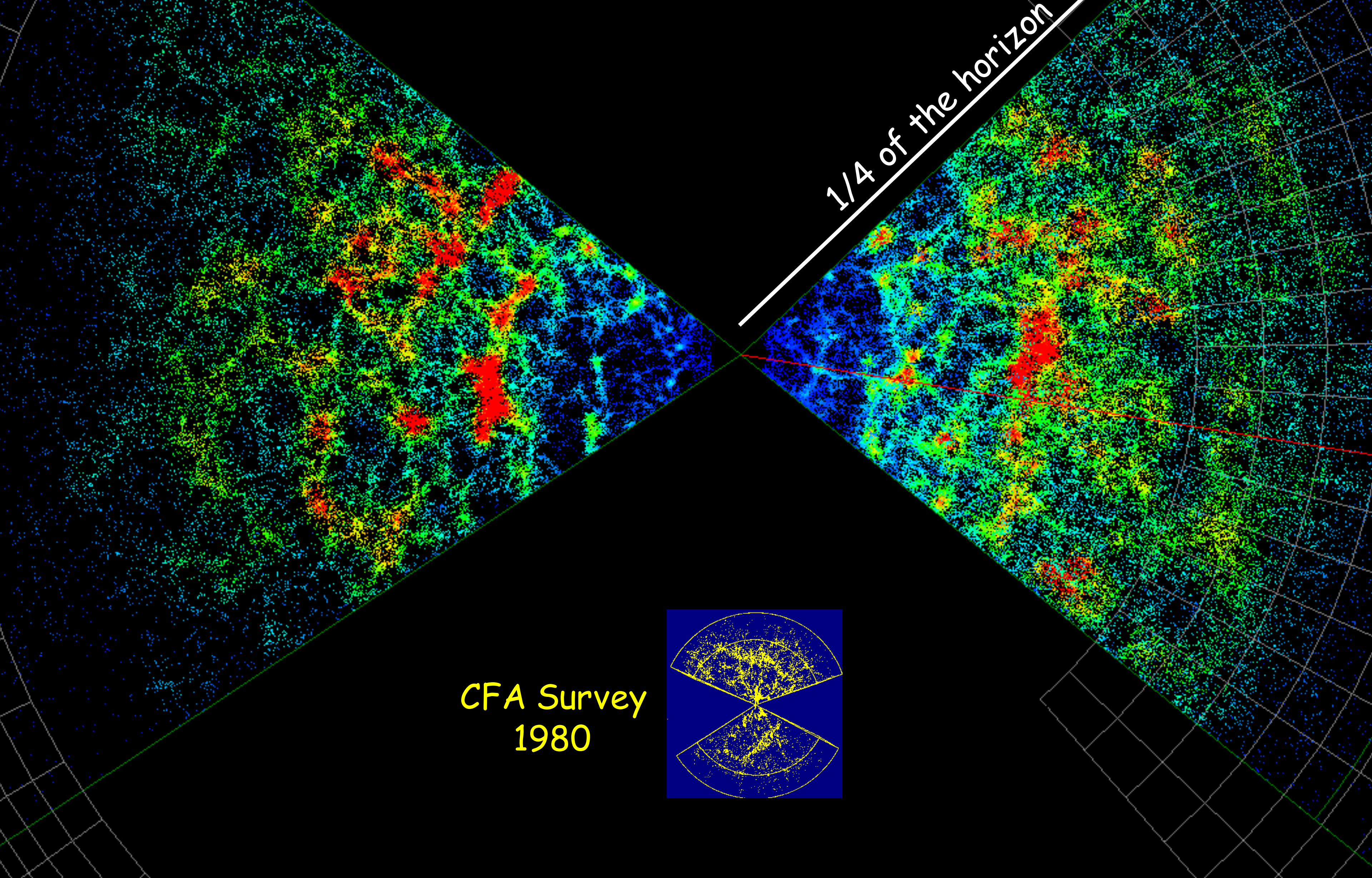
2dF Galaxy Redshift Survey
¼ M galaxies 2003



CFA Survey
1980

The Initial Fluctuations

At Inflation: Gaussian, adiabatic

fluctuation field $$\delta(x) = \frac{\rho(x) - \langle \rho \rangle}{\langle \rho \rangle}$$ a realization of an ensemble
ensembe average ~ volume average

Fourier $$\delta(\vec{x}) = \sum_k \delta_{\vec{k}}\, e^{i\vec{k}\cdot\vec{x}}$$

Power Spectrum $$P(k) \equiv \langle |\tilde{\delta}(\vec{k})|^2 \rangle \quad \propto k^n$$

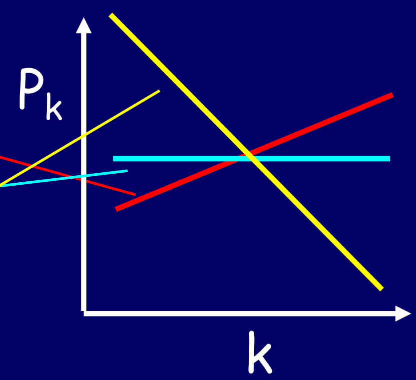
rms $$\langle \delta^2 \rangle_\lambda \sim \left\langle \int_{k=0}^{K} \int_{k'=0}^{K\sim 2\pi/\lambda} \exp[-i(k+k')\cdot x]\, d^3k'\, d^3k\ \delta_k \delta_{k'} \right\rangle \ \sim \int_{k=0}^{K} d^3k \langle \delta_k \delta_{-k} \rangle$$

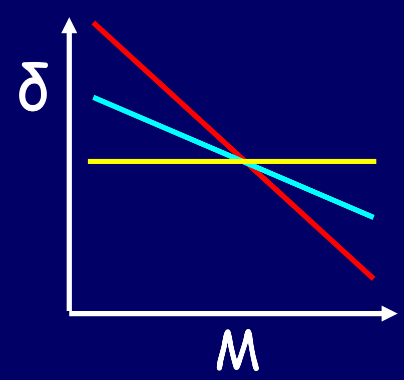
$$\leftarrow \delta_{Dirac}(k+k') \rightarrow$$

$$\langle \delta_k \delta_{-k} \rangle = \langle |\delta_k|^2 \rangle$$

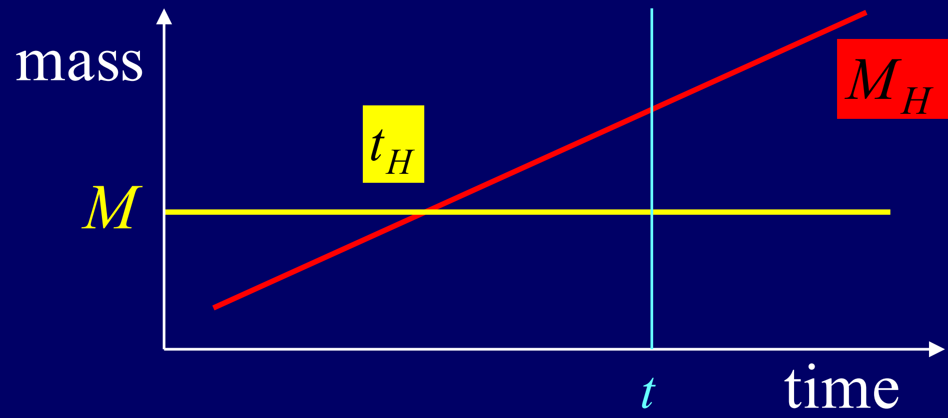
$$\langle \delta^2 \rangle_\lambda \propto \int_{k=0}^{2\pi/\lambda} P_k\, d^3k \quad \propto M^{-(n+3)/3}$$

$n = 1$	$\delta \propto M^{-2/3}$
$n = 0$	$\delta \propto M^{-1/2}$
$n = -3$	$\delta \propto const.$





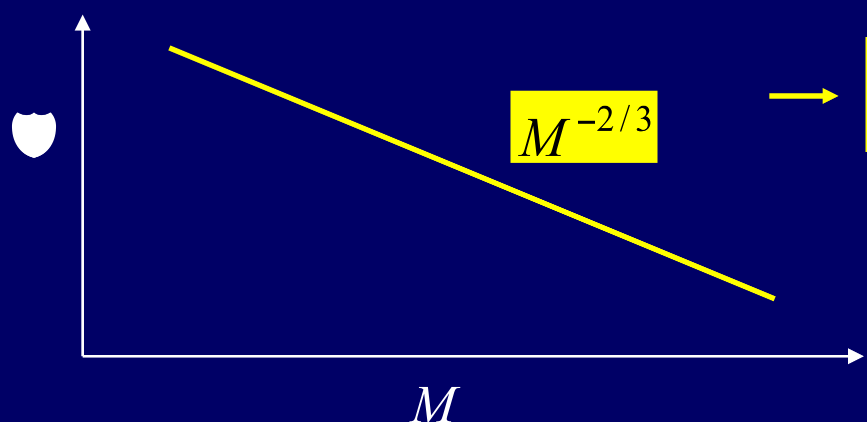
Scale-Invariant Spectrum (Harrison-Zel'dovich)



$$M_H \propto t \longleftarrow \begin{array}{l} R_{Horizon} \propto t \\ \rho_m \propto a^{-3} \propto t^{-2} \end{array}$$

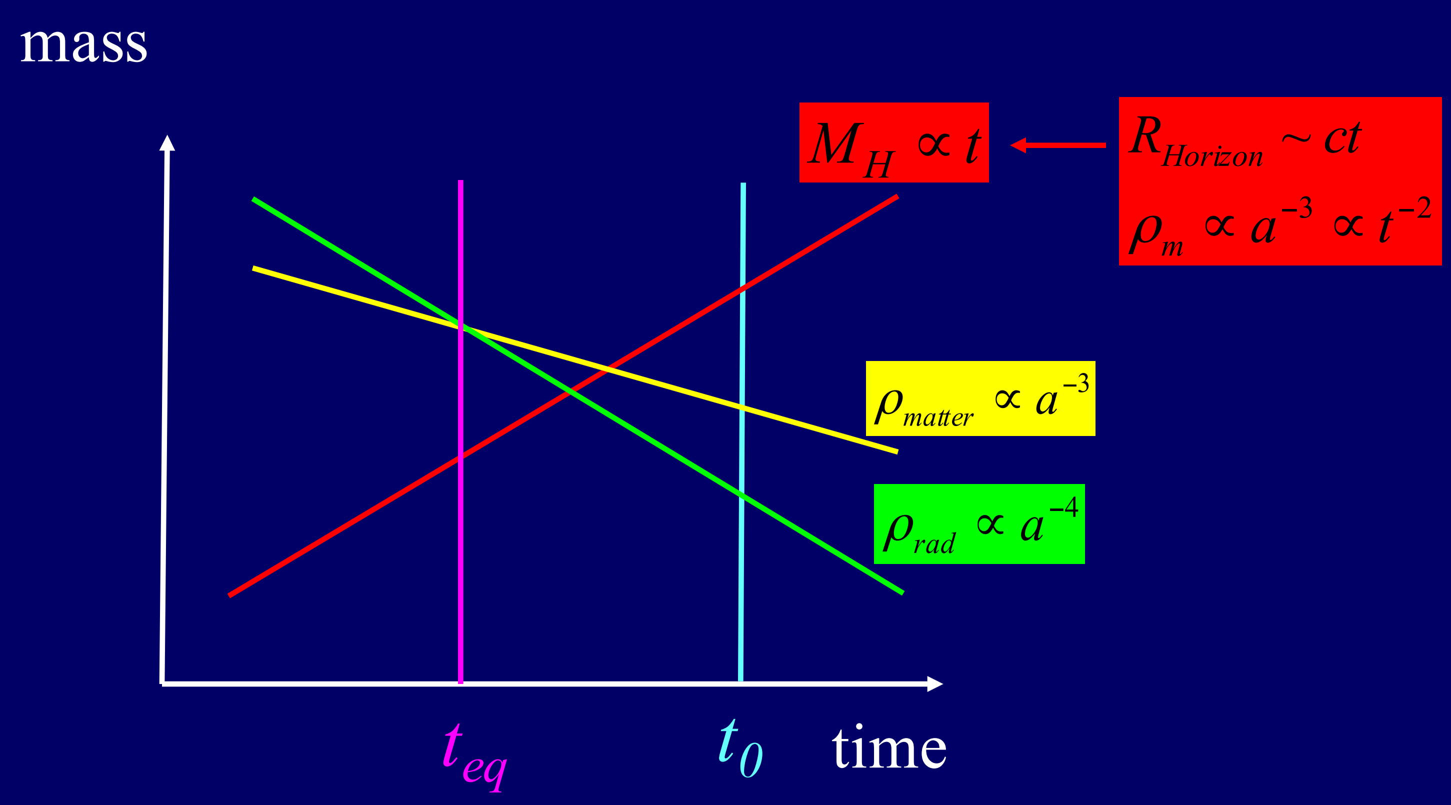
$$\delta(M,t) = \delta_H \left(\frac{t}{t_H(M)} \right)^{2/3} \propto M^{-2/3} t^{2/3}$$

