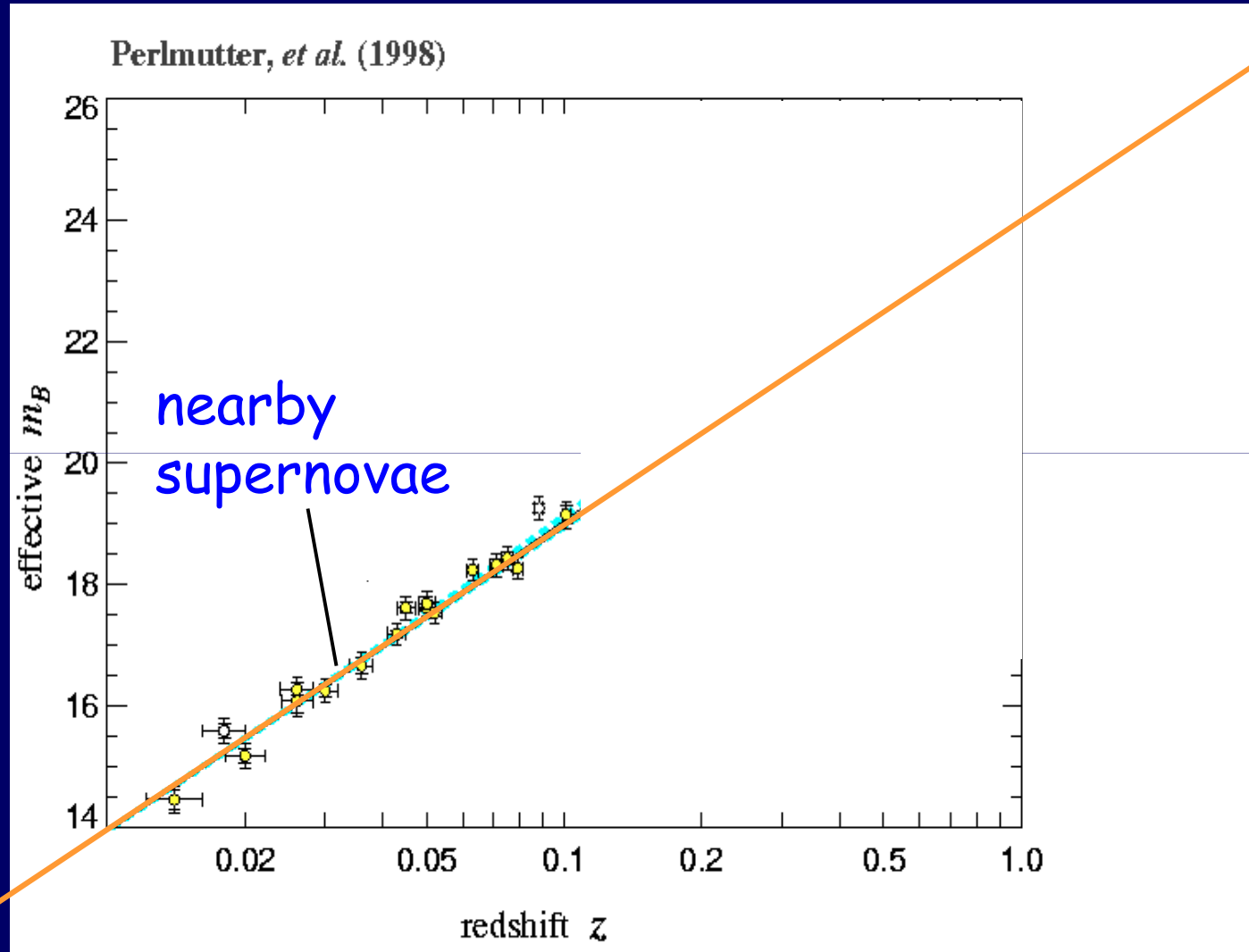
Determine M and  $H_0$  at low z

Find  $\Omega_m$  and  $\Omega_\Lambda$  by best fit at high z

# Deceleration?

Today's expansion rate  $V=H_0R$

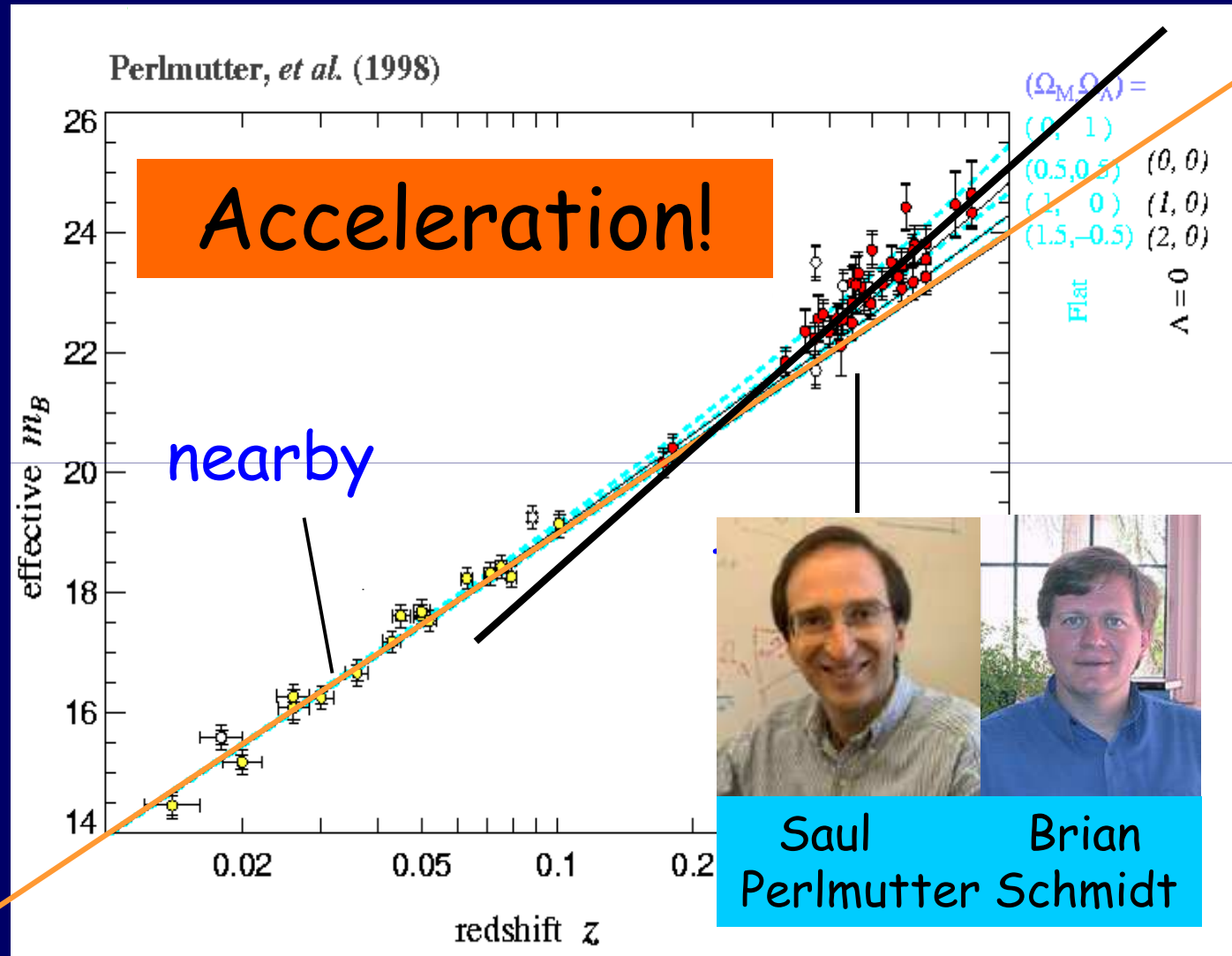
↑  
distance  
 $R$



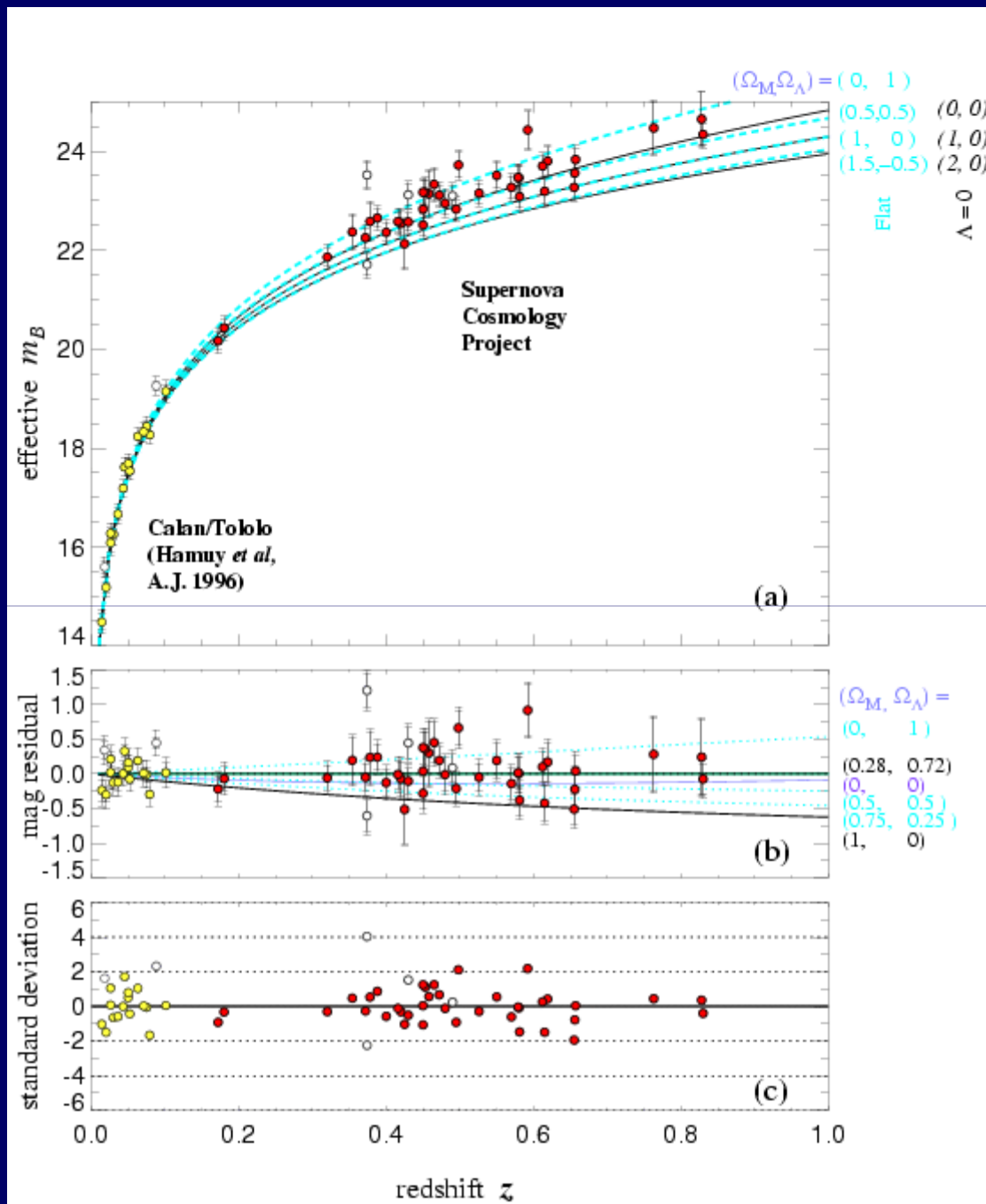
velocity  $V$  →

# Past expansion rate $V=H(t) R$

↑  
distance  
 $R$



velocity  $V$  →





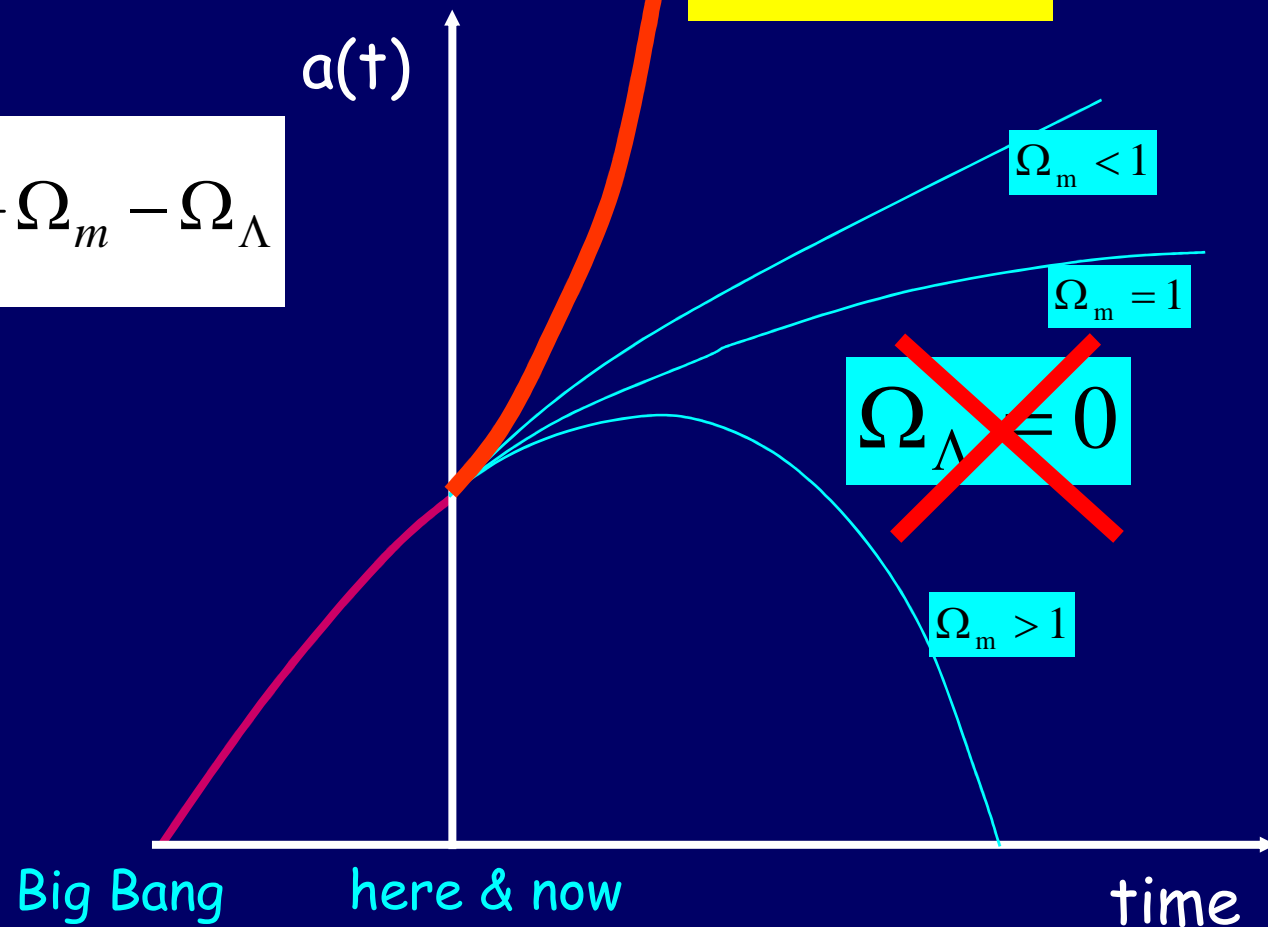
# Acceleration by a cosmological constant

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho_{m0}}{3a^3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$a \propto e^{Ht}$$

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}\Omega_m - \Omega_\Lambda$$

$$\Omega_\Lambda > 0$$



# Acceleration

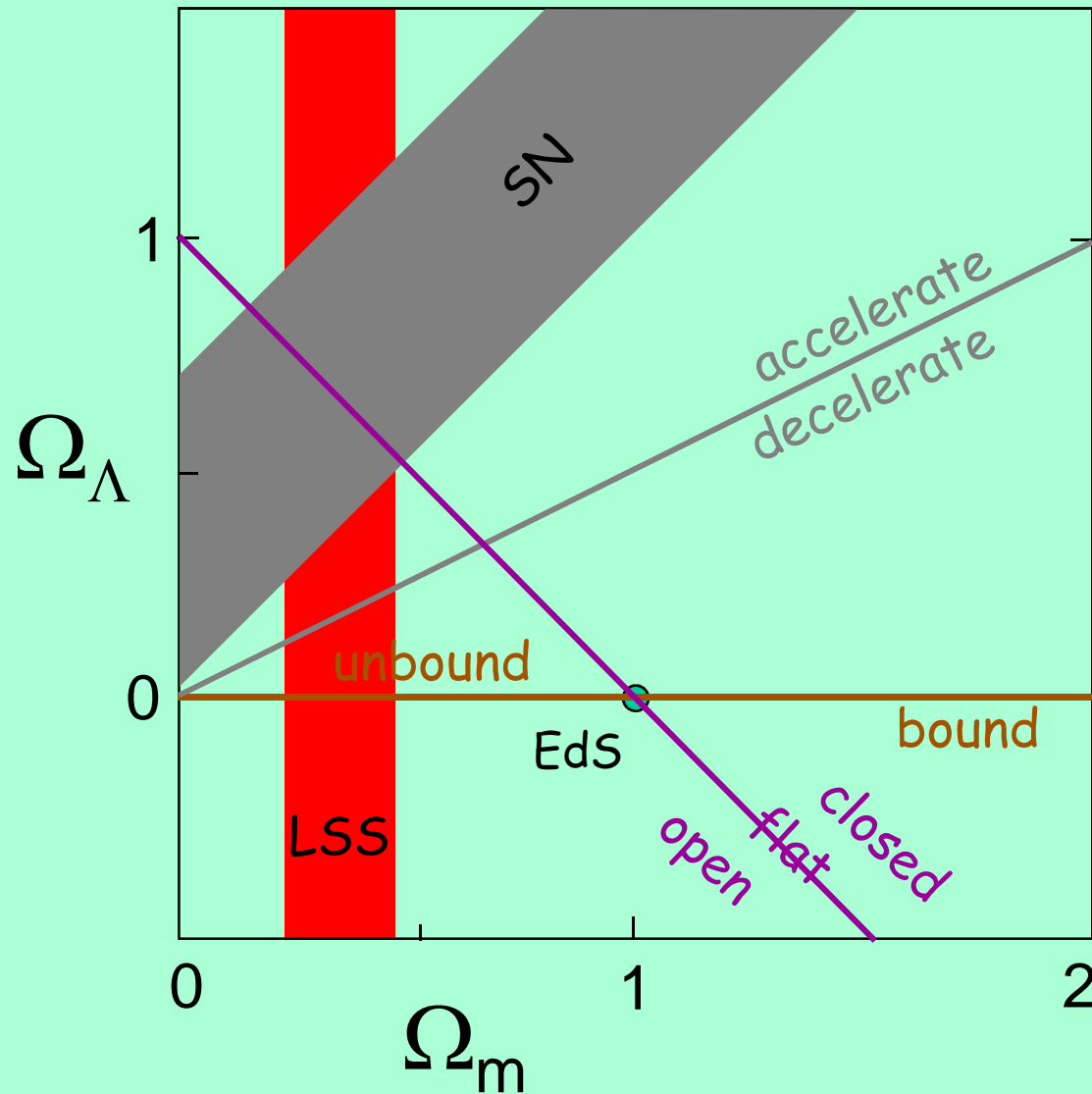
$$\Omega_{\Lambda} - \frac{1}{2}\Omega_m > 0$$



