Standard ΛCDM Model Parameters

2015: Planck (+BAO+SN)

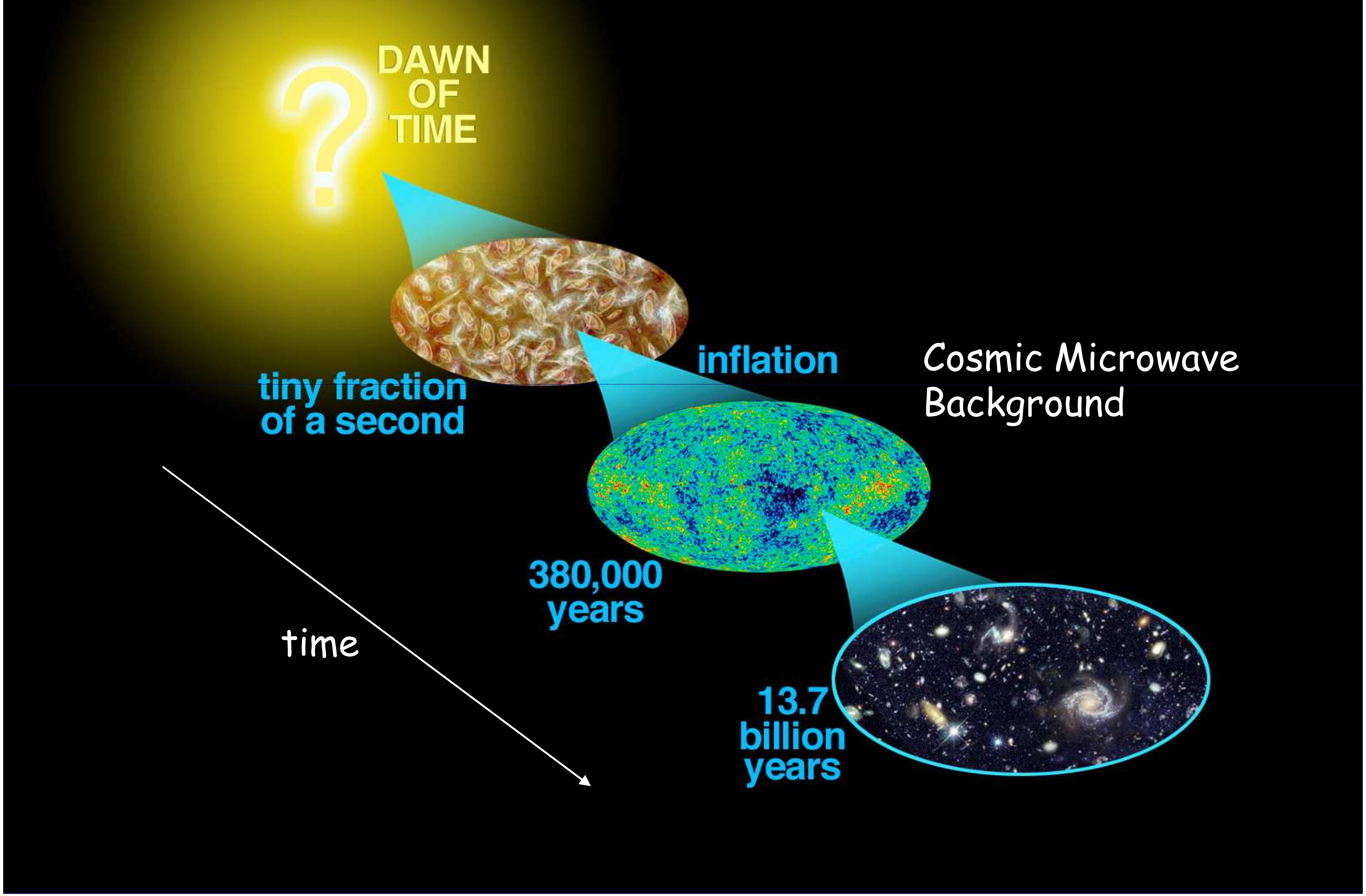
Hubble constant	$H_0 = 67.8 \pm 0.9$ km s^{-1} Mpc^{-1}
Total density	$\Omega_{m+\Lambda} = 1.000 \pm 0.005$
Dark energy density	$\Omega_\Lambda = 0.692 \pm 0.012$
Mass density	$\Omega_m = 0.308 \pm 0.012$
Baryon density	$\Omega_b = 0.0478 \pm 0.0004$
Fluctuation spectral index	$n_s = 0.968 \pm 0.006$
Fluctuation amplitude	$\sigma_8 = 0.830 \pm 0.015$
Optical depth	$\tau = 0.066 \pm 0.016$
Age of universe	$t_0 = 13.80 \pm 0.02$ Gyr

Beyond the Standard ΛCDM Model

2015: Planck (+BAO+SN)

Total density	$\Omega_{tot} = 1-\Omega_k = 1.001 \pm 0.004$
Equation of state	$w = -1.006 \pm 0.045$
Tensor/scaler fluctuations	$r < 0.11$ (95% CL)
Running of spectral index	$dn/d\ln k = -0.03 \pm 0.02$
Neutrino mass	$\sum m_\nu < 0.23$ eV (95% CL)
# of light neutrino families	$N_{eff}=3.15 \pm 0.23$

CMB


DAWN
OF
TIME
tiny fraction
of a second
inflation
Cosmic Microwave
Background
380,000
years
13.7
billion
years
time

Thermal History

p+e ↔ H   B=13.6 eV

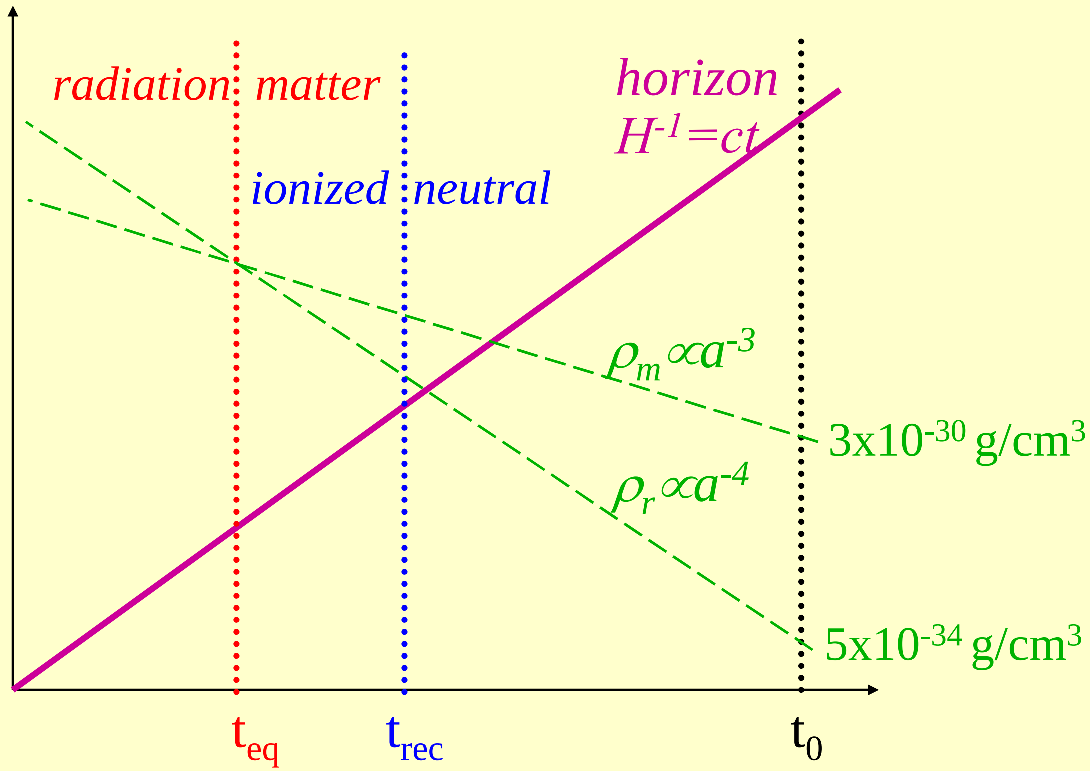
scale r

Saha equation:

$$\frac{\mathbf{x}^2}{1+\mathbf{x}^2} = \frac{(2\pi m_e k\mathbf{T})^{3/2}}{h^3\mathbf{n}} e^{-B/k\mathbf{T}}$$

$$\mathbf{x} = \frac{n_e}{n} \qquad \mathbf{T} \propto a^{-1} \quad \mathbf{n} \propto a^{-3}$$

x=1/2 at T_{rec}~ 4000 K



t~	10^4y	$5\text{x}10^5$y	$1.5\text{x}10^{10}$y
1+z~	10^4	10^3	1
T~		4000K	2.7K

CMB: Recombination

$$H+\gamma \leftrightarrow p+e^-$$

H binding energy B=13.6 eV ~2.7 kT T~60,000 K
but energetic tail of Plankian keeps it ionized

In equilibrium, for $kT \ll mc^2$ (non-relat.), Maxwell-Boltzman for each component:

$$n_x = g_x\left(\frac{m_x kT}{2\pi\hbar^2}\right)^{3/2}\exp\left(-\frac{m_x c^2}{kT}\right)$$

g=2 for p and e

→Saha eq.

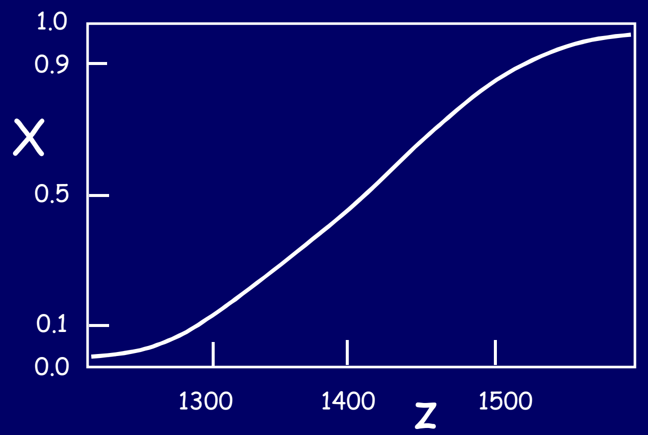
$$\frac{1-X}{X} = n_p\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2}\exp\left(\frac{B}{kT}\right) \qquad \text{ionization } X \equiv \frac{n_p}{n_p+n_H}$$

$$BB\ photons\ n_\gamma = \frac{2.404}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \qquad \eta \equiv \frac{n_b}{n_\gamma} = \frac{n_p}{X n_\gamma}$$

$$\longrightarrow \quad \frac{1-X}{X^2} = 3.84\,\eta\left(\frac{kT}{m_e c^2}\right)^{3/2}\exp\left(\frac{B}{kT}\right)$$

solve for X(T,η)

For $\eta = 5.5\times10^{-10}$ have X=0.5 at kT_{rec}=0.3eV=B/40
T_{rec}=3740K z_{rec}=1370 $t_{rec}=2.4\times10^5$y



Decoupling

$$\sigma_e \propto 1/m^2$$

Rate of photon scattering: $\Gamma(z) = n_e(z)\sigma_e c = X(z)(1+z)^3 n_{b0}\sigma_e c$

$$H(z)^2 \approx H_0^2 \Omega_{m0}(1+z)^3$$

Decoupling when Γ=H $\Omega_{m0} = 0.3 \quad \Omega_{b0} = 0.04 \quad \rightarrow \quad 1+z_{dec} = 43\, X(z_{dec})^{-2/3}$

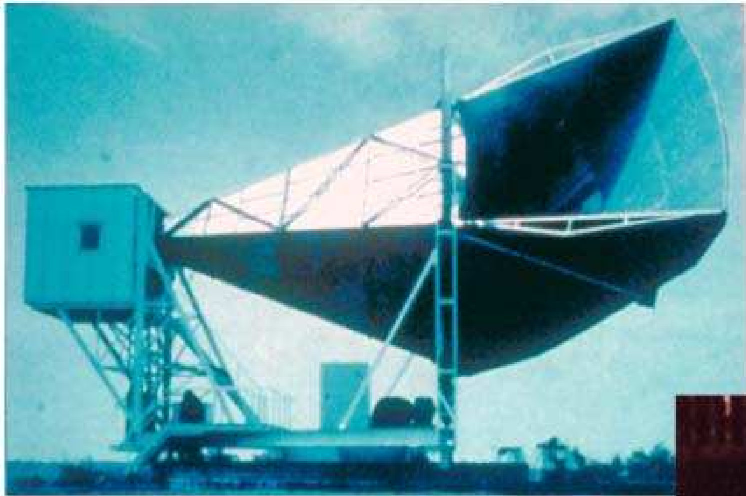
z_{dec}=1100 T_{dec}=3000K

Optical depth: $\tau(t) = \int_t^{t_0} \Gamma(t)dt = \int_0^z \frac{\Gamma(z)}{H(z)} \frac{dz}{1+z} \approx 0.035 \int_0^z X(z)(1+z)^{1/2} dz$

Last scattering at τ~1 $z \sim z_{dec}$

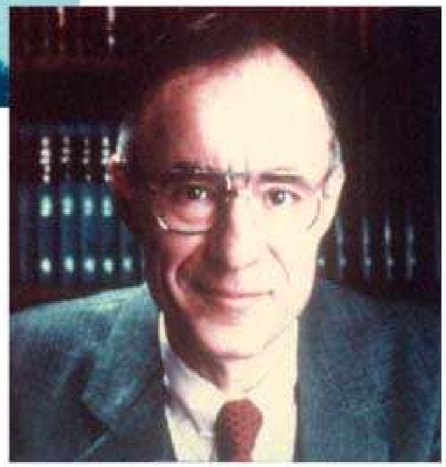
Cosmic Microwave Background Radiation

DISCOVERY OF COSMIC BACKGROUND


Microwave Receiver

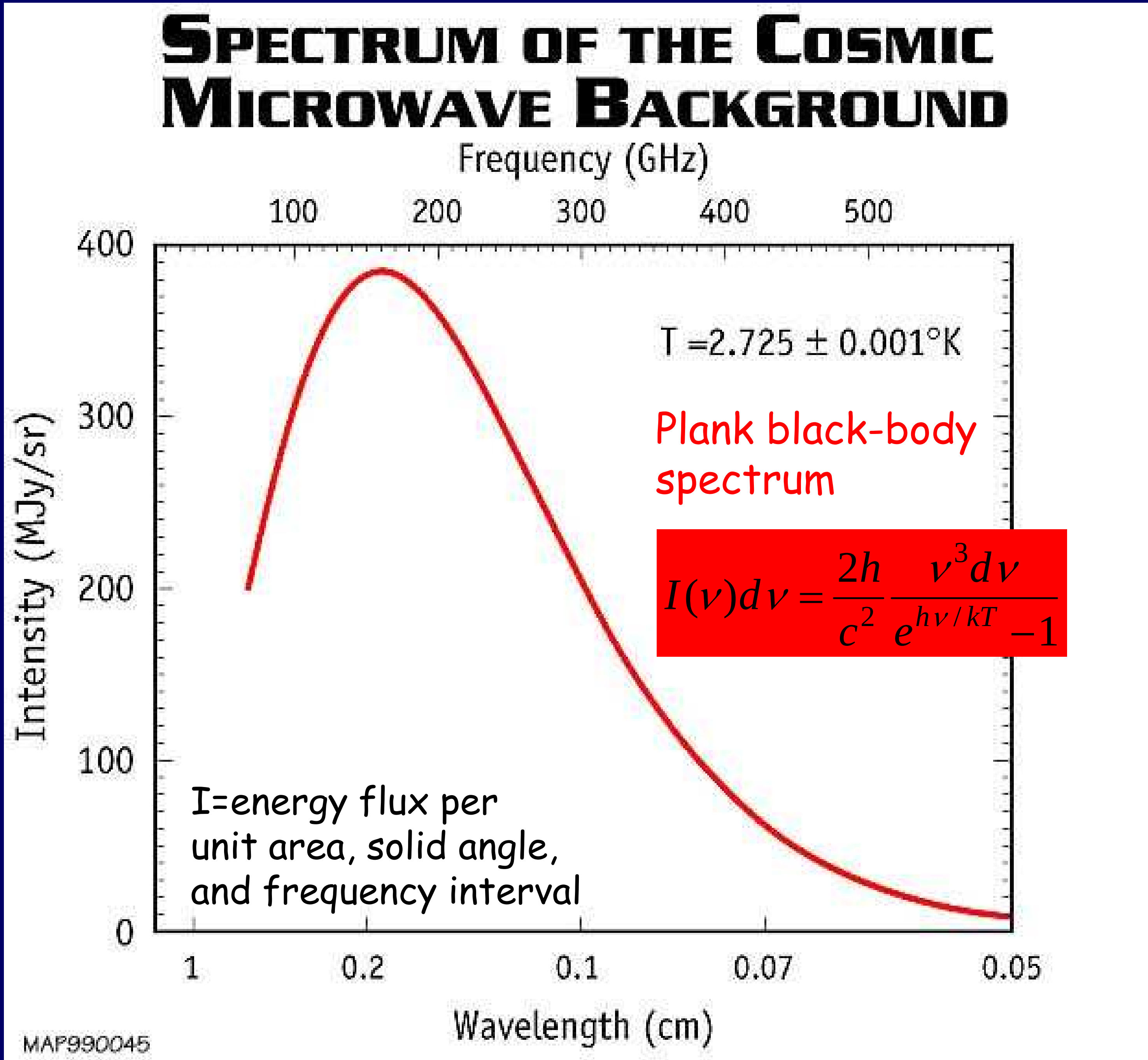
1965


Robert Wilson

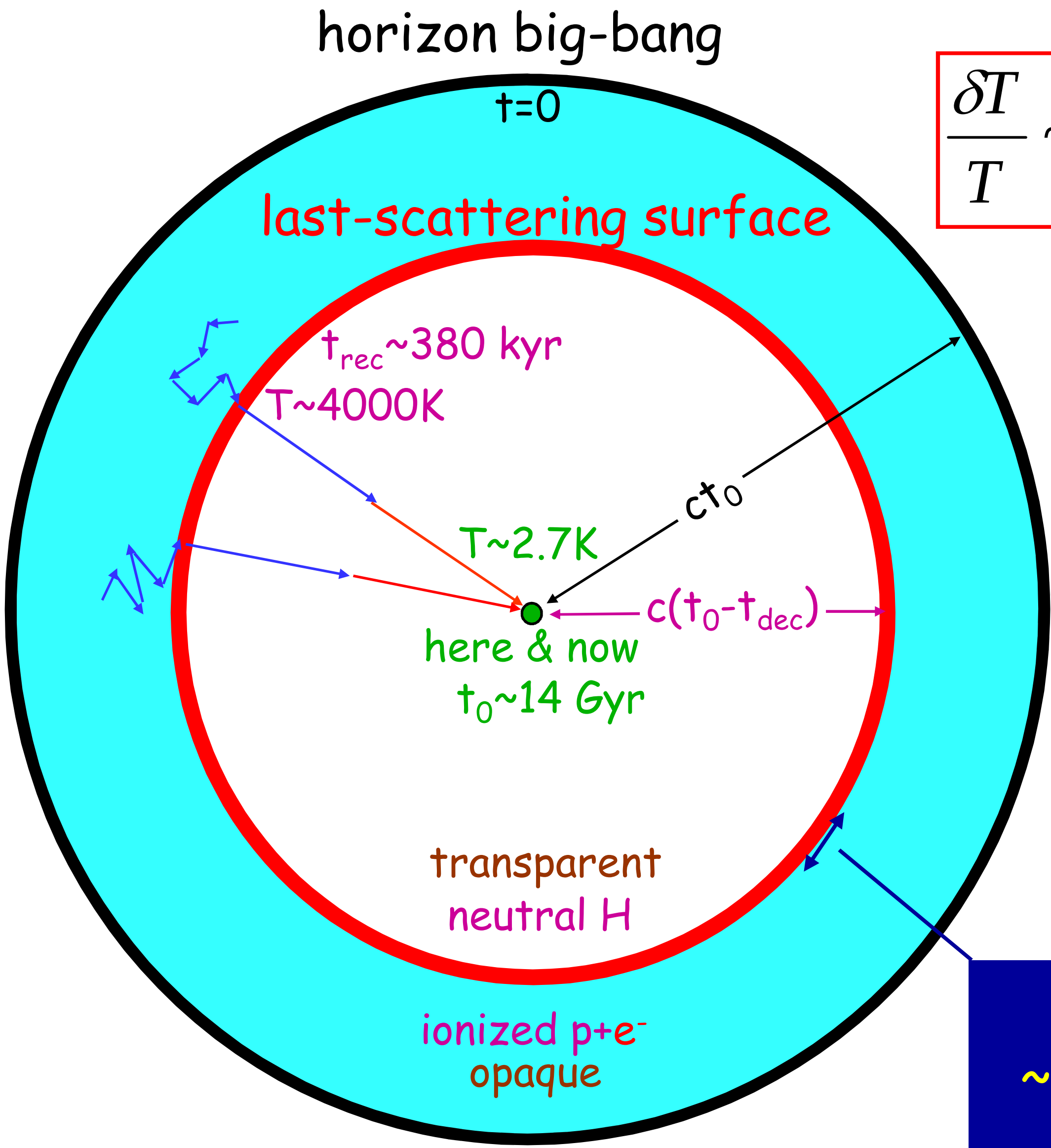

Arno Penzias

MAP990045

COBE
1992


SPECTRUM OF THE COSMIC MICROWAVE BACKGROUND
Frequency (GHz)
100
200
300
400
500
400
300
200
100
0
Intensity (MJy/sr)
T =2.725 ± 0.001°K
Plank black-body spectrum
$I(\nu)d\nu = \frac{2h}{c^2}\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}$
I=energy flux per unit area, solid angle, and frequency interval
1
0.2
0.1
0.07
0.05
Wavelength (cm)
MAP990045