$$\delta_H = const.$$



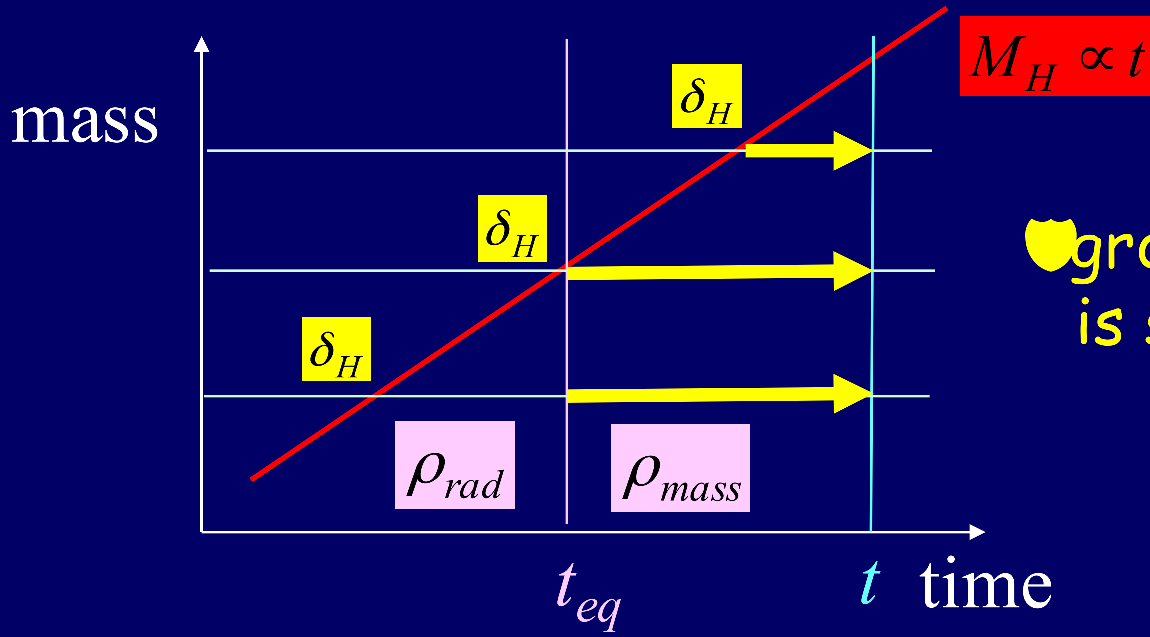
$$\longrightarrow \quad n = +1 \quad P_k \propto k$$

Cosmological Scales

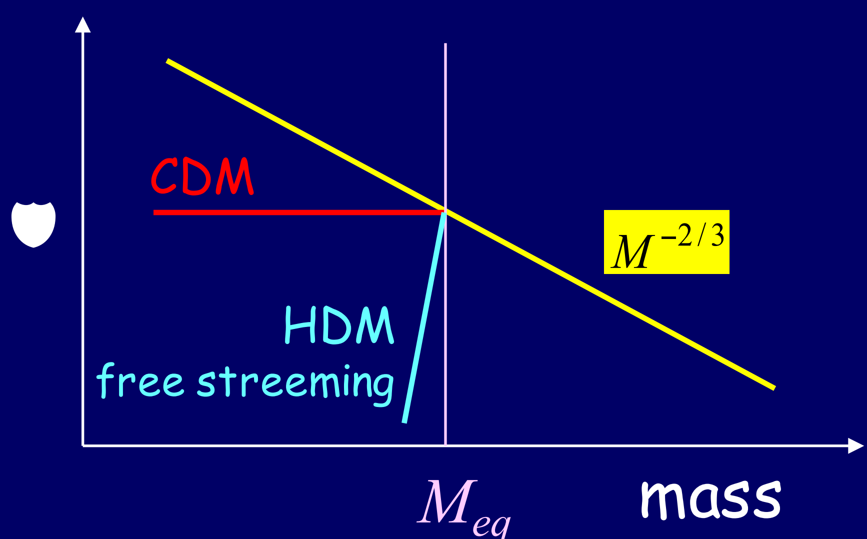


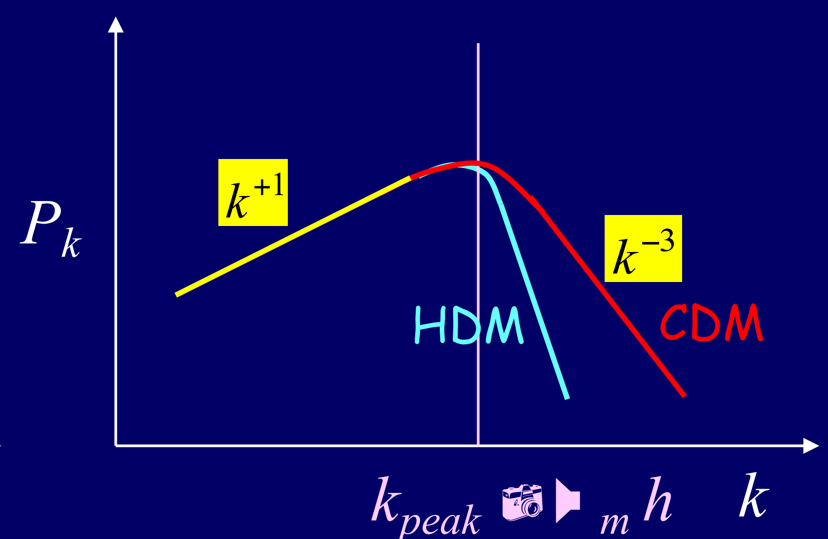
$z_{eq} \sim 10^4$

CDM Power Spectrum



growth when matter is self-gravitating





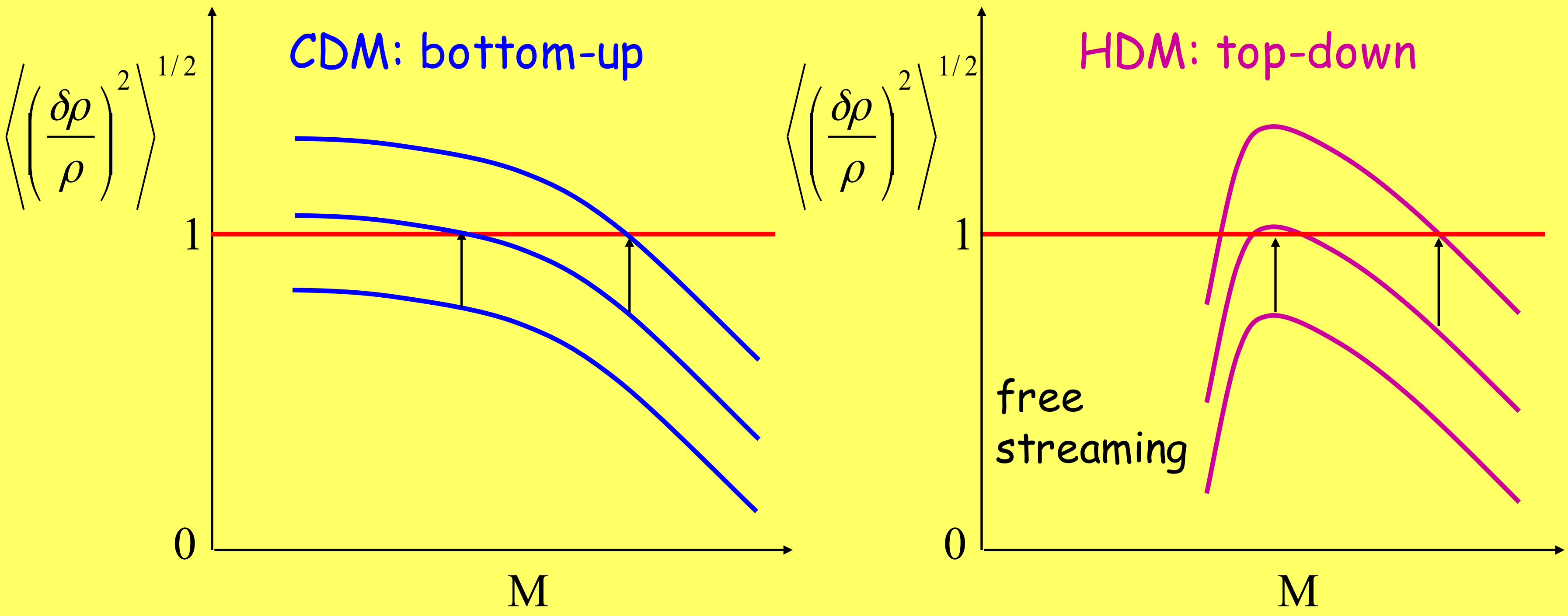
Formation of Large-Scale Structure

Fluctuation growth in the linear regime: $\delta << 1 \quad \rightarrow \quad \delta \propto a \propto t^{2/3}$

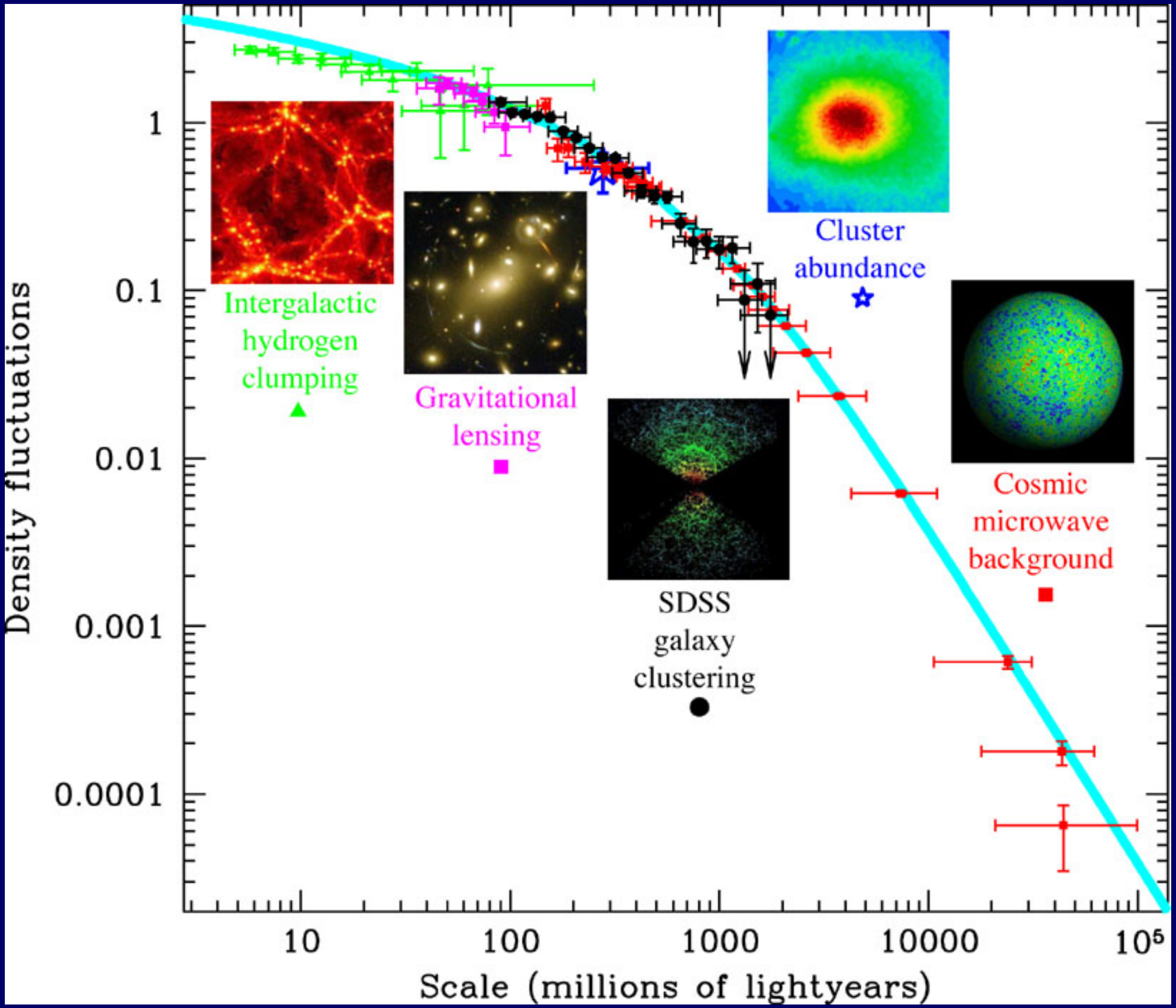
rms fluctuation at mass scale M: $\delta \propto M^{-\alpha} \quad 0 < \alpha = (n+3)/6 \leq 2/3$