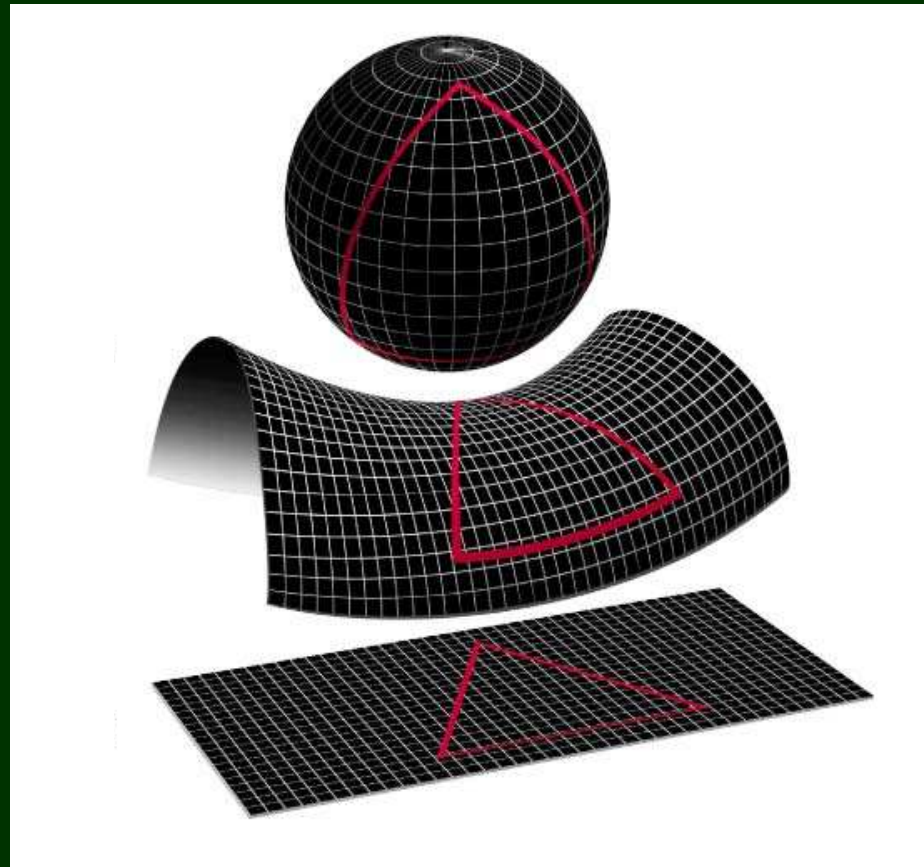
# Vacuum Energy

$$\Omega_{\Lambda} = 0.72 \pm 0.02$$

# Dark Matter and Dark Energy



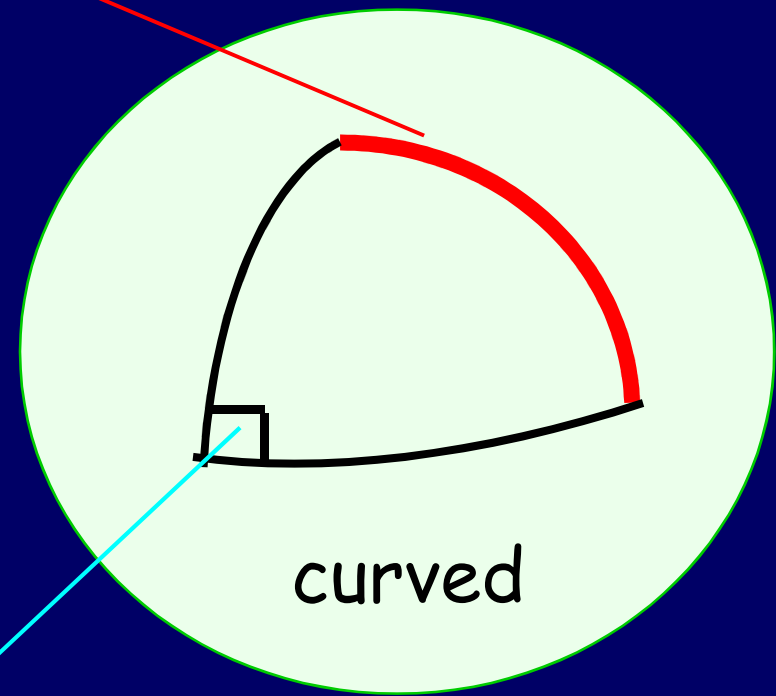
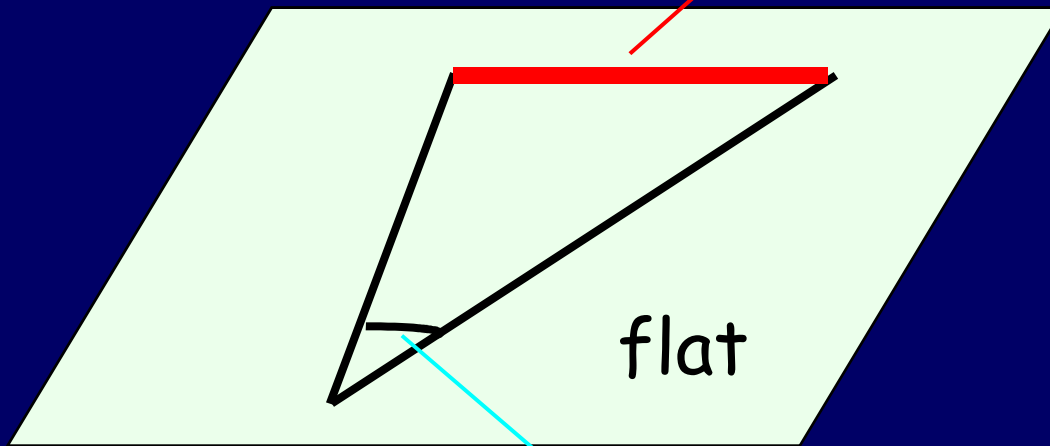
# Curvature





# Use a Standard Ruler

known intrinsic length



observed angle

$$\frac{\text{intrinsic length}}{\text{observed angle}} = \frac{x}{\theta} = d_A(z; H_0, \Omega_m, \Omega_\Lambda)$$