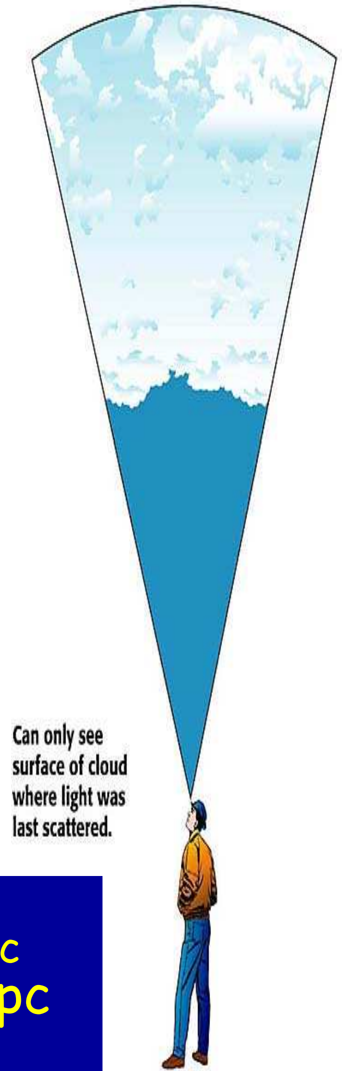
Origin of Cosmic Microwave Background



$$\frac{\delta T}{T} \sim \frac{1}{10}\frac{\delta\rho}{\rho} \sim 10^{-5}$$

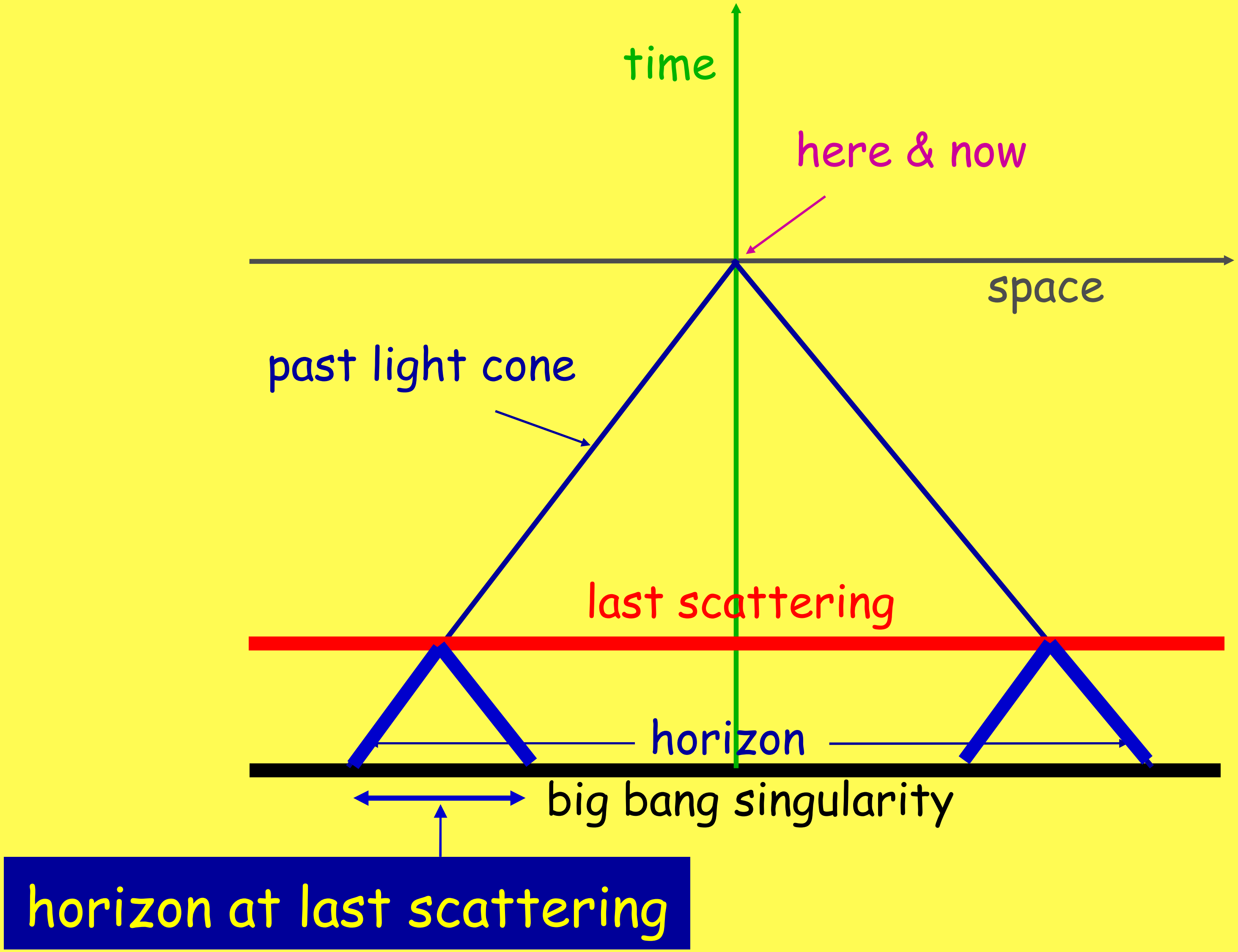
Thomson scattering

$$\sigma_T \propto m^{-2}$$

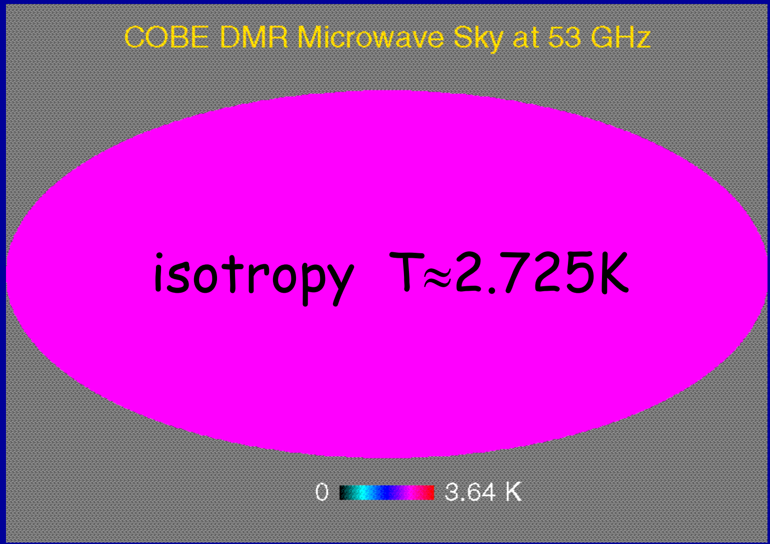


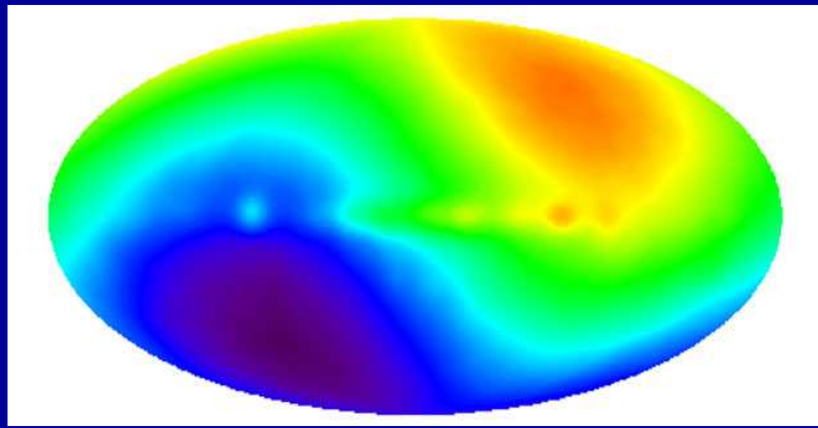
horizon at t_{rec}
~100 comoving Mpc
~1°