Typical objects forming at t: $1 \sim \delta \propto M^{-\alpha} a \quad \rightarrow M_* \propto a^{1/\alpha}$

example $n = -2 \quad \rightarrow \quad M_* \propto a^6$



Power Spectrum



ΛCDM Power Spectrum

$$P(k) \propto k\,T^2(k)$$

$$T(k) = \frac{\ln(1+2.34q)}{2.34q}\left(1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4\right)^{-1/4} \qquad q = \frac{k}{\Omega_m h^2 Mpc^{-1}}$$

normalization: $\sigma_8 \equiv \sigma_{tophat}(R = 8h^{-1}Mpc)$

Lecture

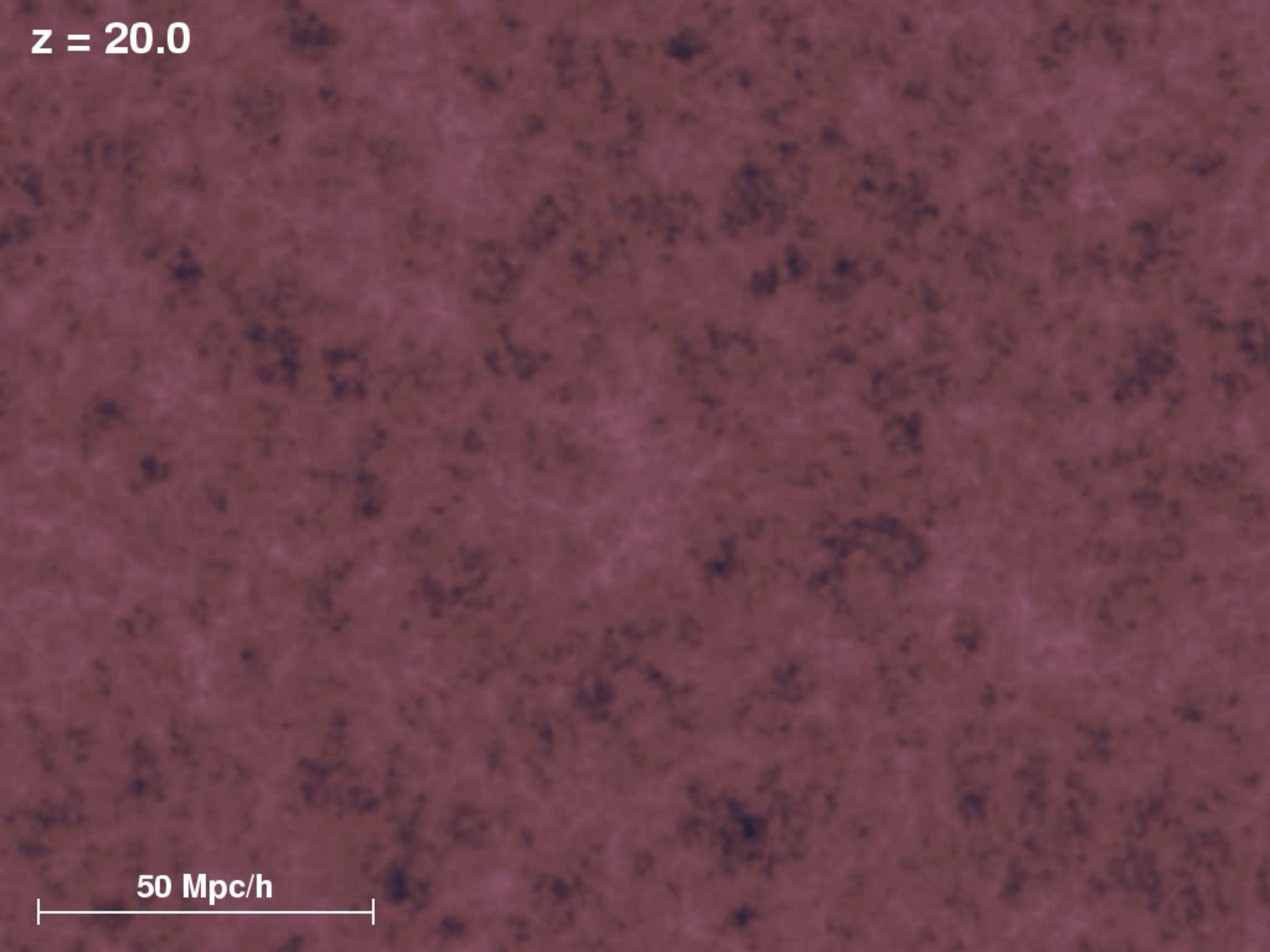
Non-linear Growth of Structure

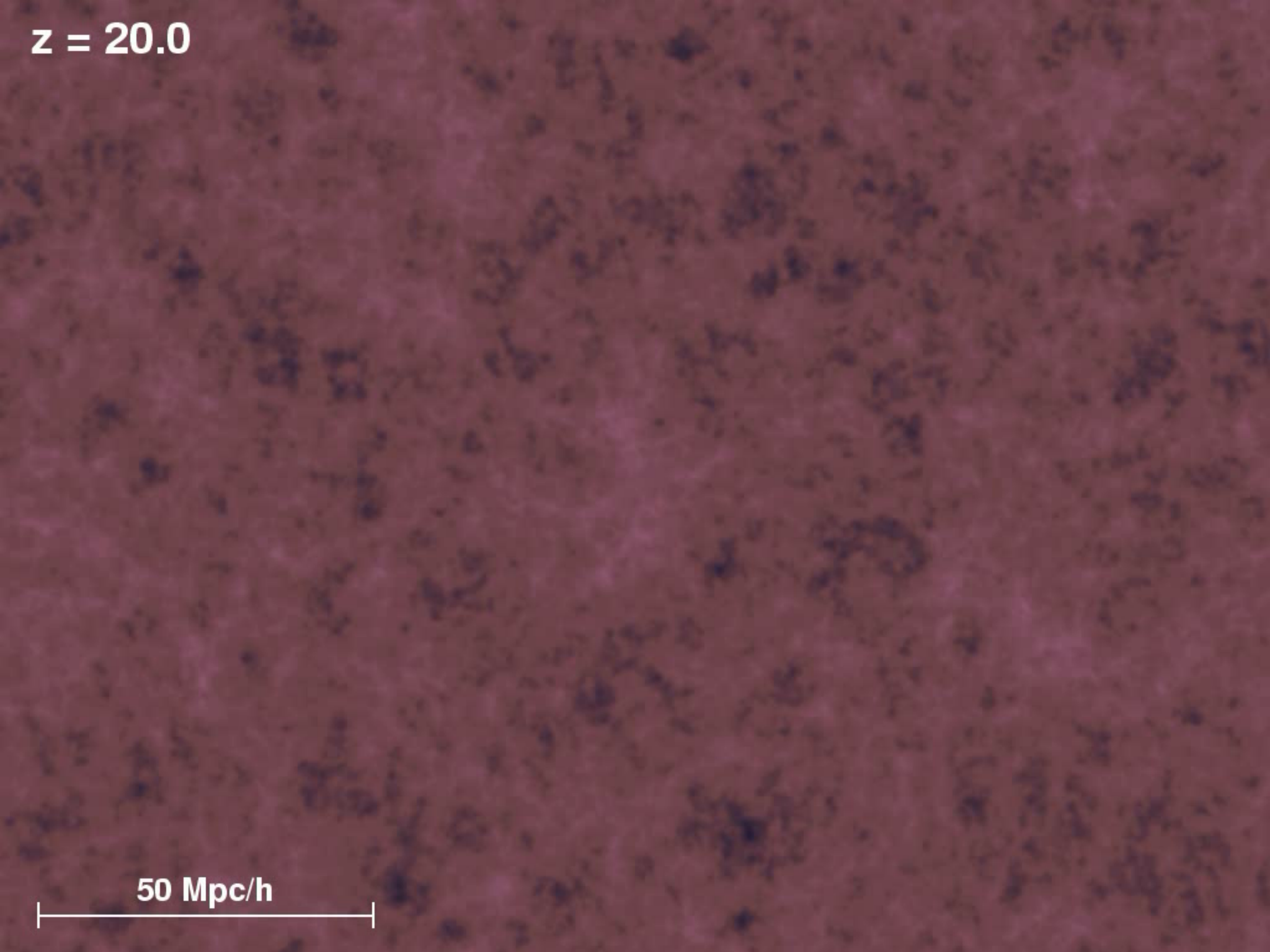
Spherical Collapse,

Virial Theorem,

Zel'dovich Approximation,

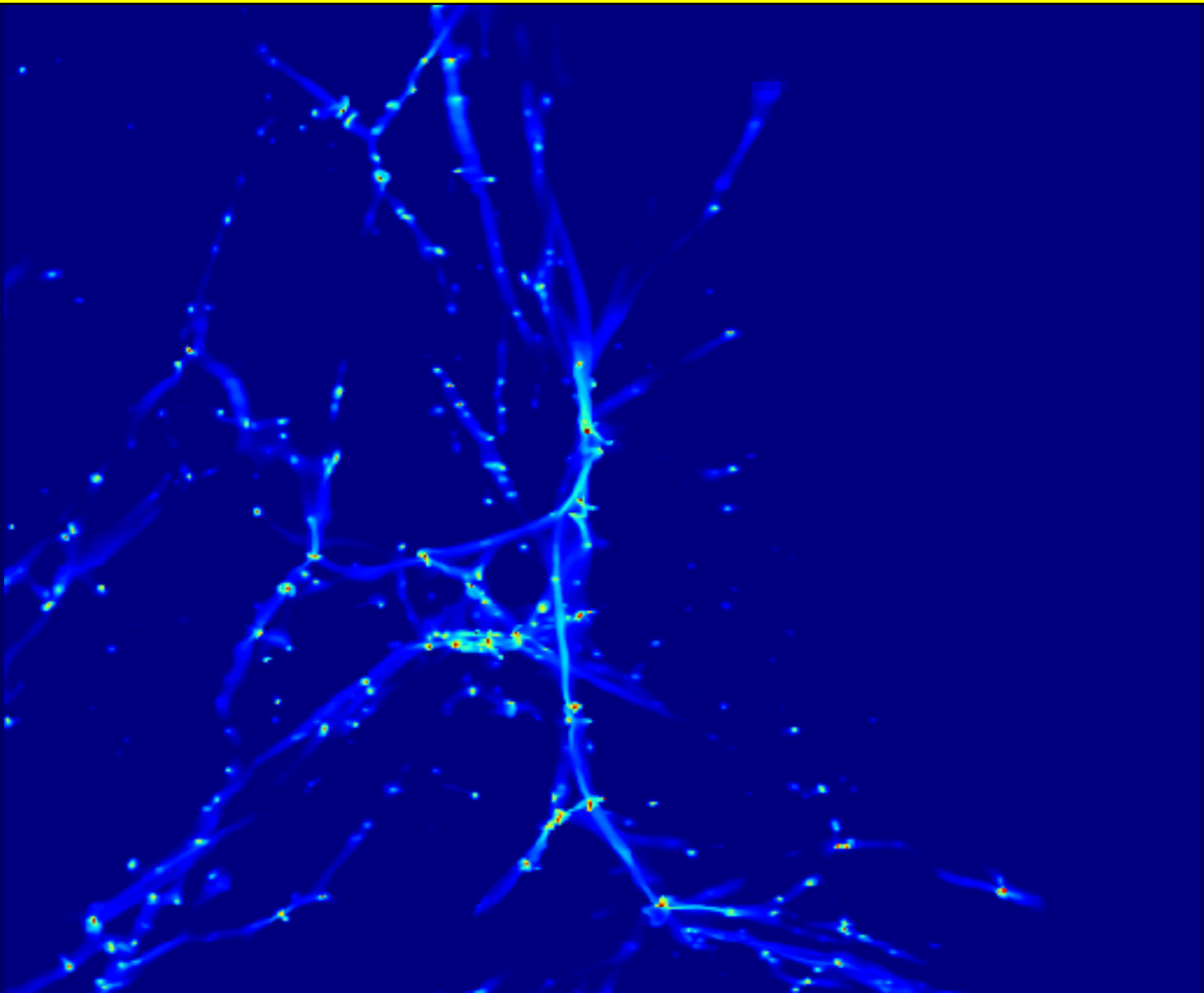
N-body Simulations


z = 20.0
50 Mpc/h

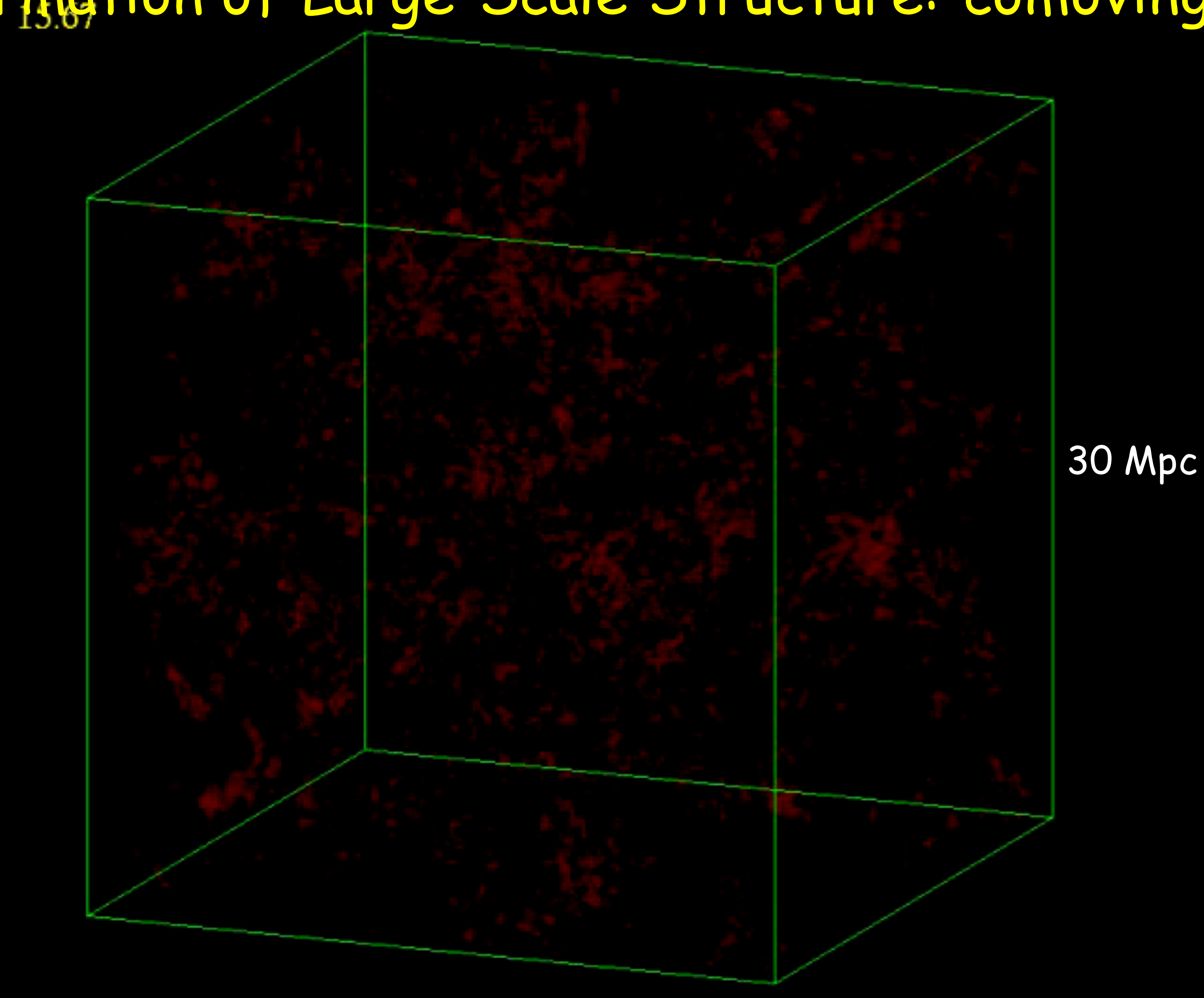

z = 20.0
50 Mpc/h

Formation of Large-Scale Structure: comoving

היווצרות הגלקסיות המוקדמות מהמארג הקוסמי



Formation of Large-Scale Structure: comoving



Filamentary Structure: Zel'dovich Approximation

Approximate the displacement from initial position

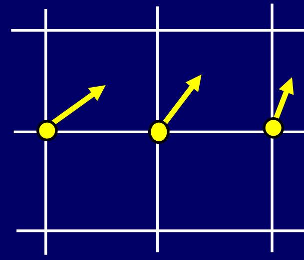
$$x(q,t) = q + D(t)\,\psi(q), \quad \psi = -\nabla\phi$$

Velocity & acceleration along displacement → trajectories straight lines

$$\dot{x} = \dot{D}\psi, \quad \ddot{x} = \ddot{D}\psi \propto \dot{x}$$

as in linear

central force → potential flow



In physical coordinates

$$r = ax, \quad v = \dot{r} = \dot{a}x + a\dot{x} = Hr + v_{pec}$$

Density (Lagrangian):

continuity $$\rho(x,t)\, d^3x = \rho_q d^3 q$$

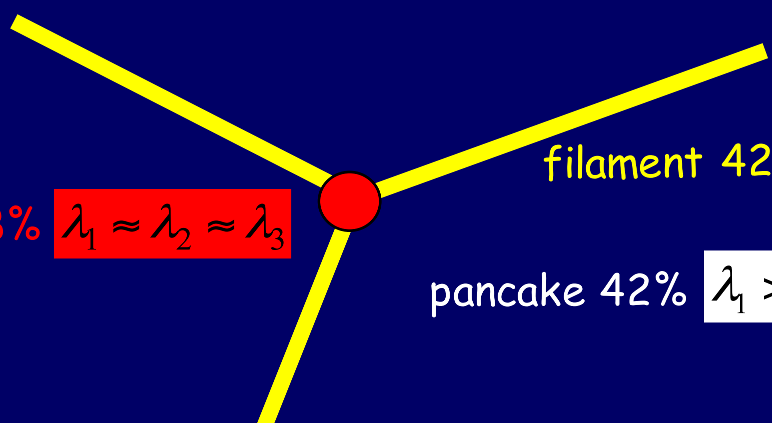
$$\rightarrow \rho(x,t) = \frac{\rho_q}{\| \partial\vec{x} / \partial\vec{q} \|} = \frac{\rho_q}{(1 - D(t)\lambda_1)(1 - D(t)\lambda_2)(1 - D(t)\lambda_3)}$$

Jacobian

→ caustics

$$\lambda_i \equiv \frac{\partial^2 \phi}{\partial^2 q_i}, \quad \lambda_1 \ge \lambda_2 \ge \lambda_3$$

deformation tensor eigenvalues



filament 42% $\lambda_1 \approx \lambda_2 >> \lambda_3$

cluster 8% $\lambda_1 \approx \lambda_2 \approx \lambda_3$

pancake 42% $\lambda_1 >> \lambda_2 \approx \lambda_3$

Zel'dovich Approximation cont'd

$$\rho(x,t) = \frac{\rho_q}{(1-D(t)\lambda_1)(1-D(t)\lambda_2)(1-D(t)\lambda_3)}$$

$$\lambda_i \equiv \frac{\partial^2 \phi}{\partial^2 q_i}, \quad \lambda_1 \geq \lambda_2 \geq \lambda_3$$

$$\delta = \frac{\rho}{\rho_q} - 1 = -D(\lambda_1 + \lambda_2 + \lambda_3) + D^2(\lambda_1\lambda_2 + \ldots) + D^3(\lambda_1\lambda_2\lambda_3) + \ldots$$

linear
$$\delta = -D(\lambda_1 + \lambda_2 + \lambda_3) = -D\,\nabla \cdot \psi = -D\,\nabla \cdot \frac{\dot{x}}{\dot{D}} = -\frac{D}{\dot{D}} \nabla \cdot v = -\frac{1}{Hf(\Omega)} \nabla \cdot v$$

→ D is the growing mode of GI obeying $\ddot{D} + 2H\dot{D} = 4\pi G \rho D$

Error:

plug density in Poisson eq. $\delta_{Poisson} \propto \nabla^2 \phi_{grav} \propto -\nabla \psi = -(\lambda_1 + \lambda_2 + \lambda_3) \propto \delta_{linear}$

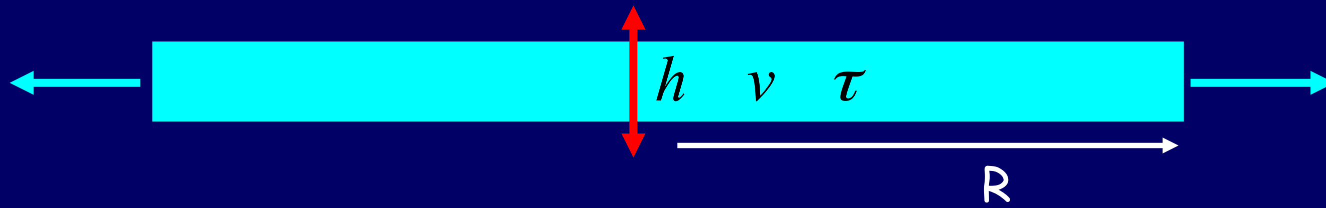
→ error is 2nd + 3rd terms
$$\frac{\Delta\rho}{\rho} = -(D\lambda_1)^2 \left(\frac{\lambda_2}{\lambda_1} + \frac{\lambda_3}{\lambda_1} + \frac{\lambda_2\lambda_3}{\lambda_1^2} \right) + 2(D\lambda_1)^3 \frac{\lambda_2\lambda_3}{\lambda_1^2}$$

error small in linear regime $D\lambda_1 << 1$

or pancakes $\lambda_1 >> \lambda_2, \lambda_3$

error big in spherical collapse $\lambda_1 \sim \lambda_2 \sim \lambda_3$

Non-dissipative Pancakes: why flat?



oscilation time << expansion time $\tau << H^{-1}$

adiabatic invariant $const. \sim \int_0^\tau \dot{x}^2 dt \sim v^2 \tau \sim h\, v$