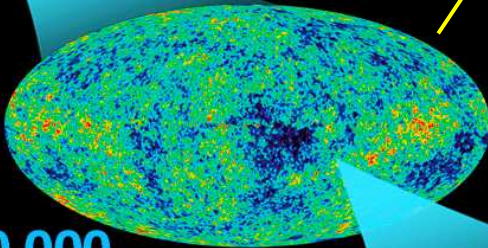
DAWN  
OF  
TIME

Cosmic Microwave  
Background



tiny fraction  
of a second

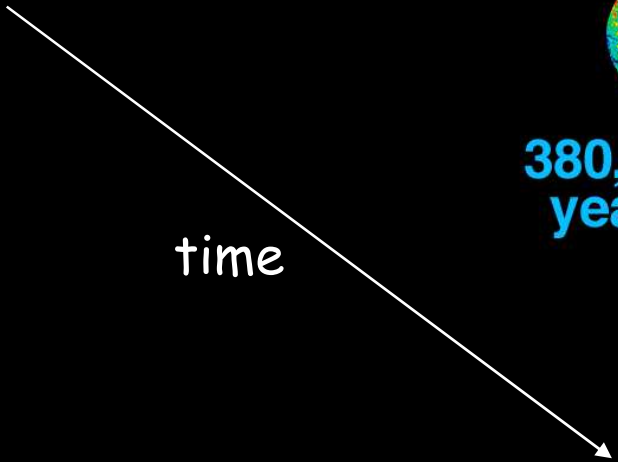
inflation



380,000  
years

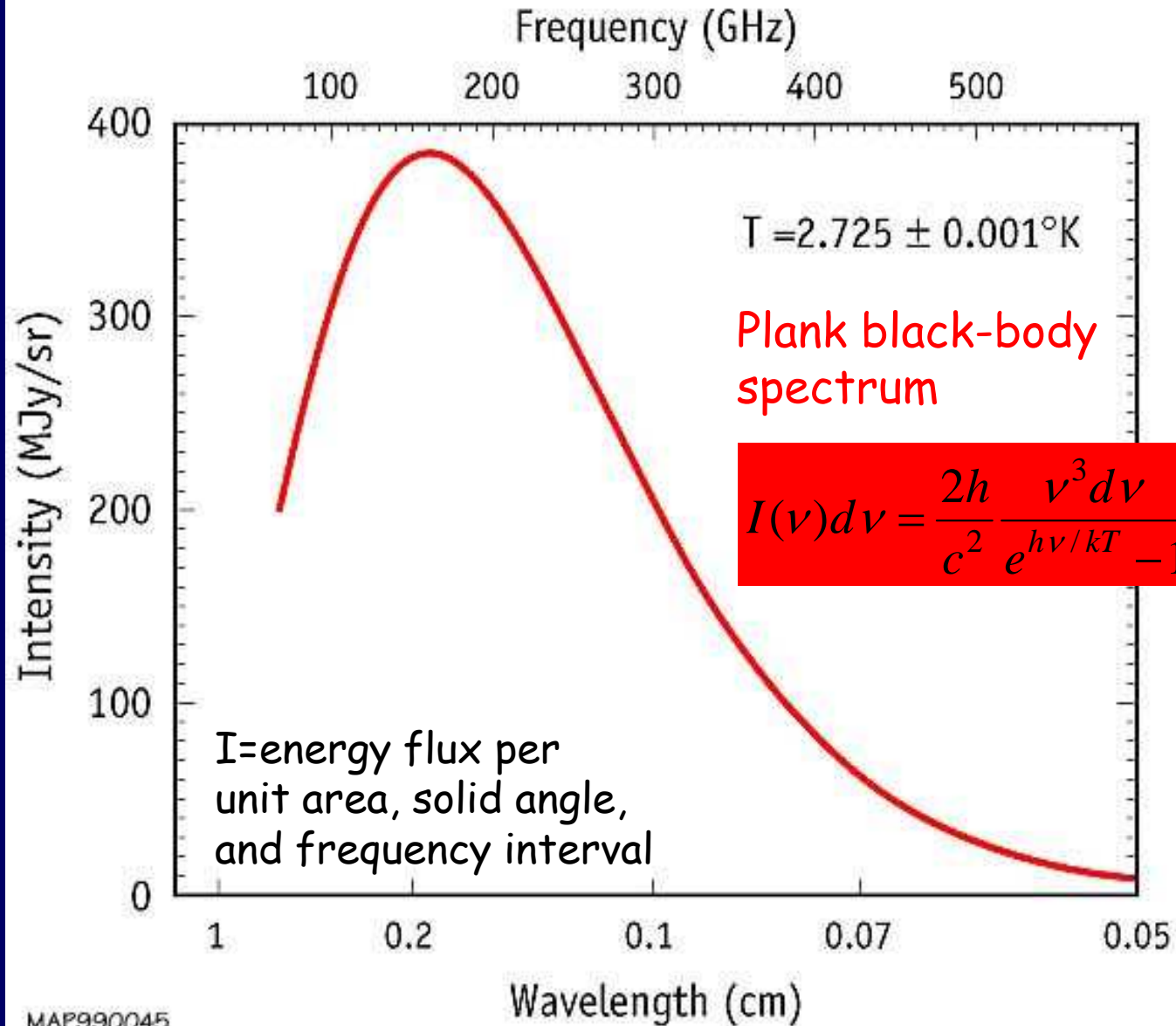


13.7  
billion  
years

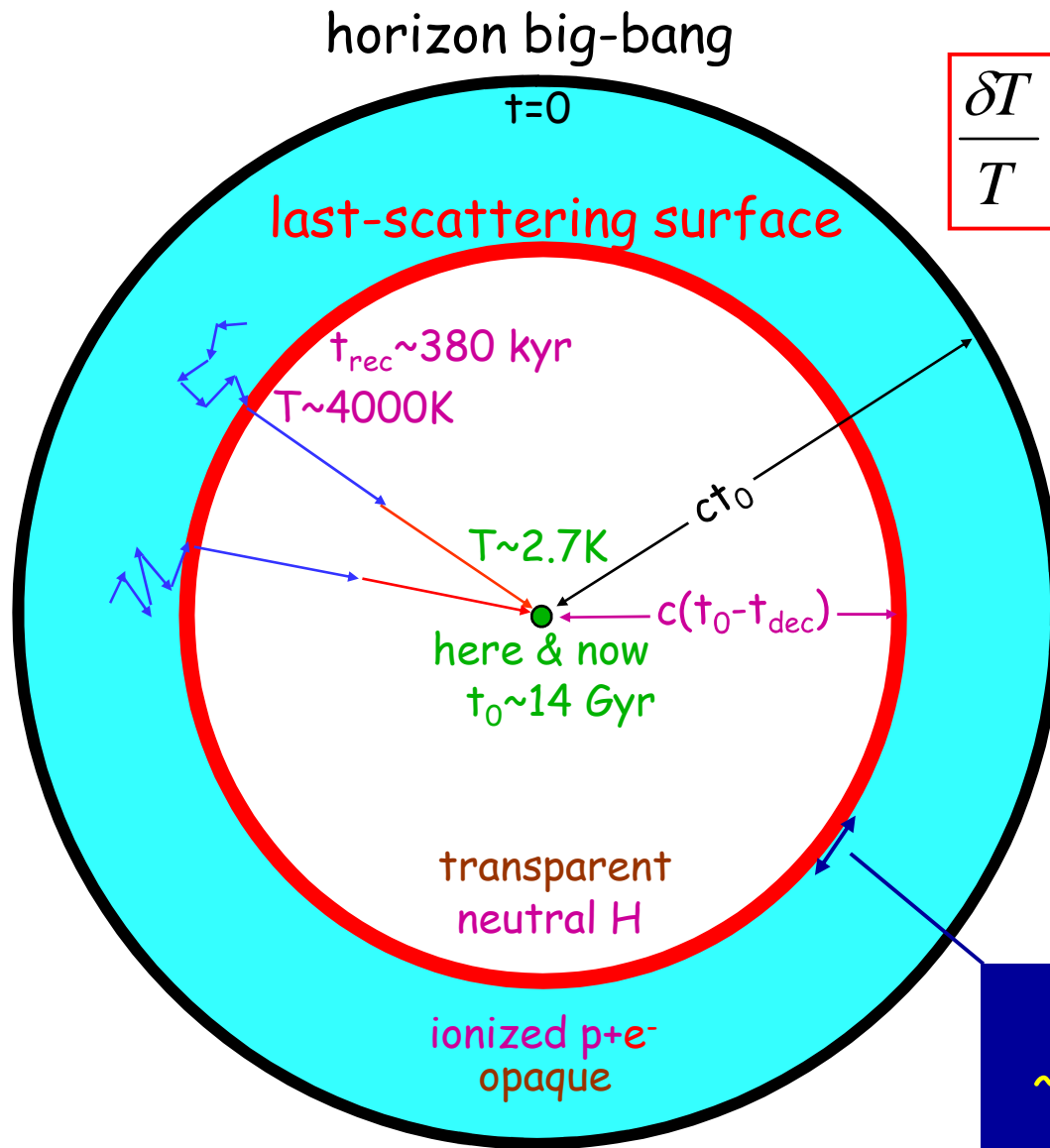


# SPECTRUM OF THE COSMIC MICROWAVE BACKGROUND

COBE  
1992



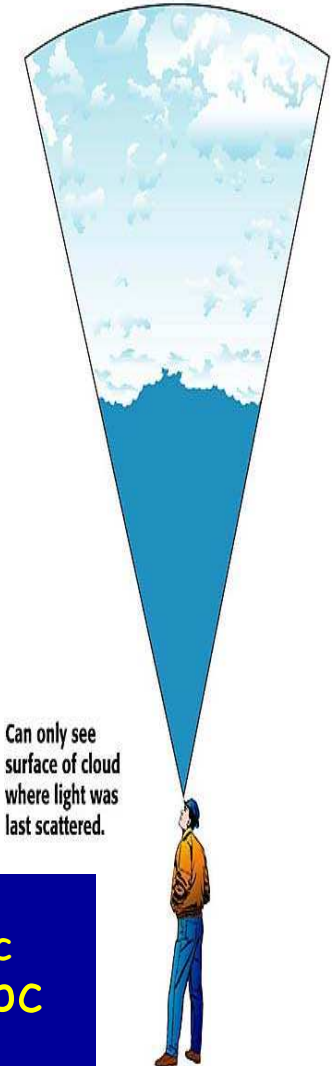
# Origin of Cosmic Microwave Background



$$\frac{\delta T}{T} \sim \frac{1}{10} \frac{\delta \rho}{\rho} \sim 10^{-5}$$

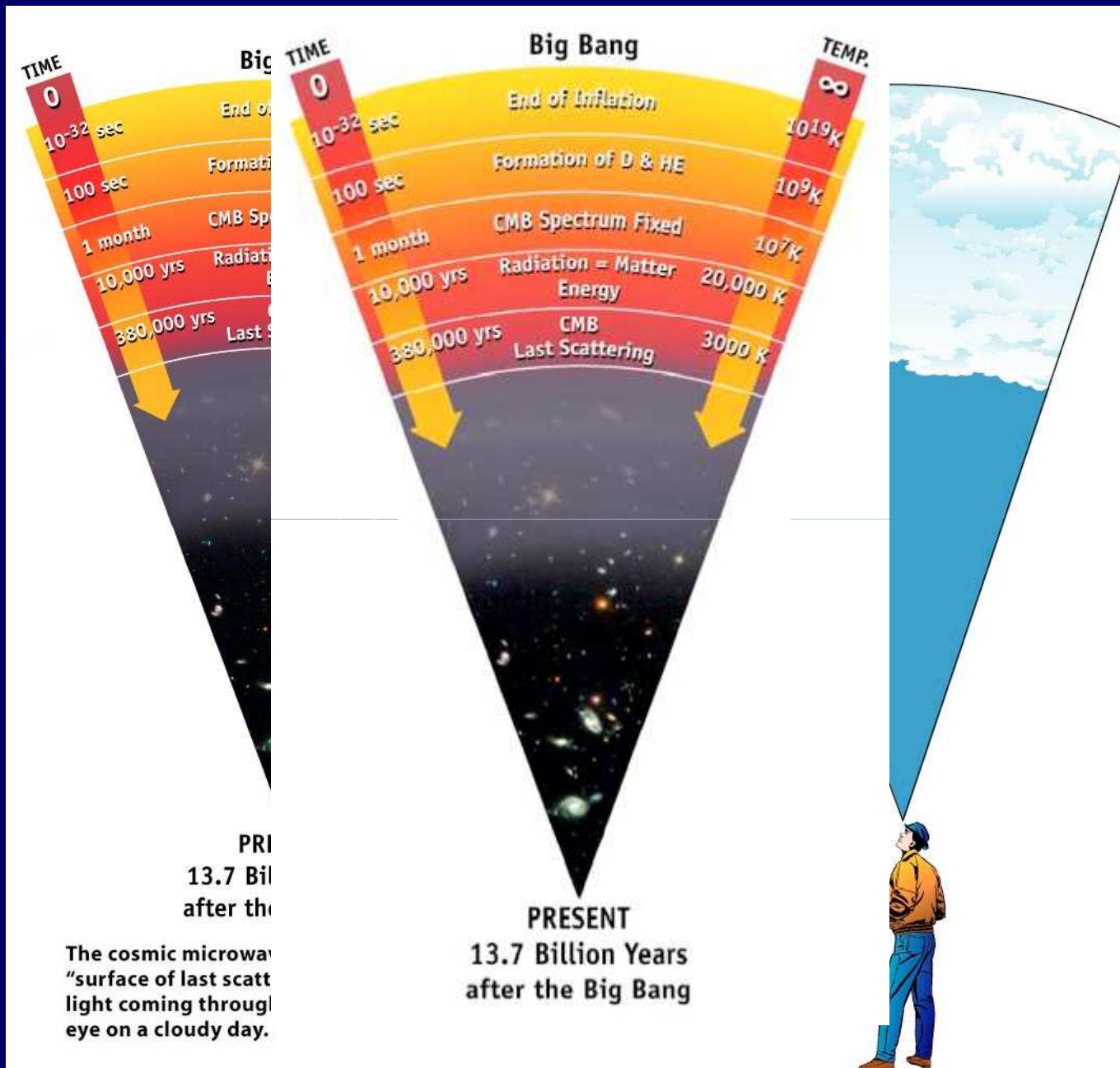
Thomson scattering

$$\sigma_T \propto m^{-2}$$

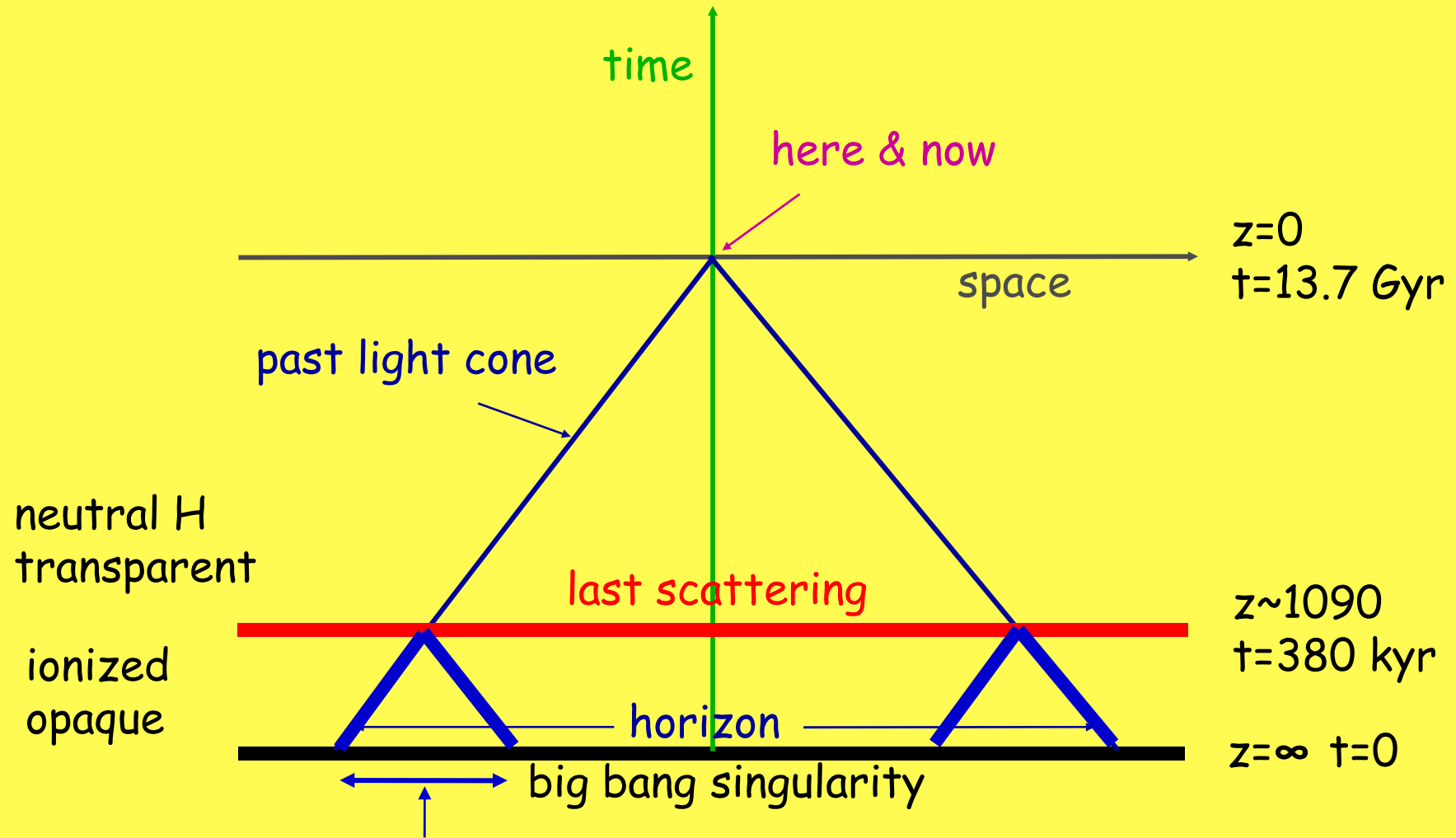


horizon at  $t_{rec}$   
 $\sim 100 \text{ comoving Mpc}$   
 $\sim 1^\circ$





# Characteristic Scale



Horizon at last scattering  $\sim 100$  comoving Mpc  $\sim$

# Acoustic Peaks

In the early hot ionized universe, photons and baryons are coupled via Thomson scattering off free electrons.

Initial fluctuations in density and curvature (quantum, Inflation) drive acoustic waves, showing as temperature fluctuations, with a characteristic scale - the sound horizon  $c_s t$ .

$$\delta T \approx \delta \rho^{1/4} \approx A(k) \cos(k c_s t)$$

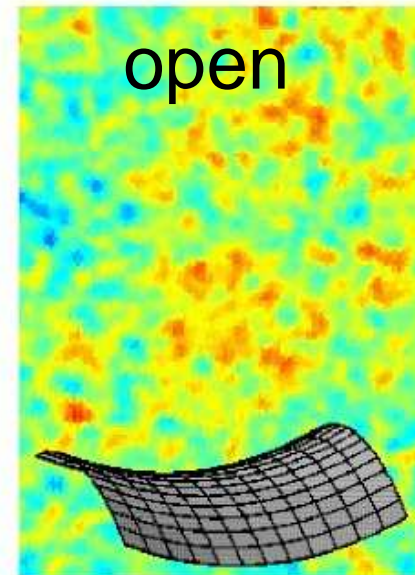
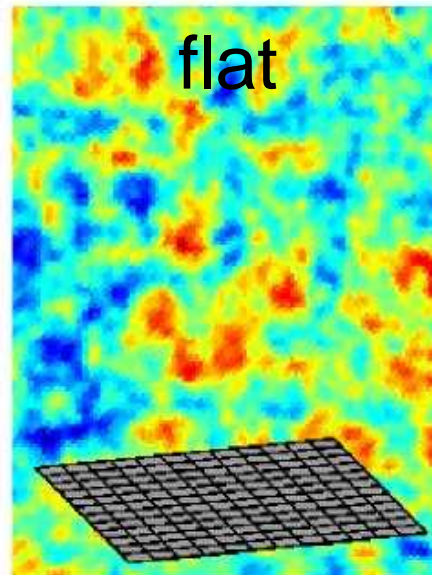
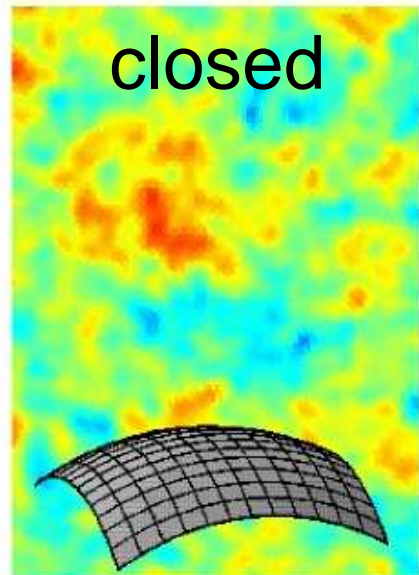
At  $z \sim 1,090$ ,  $T \sim 4,000\text{K}$ , H recombination, decoupling of photons from baryons. The CMB is a snapshot of the fluctuations at the last scattering surface.

Primary acoustic peak at  $r_{ls} \sim c t_{ls} \sim 100$  co-Mpc or  $\theta \sim 1^\circ$  ( $\ell \sim 200$ ) - the "standard ruler".

Secondary oscillations at fractional wavelengths.

# Curvature

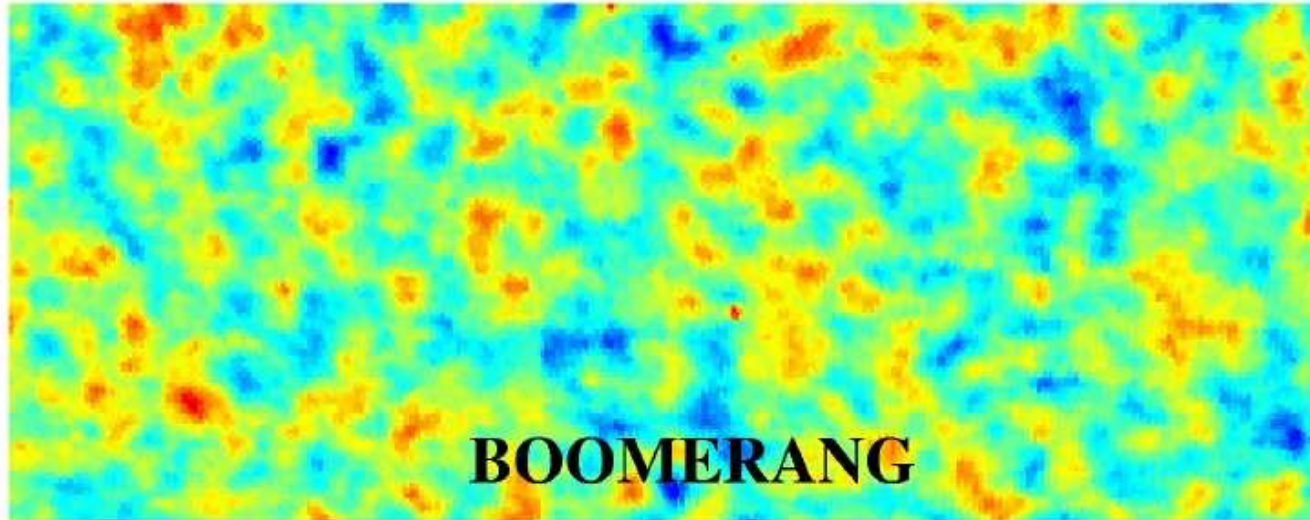
$25^\circ$



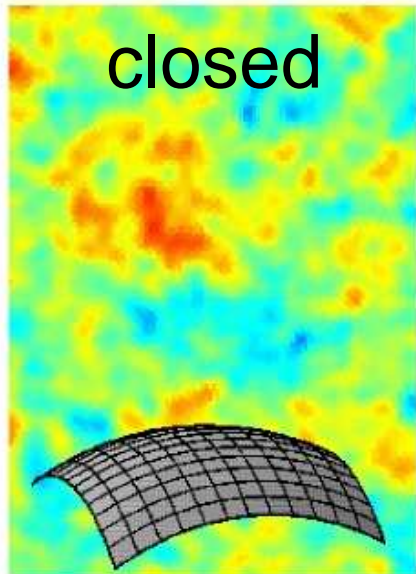


# Curvature

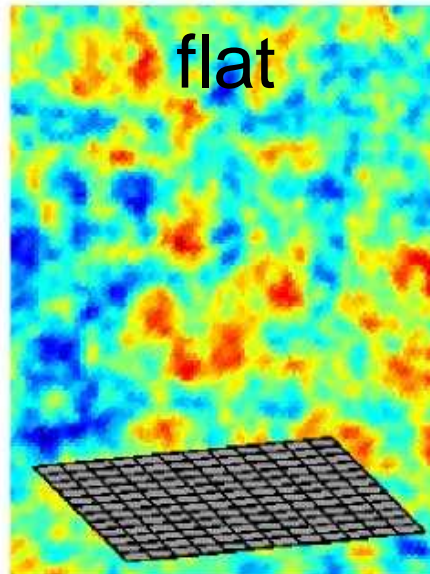
$25^\circ$



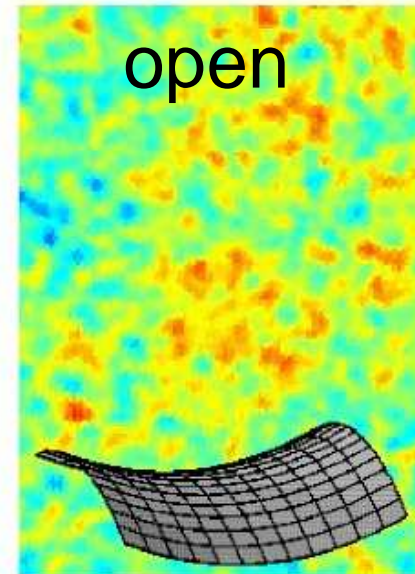
closed



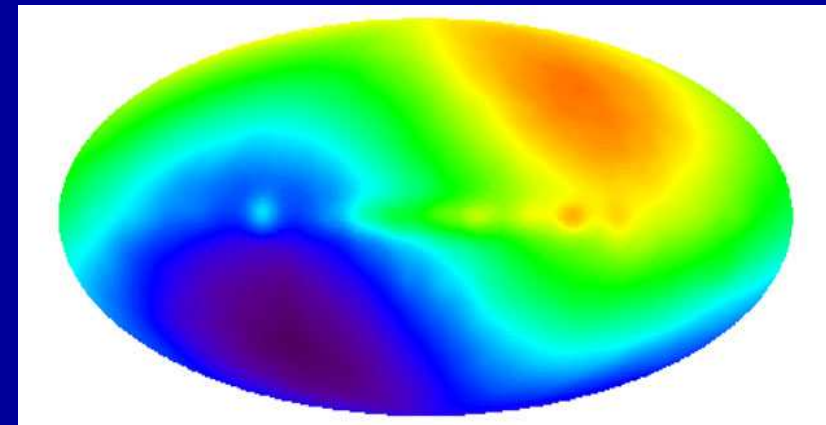
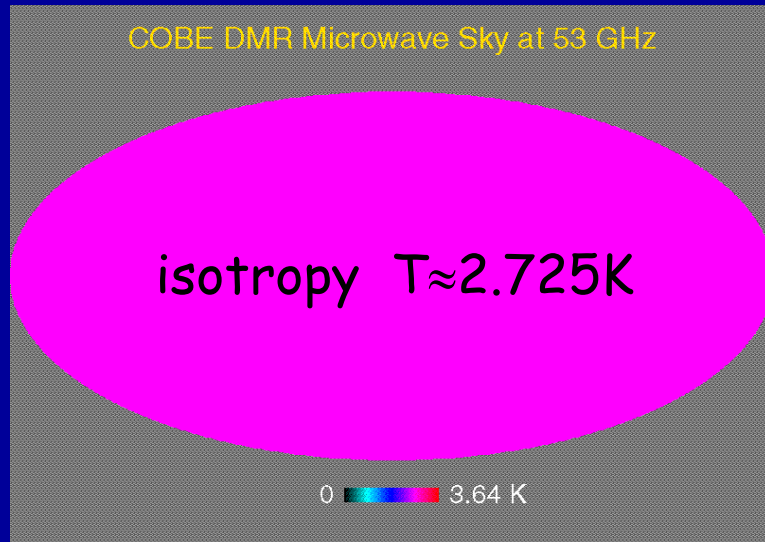
flat