Characteristic Scale

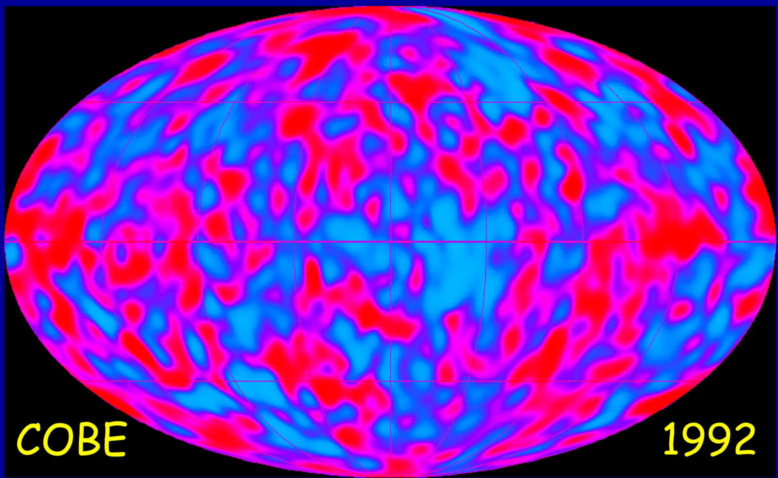


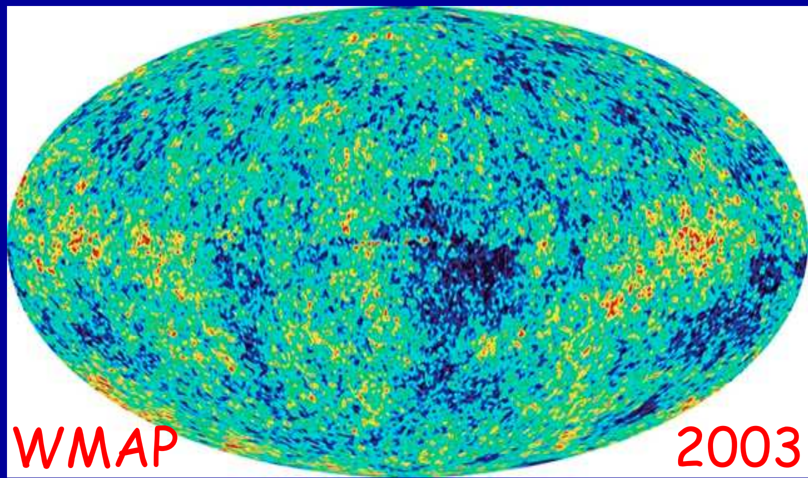
CMB Temperature Maps





$\delta T/T \sim 10^{-3}$ dipole



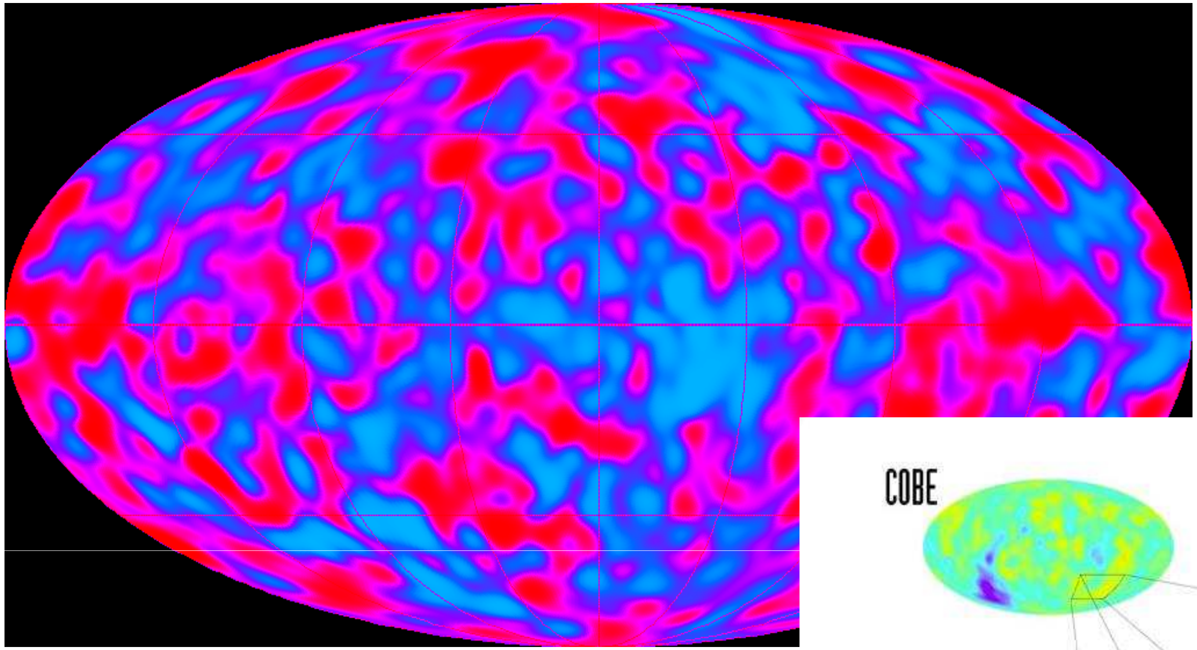


resolution ~10°

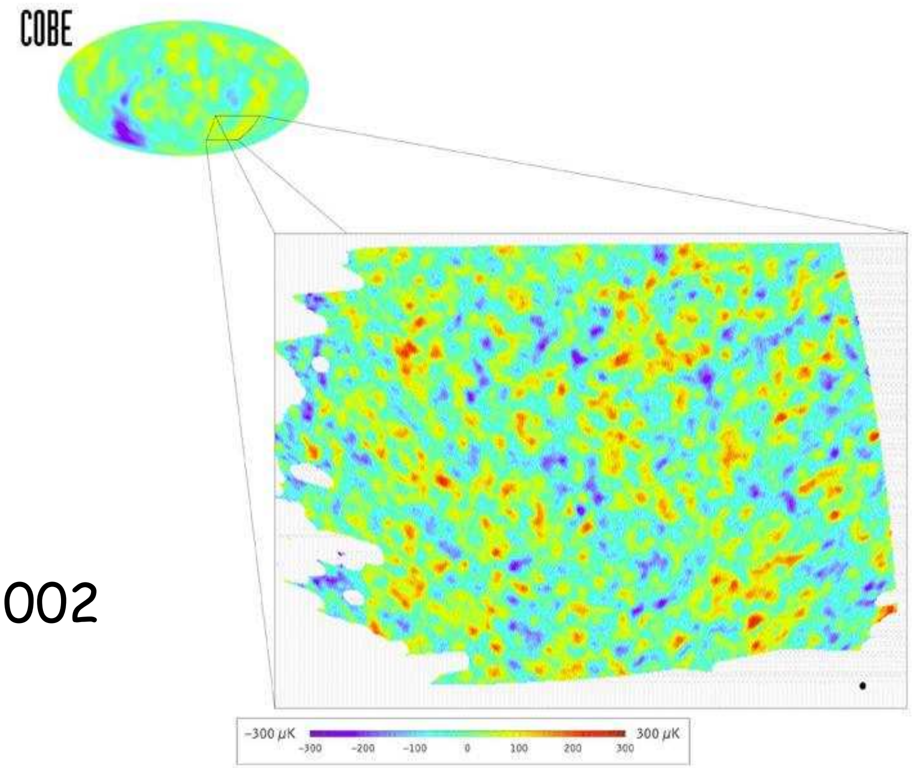
$\delta T/T \sim 10^{-5}$

resolution ~ 10′

CMB Temperature Fluctuations

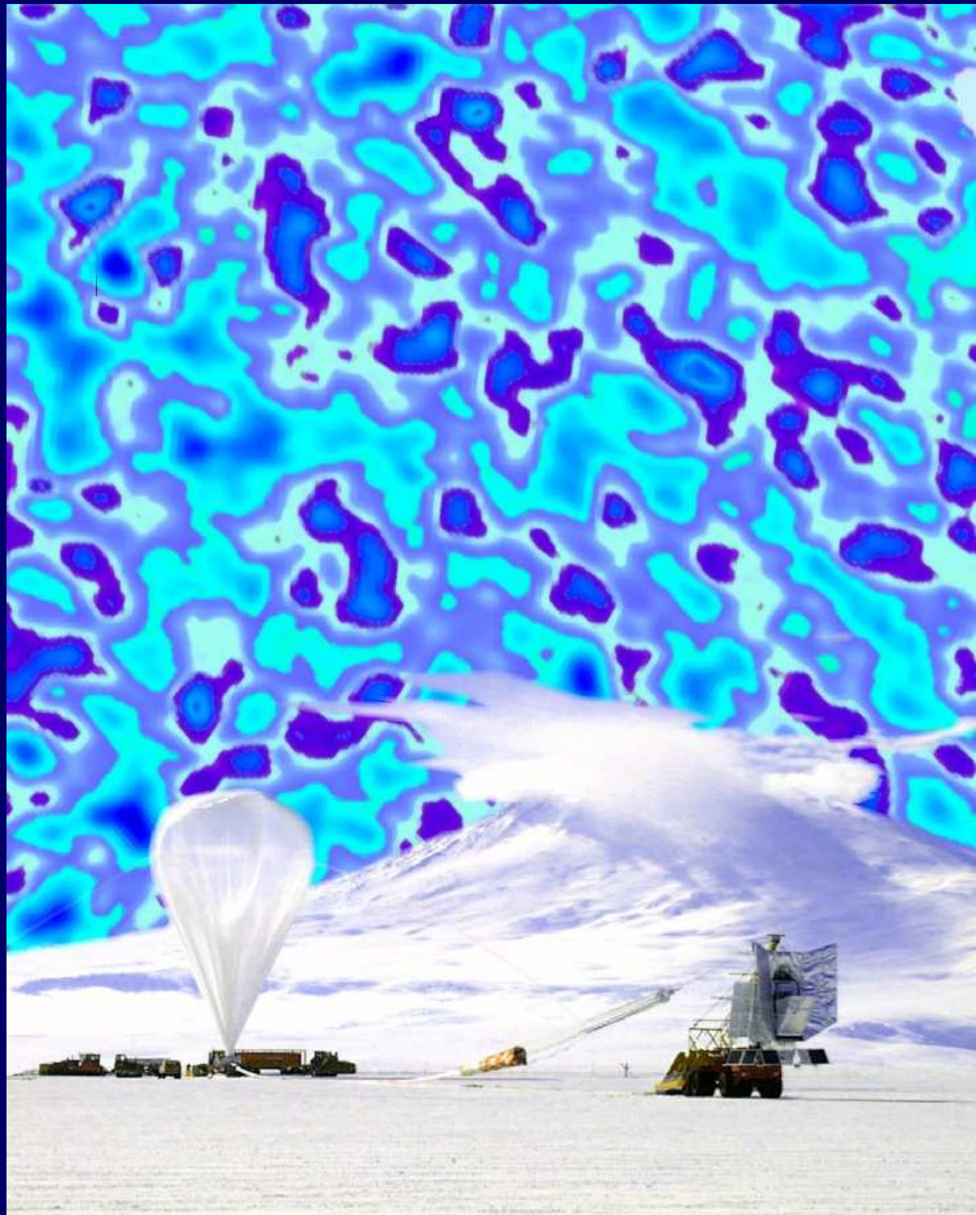


COBE 1992



BOOMERANG 2002

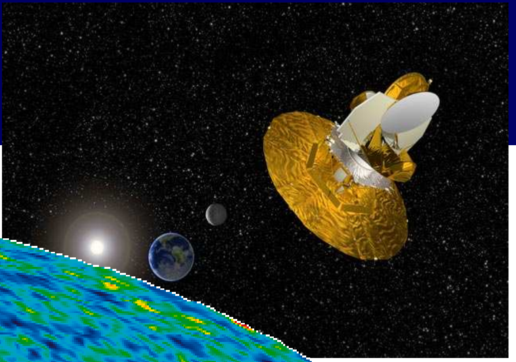
Boomerang

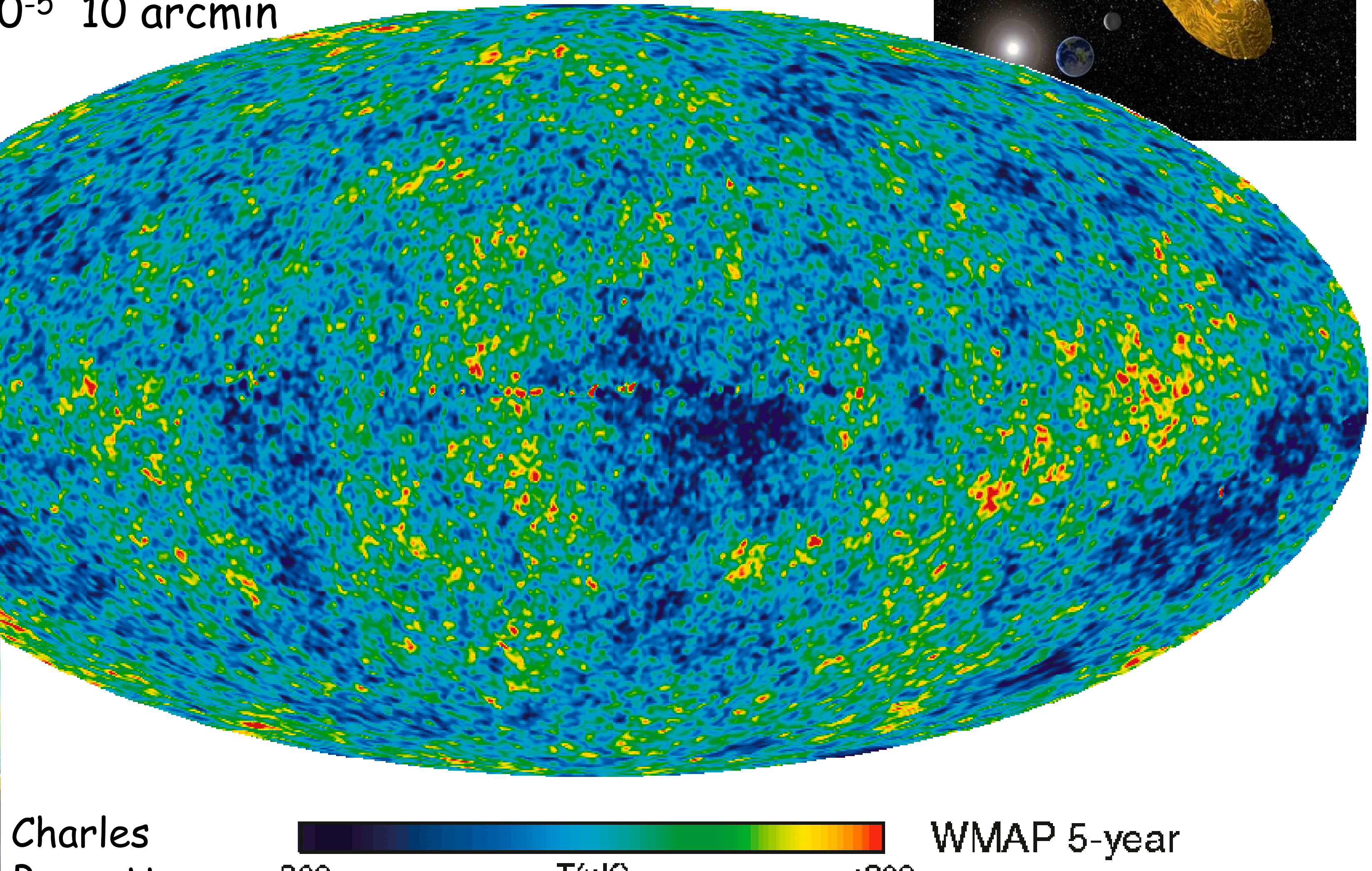


WMAP: Wilkinson Microwave Anisotropy Probe

WMAP-5 2008

$\delta T/T \sim 10^{-5}$ 10 arcmin





Charles Bennett

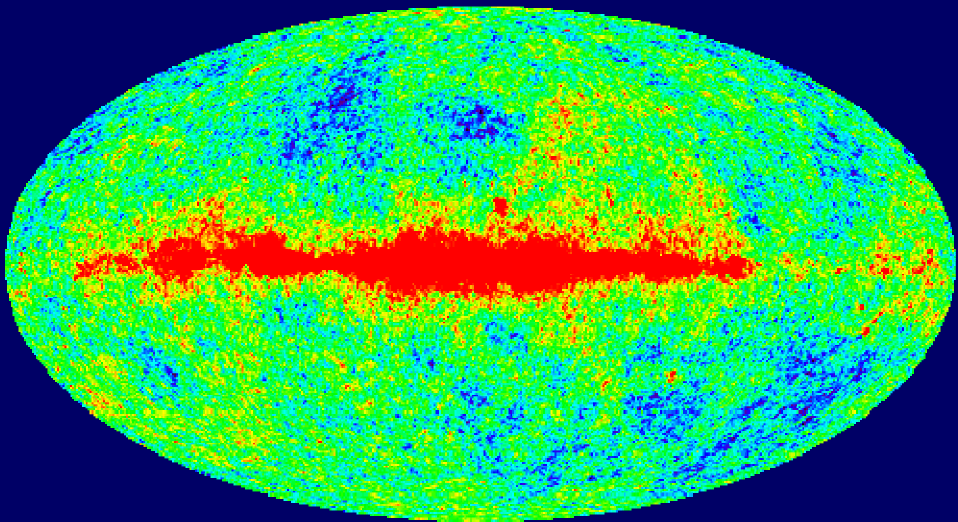
CMB anisotropy - density fluctuations

black body: $\rho_{rad} \propto T^4$

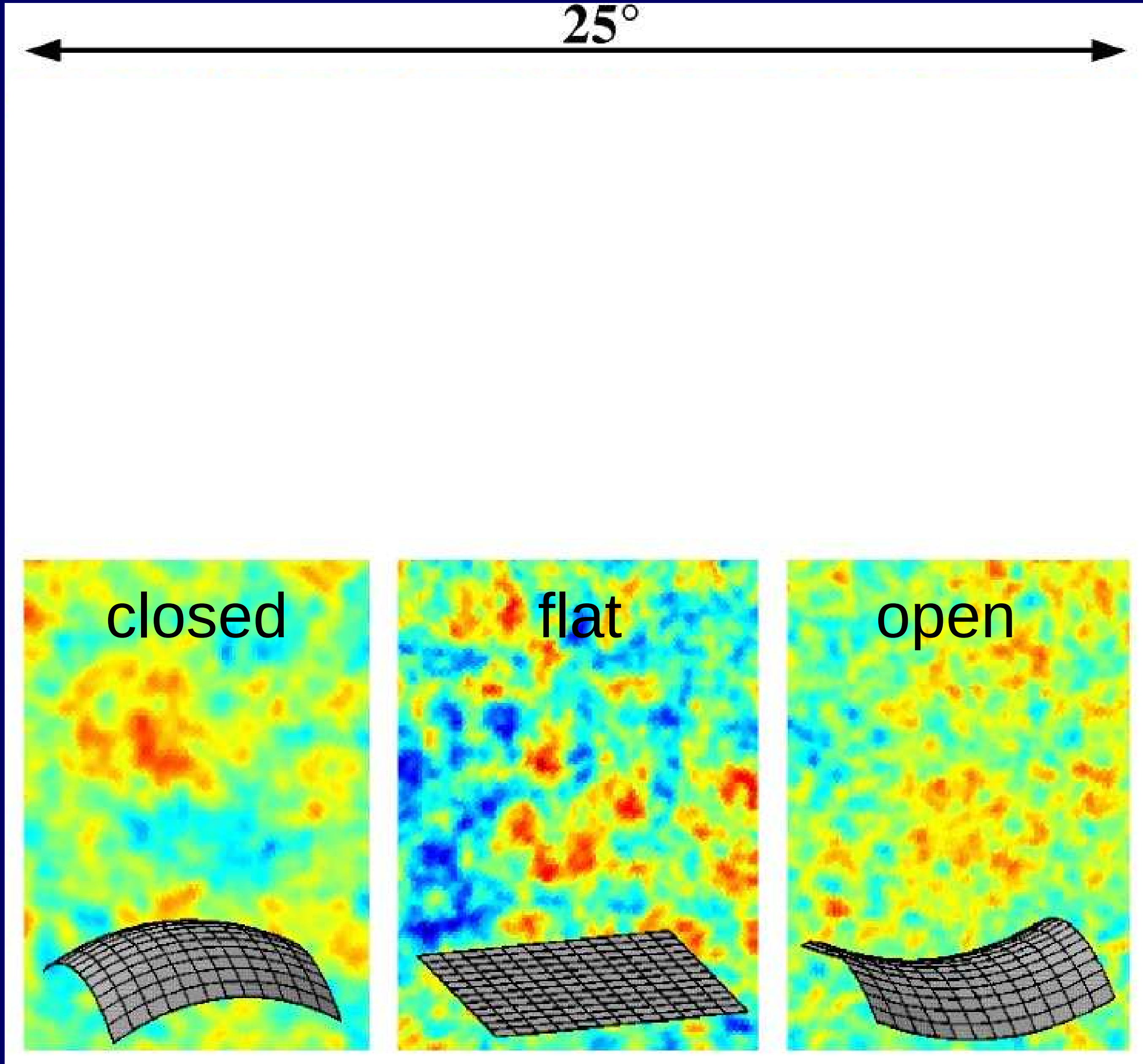
adiabatic fluctuations: $const. = \dfrac{n_\gamma}{n_m} = \dfrac{\rho_r / h\nu}{\rho_m / m} \propto \dfrac{\rho_r / T}{\rho_m}$

$$\longrightarrow \quad \frac{\delta\rho_m}{\rho_m} \approx 3\frac{\delta T}{T} \quad \to \quad \sim 10\frac{\delta T}{T}$$

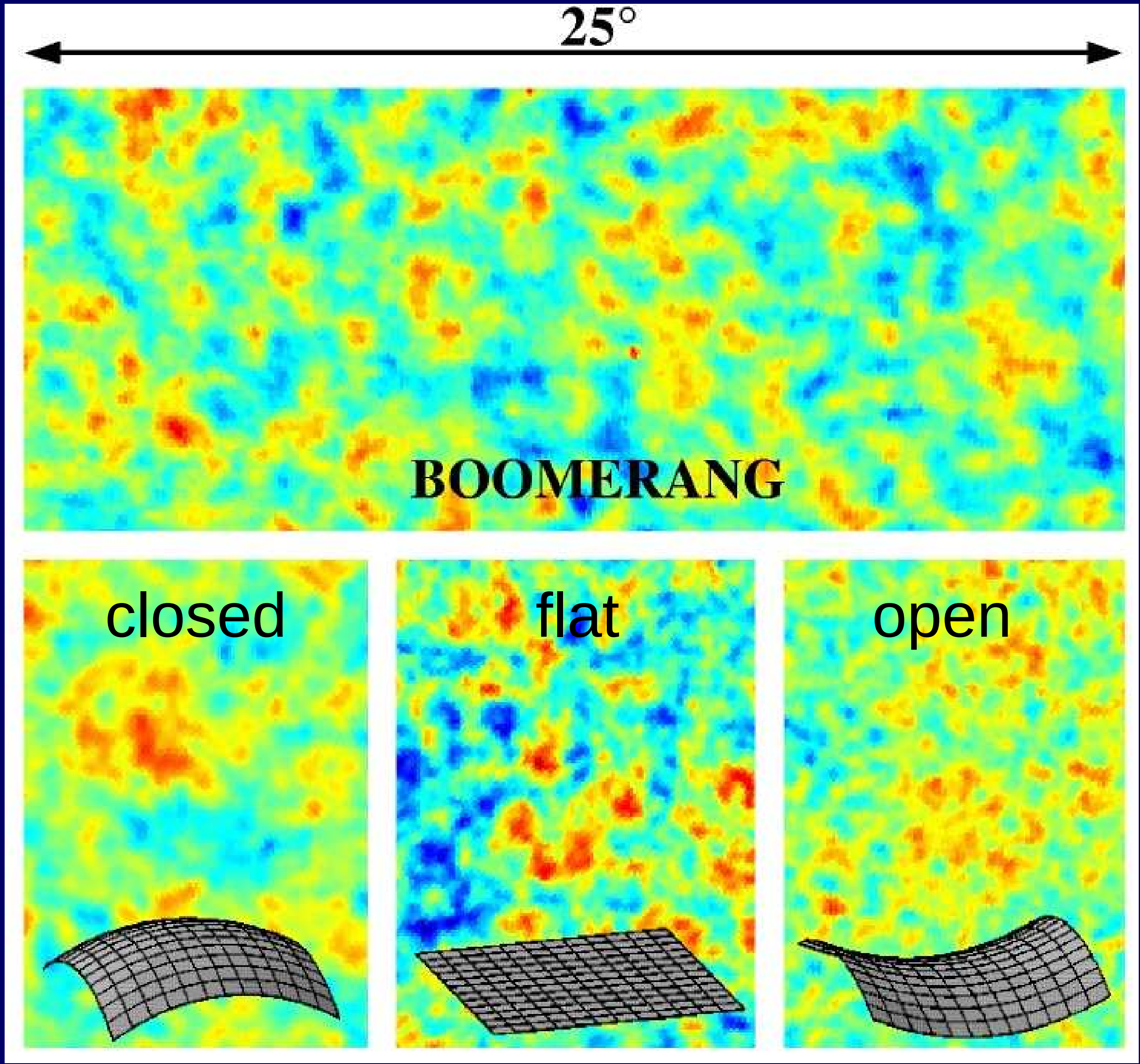
$$\frac{\delta T}{T} \sim 10^{-5} \quad \to \quad \frac{\delta\rho}{\rho} \sim 10^{-4}$$

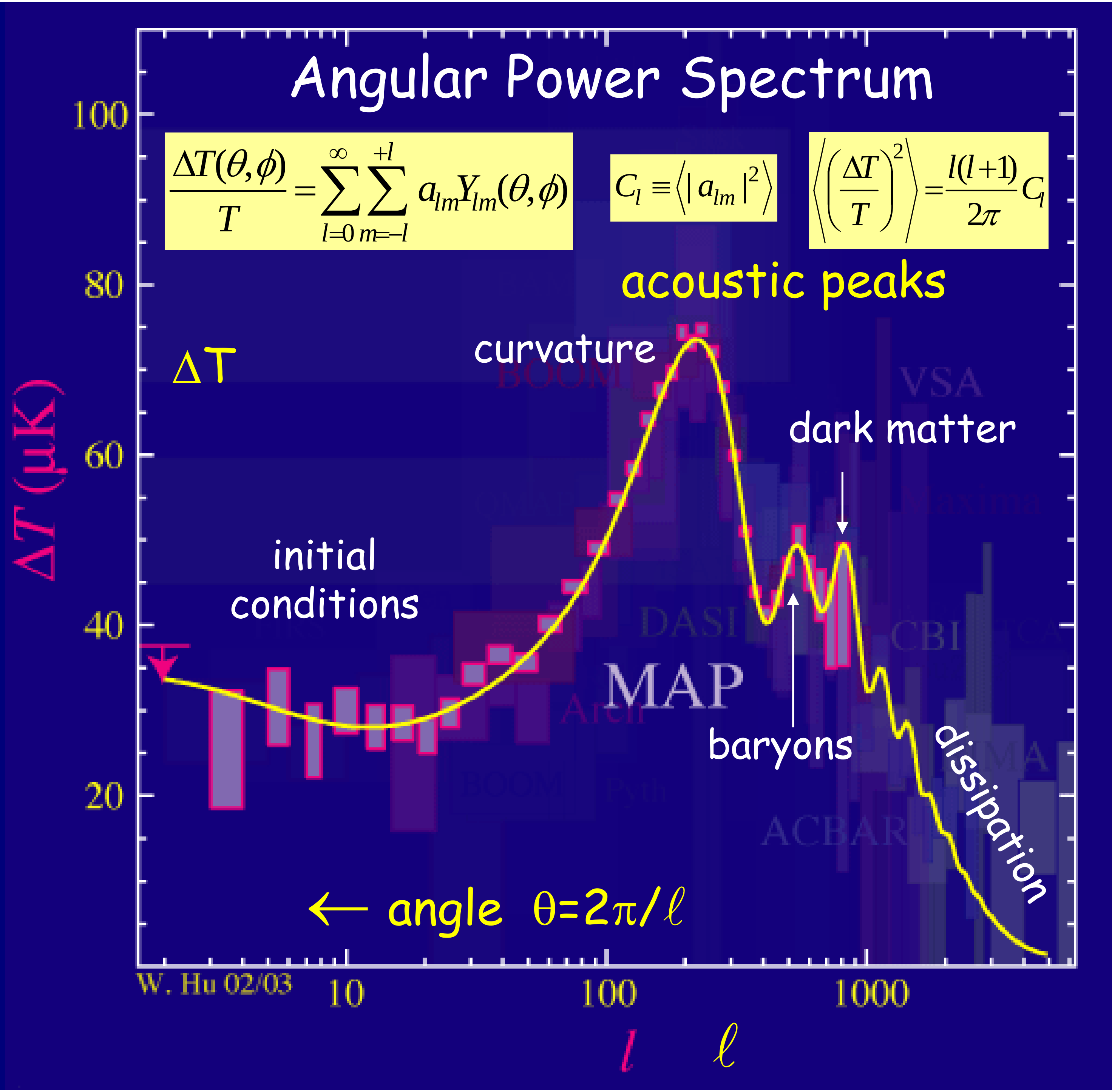


Curvature

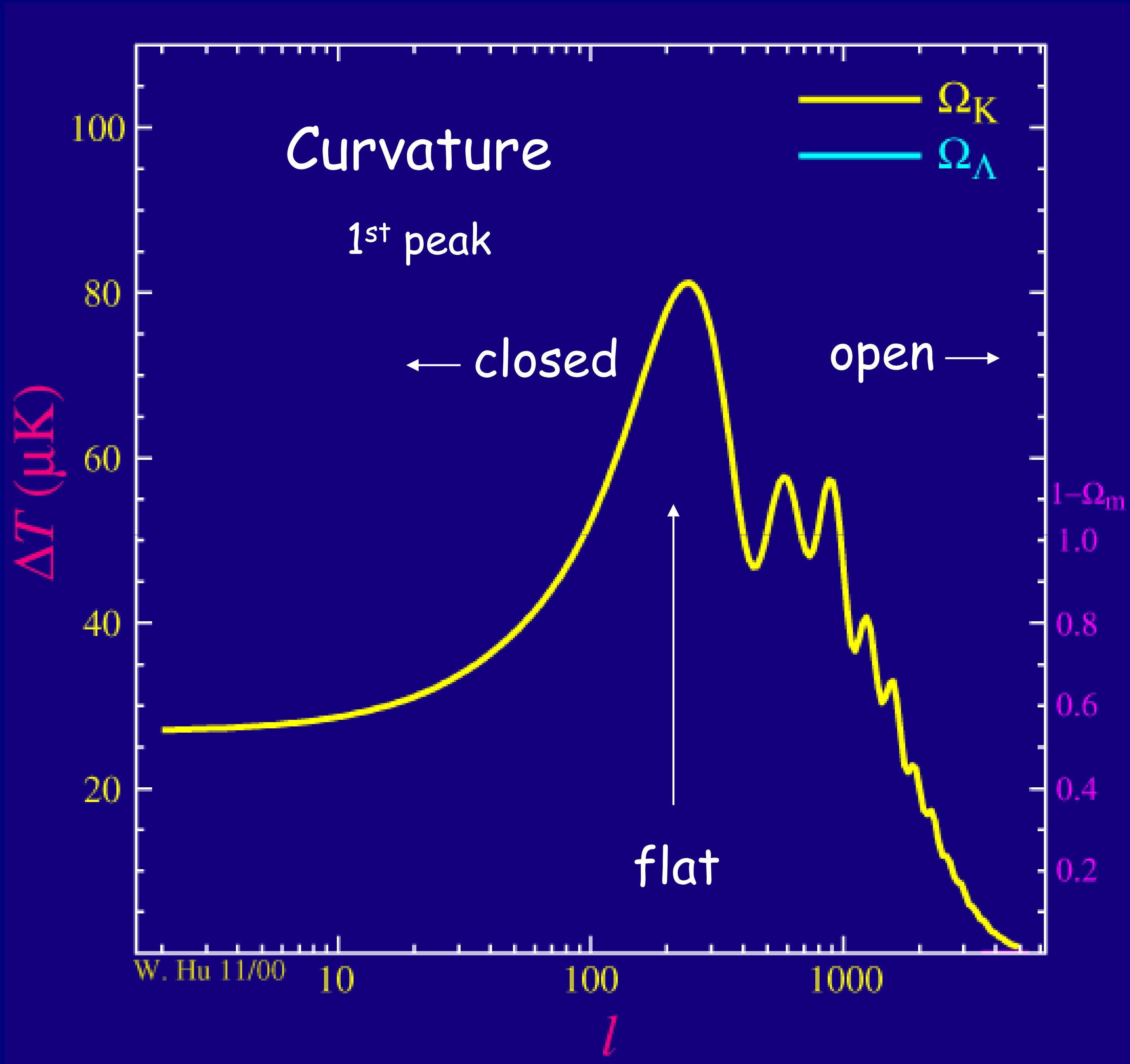


Curvature





Angular Power Spectrum
$\frac{\Delta T(\theta,\phi)}{T} = \sum_{l=0}^{\infty}\sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta,\phi)$
$C_l \equiv \langle | a_{lm} |^2 \rangle$
$\left\langle \left(\frac{\Delta T}{T} \right)^2 \right\rangle = \frac{l(l+1)}{2\pi} C_l$
acoustic peaks
curvature
dark matter
initial conditions
baryons
dissipation
ΔT
ΔT (μK)
MAP
BOOM
VSA
DASI
CBI
ACBAR
← angle θ=2π/ℓ
W. Hu 02/03
100
80
60
40
20
10
100
1000
l
ℓ


Curvature
1st peak
closed
open
flat
Ω_K
Ω_Λ
ΔT (μK)
100
80
60
40
20
$1-\Omega_m$
1.0
0.8
0.6
0.4
0.2
W. Hu 11/00
10
100
1000
l

Origin of Fluctuations in CMB Temperature

Large angles l<180: Sachs-Wolf $\frac{\delta T}{T} = \frac{1}{3}\frac{\delta\phi}{c^2}$ gravitational redshift

$$\nabla^2(\delta\phi) = \frac{4\pi G}{c^2}\delta\rho$$

On scales of the horizon at decoupling (l~180) and below: acousitic oscilations of photon-baryon fluid in the dark-matter potential wells.

in eq. of state w = 0 (cold baryons) to 1/3 (photons), depending on n_b/n_γ

maximum T where the oscilation is at maximum compression: $T \propto \rho^{1/4}$
minimum T at maximun expansion.

Peak angular scale at l~180. At this scale max compression at $t_{dec.}$
Exact l_{max} tells Ω_k.

Peak amplitude depends on sound speed $c_s = w\,c$ ⟶ Ω_b

Origin of Peaks

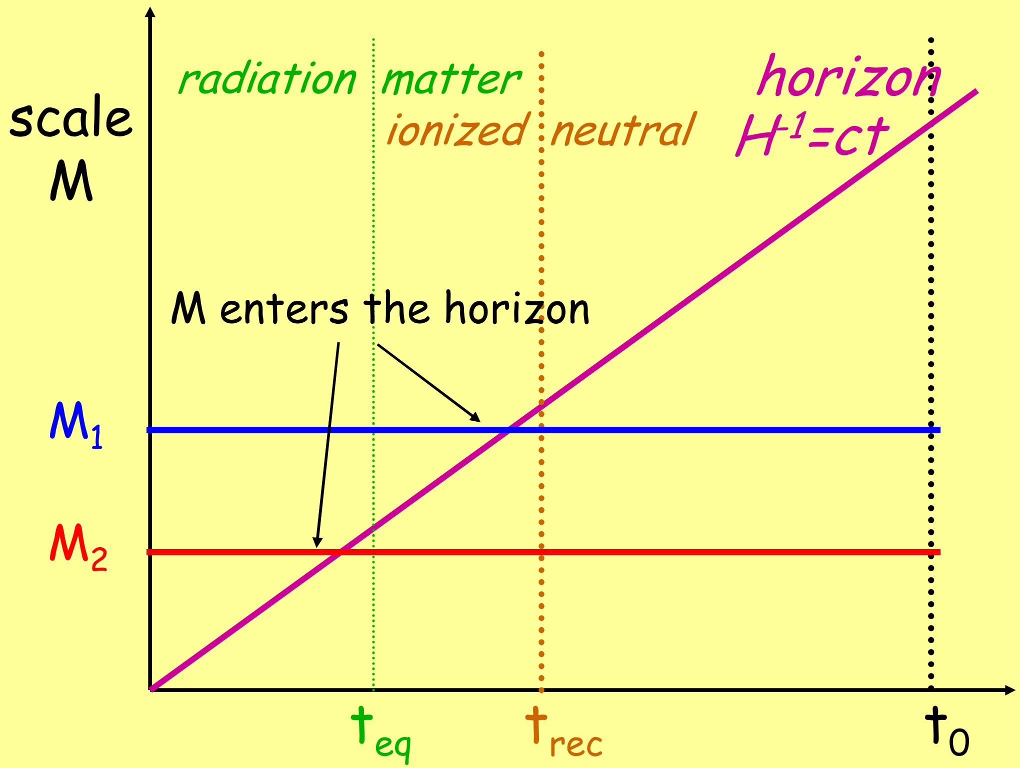
Horizon:

$r_h \propto t$

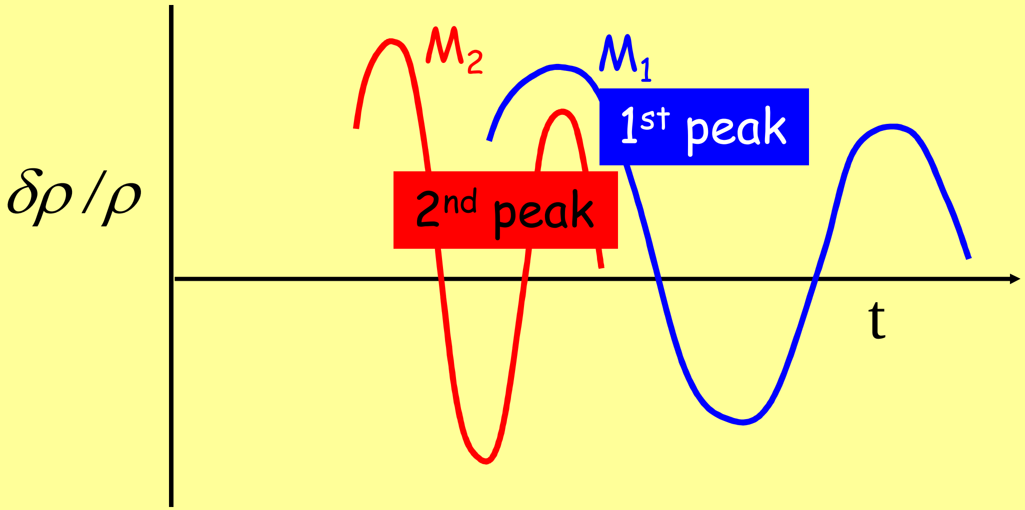
$M_h \propto \rho\, r_h^{\ 3} \propto (t^{2/3})^{-3} t^3 \propto t$

Comoving sphere:

$a \propto t^{2/3} \quad M = const.$



Fluctuations grow after entering the horizon



Acoustic Peaks

In the early hot ionized universe, photons and baryons are coupled via Thomson scattering off free electrons.