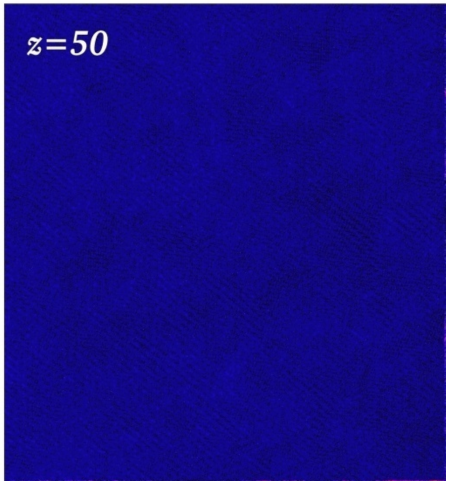
$$v \sim f\tau \propto f v^{-2} \quad \rightarrow v \propto f^{1/3} \propto R^{-2/3}$$

$$f \propto \Sigma \propto R^{-2}$$

$$h \propto v^{-1} \propto R^{2/3}$$

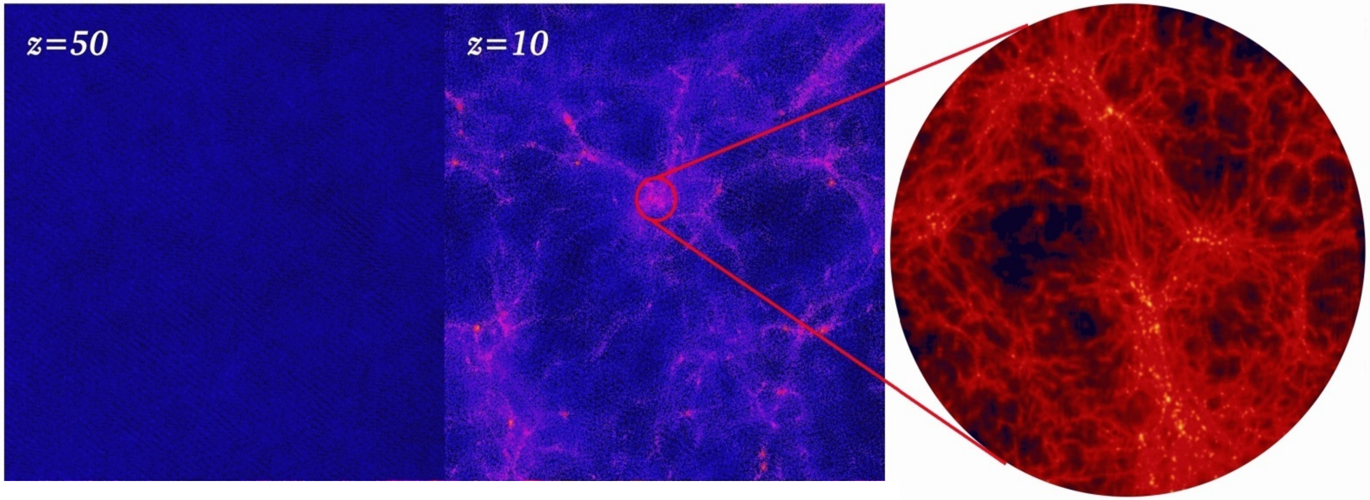
$$\frac{h}{R} \propto R^{-1/3} \propto a^{-1/3}$$