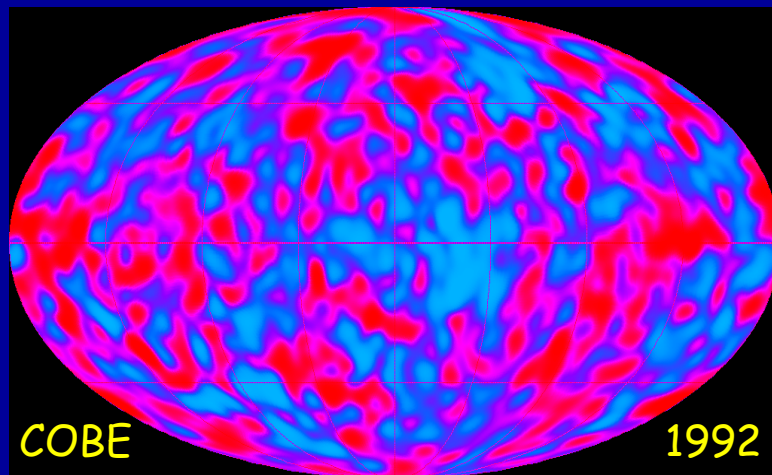
open



# CMB Temperature Maps



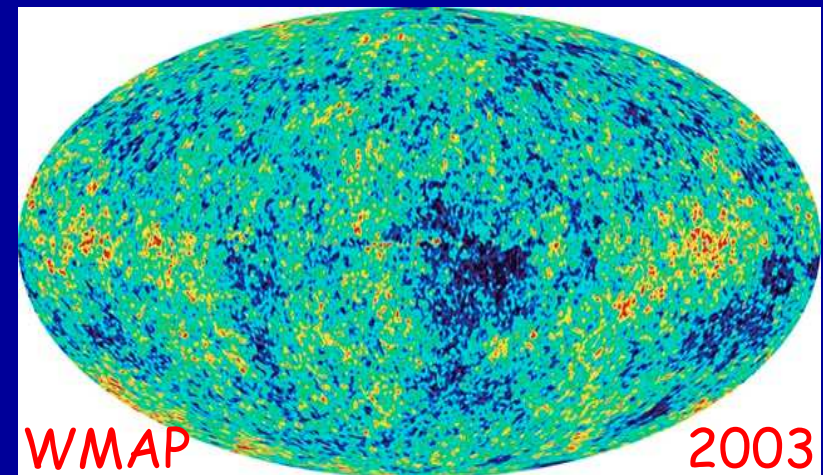
$\delta T/T \sim 10^{-3}$  dipole



COBE

1992

resolution  $\sim 10^\circ$



WMAP

2003

$\delta T/T \sim 10^{-5}$

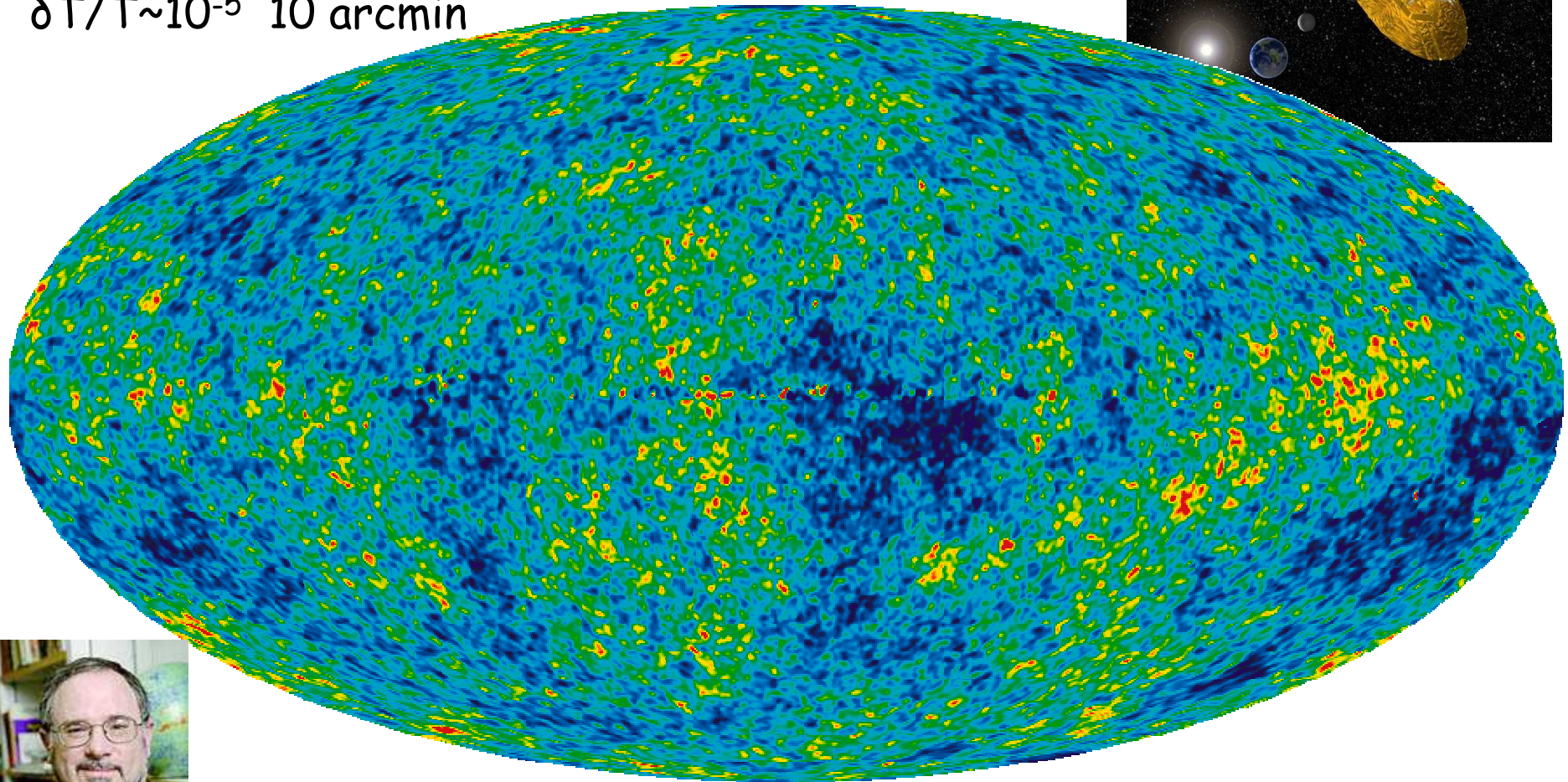
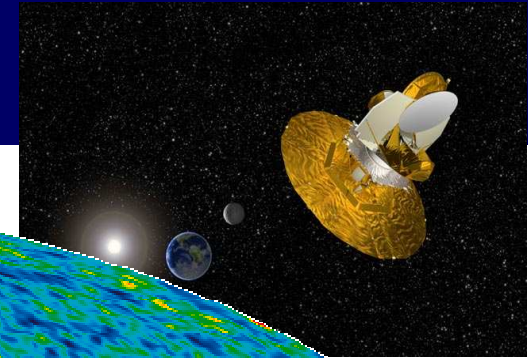
resolution  $\sim 10'$



# WMAP: Wilkinson Microwave Anisotropy Probe

WMAP-5 2008

$\delta T/T \sim 10^{-5}$  10 arcmin

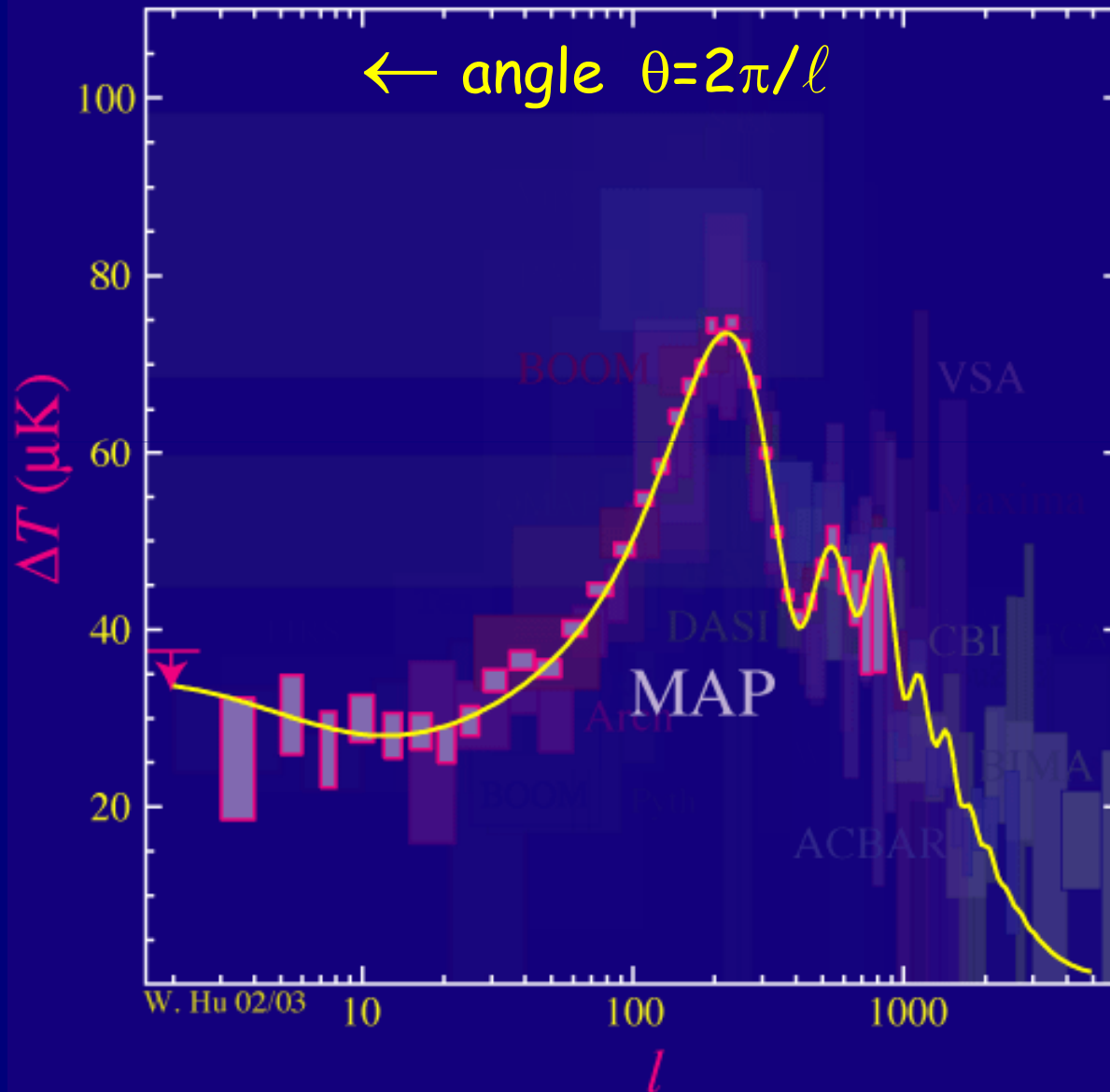


Charles  
Bennett



WMAP 5-year

# Angular Power Spectrum

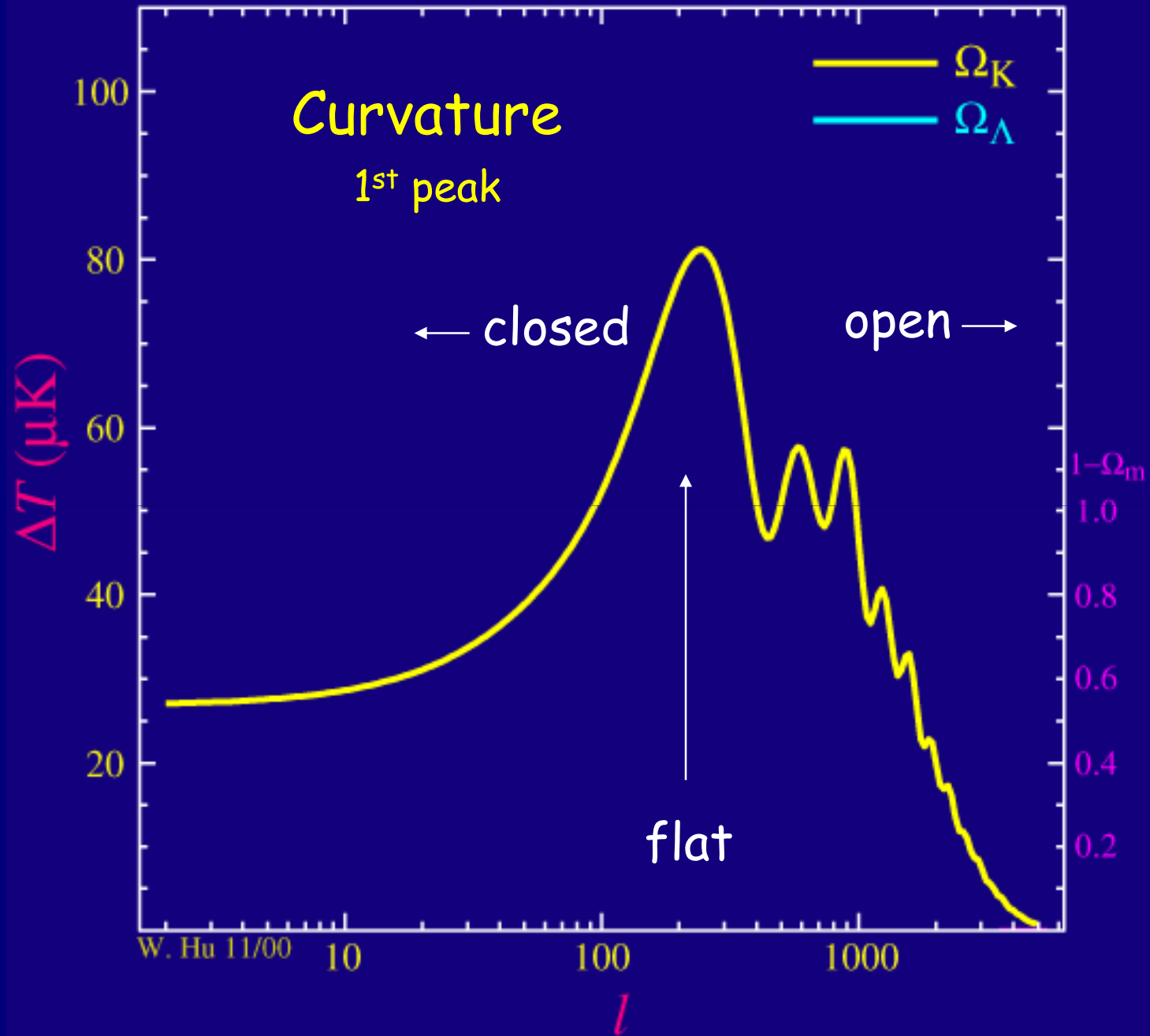


$$\frac{\Delta T(\theta, \phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta, \phi)$$

$$C_l \equiv \langle |a_{lm}|^2 \rangle$$

$$\left\langle \left( \frac{\Delta T}{T} \right)^2 \right\rangle = \frac{l(l+1)}{2\pi} C_l$$





# Curvature

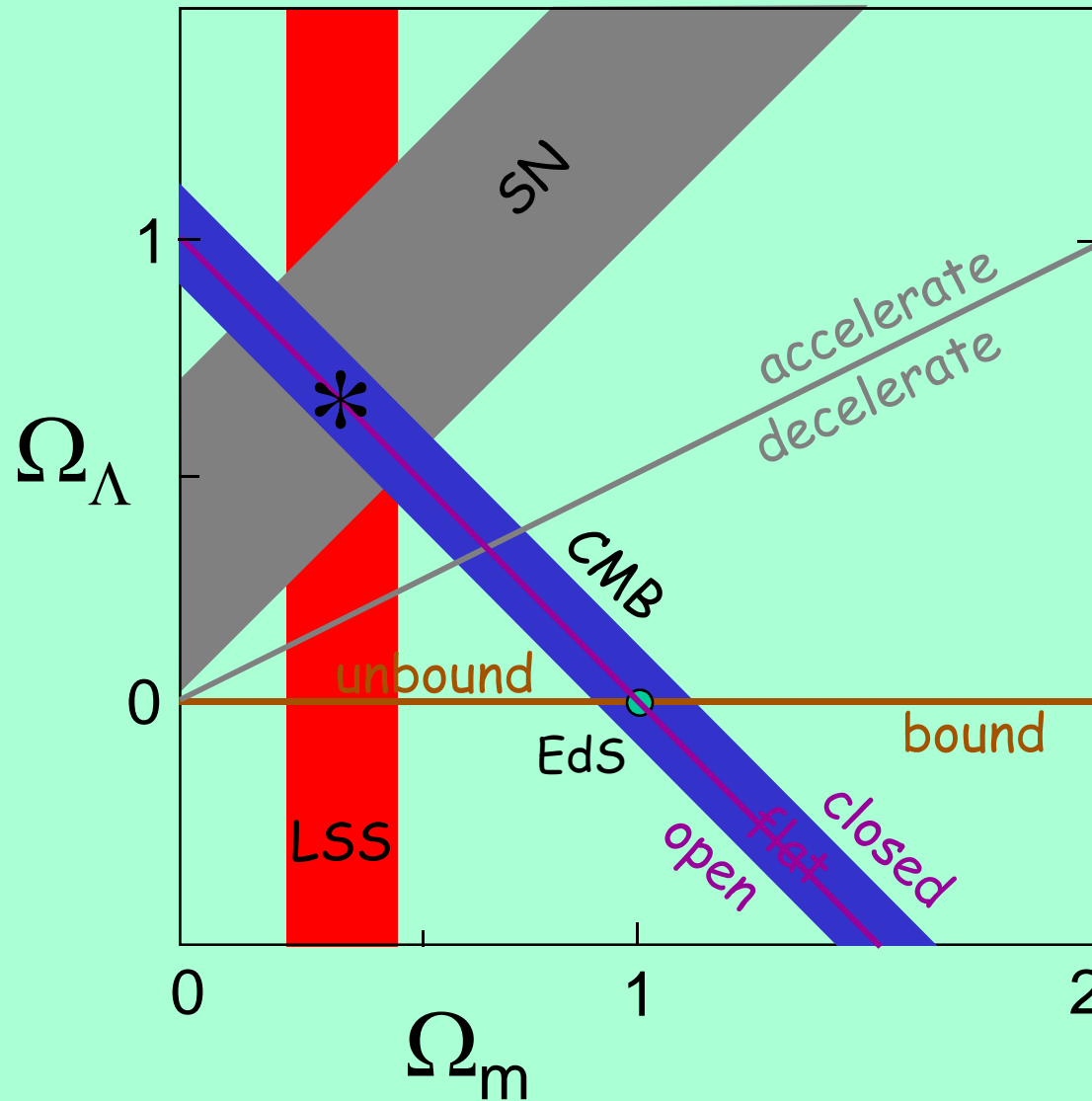
The Universe is nearly flat:

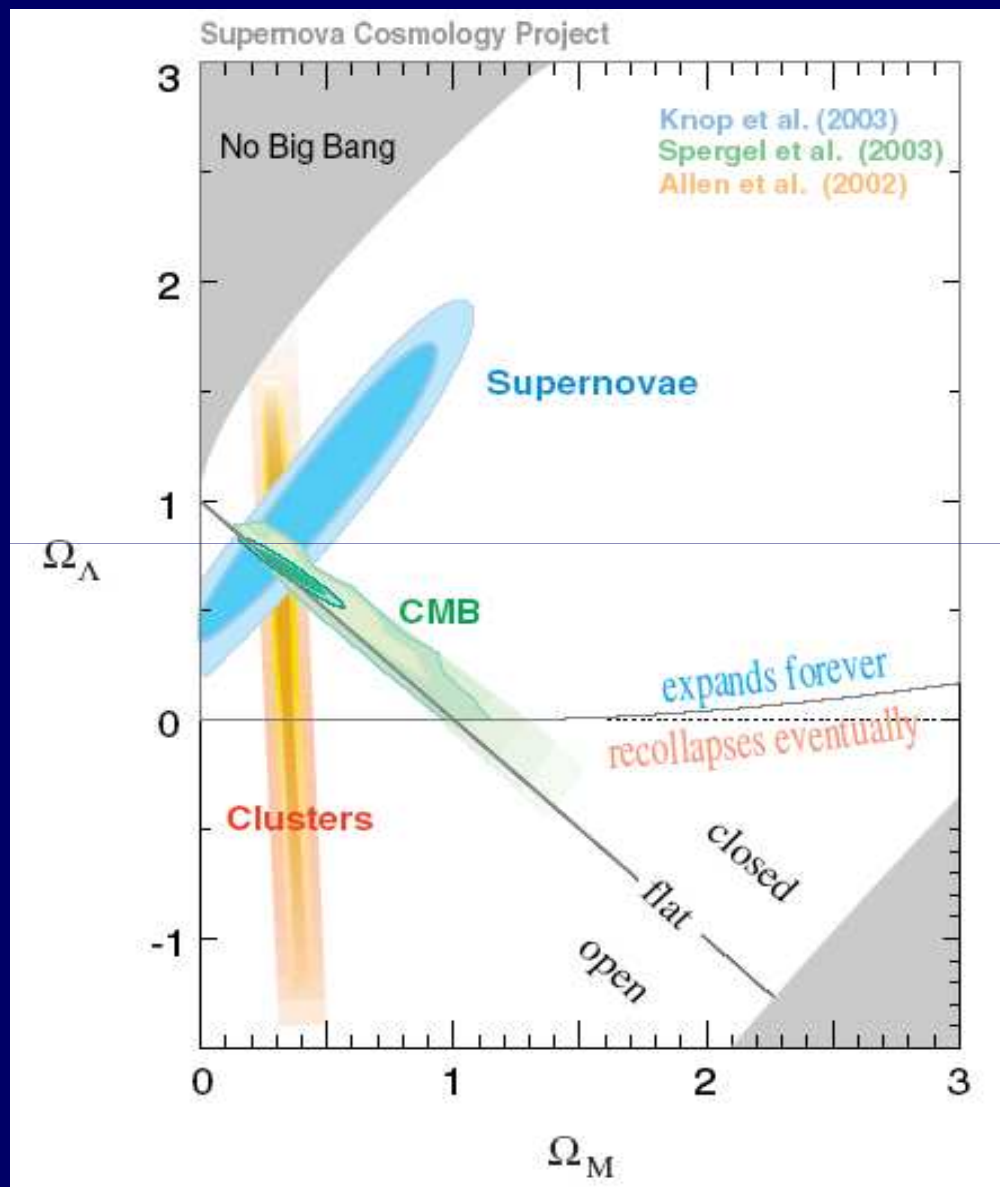
$$1 - \Omega_k = \Omega_m + \Omega_\Lambda = 1.005 \pm 0.006$$

Open? Closed?

Surely much larger than our horizon!

# Dark Matter and Dark Energy

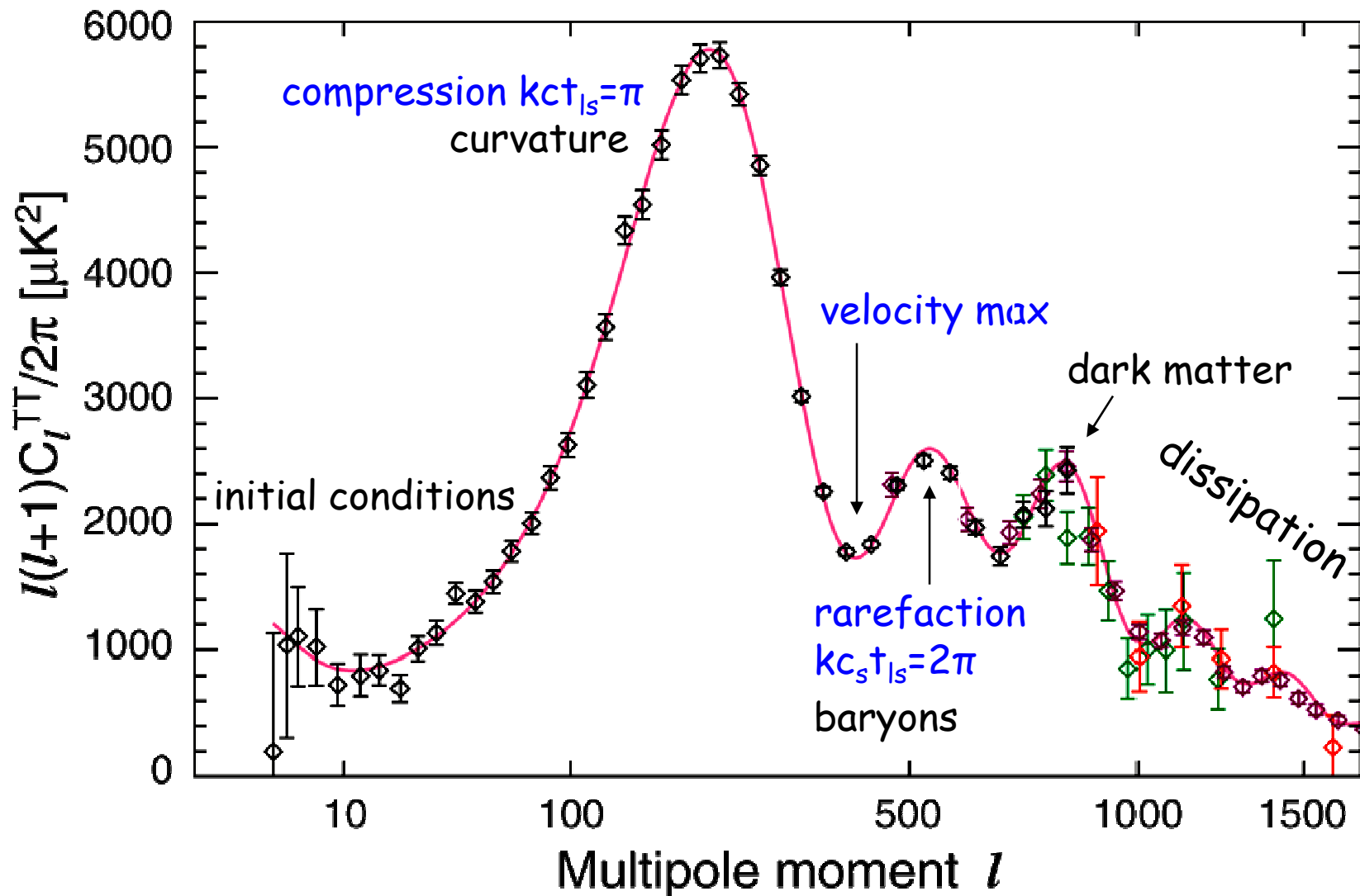




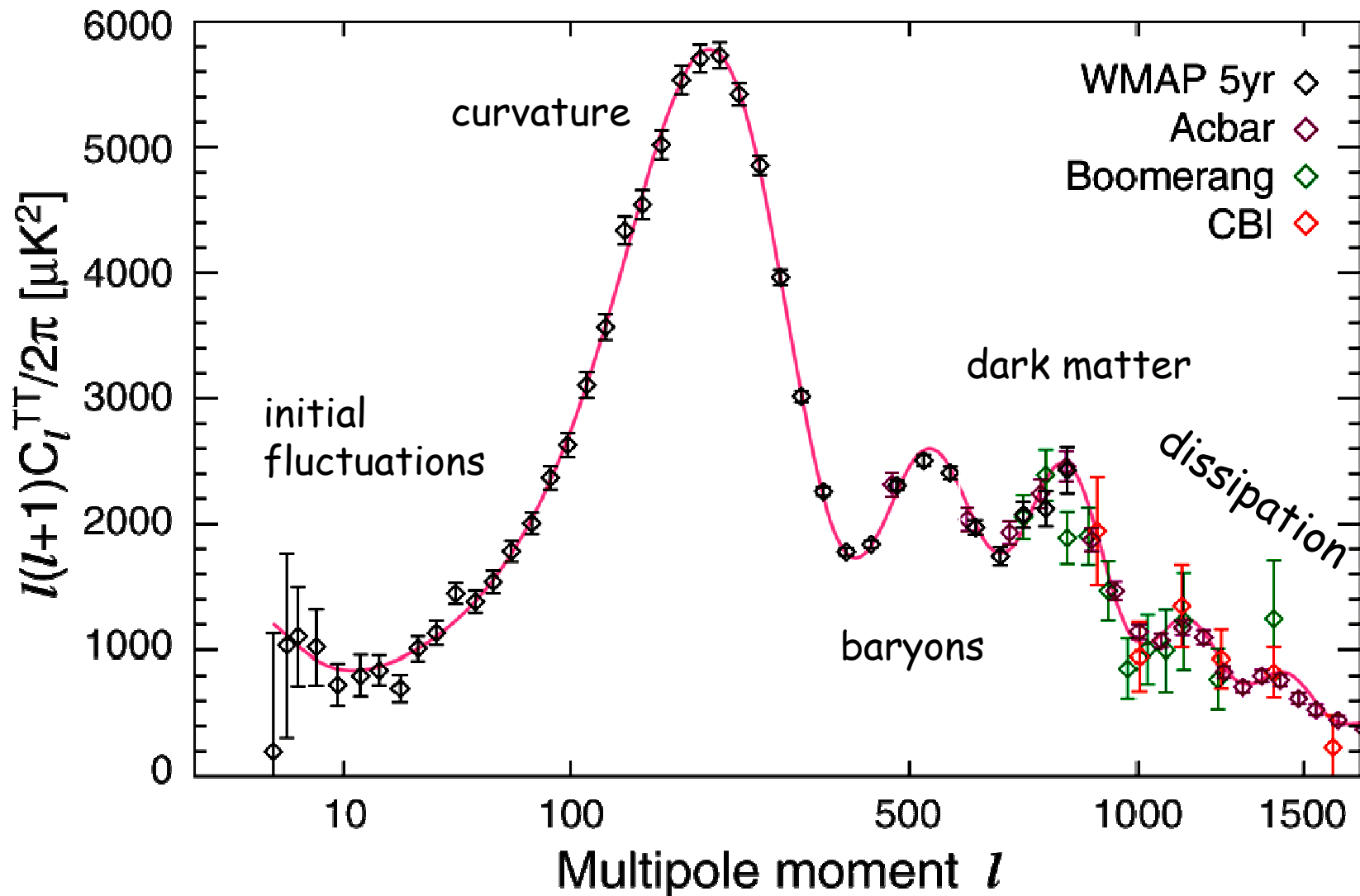


Other Parameters:  
Baryons, Fluctuations

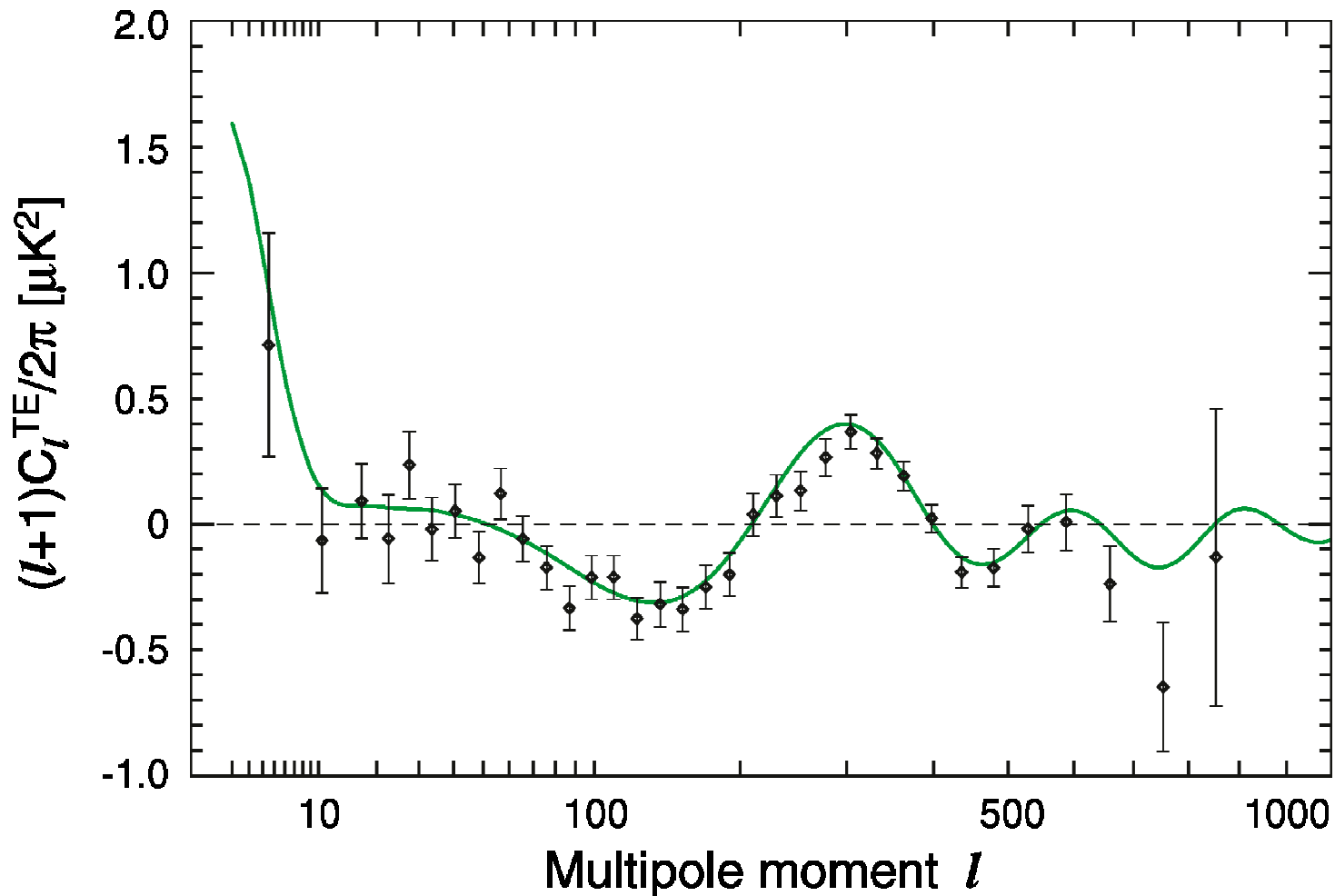
# CMB Acoustic Oscillations explore all parameters