Initial fluctuations in density and curvature (quantum, Inflation) drive acoustic waves, showing as temperature fluctuations, with a characteristic scale - the sound horizon $c_s t$.

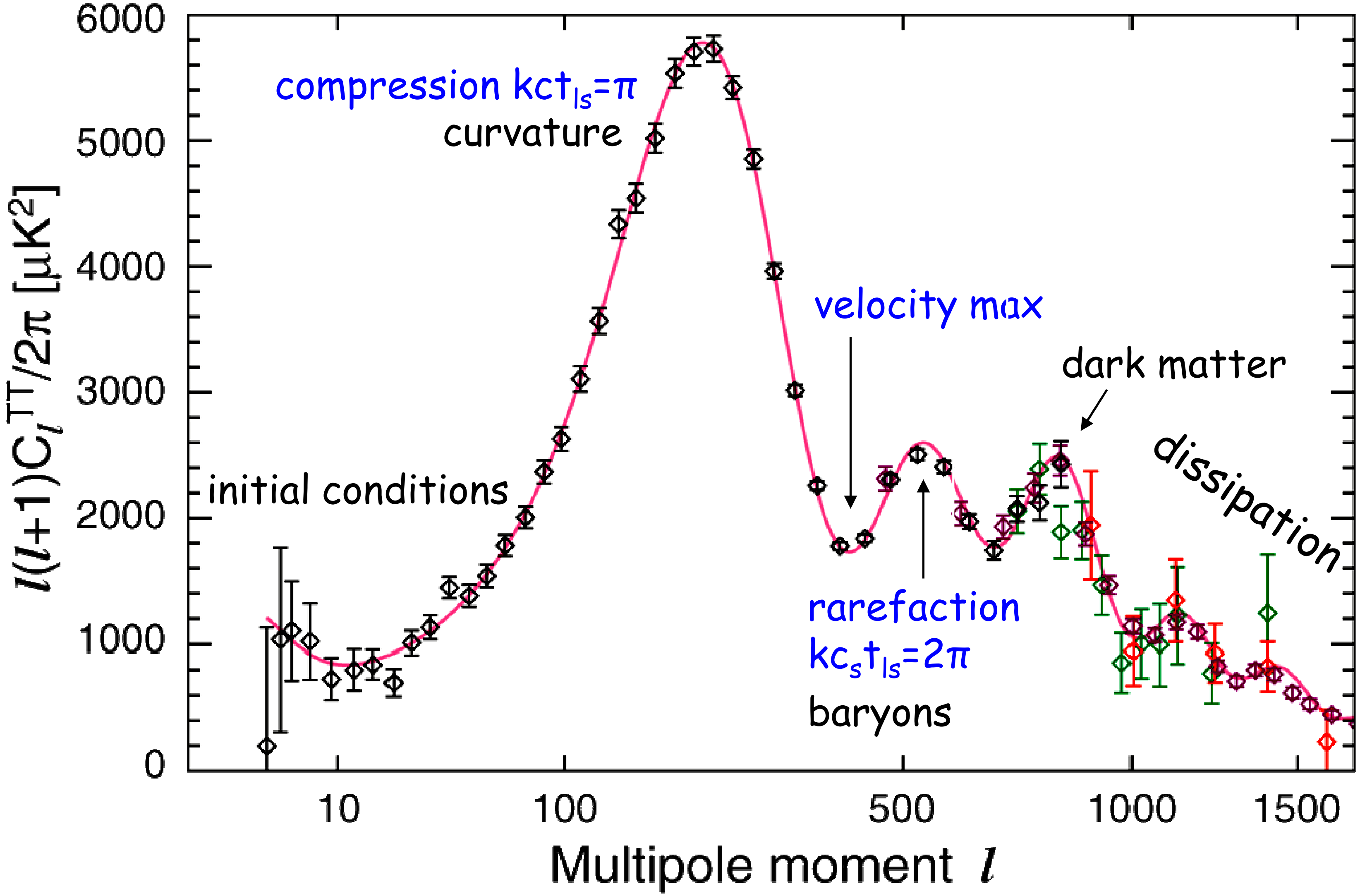
$$\delta T \approx \delta\rho^{1/4} \approx A(k)\cos(kc_s t)$$

At z~1,090, T~4,000K, H recombination, decoupling of photons from baryons. The CMB is a snapshot of the fluctuations at the last scattering surface.

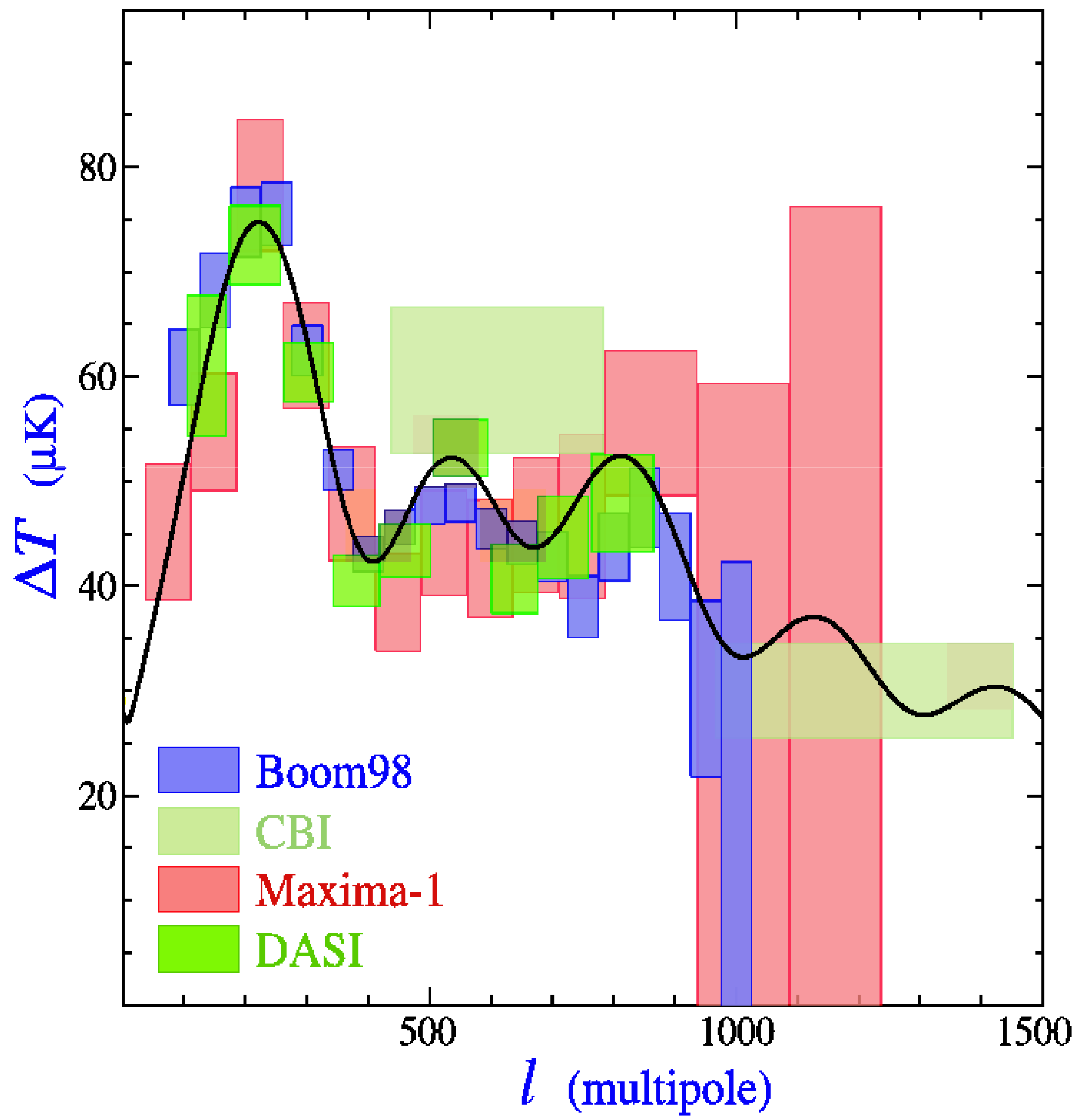
Primary acoustic peak at $r_{ls} \sim ct_{ls} \sim 100$ co-Mpc or $\theta \sim 1°$ ($\ell \sim 200$) – the "standard ruler".

Secondary oscillations at fractional wavelengths.

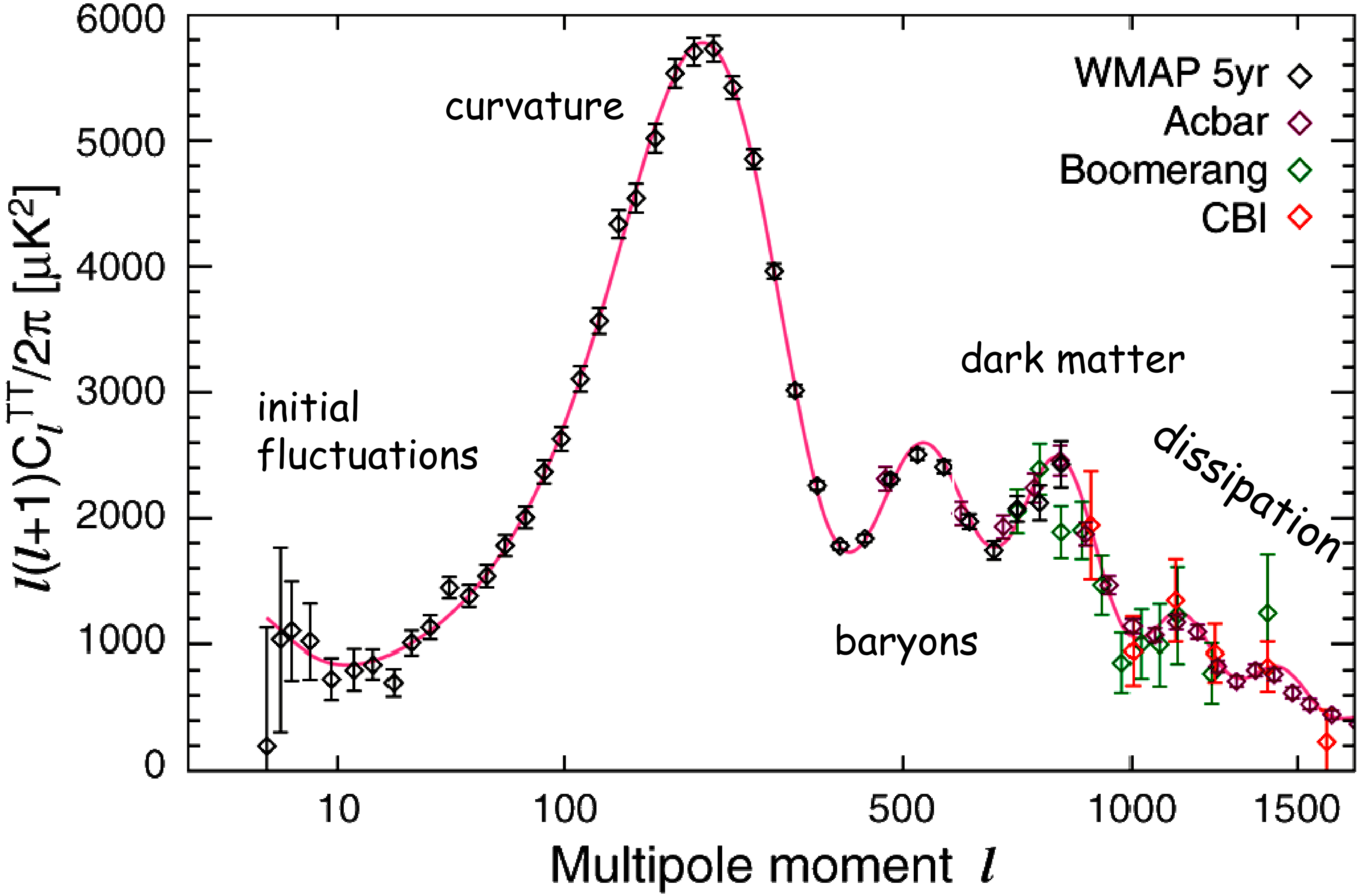
CMB Acoustic Oscillations explore all parameters



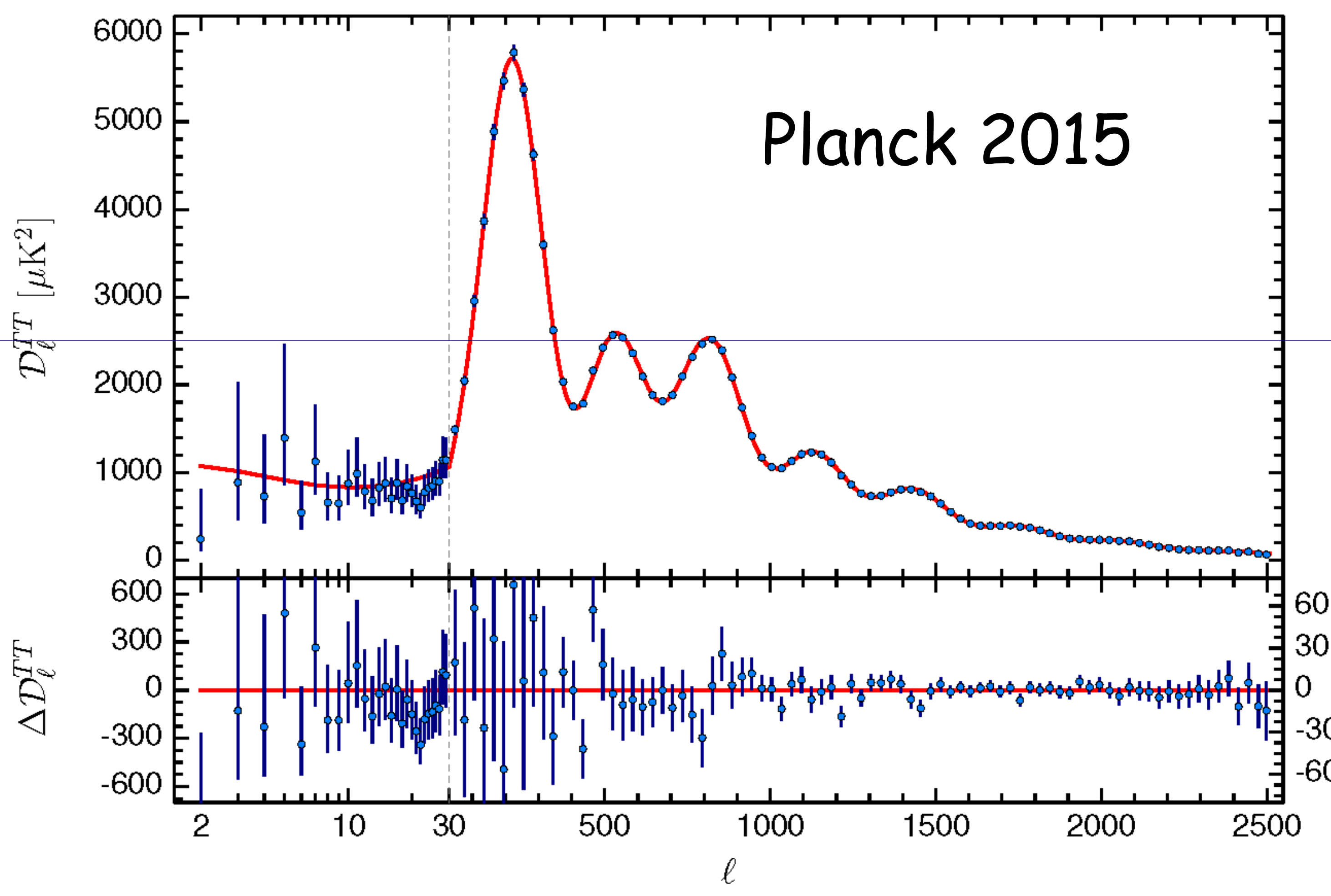
Pre-WMAP CMB Anisotropy spectrum



The ΛCDM model is very successful
Accurate parameter determination



The ΛCDM model is very successful
Accurate parameter determination



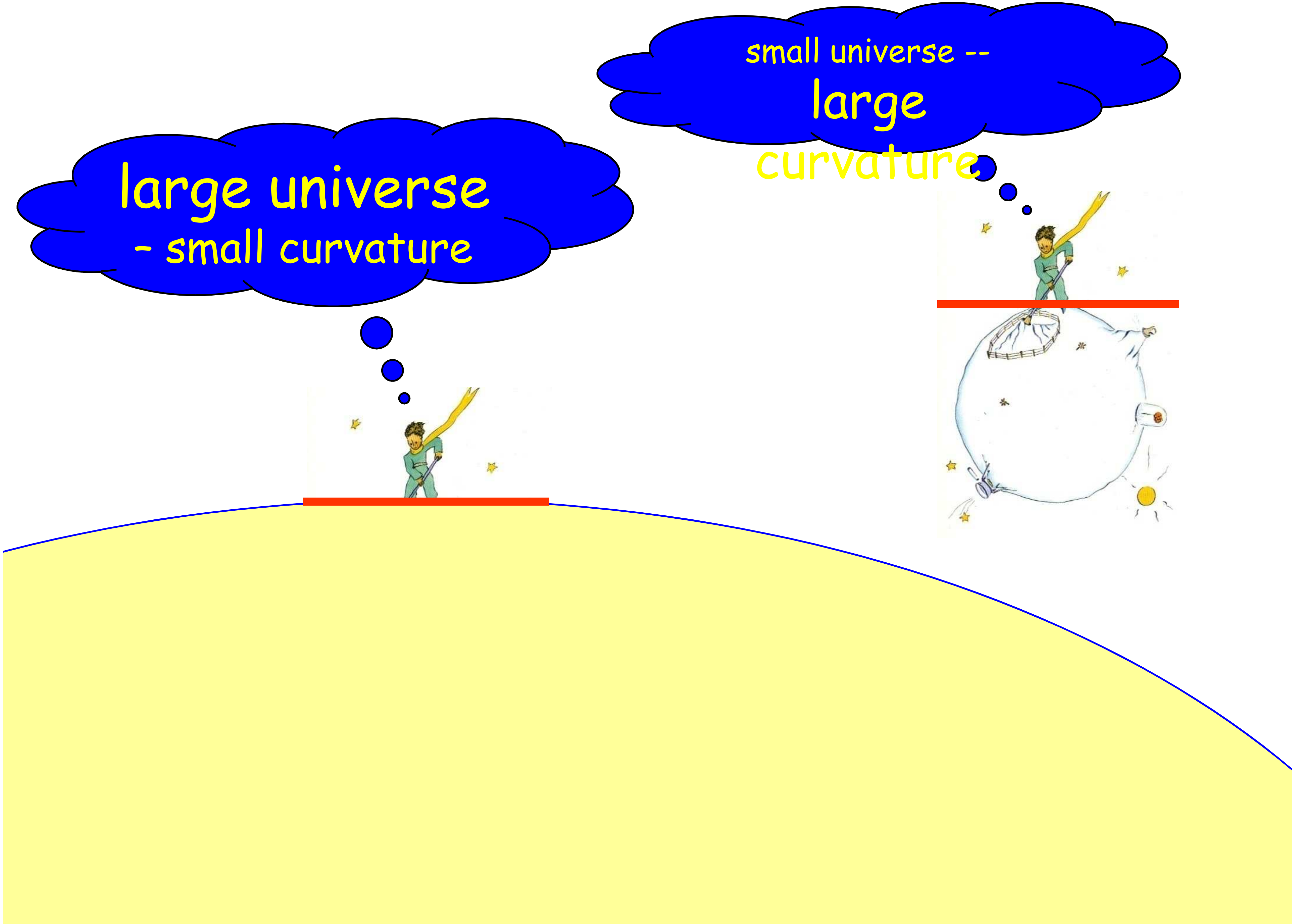
Curvature

The Universe is nearly flat:

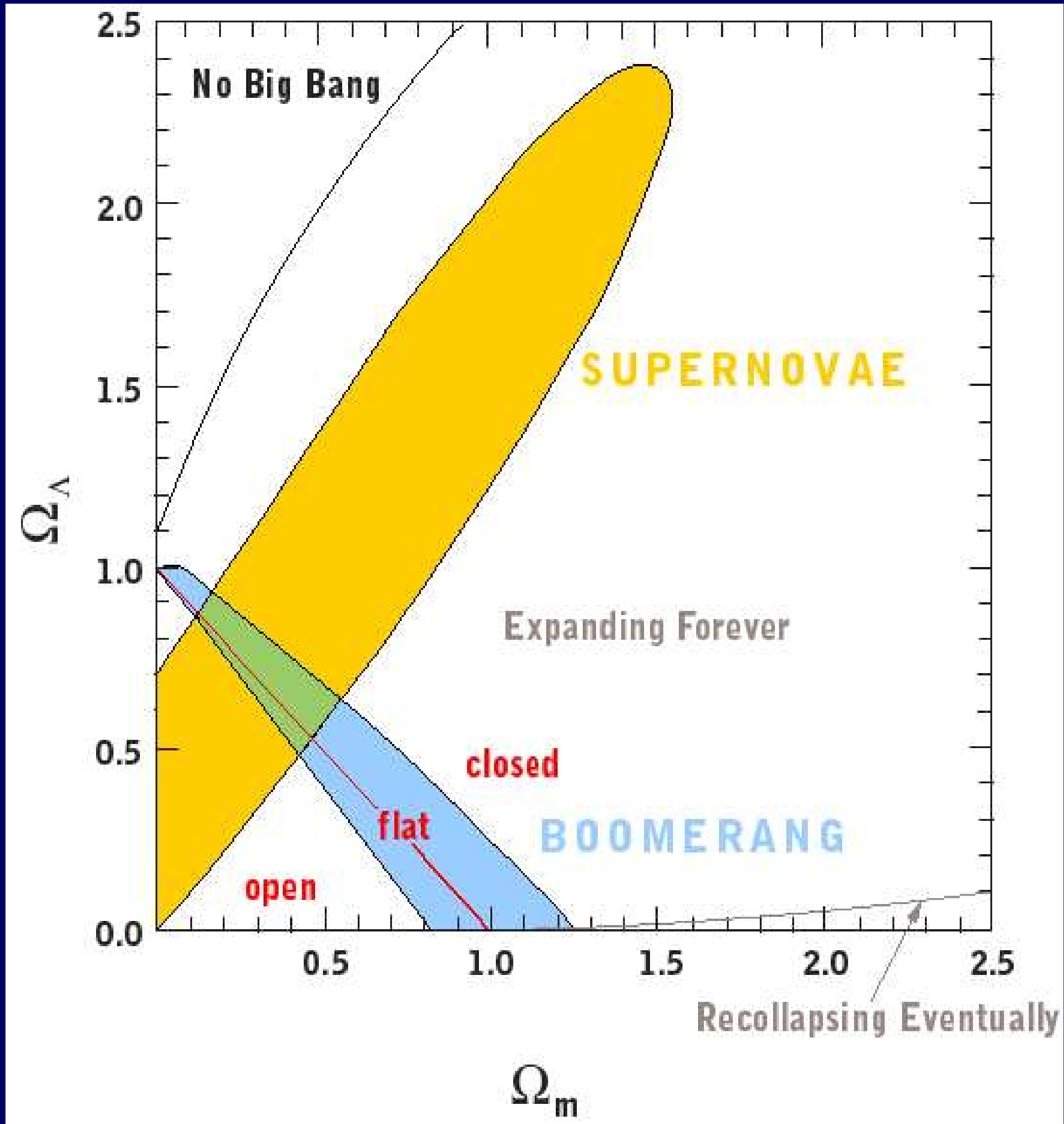
$$1-\Omega_k = \Omega_{\mathrm{m}} + \Omega_\Lambda = 1.02 \pm 0.02$$

Open? Closed?

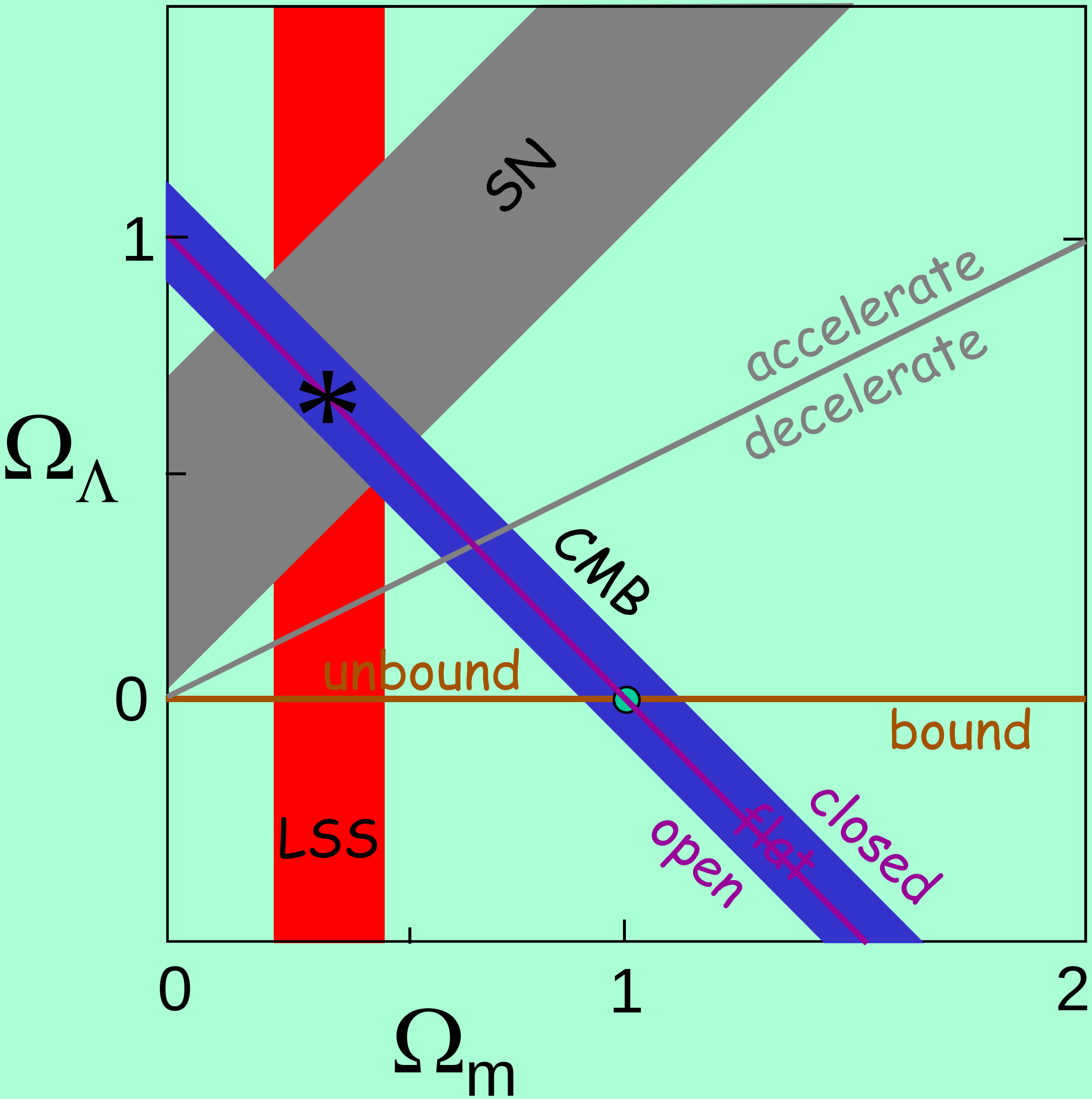
Surely much larger than our horizon!


large universe
- small curvature
small universe --
large
curvature

Cosmological Parameters CMB+SN



Cosmological Parameters



Our Universe

Nearly flat: $\Omega_{tot} = 1.02 \pm 0.02$

but a bizzare mixture:

$$\Omega_{luminous} \approx 0.01$$

$$\Omega_{baryons} = 0.044 \pm 0.004$$

$$\Omega_{mass} = 0.30 \pm 0.05$$

$$\Omega_{\Lambda} = 0.70 \pm 0.05$$

5% baryons, 25% dark matter, 70% dark energy

Our Universe

- Luminous matter 1%
- Dark baryonic matter 4%
- Dark matter – exotic particles 25%
- Dark energy 70%

(Luminous matter, dark baryonic matter and dark matter: attractive; dark energy: repulsive)

Expansion forever!
accelerated by the repulsion of the vacuum

From measurements of anisotropy in the Cosmic Microwave Background:
Euclidean geometry in the observable volume
–the universe is open or closed but very BIG!

Cosmological Parameters by WMAP

Old Universe – *New* Numbers

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$ curvature

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$ dark energy

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$ baryons

$n_b = 2.5 \times 10^{-7}\,{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$ dark matter

$\Omega_\nu h^2 < 0.0076$ (95%CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}\,{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

$n_s = 0.93^{+0.03}_{-0.03}$

$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr age

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

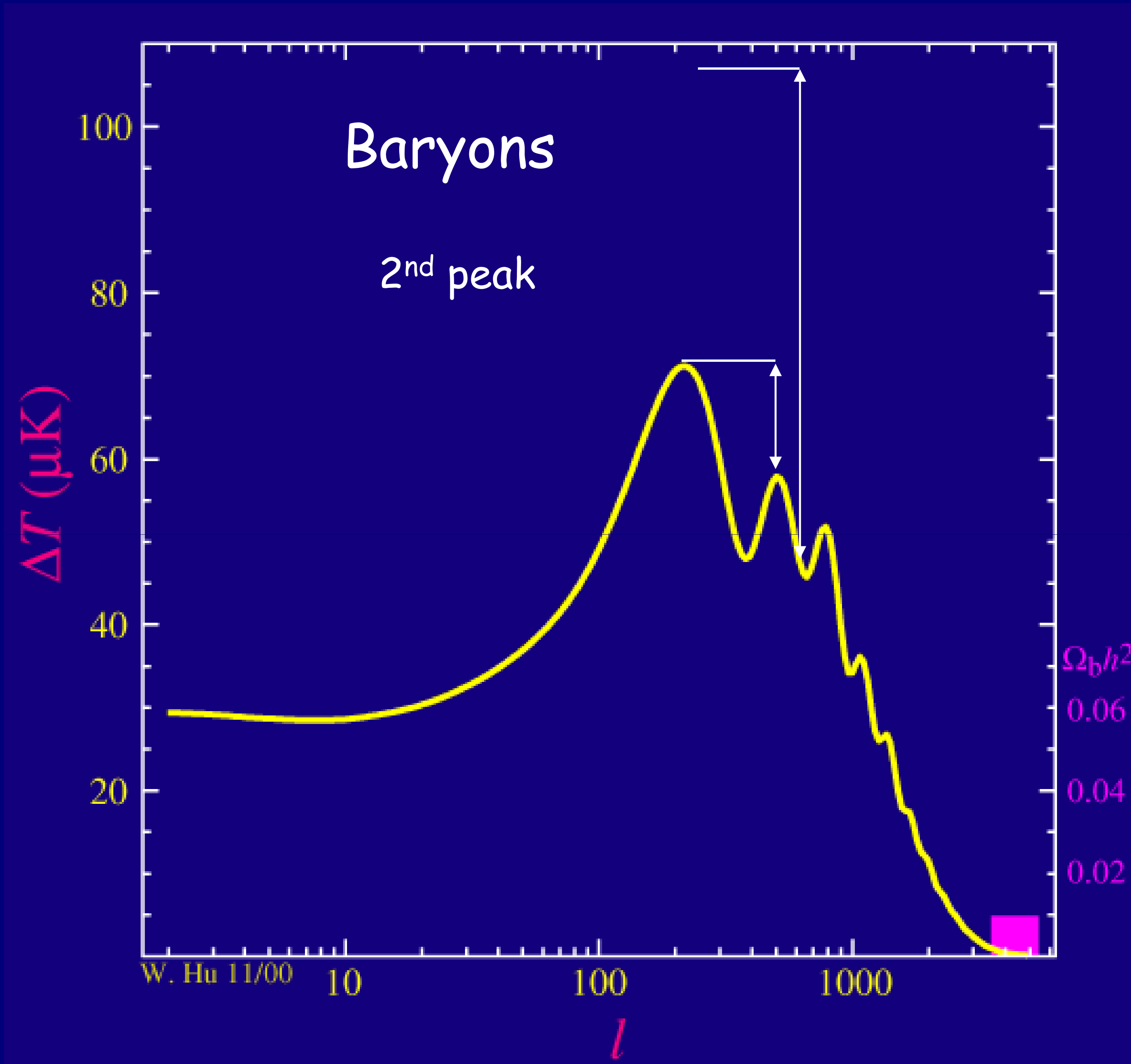
$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0^\circ.598^{+0.002}_{-0.002}$

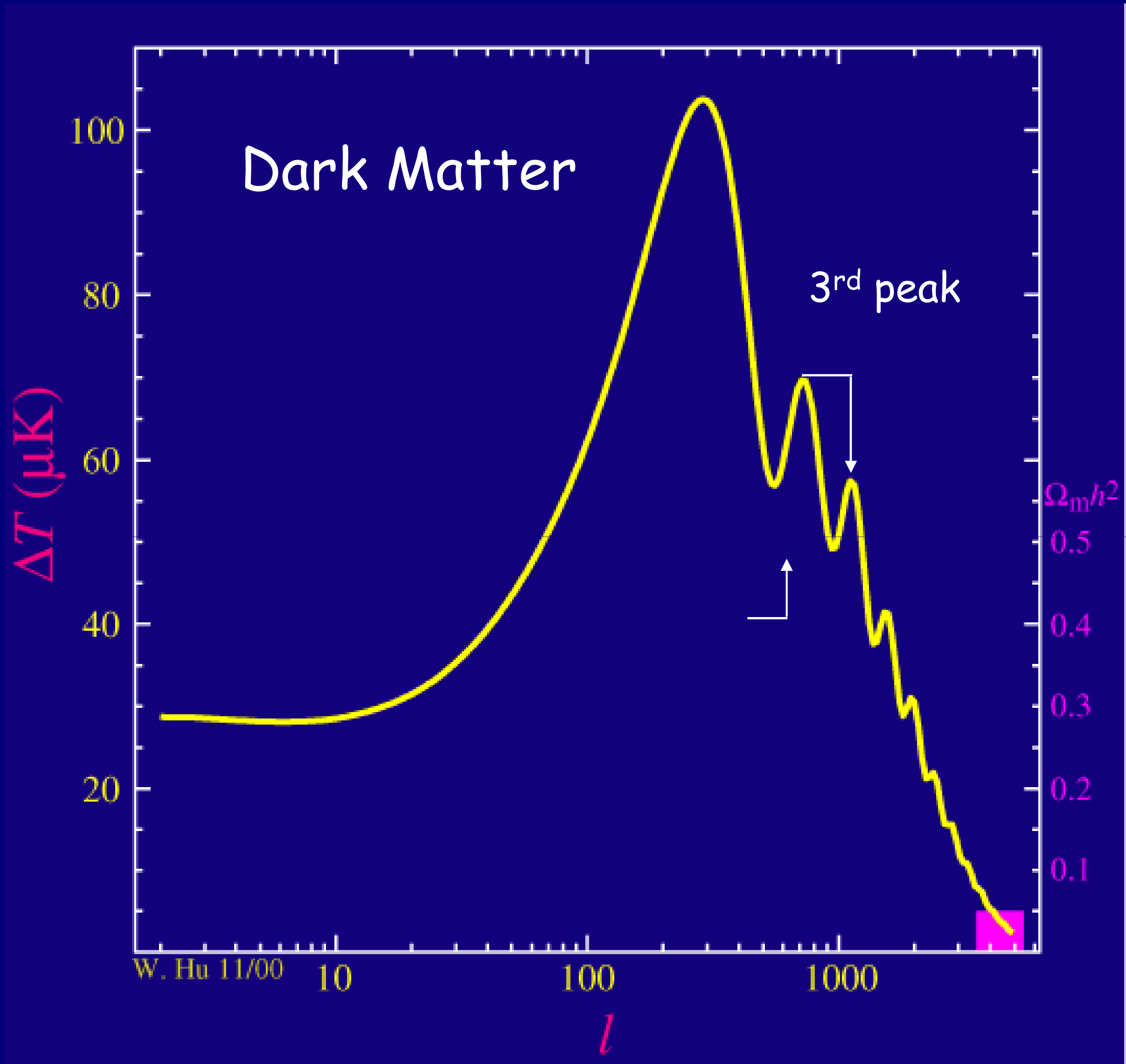
$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

$l_A = 301^{+1}_{-1}$

$r_s = 147^{+2}_{-2}$ Mpc

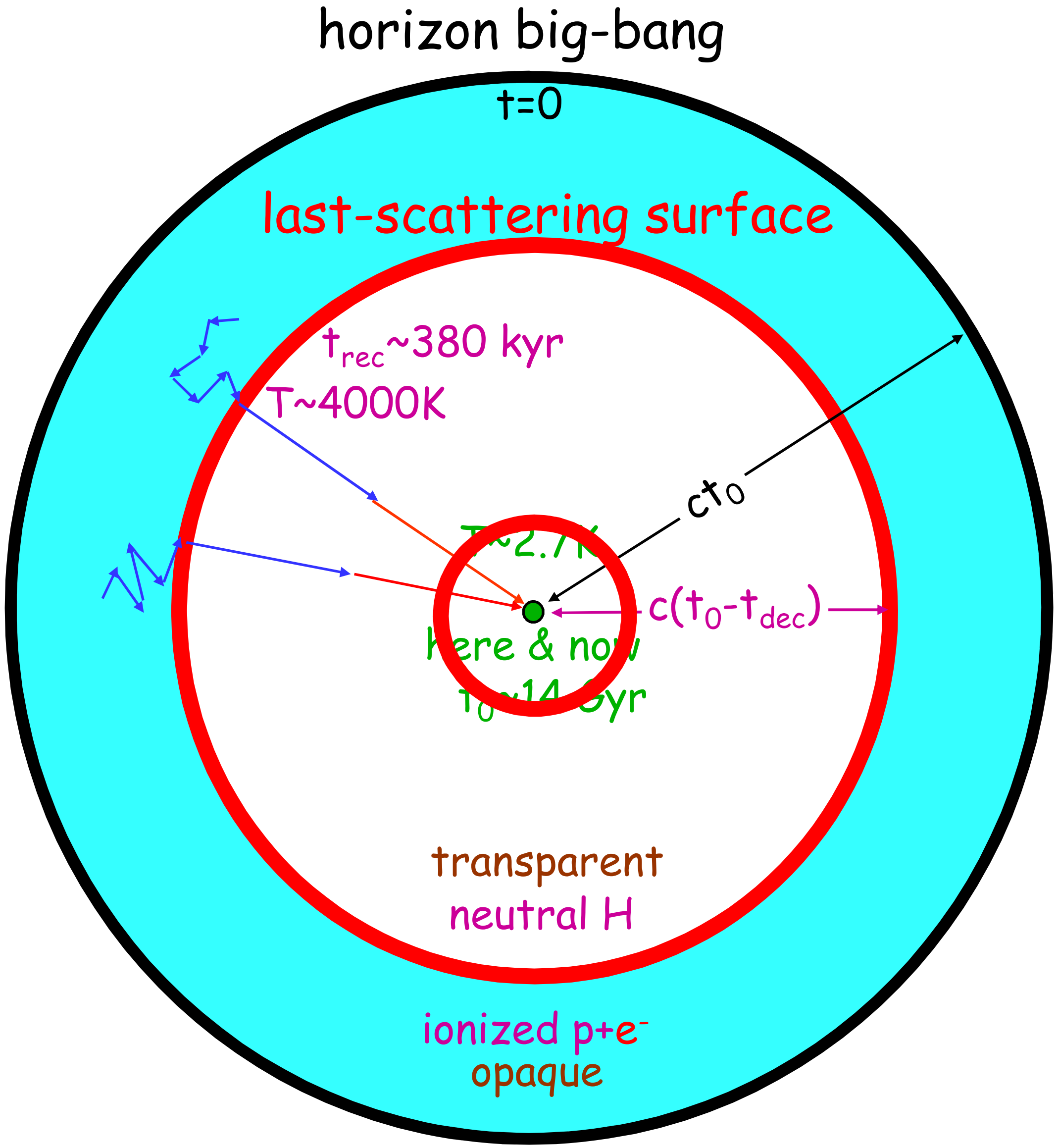


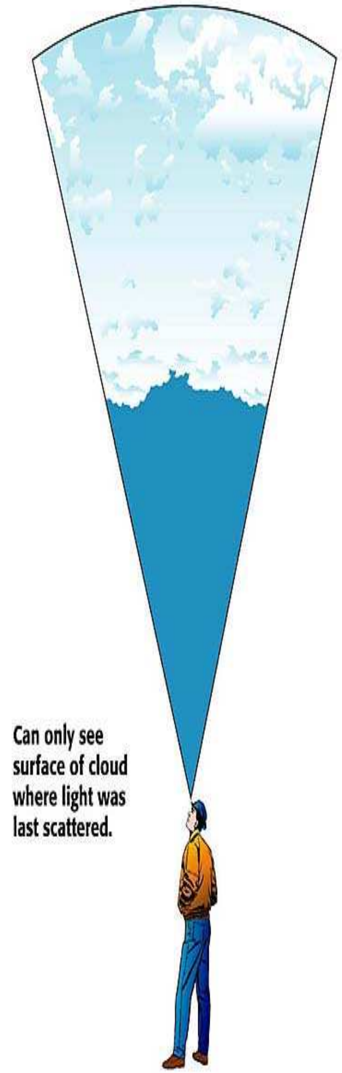
Baryons
2nd peak
100
80
60
40
20
ΔT (μK)
$\Omega_b h^2$
0.06
0.04
0.02
W. Hu 11/00
10
100
1000
l