pancake becomes flatter in time

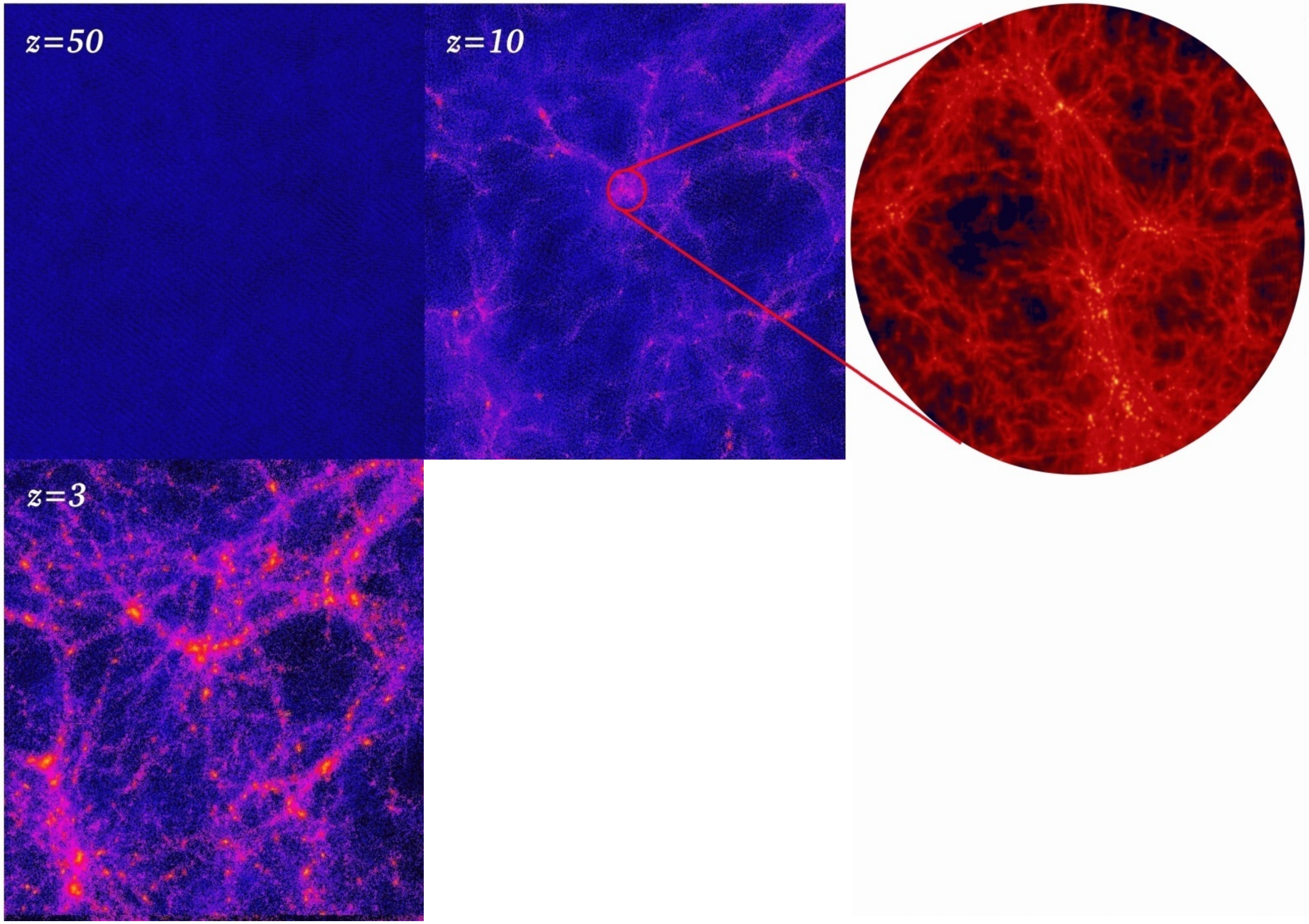


N-body simulation

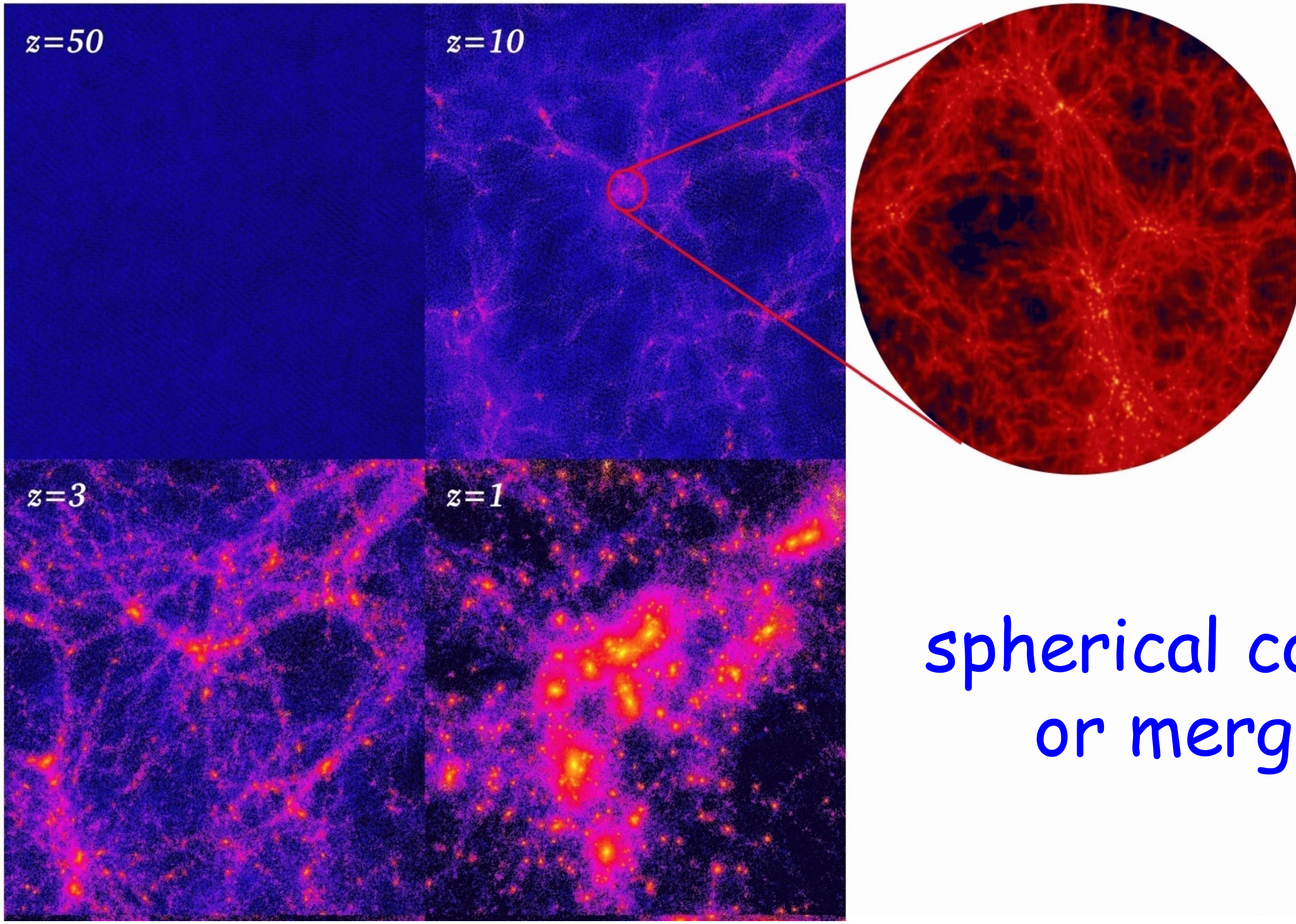
ΛCDM



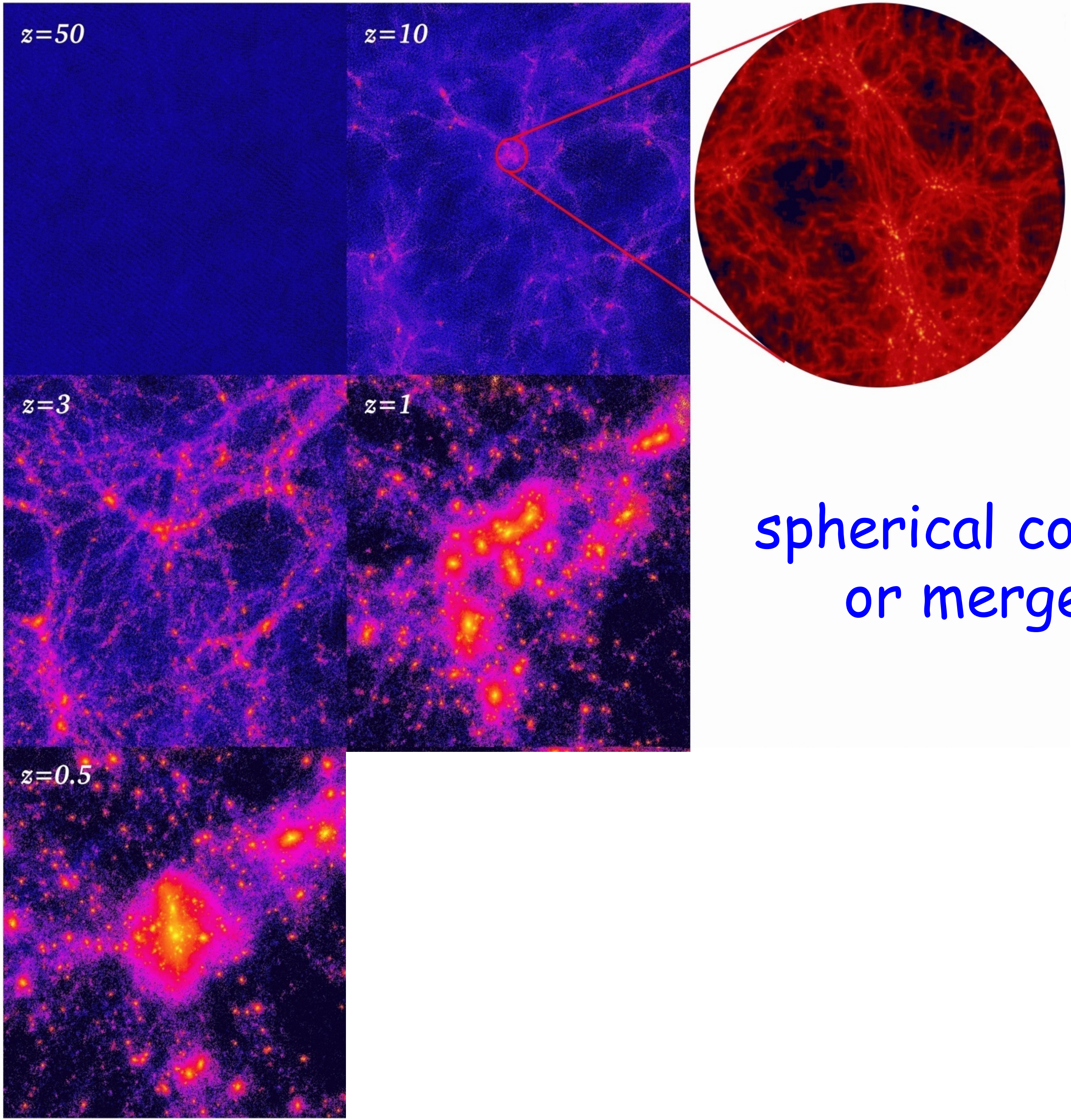
z=50
z=10



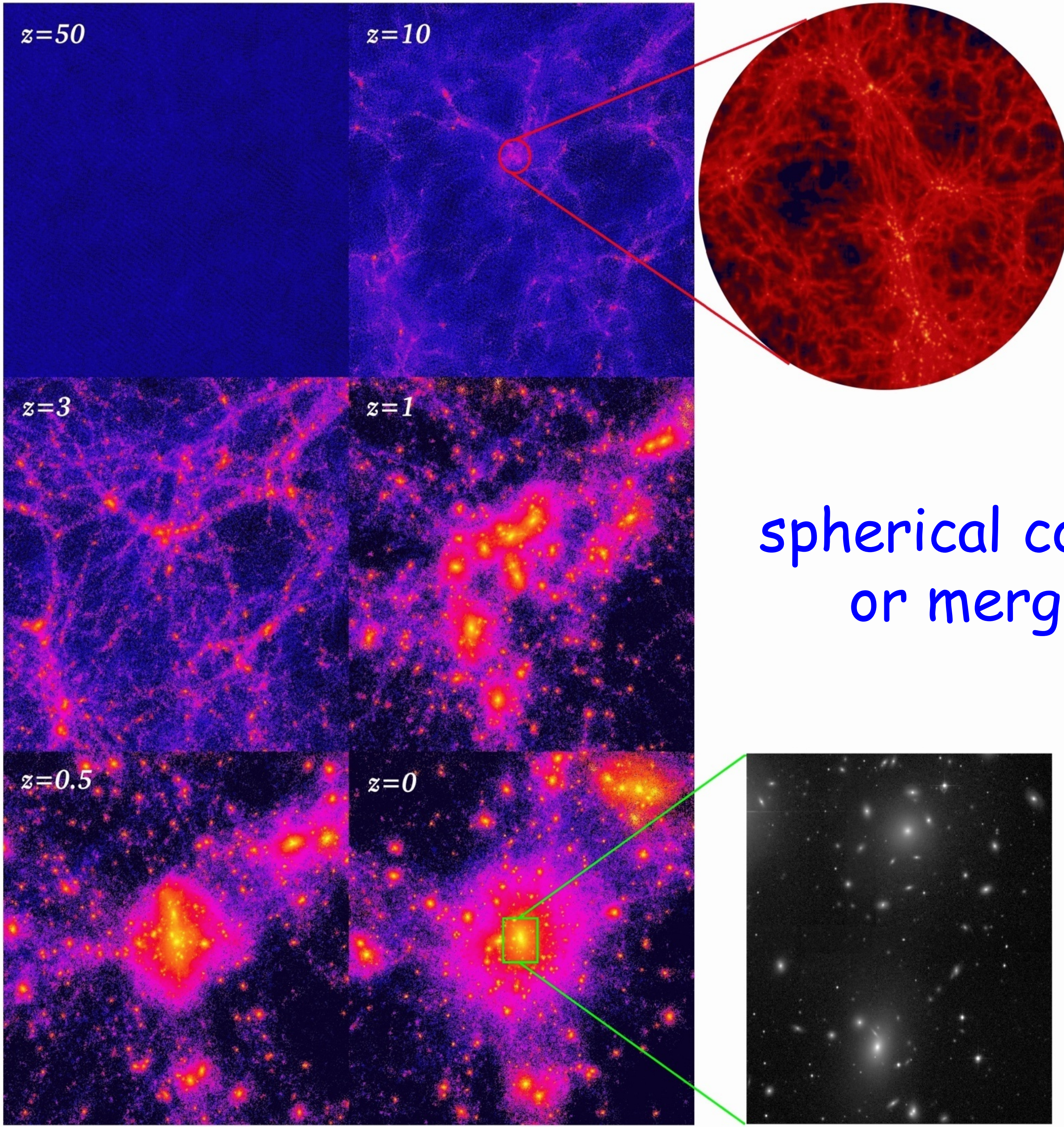
z=50
z=10
z=3



spherical collapse
or mergers

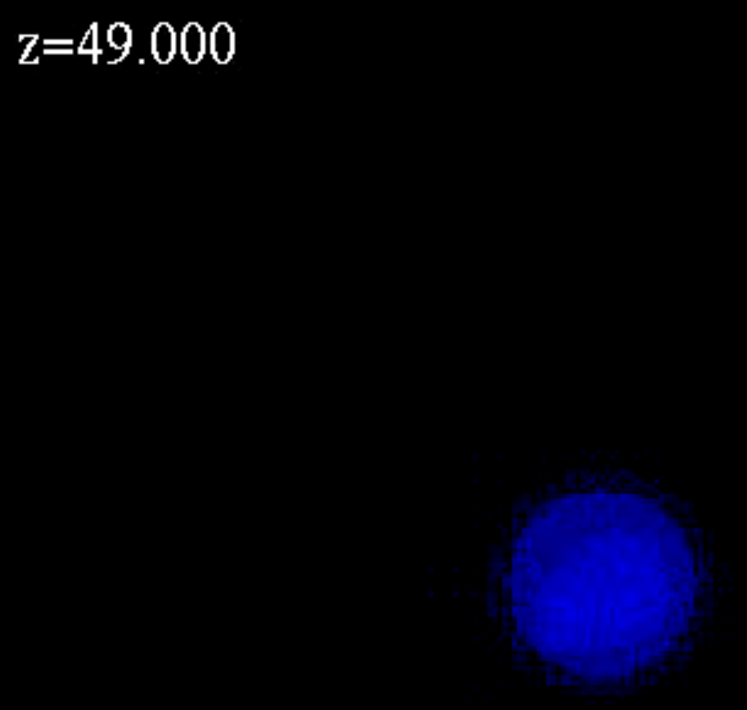


spherical collapse
or mergers

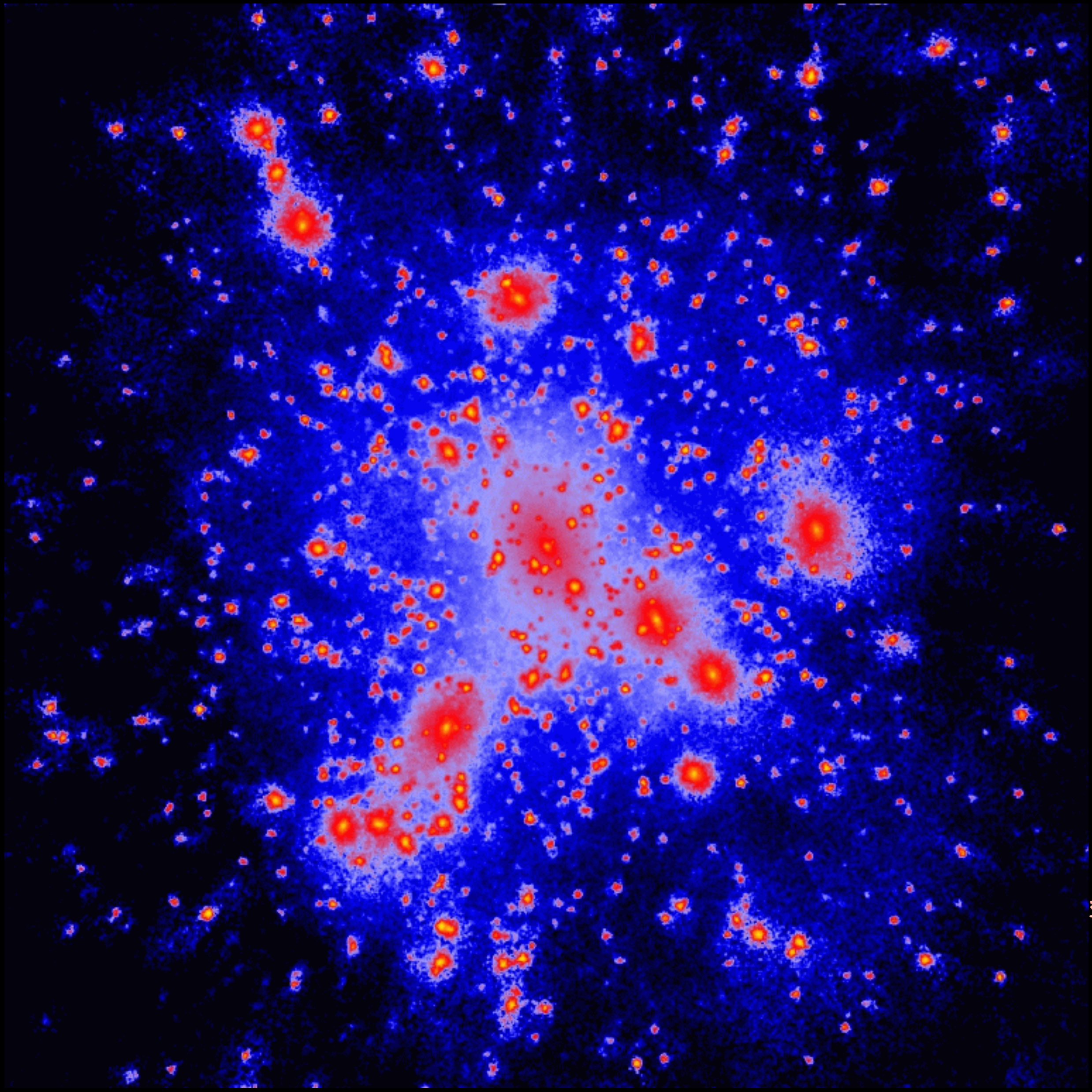


spherical collapse
or mergers

N-body simulation of Halo Formation



N-body simulation of Halo Formation



The Friedman Equation

Newton's gravity: space fixed, external force determining motions $\nabla^2\phi = 4\pi G\,\rho$

Einstein's equations $G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi\, G\, T_{\mu\nu}$ $G = c = 1$

Gravity is an intrinsic property of space-time. geometry <-> energy density.
Particles move on geodesics (local straight lines) determined by the local curvature.

left side of E's eq. is the most general function of g and its 1st and 2nd time derivatives that reduces to Newton's equation

For the isotropic RW metric $ds^2 = dt^2 - a^2(t)[du^2 + S_k(u)\,d\gamma^2]$

Einstein's tensor $G_{tt} = 3\frac{\dot{a}^2}{a^2} + \frac{3k}{a^2}$ $G_{uu} = G_{\vartheta\vartheta} = G_{\varphi\varphi} = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2}$

Stress-energy tensor $T_{tt} = \rho$ $T_{uu} = T_{\vartheta\vartheta} = T_{\varphi\varphi} = P$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$ energy conservation

$$\frac{2\ddot{a}}{a} = -\frac{\dot{a}^2}{a^2} - \frac{k}{a^2} - 8\pi P + \Lambda$$ eq. of motion

mass conservation

$$\rho_m V = const. \quad \rightarrow \quad \rho_m \propto a^{-3}$$

conservation of number of photons

$$N = \frac{\rho_r V}{h\nu} \propto \frac{\rho_r a^3}{a^{-1}} = const. \quad \rightarrow \quad \rho_r \propto a^{-4}$$