The  $\Lambda$ CDM model is very successful  
Accurate parameter determination

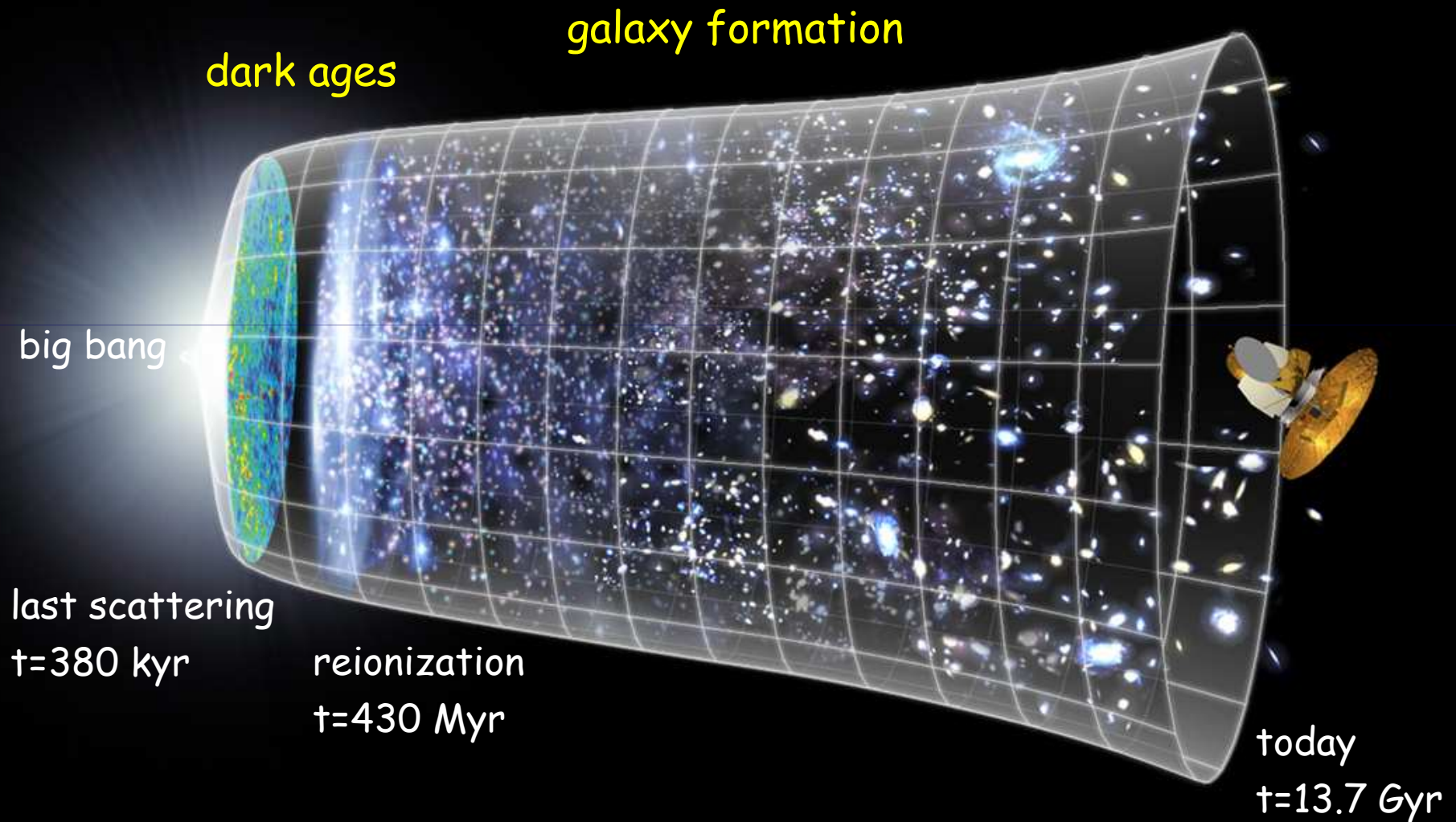


Polarization by scattering off electrons;  
re-ionization by stars & quasars at  $z \sim 10$

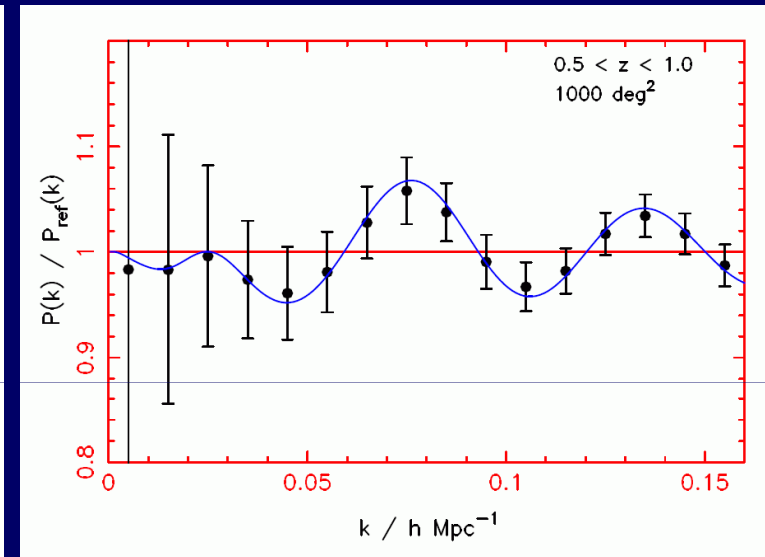
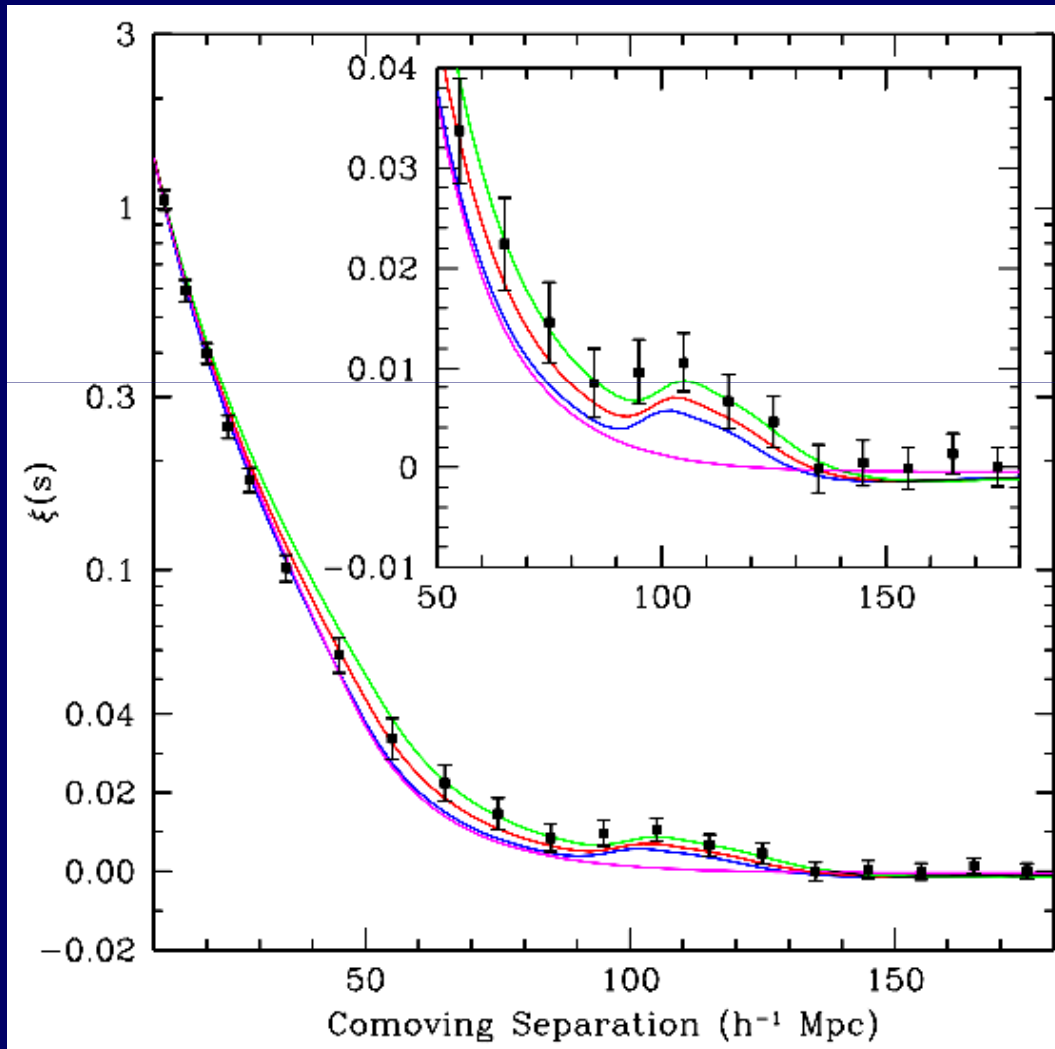




# Cosmic History

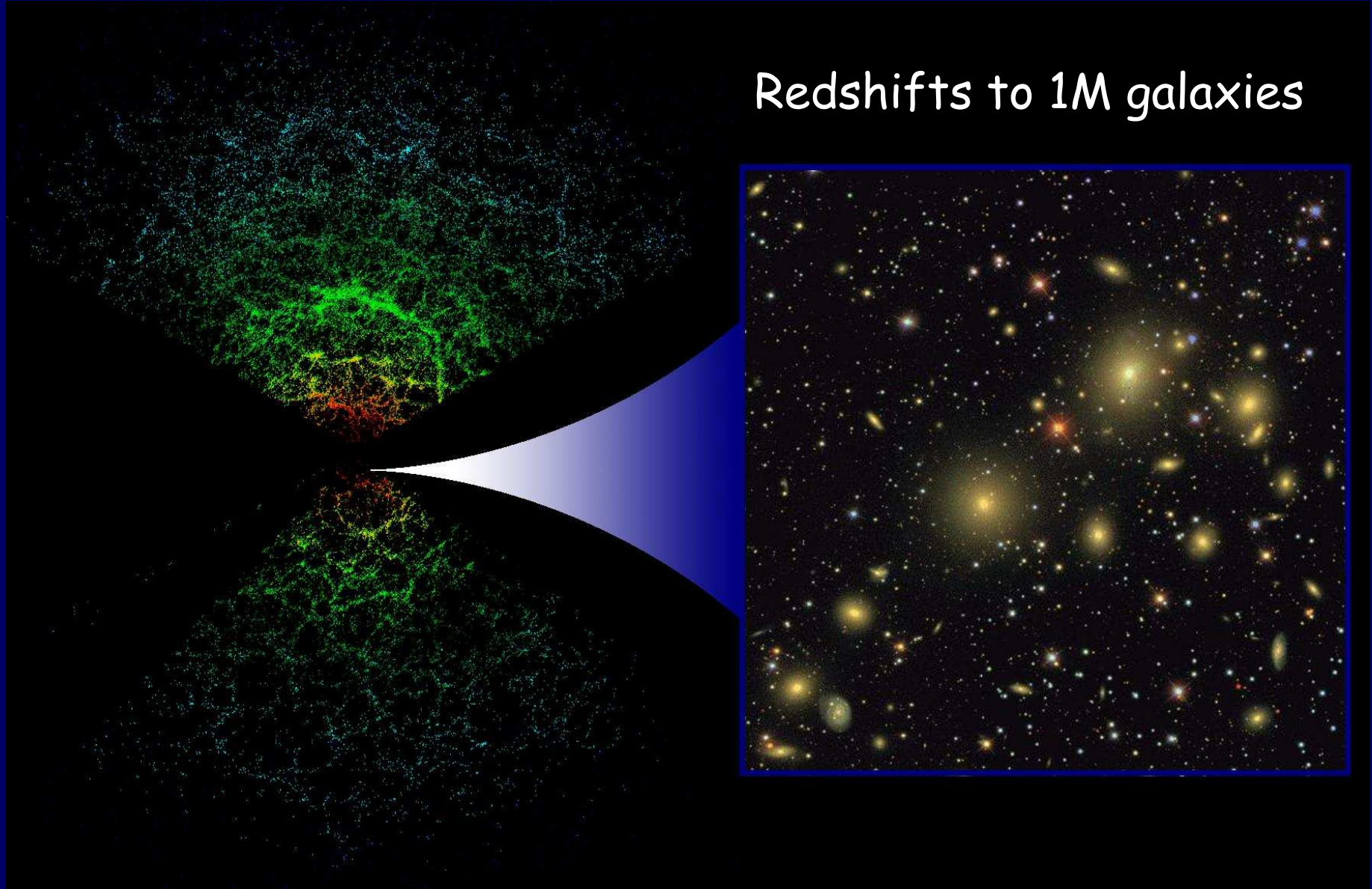


# Baryonic Acoustic Oscillations observed in the galaxy-galaxy correlation function (SDSS, $z=0.35$ )



# The Sloan Digital Sky Survey

Redshifts to 1M galaxies



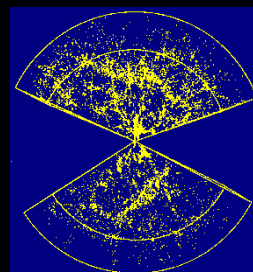


# 2dF Galaxy Redshift Survey

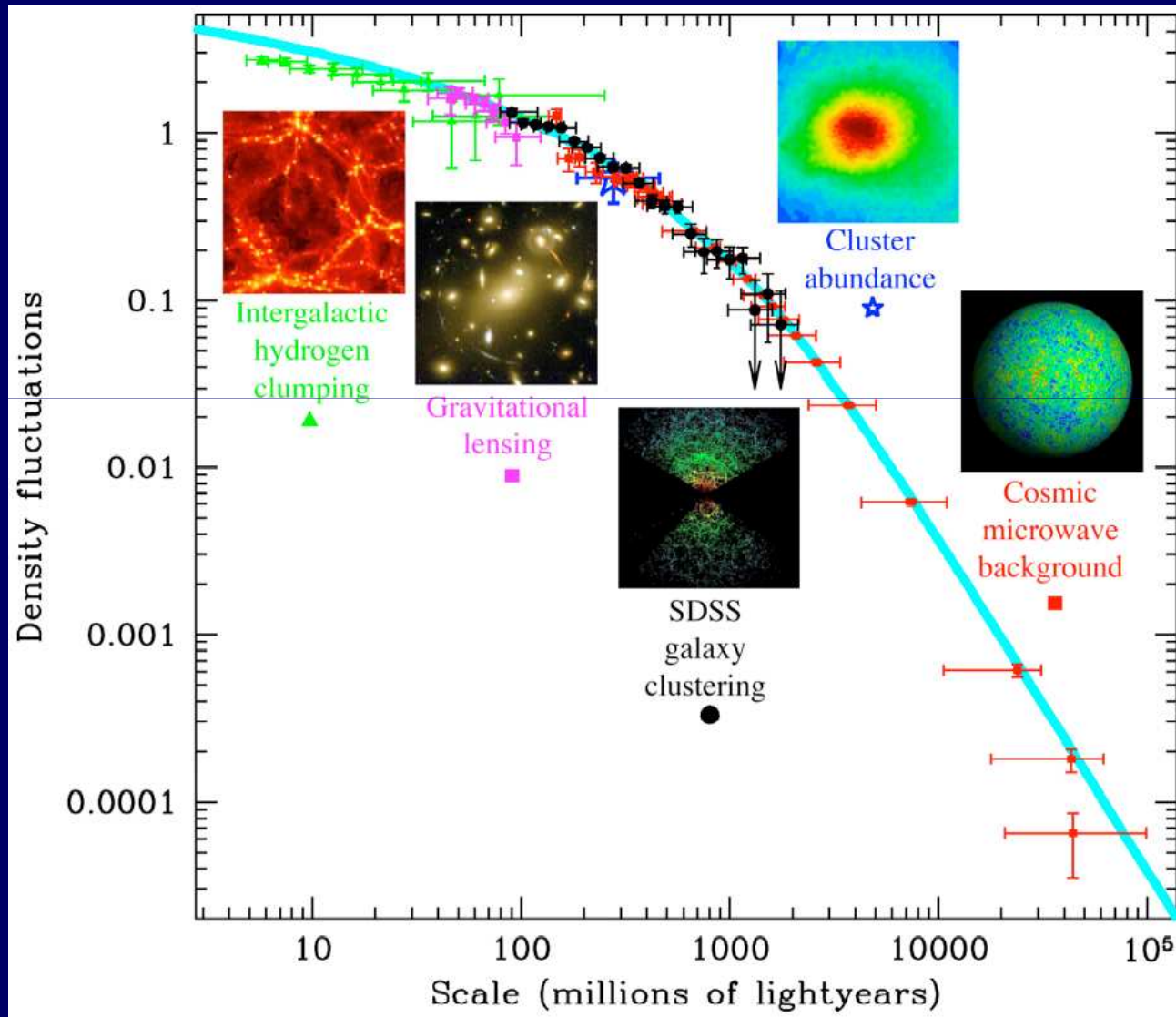
$\frac{1}{4}$  M galaxies 2003

1/4 of the horizon

CFA Survey  
1980

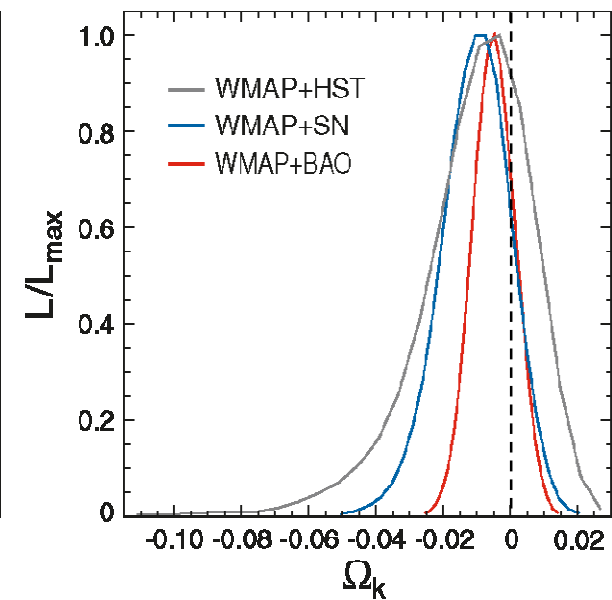
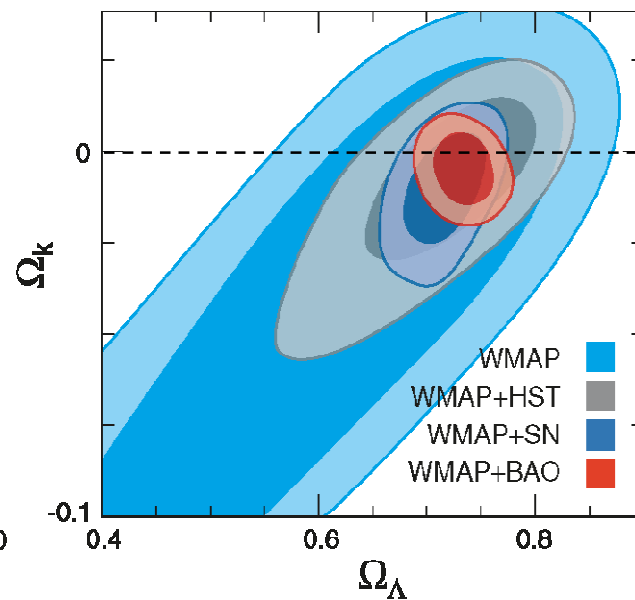
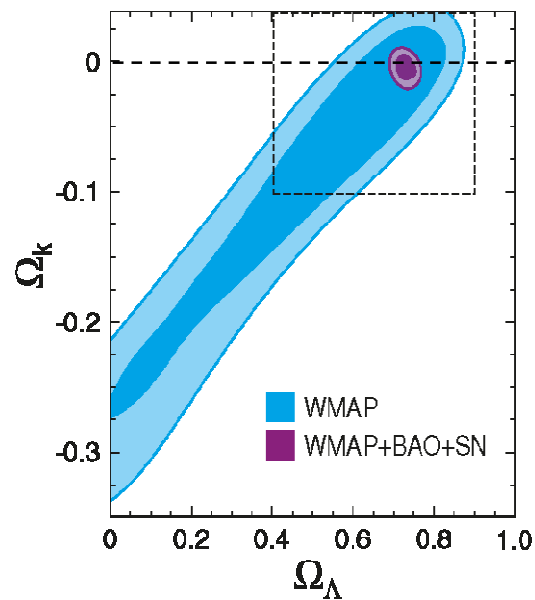