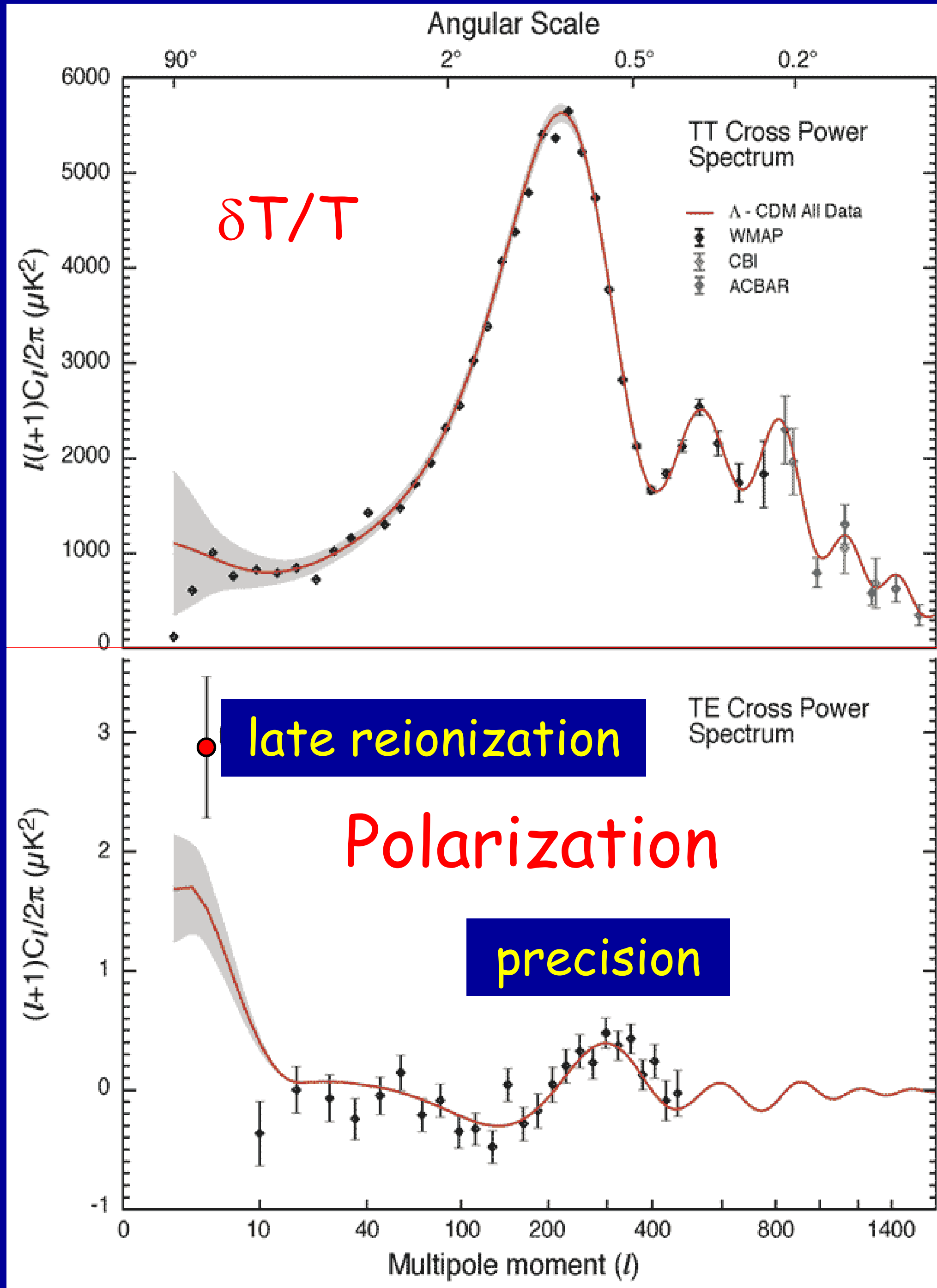
Dark Matter
3rd peak
100
80
60
40
20
ΔT (μK)
Ωmh2
0.5
0.4
0.3
0.2
0.1
W. Hu 11/00
10
100
1000
l

Late Re-ionization → Polarization of CMB

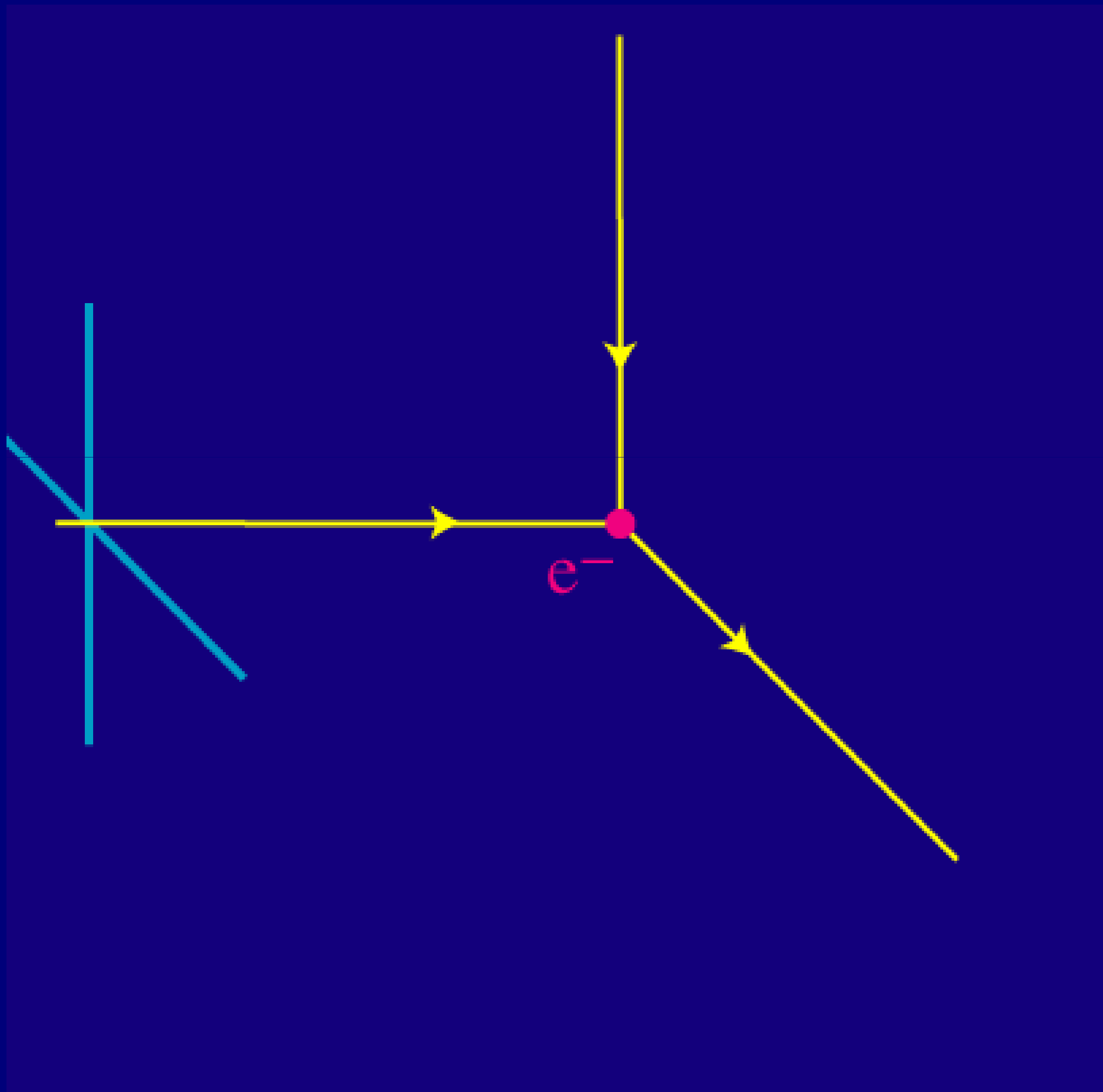




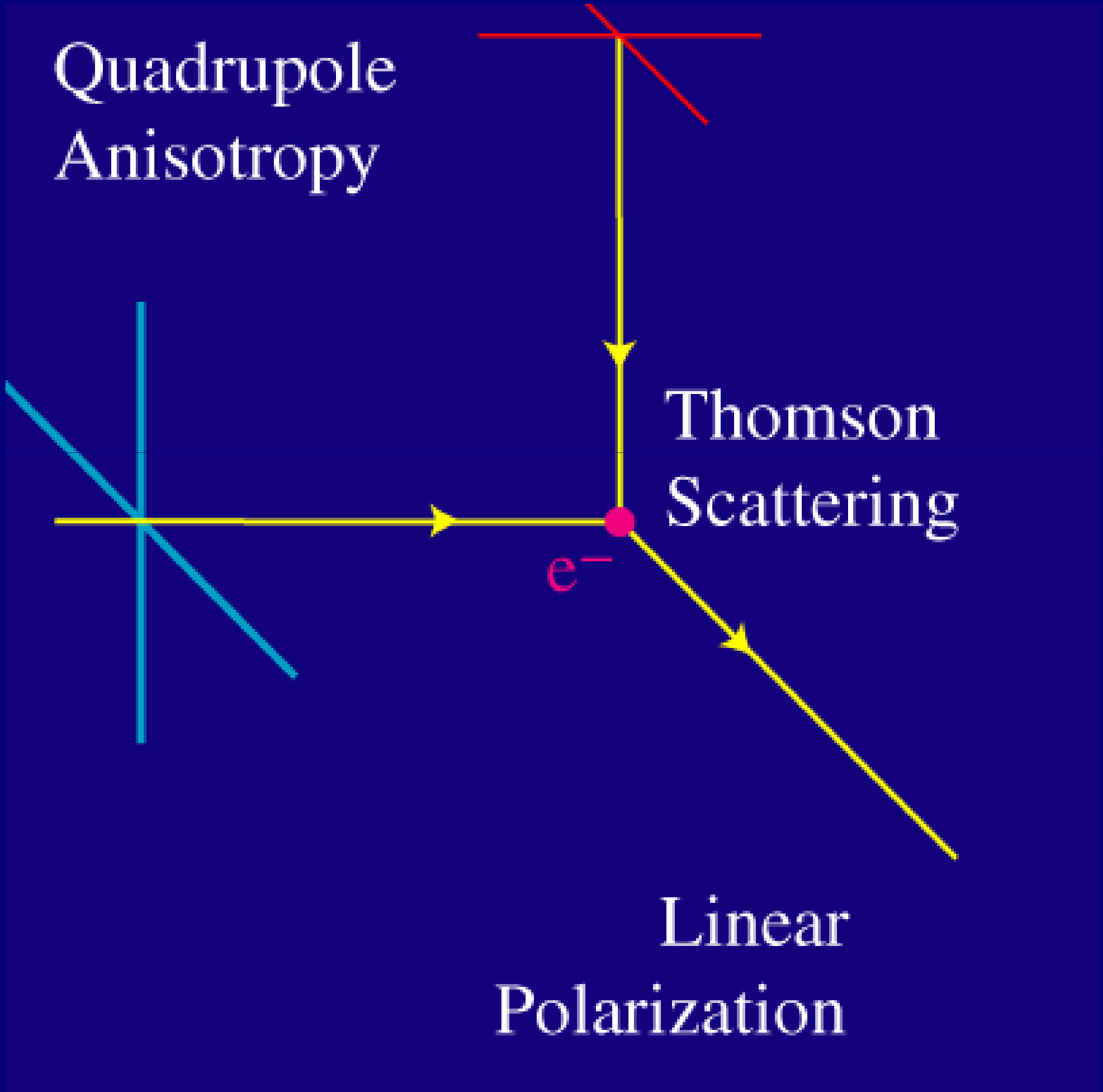
Anisotropy power spectrum by WMAP



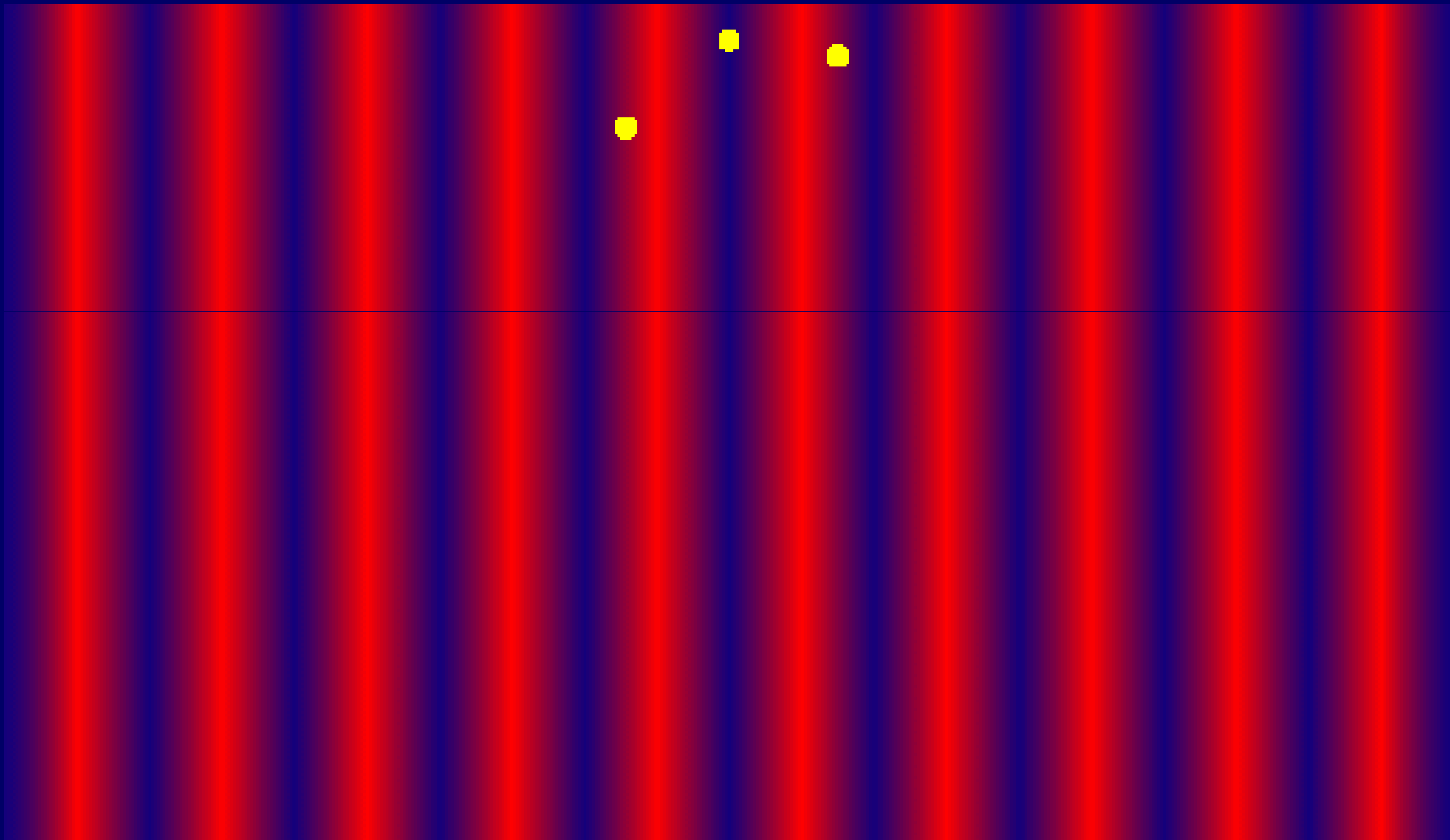
Polarization by scattering off free e^-



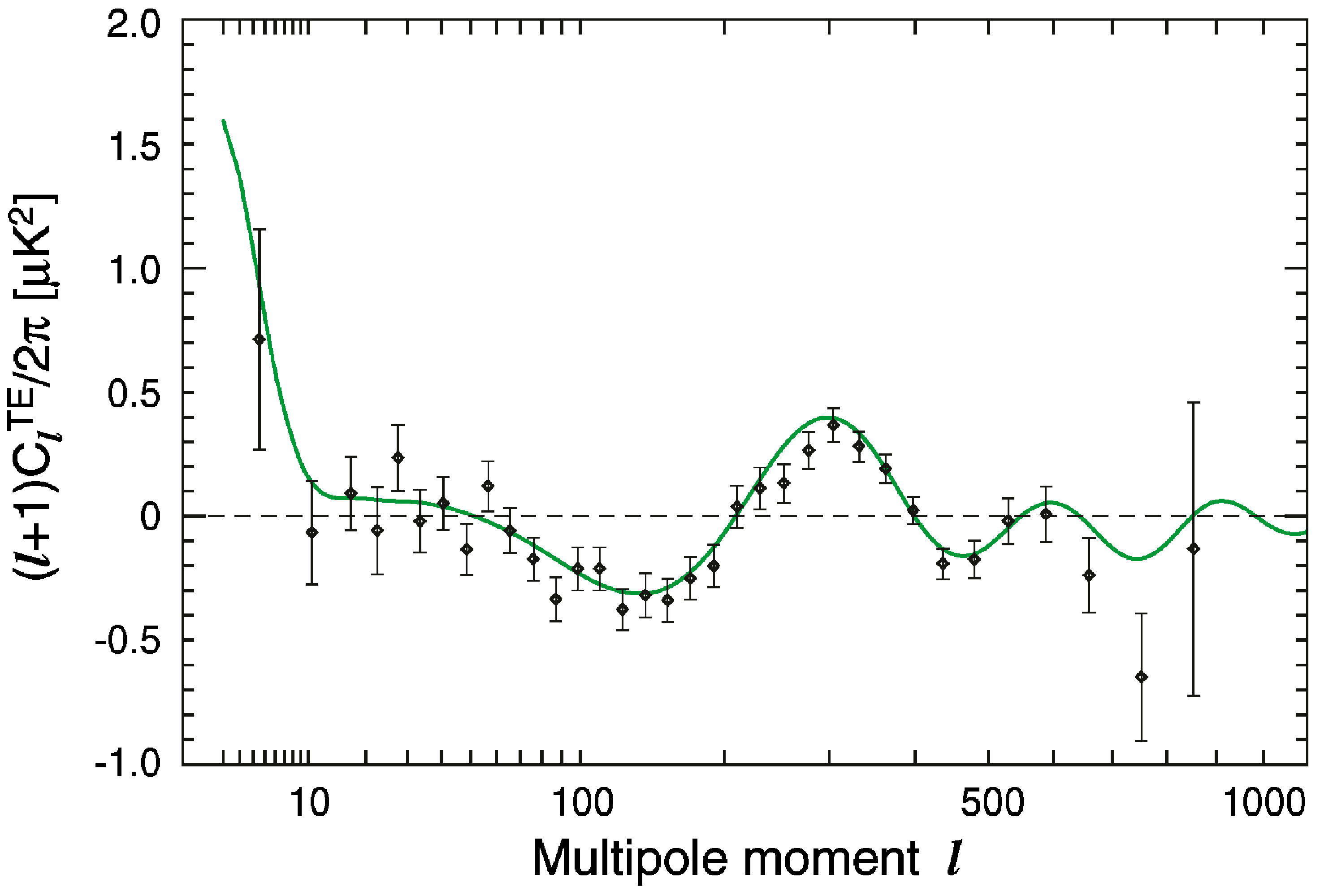
Polarization by scattering off free e^-



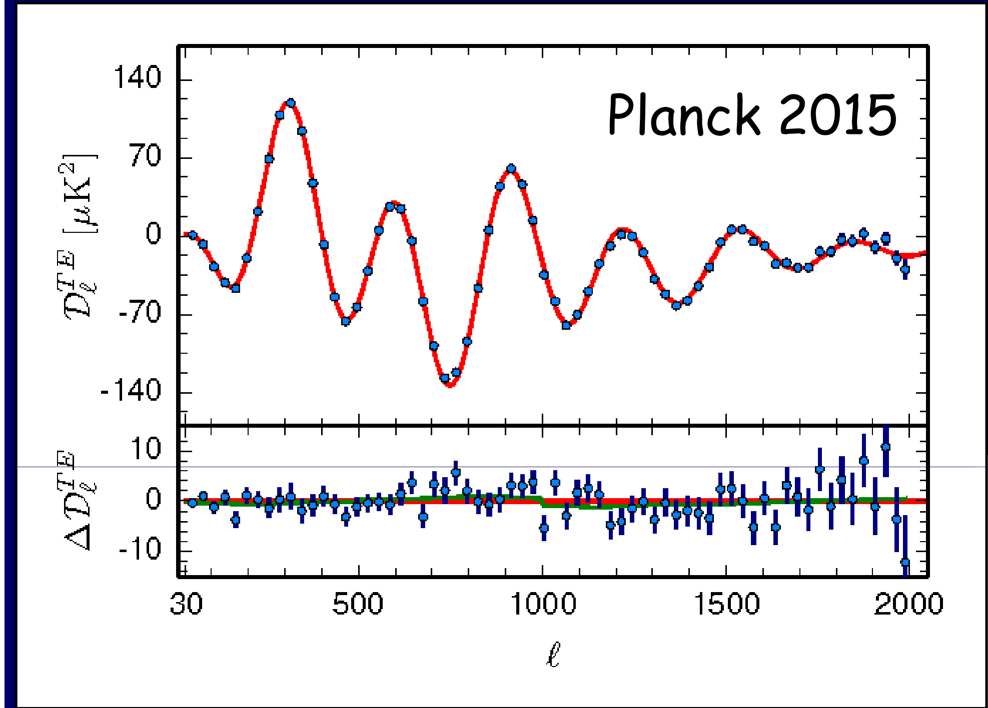
Polarization anisotropy due to quadrupole on last-scattering surface