A differential equation for a(t)

Solutions of Friedman eq. (matter era)

$$\frac{\dot a^2}{a^2} = \frac{2a^*}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$a^* \equiv \frac{4\pi G \rho_{m0}}{3} = const.$$

radiation era

$$\dot a^2 \propto a^{-2} \quad \rightarrow \quad a \propto t^{1/2}$$

$$a \to 0, \text{any } k: \quad \dot a^2 = \frac{2a^*}{a} \quad \rightarrow \quad a \propto t^{2/3}$$

$$\Lambda = 0 \quad \dot a^2 = \frac{2a^*}{a} - k \quad \rightarrow \quad \dot a^2 \downarrow$$

$$k = 0 \quad \dot a^2 = \frac{2a^*}{a} \quad \rightarrow \quad a \propto t^{2/3}$$

$$k = -1 \quad \dot a^2 = 1 + \frac{2a^*}{a} \quad a \propto t \ \ (a \to \infty)$$

$$k = +1 \quad \dot a^2 = -1 + \frac{2a^*}{a} \quad \rightarrow \quad \text{turnaround}$$

conformal time

$$d\eta \equiv dt / a(t)$$

$$a = a^*[1 - \cos(\eta)]$$

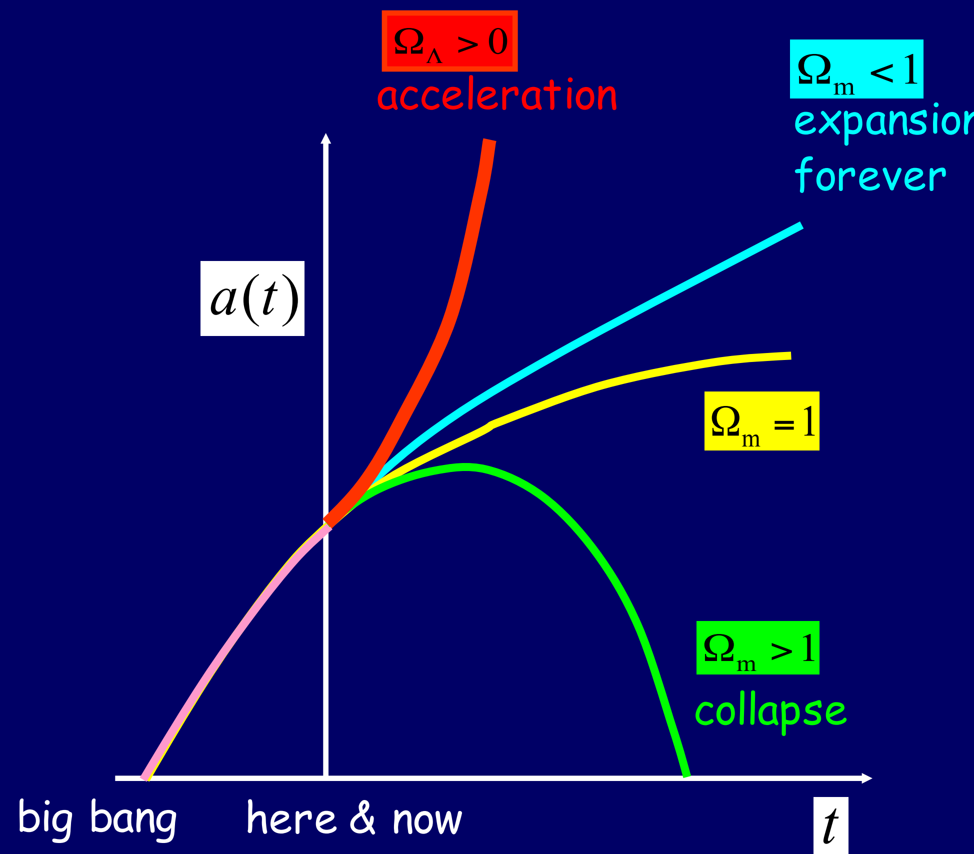
$$t = a^*[\eta - \sin(\eta)]$$

critical density

$$\rho = \frac{3H^2}{8\pi G} + \frac{3k}{8\pi G a^2}$$

$$\Omega \equiv \frac{\rho}{3H^2 / 8\pi G}$$

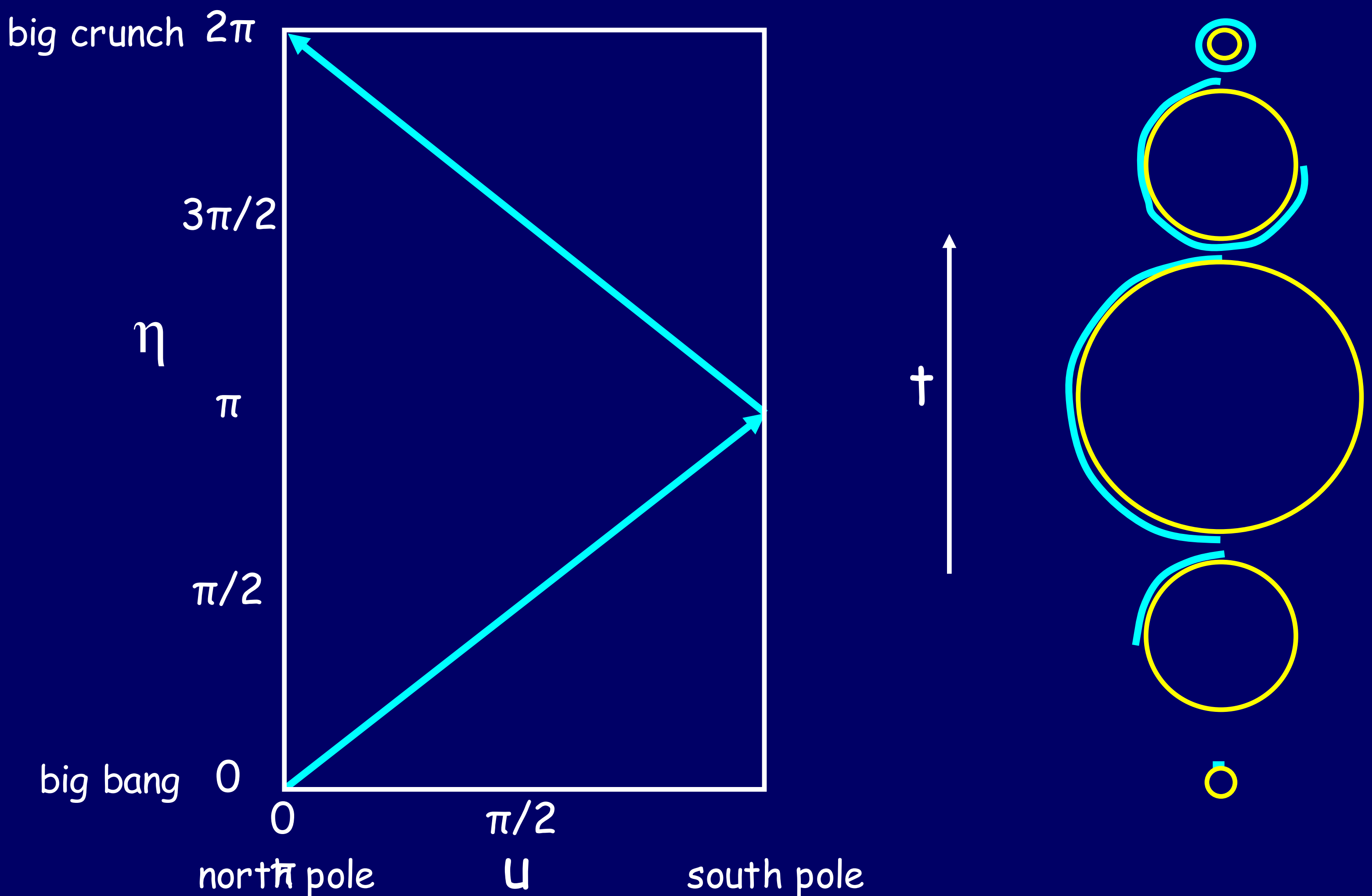
$$\Lambda > 0 \quad H^2 \equiv \frac{\dot a^2}{a^2} = \frac{\Lambda c^2}{3} \quad (a \to \infty) \quad \rightarrow \quad a \propto e^{Ht}$$



Light travel in a closed universe

A photon is emitted at the origin (u_e=0) right after the big-bang (t_e=0)

conformal time $d\eta \equiv \frac{dt}{a(t)}$ photon: $ds = 0 \quad \rightarrow \quad d\eta = du$



Top-Hat Model (Λ=0, matter era)

a bound sphere (k=1) in EdS universe (k=0) $\dot{a}^2 = \frac{2a^*}{a} - k \quad a^* \equiv (4\pi/3)G\rho a^3 = const.$

conformal time $d\eta \equiv \frac{dt}{a(t)}$

$$a = (a^*/2)\eta^2 \quad t = (a^*/6)\eta^3$$

$$a_p = a_p^*(1-\cos\eta_p) \quad t = a_p^*(\eta_p - \sin\eta_p)$$

$$t_p = t \rightarrow \eta^3(\eta_p) = \frac{6a_p^*}{a^*}(\eta_p - \sin\eta_p) \rightarrow a(\eta_p) = \frac{1}{2}\left(\frac{6a_p^*}{a^*}(\eta_p - \sin\eta_p)\right)^{2/3}$$

overdensity:

$$a^* \propto \rho a^3 \quad a_p^* \propto \rho_p a_p^3 \rightarrow \frac{\rho_p}{\rho} = \frac{a_p^*}{a^*}\left(\frac{a}{a_p}\right)^3 = \frac{9(\eta_p - \sin\eta_p)^2}{2(1-\cos\eta_p)^3}$$

linear perturbation $\delta\rho/\rho << 1 \quad \eta_p << 1$

Taylor $\cos\eta \approx 1 - \frac{1}{2}\eta^2 + \frac{1}{24}\eta^4 \quad \sin\eta \approx \eta - \frac{1}{6}\eta^3 + \frac{1}{120}\eta^5$

$$\frac{\delta\rho}{\rho} \equiv \frac{\rho_p - \rho}{\rho} \approx 0.15\eta_p^2 \propto a \propto t^{2/3}$$

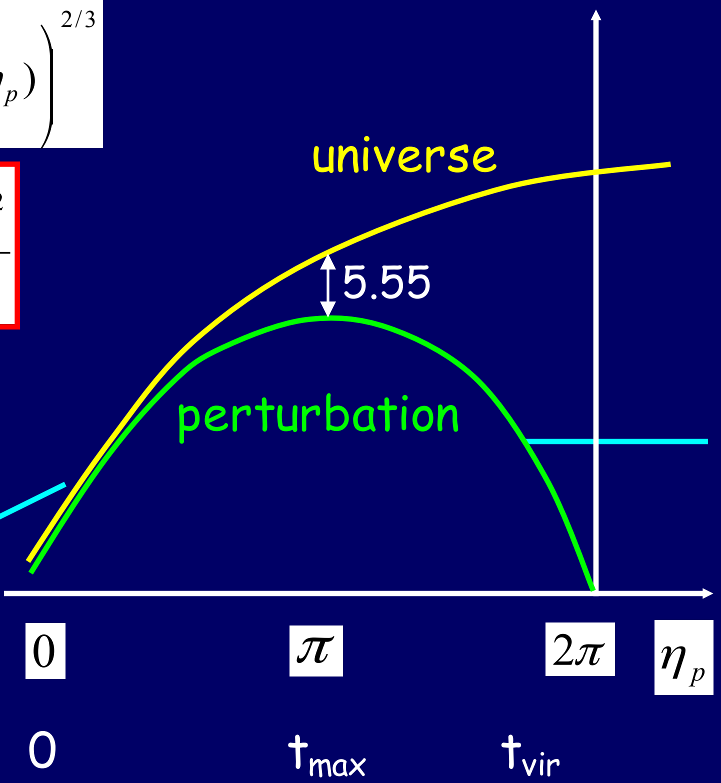
$$\delta \propto a$$

turnaround $\eta_p = \pi$

$$\frac{\rho_p}{\rho} = \frac{9\pi^2}{16} \approx 5.55$$

linear equivalent to collapse

$$\delta_{2\pi} = \delta(\eta_p << 1)\left(\frac{t(\eta_p = 2\pi)}{t(\eta_p << 1)}\right)^{2/3} = 0.15\eta_p^2\left(\frac{2\pi}{\eta_p^3/6}\right)^{2/3} \approx 1.68 \equiv \delta_c$$