# Success of the Standard Model: Fluctuation Power Spectrum

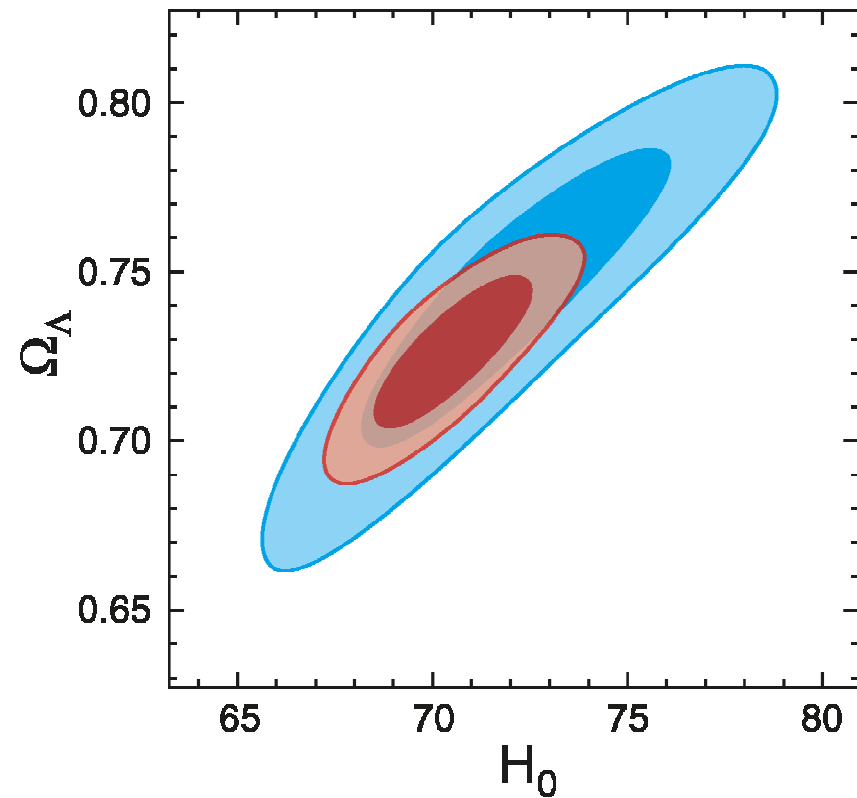
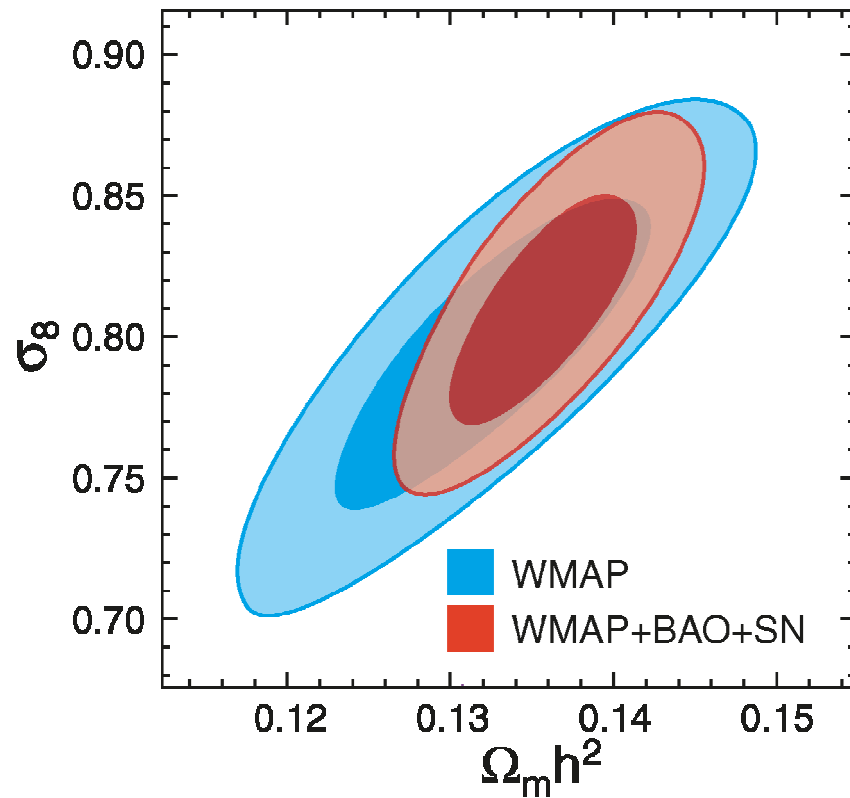




# Constraints on Curvature



# Correlated Constraints on Parameters



# Standard $\Lambda$ CDM Model Parameters

2008: WMAP5+BAO+SN

Hubble constant  $H_0 = 70.1 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$

Baryon density  $\Omega_b = 0.0462 \pm 0.0015$

Cold dark matter density  $\Omega_c = 0.233 \pm 0.013$

Dark energy density  $\Omega_\Lambda = 0.721 \pm 0.015$

Fluctuation spectral index  $n_s = 0.960 \pm 0.014$

Fluctuation amplitude  $\sigma_8 = 0.817 \pm 0.026$

Age of universe  $t_0 = 13.73 \pm 0.12 \text{ Gyr}$

Total density  $\Omega_{\text{tot}} = 1 - \Omega_k = 1.005 \pm 0.006$

# Standard $\Lambda$ CDM Model Parameters

2015: Planck (+BAO+SN)

Hubble constant	$H_0 = 67.8 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$
Total density	$\Omega_{m+\Lambda} = 1.000 \pm 0.005$
Dark energy density	$\Omega_{\Lambda} = 0.692 \pm 0.012$
Mass density	$\Omega_m = 0.308 \pm 0.012$
Baryon density	$\Omega_b = 0.0478 \pm 0.0004$
Fluctuation spectral index	$n_s = 0.968 \pm 0.006$
Fluctuation amplitude	$\sigma_8 = 0.830 \pm 0.015$
Optical depth	$\tau = 0.066 \pm 0.016$
Age of universe	$t_0 = 13.80 \pm 0.02 \text{ Gyr}$



# Parameters for Standard Model

		WMAP5	WMAP+BAO+SN
Age of universe	$t_0$	$13.69 \pm 0.13$ Gyr	$13.73 \pm 0.12$ Gyr
Hubble constant	$H_0$	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	$70.1 \pm 1.3$ km/s/Mpc
Baryon density	$\Omega_b$	$0.0441 \pm 0.0030$	$0.0462 \pm 0.0015$
Physical baryon density	$\Omega_b h^2$	$0.02273 \pm 0.00062$	$0.02265 \pm 0.00059$
Dark matter density	$\Omega_c$	$0.214 \pm 0.027$	$0.233 \pm 0.013$
Physical dark matter density	$\Omega_c h^2$	$0.1099 \pm 0.0062$	$0.1143 \pm 0.0034$
Dark energy density	$\Omega_\Lambda$	$0.742 \pm 0.030$	$0.721 \pm 0.015$
Curvature fluctuation amplitude, $k_0 = 0.002 \text{ Mpc}^{-1}$ <sup>b</sup>	$\Delta_{\mathcal{R}}^2$	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	$\sigma_8$	$0.796 \pm 0.036$	$0.817 \pm 0.026$
$l(l+1)C_{220}^{TT}/2\pi$	$C_{220}$	$5756 \pm 42 \mu\text{K}^2$	$5748 \pm 41 \mu\text{K}^2$
Scalar spectral index	$n_s$	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
Redshift of matter-radiation equality	$z_{\text{eq}}$	$3176^{+151}_{-150}$	$3280^{+88}_{-89}$
Angular diameter distance to matter-radiation eq. <sup>c</sup>	$d_A(z_{\text{eq}})$	$14279^{+186}_{-189}$ Mpc	$14172^{+141}_{-139}$ Mpc
Redshift of decoupling	$z_*$	$1090.51 \pm 0.95$	$1091.00^{+0.72}_{-0.73}$
Age at decoupling	$t_*$	$380081^{+5843}_{-5841}$ yr	$375938^{+3148}_{-3115}$ yr
Angular diameter distance to decoupling <sup>c,d</sup>	$d_A(z_*)$	$14115^{+188}_{-191}$ Mpc	$14006^{+142}_{-141}$ Mpc
Sound horizon at decoupling <sup>d</sup>	$r_s(z_*)$	$146.8 \pm 1.8$ Mpc	$145.6 \pm 1.2$ Mpc
Acoustic scale at decoupling <sup>d</sup>	$l_A(z_*)$	$302.08^{+0.83}_{-0.84}$	$302.11^{+0.84}_{-0.82}$
Reionization optical depth	$\tau$	$0.087 \pm 0.017$	$0.084 \pm 0.016$
Redshift of reionization	$z_{\text{reion}}$	$11.0 \pm 1.4$	$10.8 \pm 1.4$
Age at reionization	$t_{\text{reion}}$	$427^{+88}_{-65}$ Myr	$432^{+90}_{-67}$ Myr

# Beyond the Standard Model

# What is the Dark Energy?

*“ ‘Most embarrassing observation in physics’ – that’s the only quick thing I can say about dark energy that’s also true.”*

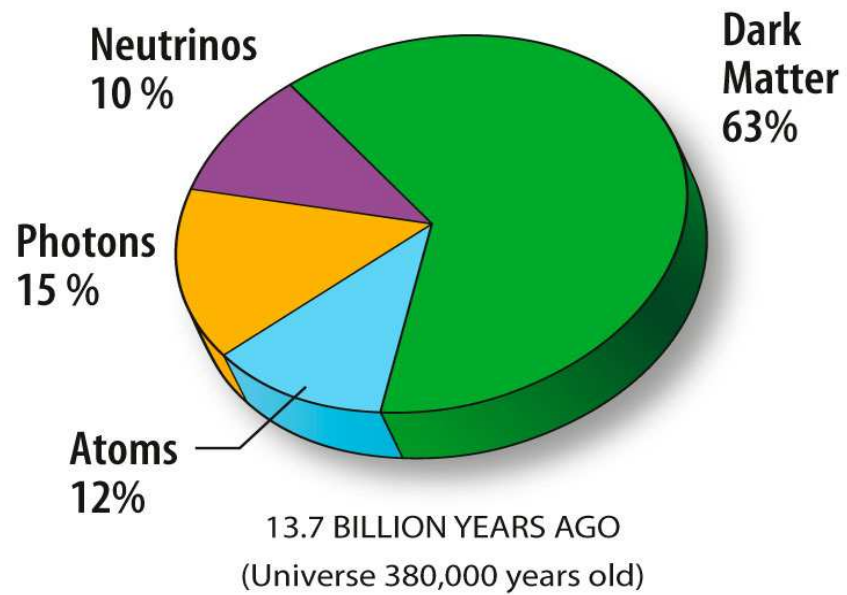
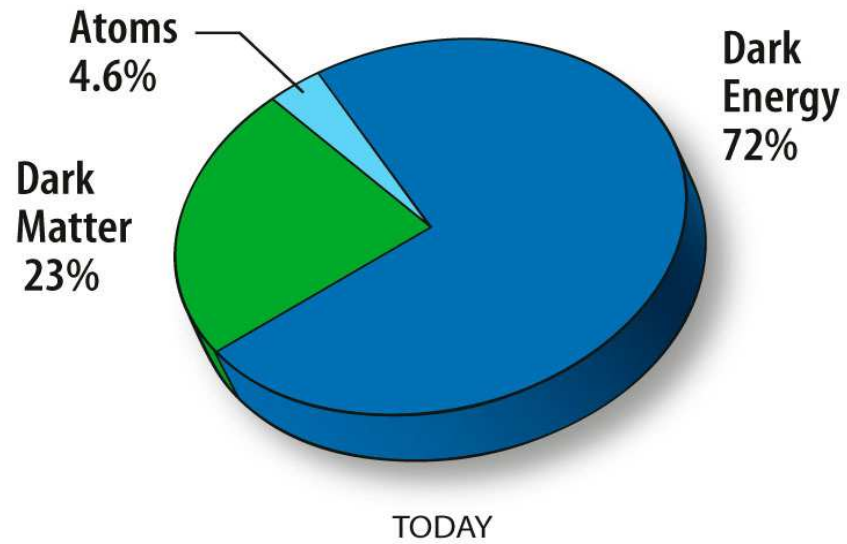
Edward Witten

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

or

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu} + \rho_{\text{vacuum}} g_{\mu\nu})$$

- Cosmological constant in GR?
- Failure of GR? Quintessence? Novel property of matter?
- Why so small? in cosmology  $10^{-48} \text{ GeV}^4$   
vs QFT:  $10^8 \text{ GeV}^4$  (ElectroWeak) or  $10^{72} \text{ GeV}^4$  (Planck)
- Why becoming dominant now? (Anthropic principle?)



# Generalized Dark Energy

Cosmological constant

$$\rho_{tot} = \rho_{\Lambda} = \text{const.}$$

Energy conservation during expansion

$$d(\rho c^2 a^3) = -p d(a^3)$$

→ Equation of state

$$p = -\rho c^2$$

Generalized eq. of state  
e.g. Quintessence

$$p \equiv w \rho c^2$$

$$w(x, t)?$$

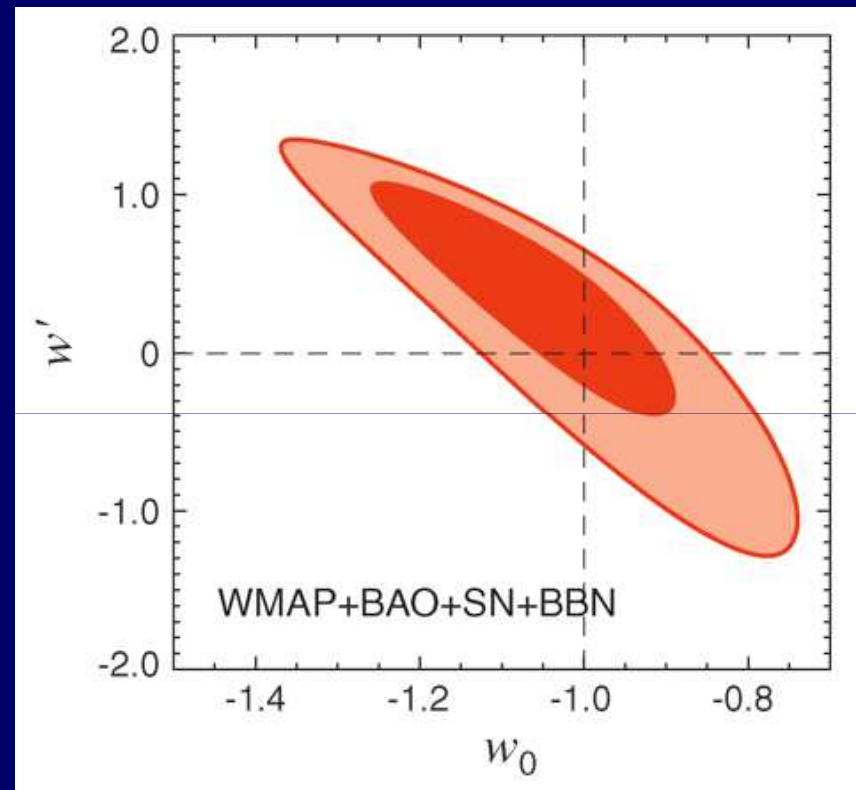
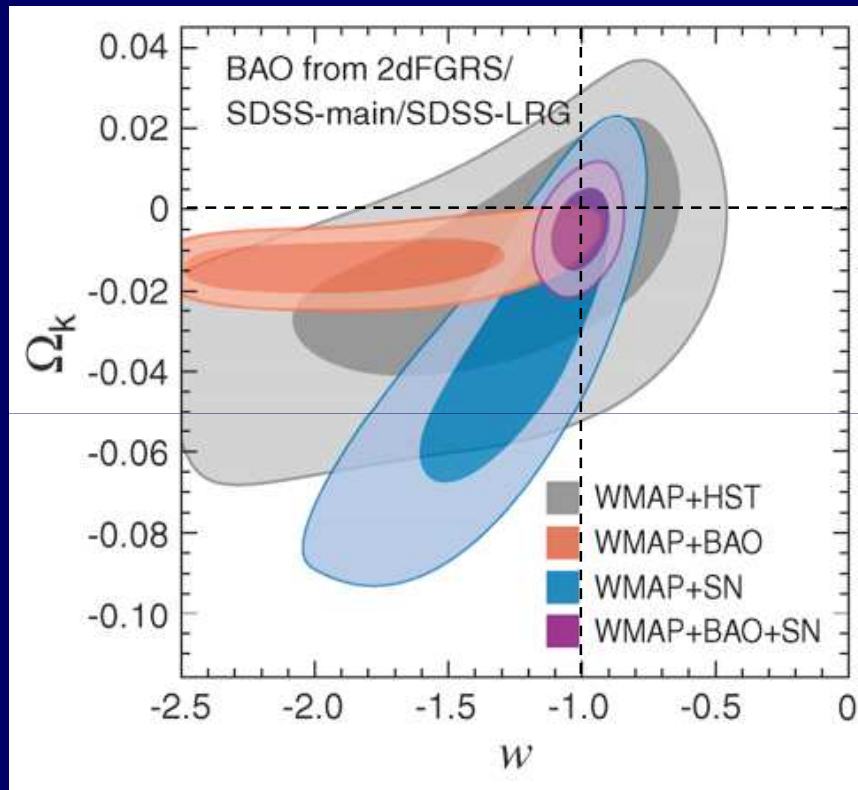
$$\ddot{a} > 0 \quad \leftrightarrow \quad w < -1/3$$

$$\Lambda \quad \leftrightarrow \quad w = -1$$

Friedman eq. and the fluctuation growth-rate eq. probe different parts of the theory and can constrain  $w(t)$