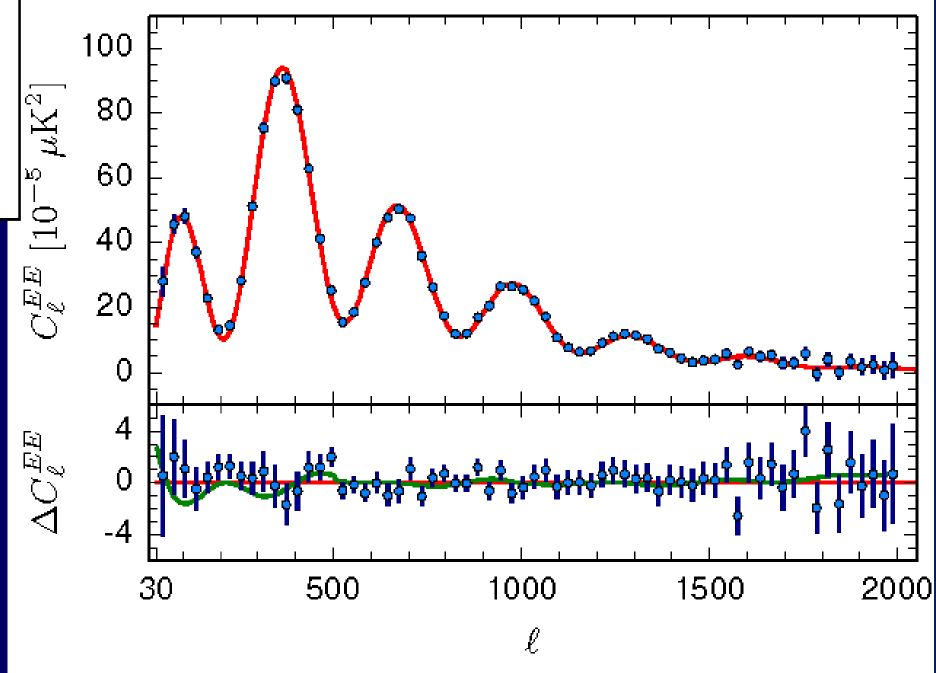


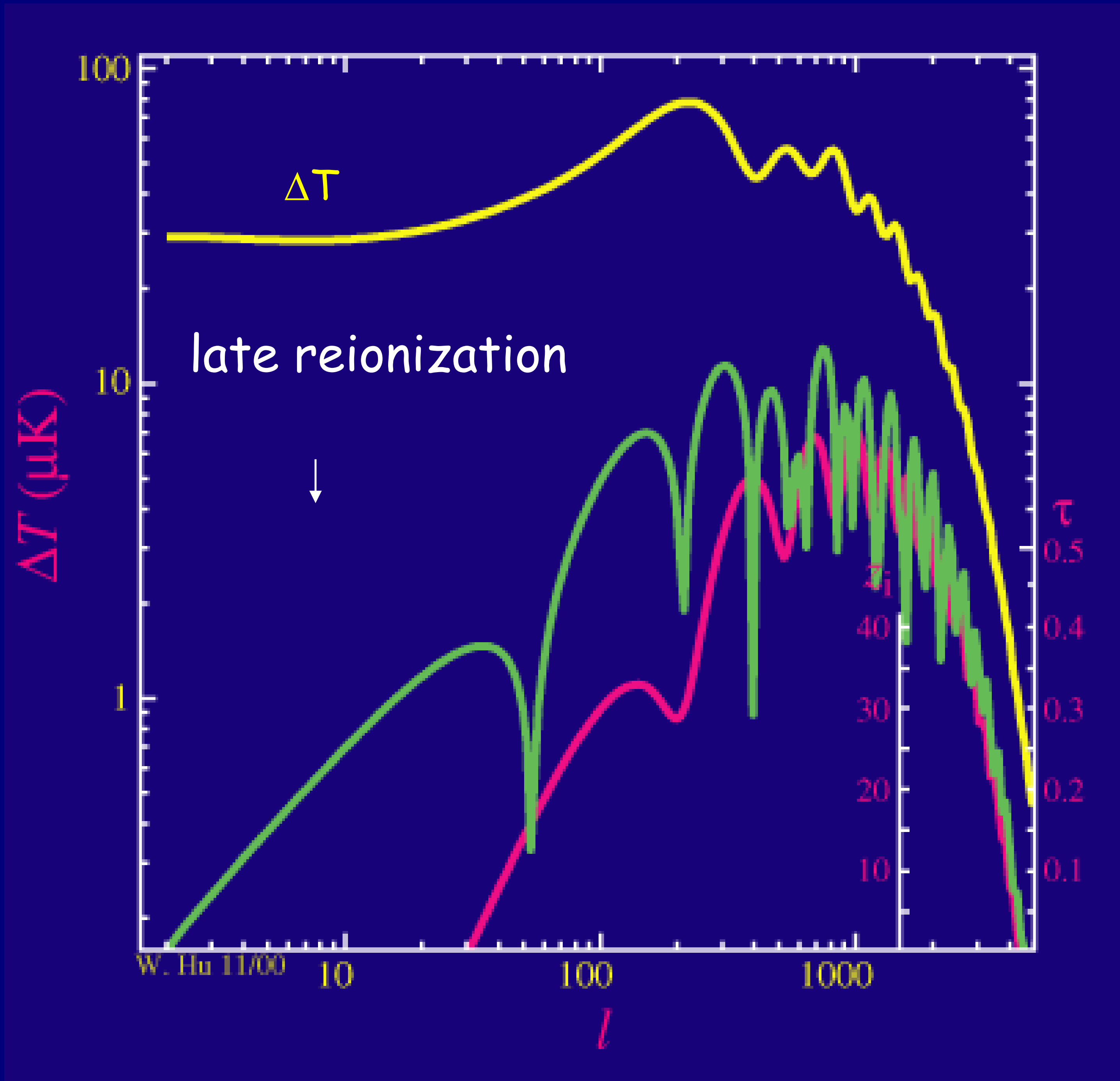
Polarization by scattering off electrons; re-ionization by stars & quasars at z~10



Polarization by scattering off electrons; re-ionization by stars & quasars at z~10





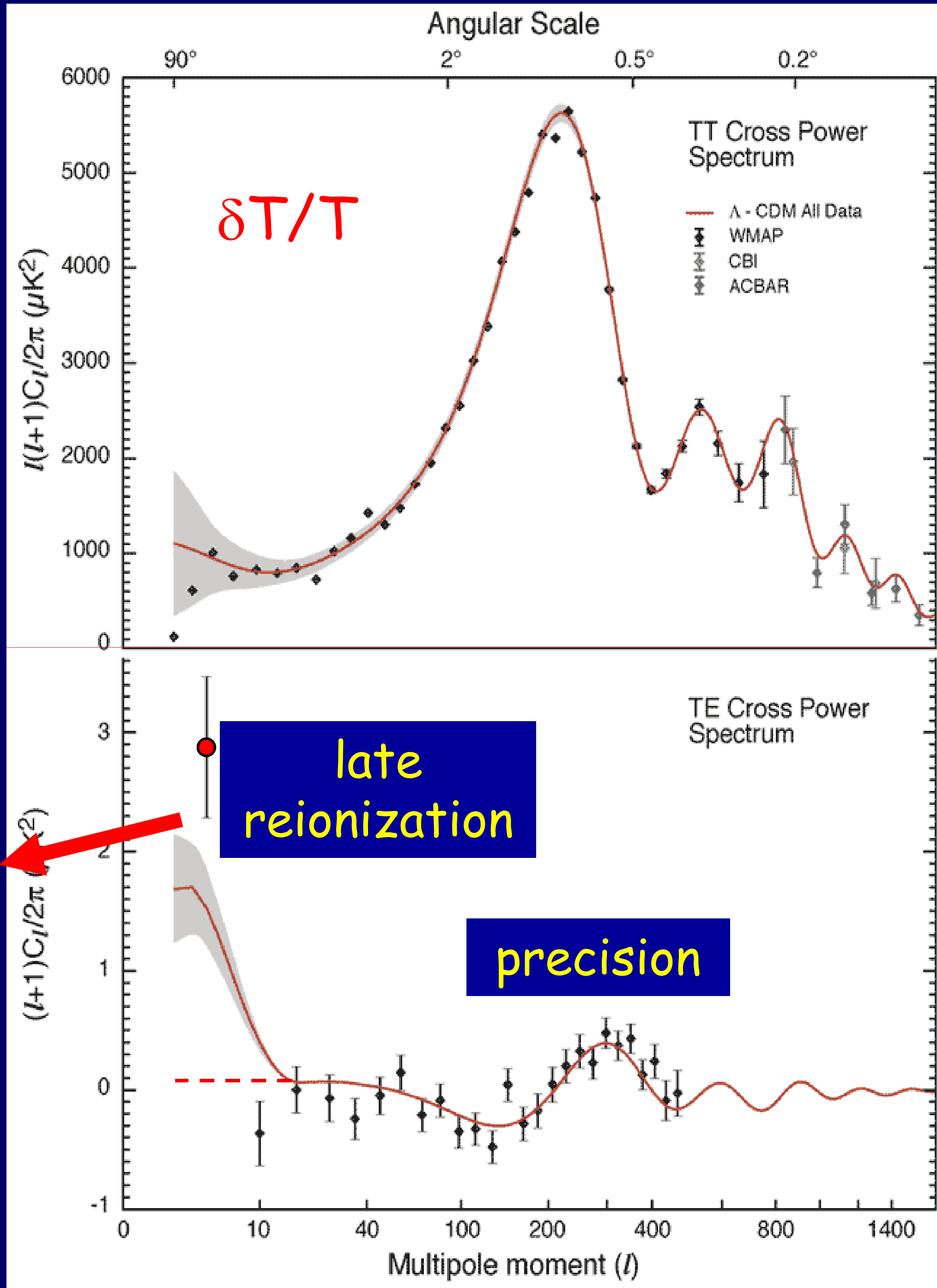


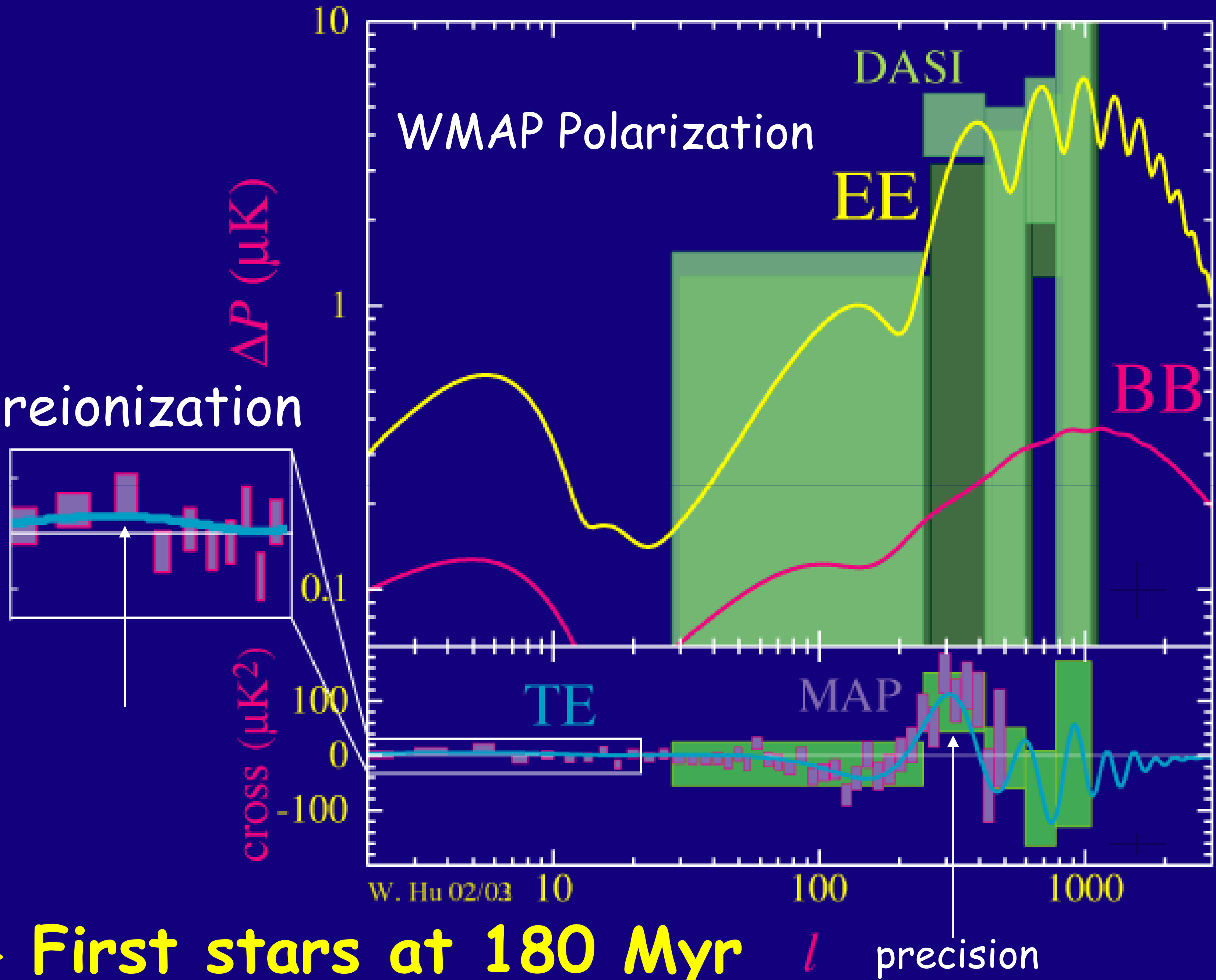
ΔT
late reionization
100
10
1
ΔT (μK)
W. Hu 11/00
10
100
1000
l
τ
0.5
0.4
0.3
0.2
0.1
z_i
40
30
20
10

Anisotropy power spectrum by WMAP

Polarization

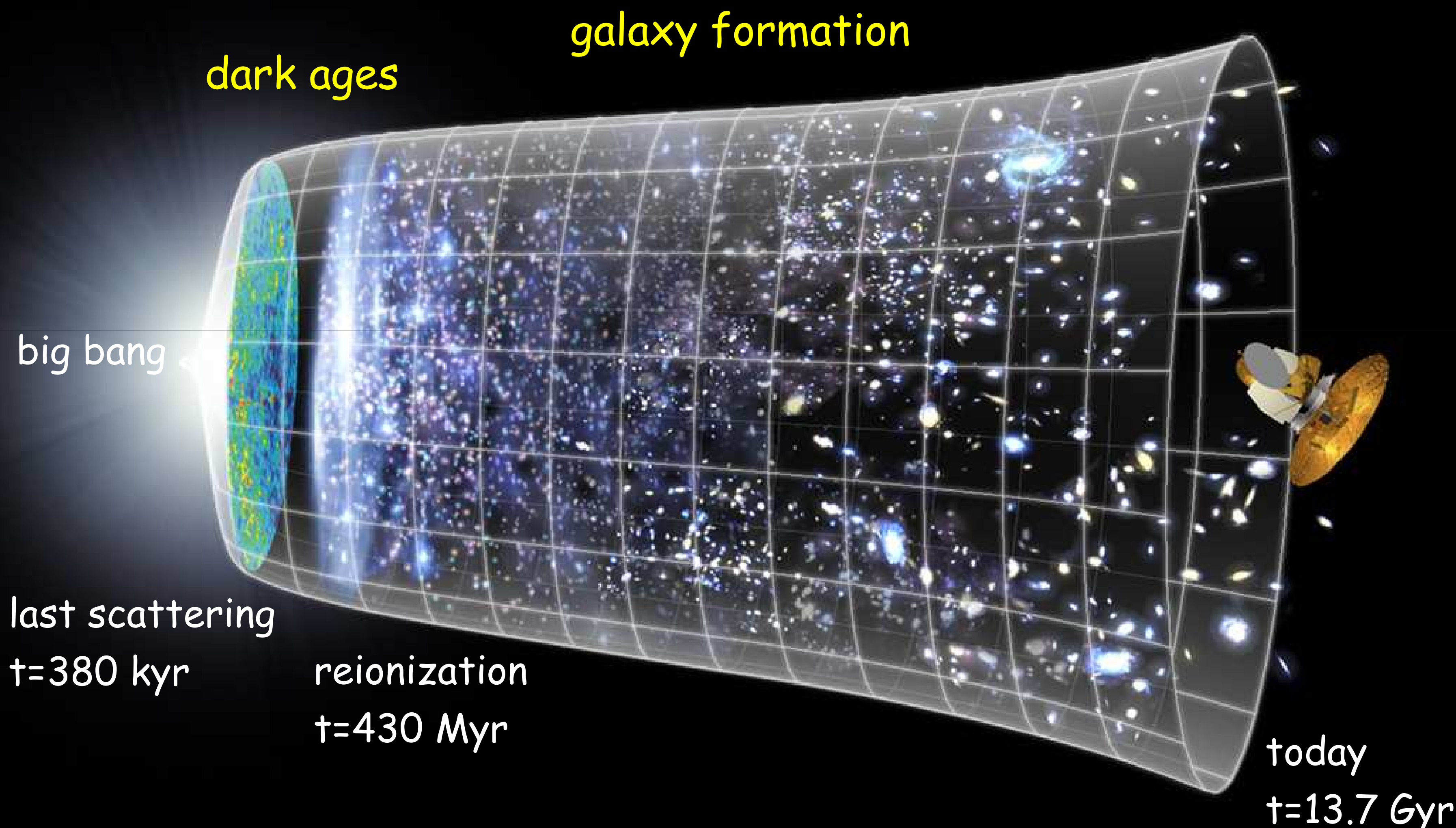
First stars at 180 Myr



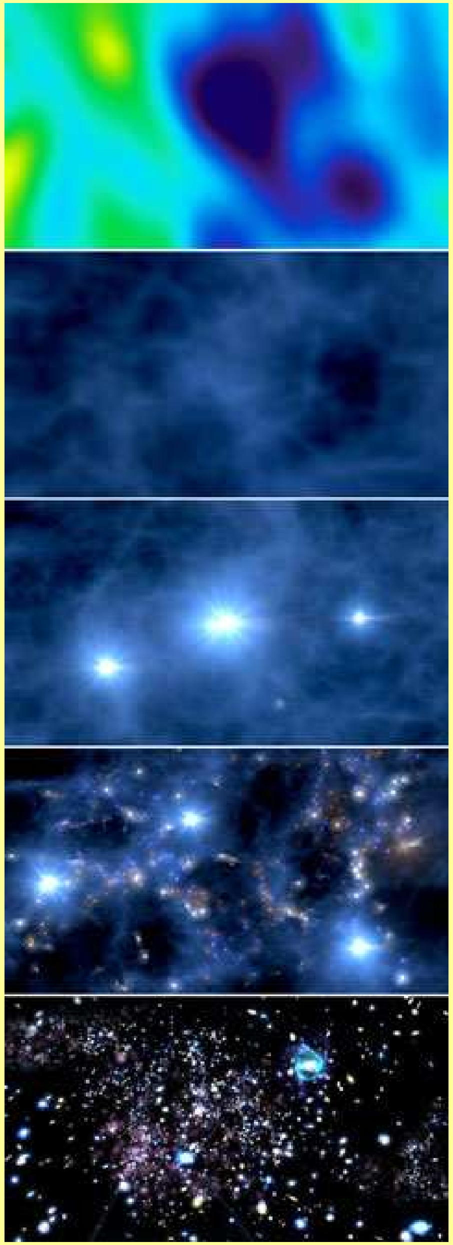


→ **First stars at 180 Myr**

Cosmic History



Cosmological Epochs



380 kyr z~1000 — recombination last scattering

dark ages

180 Myr z=8.8±1.5 — first stars reionization

galaxy formation

13.8 Gyr — today

Cosmological Parameters by WMAP

Old Universe – *New* Numbers

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$

$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$

$\Omega_\nu h^2 < 0.0076$ (95%CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

baryons/photons

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

amplitude of fluctuations

$n_s = 0.93^{+0.03}_{-0.03}$

initial power spectrum

$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0.598^{\circ}{}^{+0.002}_{-0.002}$

$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

$l_A = 301^{+1}_{-1}$

$r_s = 147^{+2}_{-2}$ Mpc

Inflation

Problems with standard hot-big-bang:

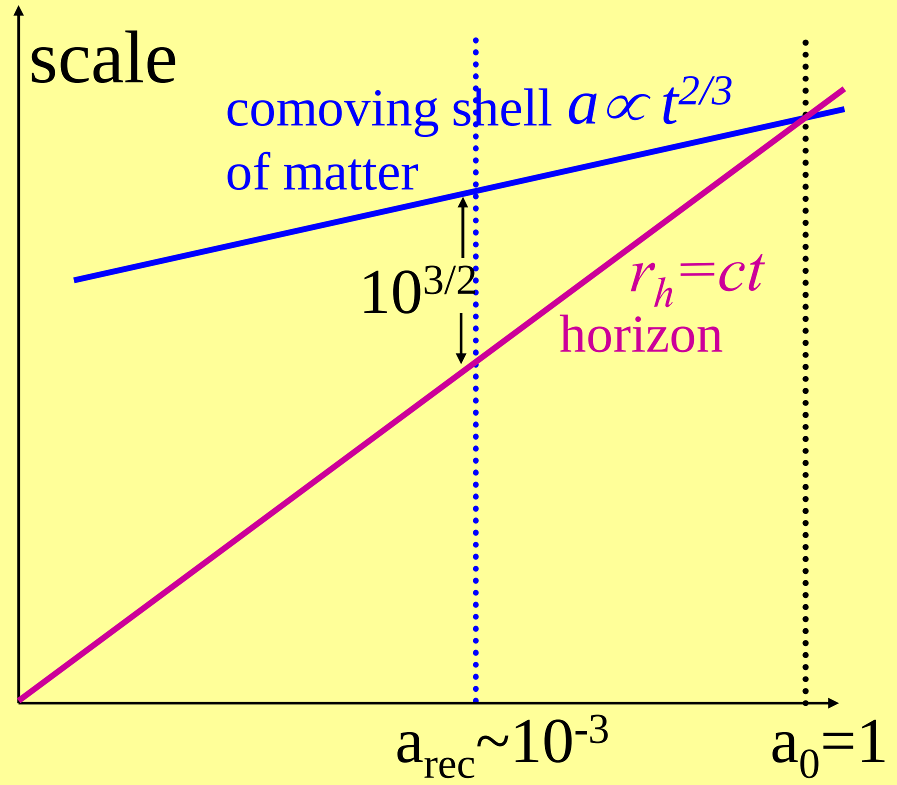
1. horizon-causality

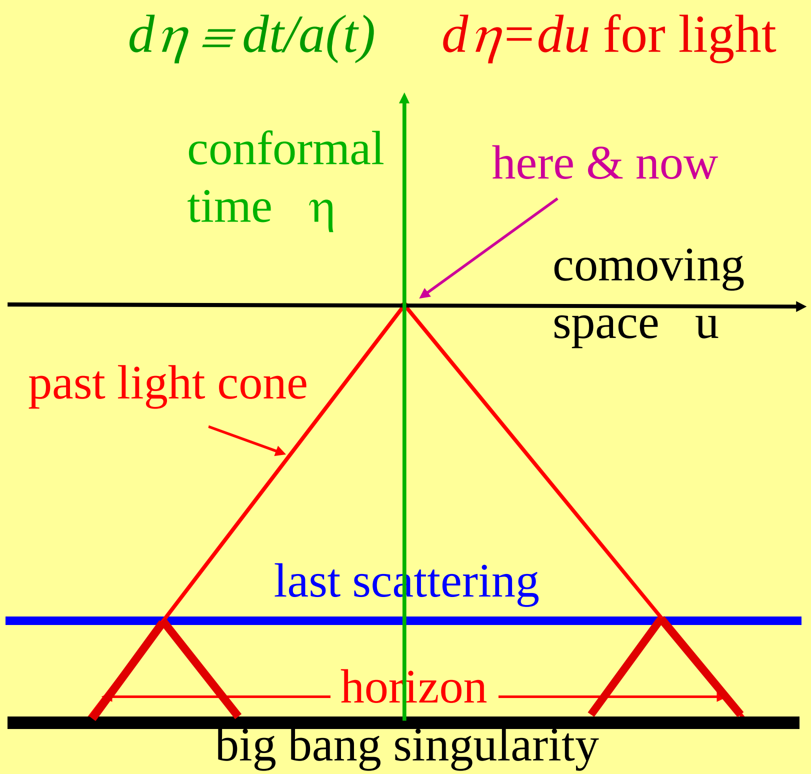
2. flattness

3. origin of expansion

4. origin of fluctuations

Horizon Causality Problem





$$\eta = \int \frac{dt}{a} \propto t^{1/2} \propto a \quad \text{radiation}$$

$$\propto t^{1/3} \propto a^{1/2} \quad \text{matter}$$

$$\theta_{causal} \sim \frac{u_h(t_{ls})}{u_h(t_0)} \sim \frac{\eta_{ls}}{\eta_0} \sim \left(\frac{a_{ls}}{a_0}\right)^{1/2} \sim \frac{1}{1100^{1/2}} \sim \frac{1}{30} \sim 2^\circ$$

Flattness problem

Friedman equation: $$\dot{a}^2 - \frac{8\pi G}{3}\rho_{tot} a^2 = -kc^2$$

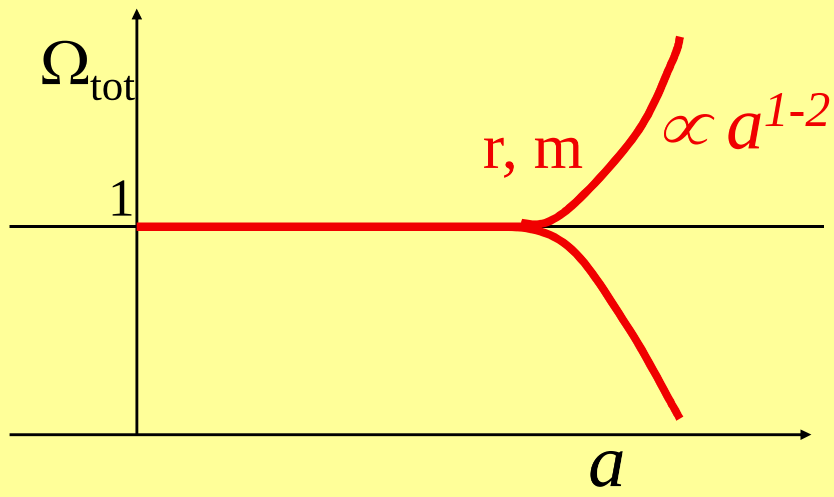
density parameter $$\Omega_{tot} \equiv \frac{\rho_{tot}}{3H^2/8\pi G}$$

$$\rho_{tot} = \rho_r + \rho_m + \rho_\Lambda$$

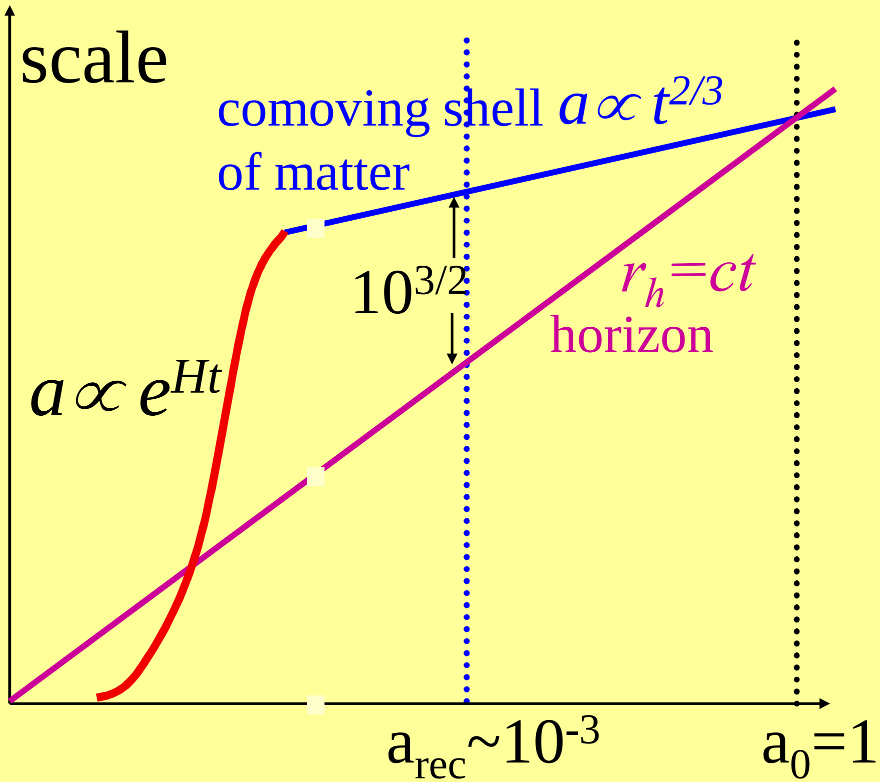
$$\rho_r \propto a^{-4} \quad \rho_m \propto a^{-3}$$

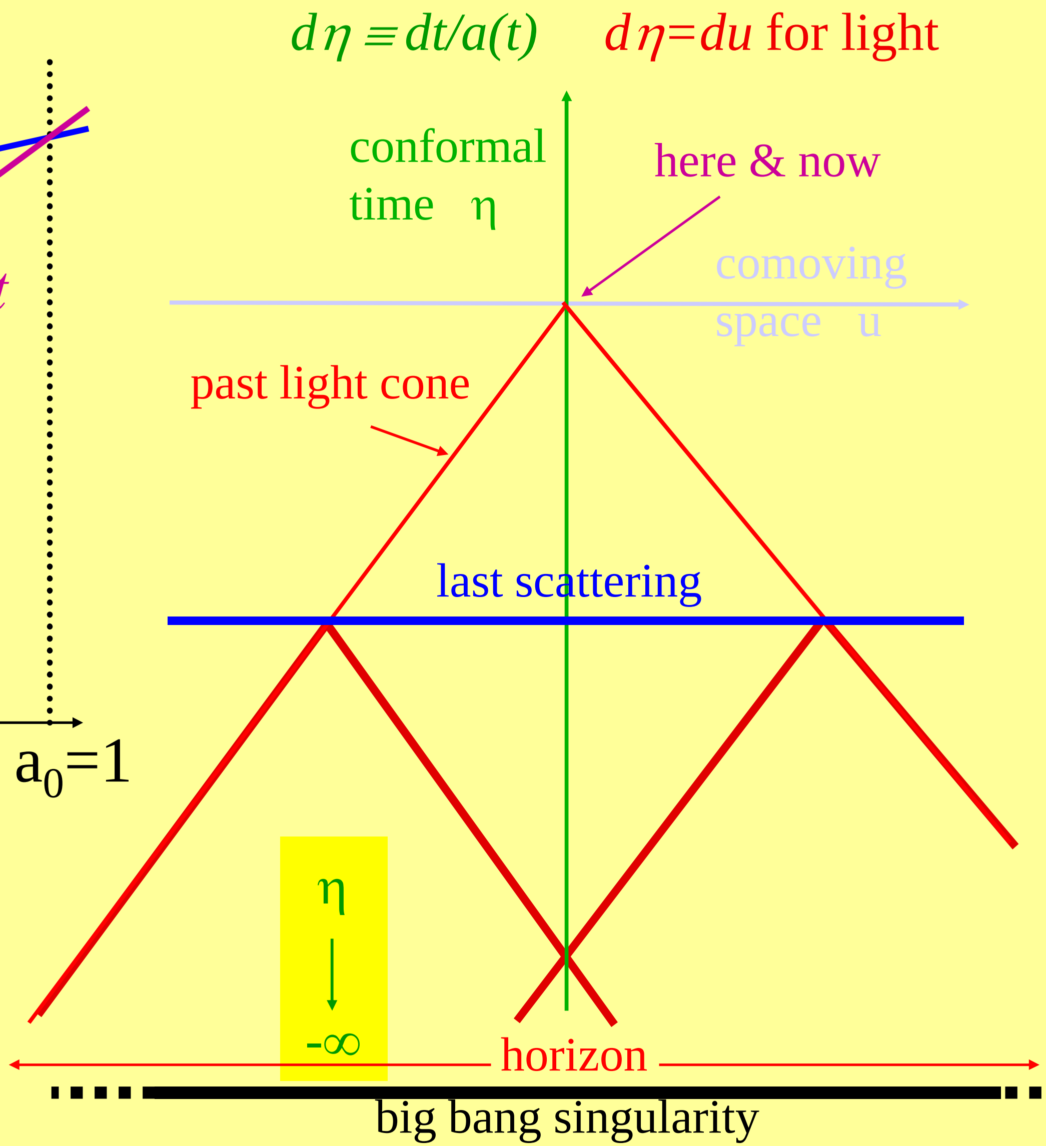
$$\rho_\Lambda \propto \Lambda = const.$$

$$\Omega_{tot}^{-1} - 1 = -\frac{3k}{8\pi G \rho a^2}$$



Inflation → Causality





$$\eta = \int \frac{dt}{a} \propto t^{1/2} \propto a \quad \text{radiation}$$

$$\propto t^{1/3} \propto a^{1/2} \quad \text{matter}$$

$$\propto -e^{-Ht} \propto -a^{-1} \quad \text{vacuum}$$

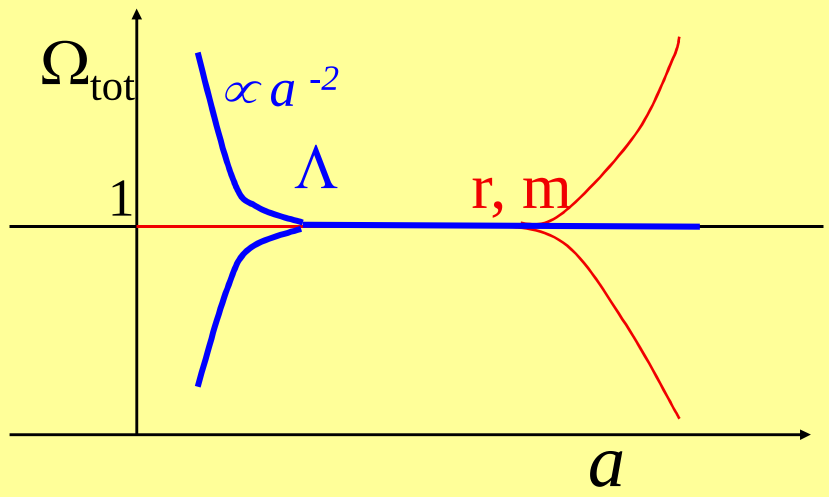
Inflation → Flattness

Friedman equation: $$\dot{a}^2 - \frac{8\pi G}{3}\rho_{tot}a^2 = -kc^2$$

density parameter $$\Omega_{tot} \equiv \frac{\rho_{tot}}{3H^2/8\pi G}$$

$$\rho_{tot} = \rho_r + \rho_m + \rho_\Lambda \qquad \rho_r \propto a^{-4} \quad \rho_m \propto a^{-3} \quad \rho_\Lambda \propto \Lambda = const.$$

$$\Omega_{tot}^{-1} - 1 = -\frac{3k}{8\pi G\rho a^2}$$



Inflation with Λ

FRW: $\dot{a}^2 - \frac{8\pi G}{3}\rho_{tot} a^2 = -kc^2$

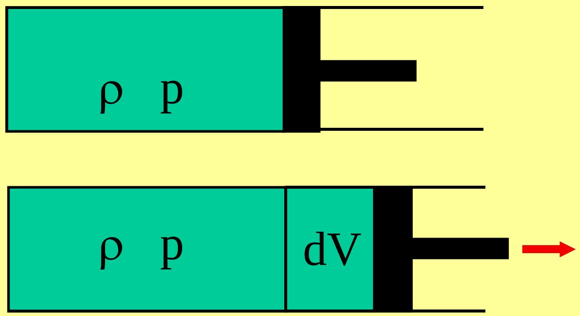
$$\frac{8\pi G}{3}\rho = H_0^{\,2}(\Omega_{\Lambda 0} + \Omega_{m0}a^{-3} + \Omega_{r0}a^{-4})$$

if $\rho_{tot} \approx \rho_\Lambda \approx const.$ $k = 0$ $\longrightarrow$ $H \equiv \frac{\dot{a}}{a} = \left(\frac{\Lambda c^2}{3}\right)^{1/2}$ $a \propto e^{Ht}$

Energy conservation:

$$d(\rho c^2 a^3) = -p\,d(a^3)$$

$\longrightarrow$ $\ddot{a} = -\frac{4\pi G}{3} a \left(\rho + \frac{3p}{c^2}\right)$



if ρ_m dominates $p_m \approx 0$ $\rho_m \propto a^{-3}$ $\ddot{a} = -GV\rho_m / a^2$

if ρ_r dominates $p_r / c^2 = \rho / 3$ $\rho_r \propto a^{-4}$ $\ddot{a} = -2GV\rho_r / a^2$

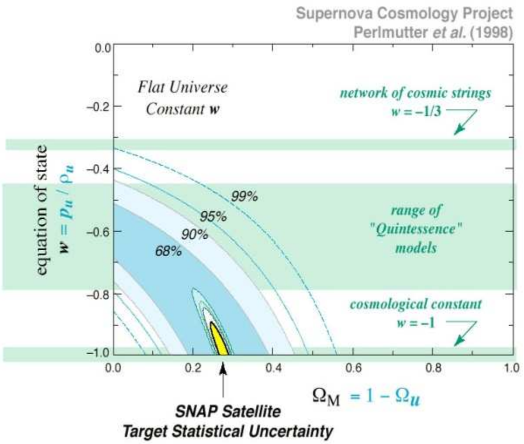
if ρ_Λ dominates $p_\Lambda / c^2 = -\rho$ $\rho_\Lambda = const.$ $\ddot{a} = 2GV\rho_\Lambda / a^2$ repulsion possible

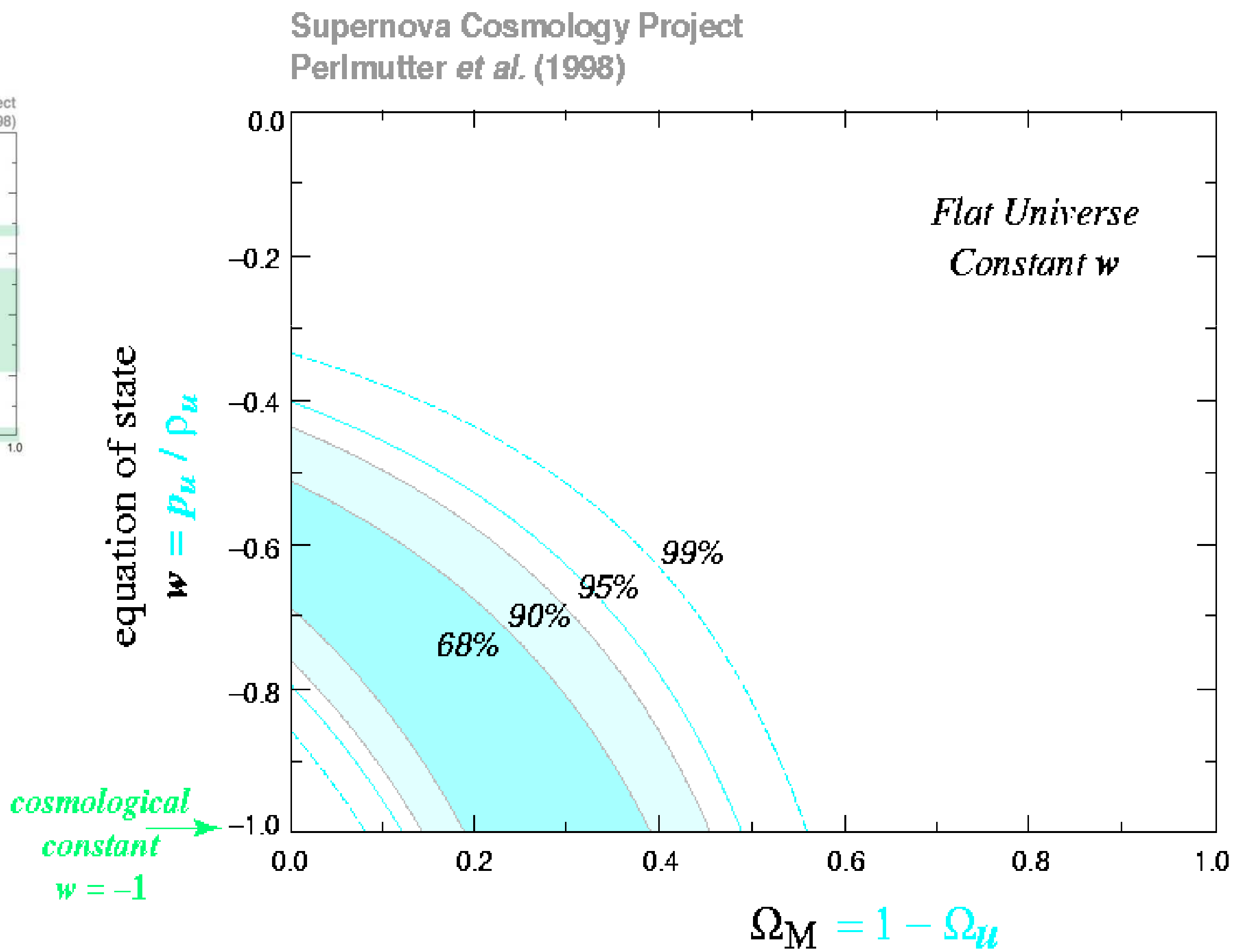
Quintessence: $p / c^2 \equiv \omega\rho$ $\Lambda \leftrightarrow \omega = -1$

for inflation need $\ddot{a} > 0$ to exceed $r_h \propto ct \rightarrow \ddot{r}_h = 0$ $\longrightarrow$ $\omega < -1/3$

Dark Energy by SN

Unknown Component
Ω_u
of Energy Density





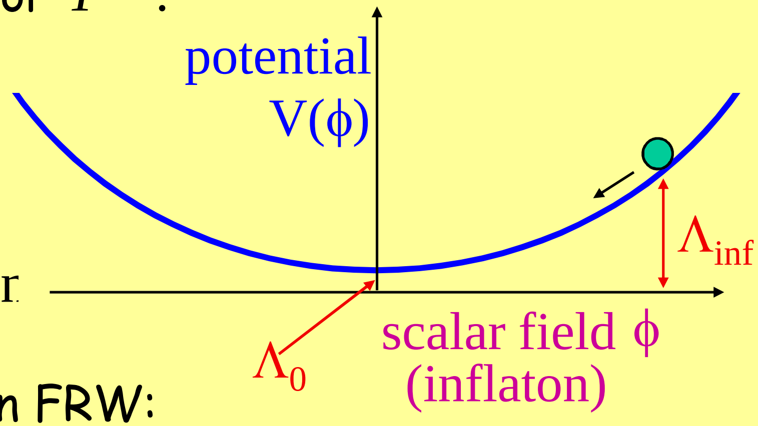
Inflation in field theory

From Lagrangian and energy-momentum tensor $T^{\mu\nu}$:

$$\rho = \dot{\phi}^2/2 + V(\phi) + (\nabla\phi)^2/2$$ fluctuations

$$p = \dot{\phi}^2/2 - V(\phi) + (\nabla\phi)^2/6$$

if $\dot{\phi}^2 << V$ and $\nabla\phi \approx 0 \rightarrow p \approx -\rho \approx -V \rightarrow$ inflation



When slow roll? Fluid eq. for inflaton field in FRW:

$$\dot{\rho} + 3H(\rho + p) = 0$$

example: $V(\phi) = m^2\phi^2/2$

→Equation of motion: $\ddot{\phi} + 3H\dot{\phi} = \nabla^2\phi - \partial V/\partial\phi$

slow roll approximation: $\ddot{\phi}$ negligible (and $\nabla^2\phi \approx 0$)

"friction" leads to terminal constant "velocity"

$$3H\dot{\phi} = -\partial V/\partial\phi \quad \rightarrow \quad V \approx const \quad \rightarrow \quad \text{inflation}$$

$$\dot{\phi}^2 << V \quad \leftrightarrow \quad \left(\frac{\partial V}{\partial\phi}\right)^2 << \frac{9H^2V}{\hbar c^3} = \frac{24\pi G V^2}{\hbar c^5} \quad \left(H^2 = \frac{8\pi G V}{3c^2}\right) \quad \longrightarrow \quad \left(\frac{E_P}{V}\frac{\partial V}{\partial\phi}\right)^2 << 1$$

FRW (k=0): $H^2 = \frac{8\pi G}{3}(\dot{\phi}^2/2 + V(\phi) + \rho_m + \rho_r)$

$H = \dot{a}/a \approx const \qquad a \propto e^{Ht}$

End of inflation

Reheating

Scale-invariant large-scale density fluctuations from small quantum fluctuations in ϕ

Planck Scale

In the early universe gravity dominates (asymptotic freedom for GUTs)

Strength of gravity compared to quantum effects:

$$\Delta E_{QM} \cdot t \approx h \quad E_G \approx mc^2 \approx \frac{Gm^2}{l} \quad l \approx ct \text{ horizon}$$

$$\alpha_G = \frac{E_G}{\Delta E_{QM}} \approx \frac{Gm^2/ct}{h/t}$$

In analogy to strength of E&M interaction compared to quantum effects:

$$\alpha_{EM} = \frac{E_{EM}}{\Delta E_{QM}} = \frac{e^2/ct}{h/t} = \frac{e^2}{ch} \approx \frac{1}{137}$$

Quantum gravity when $\alpha_G \sim 1$

$$m_P = \left(\frac{hc}{G}\right)^{1/2} \approx 2.5\times10^{-5}\,g$$

$$E_P = \left(\frac{hc^5}{G}\right)^{1/2} \approx 1.2\times10^{19}\,GeV$$

$$t_P = \left(\frac{hG}{c^5}\right)^{1/2} \approx 10^{-43}\,s$$

$$l_P = \left(\frac{Gh}{c^3}\right)^{1/2} \approx 1.7\times10^{-33}\,cm$$

$$\frac{a_P}{a_0} \approx \left(\frac{t_P}{t_0}\right)^{1/2} \approx 5\times10^{-31}$$

$$\rho_P \approx 1.2\times10^{93}\,g/cm^3$$

$$T_P \approx 6\times10^{30}\,K$$

יקומים רבים ?