# Equation of State and its Time Variation



Very close to standard GR  
with a cosmological constant

# Beyond the Standard $\Lambda$ CDM Model

2008: WMAP5+BAO+SN

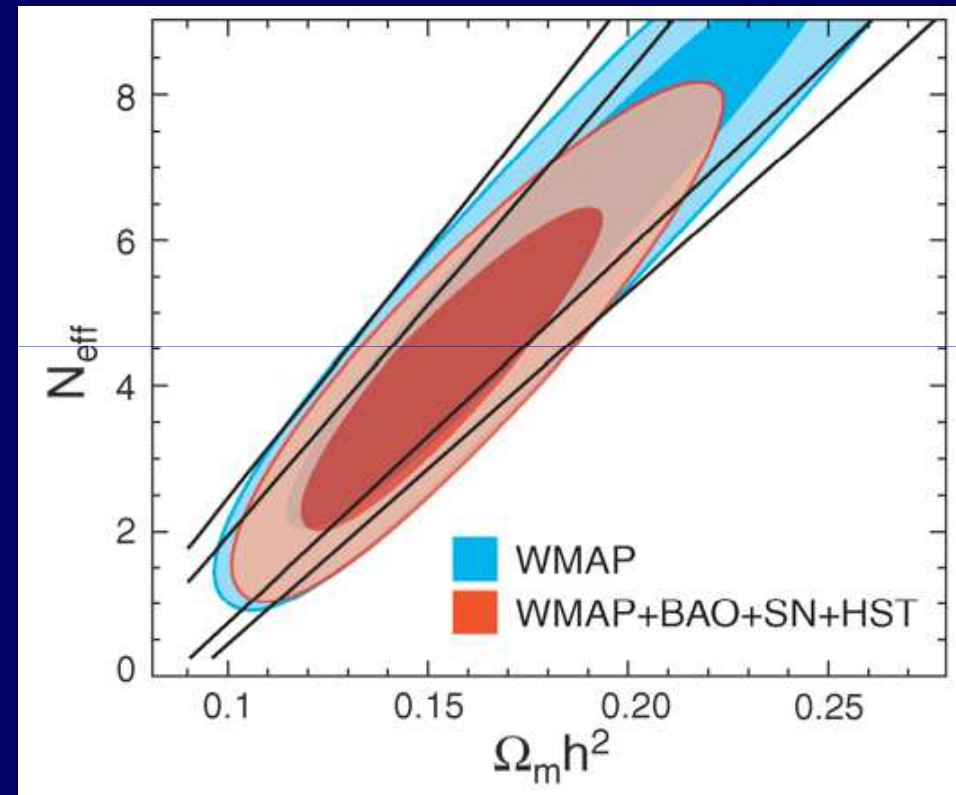
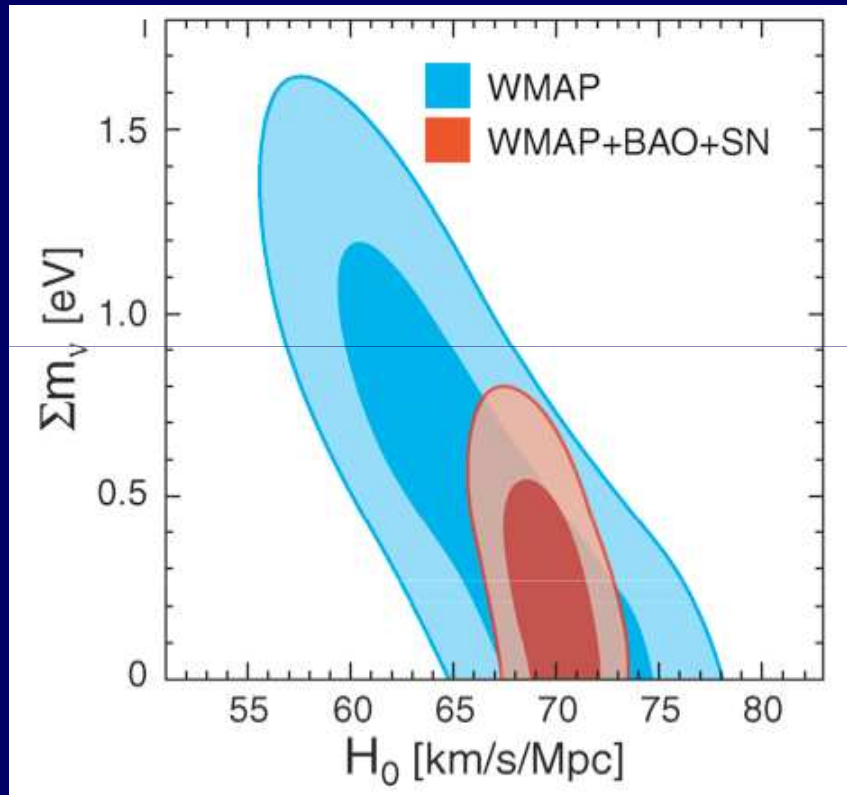
Total density	$\Omega_{\text{tot}} = 1 - \Omega_k = 1.005 \pm 0.006$
Equation of state	$w = -0.97 \pm 0.06$
Tensor/scalar fluctuations	$r < 0.20$ (95% CL)
Running of spectral index	$dn/d\ln k = -0.03 \pm 0.02$
Neutrino mass	$\sum m_\nu < 0.61$ eV (95% CL)
# of light neutrino families	$N_{\text{eff}} = 4.4 \pm 1.5$

# Beyond the Standard $\Lambda$ CDM Model

2015: Planck (+BAO+SN)

Total density	$\Omega_{\text{tot}} = 1 - \Omega_k = 1.001 \pm 0.004$
Equation of state	$w = -1.006 \pm 0.045$
Tensor/scalar fluctuations	$r < 0.11$ (95% CL)
Running of spectral index	$dn/d\ln k = -0.03 \pm 0.02$
Neutrino mass	$\sum m_\nu < 0.23$ eV (95% CL)
# of light neutrino families	$N_{\text{eff}} = 3.15 \pm 0.23$

# Neutrino Mass and # of Families



# Open Questions

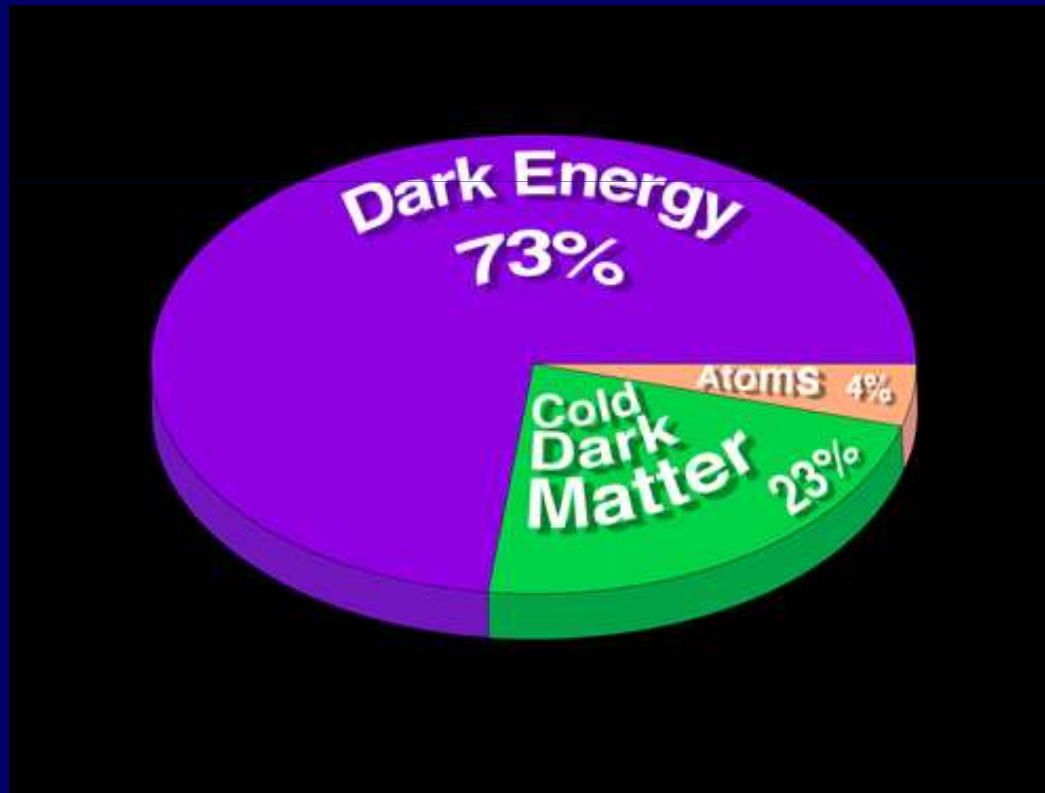
- The Big Bang? Inflation?
- What is the dark energy?
- What is the dark matter particle?
- How do galaxies form from the cosmic web?
- How do stars form in galaxies



# Conclusions

- Cosmology has a Standard Model:  $\Lambda$ CDM
- The basic parameters are accurately measured using multiple techniques
- Mysteries: dark matter, dark energy, big-bang, inflation
- Next step: probe physics beyond the standard model
- Current effort: galaxy formation

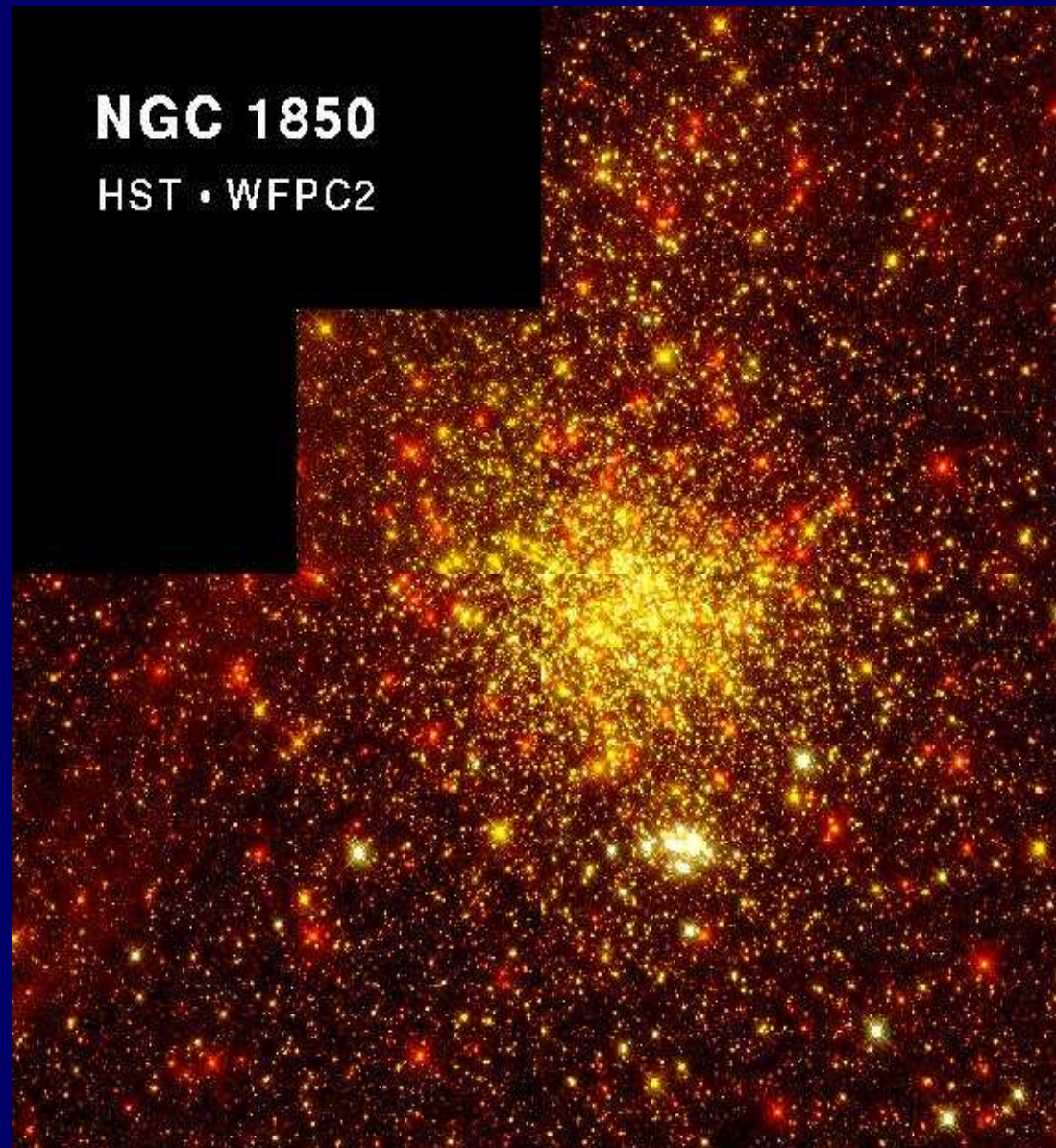
# Thank You



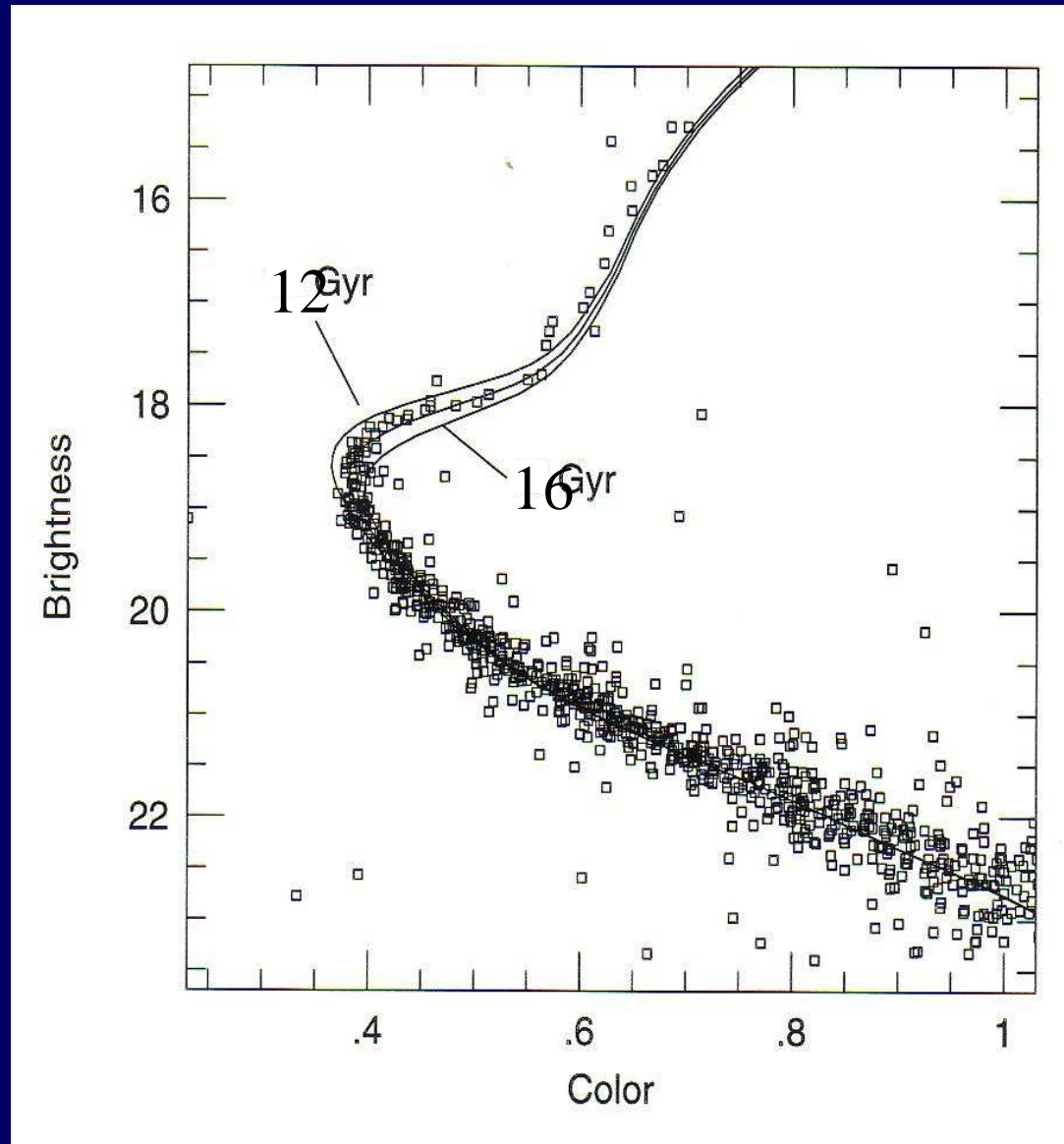
# Parameters for Extended Models

		WMAP5	WMAP+BAO+SN
Total density <sup>f</sup>	$\Omega_{\text{tot}}$	$1.099^{+0.100}_{-0.085}$	$1.0052 \pm 0.0064$
Equation of state <sup>g</sup>	$w$	$-1.06^{+0.41}_{-0.42}$	$-0.972^{+0.061}_{-0.060}$
Tensor to scalar ratio, $k_0 = 0.002 \text{ Mpc}^{-1}$ <sup>b,h</sup>	$r$	$< 0.43$ (95% CL)	$< 0.20$ (95% CL)
Running of spectral index, $k_0 = 0.002 \text{ Mpc}^{-1}$ <sup>b,i</sup>	$dn_s/d \ln k$	$-0.037 \pm 0.028$	$-0.032^{+0.021}_{-0.020}$
Neutrino density <sup>j</sup>	$\Omega_\nu h^2$	$< 0.014$ (95% CL)	$< 0.0065$ (95% CL)
Neutrino mass <sup>j</sup>	$\sum m_\nu$	$< 1.3 \text{ eV}$ (95% CL)	$< 0.61 \text{ eV}$ (95% CL)
Number of light neutrino families <sup>k</sup>	$N_{\text{eff}}$	$> 2.3$ (95% CL)	$4.4 \pm 1.5$

# Age of the oldest globular star clusters



# Age of an old star cluster





# The Age of the Universe