- **Collapse to Virial Equilibrium**

$$E_{max} \simeq -\frac{GM^2}{R_{max}} \quad (E_k \simeq 0) \qquad E_{vir} \simeq \frac{1}{2} E_{grav} \simeq -\frac{1}{2}\frac{GM^2}{R_{vir}}$$

$$\text{E conserved} \quad \rightarrow \quad \frac{R_{vir}}{R_{max}} \simeq \frac{1}{2} \quad \rightarrow \quad \frac{\rho_{vir}}{\rho_{max}} \simeq 8$$

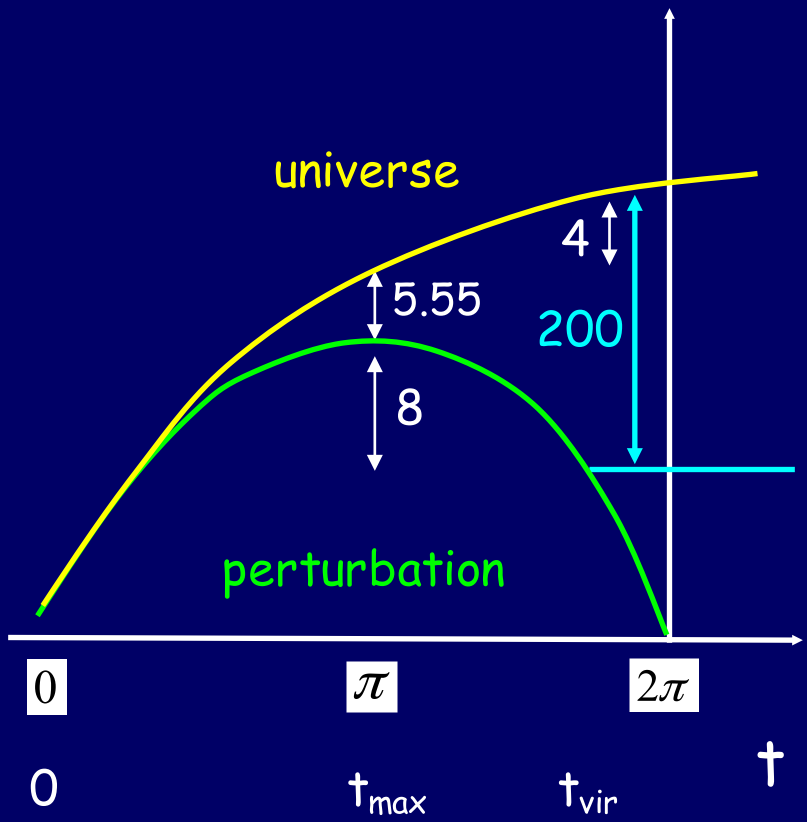
- **Virial density:**

$$\frac{\rho_{vir}}{\rho_{univ}} \simeq 5.55 \times 8 \times \left(\frac{a_{vir}}{a_{max}}\right)^3$$

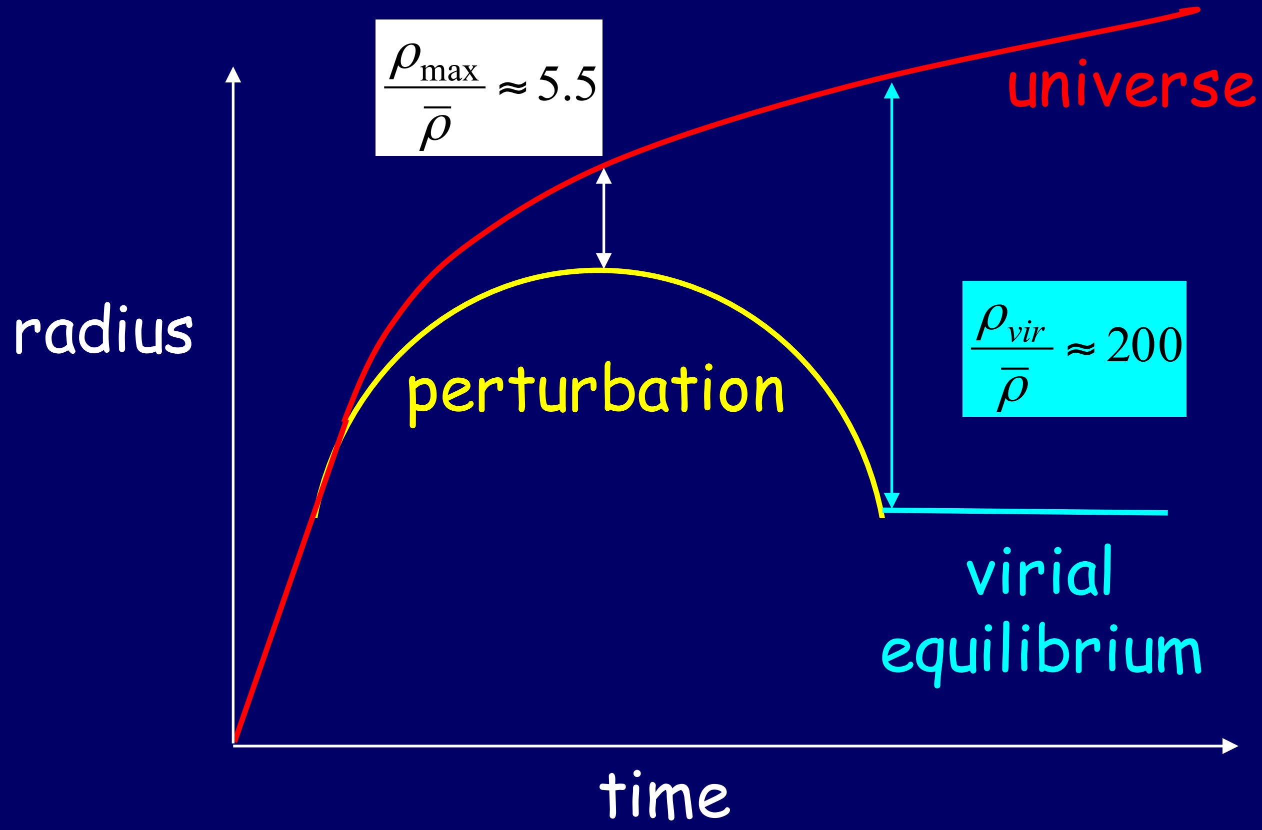
Assume virialization at collapse, $\eta_p \simeq 2\pi$,

$$\frac{t_{col}}{t_{max}} \simeq \frac{2\pi}{\pi} = 2 \quad \rightarrow \frac{a_{vir}}{a_{max}} = \left(\frac{t_{col}}{t_{max}}\right)^{2/3} \simeq 2^{2/3}$$

$$\rightarrow \frac{\rho_{vir}}{\rho_{univ}} \simeq 5.55 \times 8 \times 4 \simeq 178 \sim 200$$



Spherical Collapse



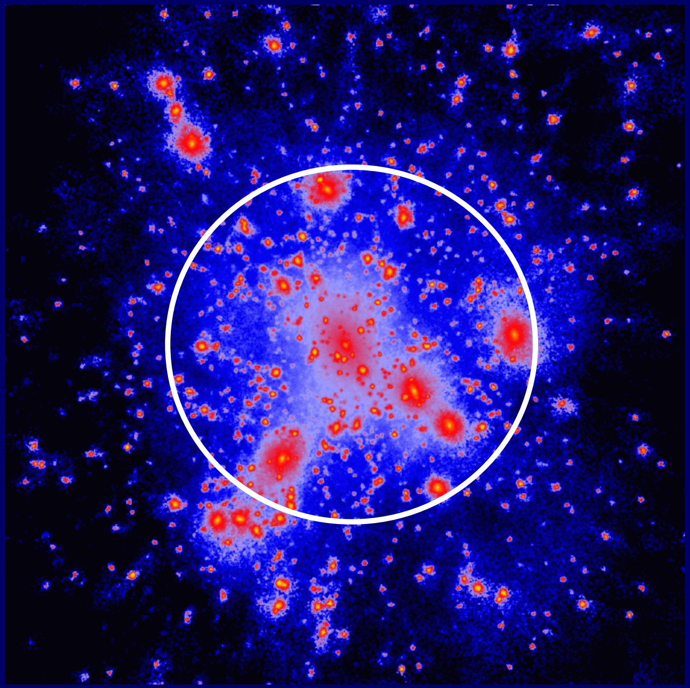
virial equilibrium: $E = -\frac{1}{2}\frac{GM}{R_{vir}} = -\frac{GM}{R_{\max}}$

Virial Scaling Relations

Virial equilibrium: $$V^2 = \frac{GM}{R}$$

Spherical collapse: $$\frac{M}{(4\pi/3)R^3} = \Delta\rho_u = \Delta\rho_{u0}a^{-3}$$ $\Delta \approx 200$

$$\rightarrow M \propto V^3 a^{3/2} \propto R^3 a^{-3}$$



Weak dependence on time of formation:

$$D(a)\,\delta_0(M) \approx 1 \quad \rightarrow \quad a \propto M^{\alpha} \quad \alpha = (n+3)/6 \approx 0.1-0.2$$

$$M \propto V^4 \quad \text{for } n = -2$$

Practical formulae:

$$\rho_u \approx 2.76\times10^{-30}\, g\ cm^{-3}\ \Omega_{m0.3} h_{0.7}^2 a^{-3}$$

$$M_{11} \approx V_{100}^3 A^{-3/2} \approx R_{Mpc}^3 A^{-3}$$

$$A \equiv a\,(\Delta_{200}\Omega_{m0.3}h_{0.7}^2)^{-1/3}$$

$$\Delta(a) \approx [18\pi^2 - 82\Omega_\Lambda(a) - 39\Omega_\Lambda(a)^2]/\Omega_m(a) \quad \Delta(a<<1) \approx 178 \quad \Delta_0 \approx 340$$

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{\Omega_\Lambda + \Omega_m a^{-3}} \quad \Omega_m(a) + \Omega_\Lambda(a) = 1$$

Lecture 6

Hierarchical Clustering

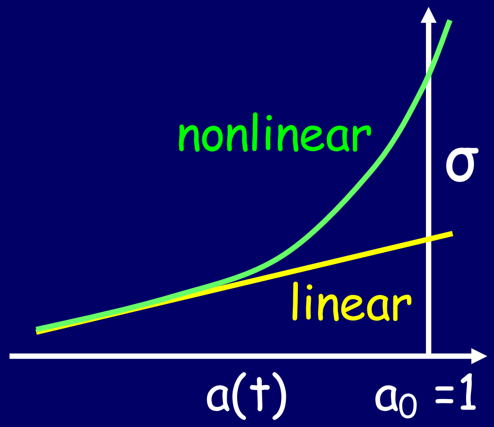
Press Schechter Formalism

Press Schechter Formalism   halo mass function n(M,a)

Gaussian random field $P(\delta) = (2\pi\sigma^2)^{-1/2} \exp(-\delta^2 / 2\sigma^2)$

random spheres of mass M

linear-extrapolated δ_{rms} at a: $\sigma(M,a) = \sigma_0(M)\, D(a)$

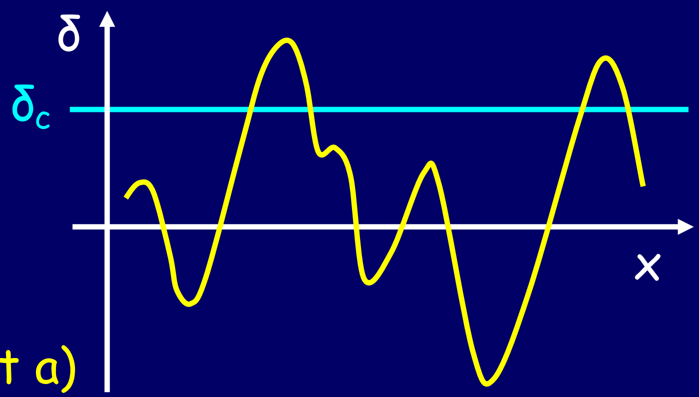


fraction of spheres with $\delta > \delta_c = 1.68$:

$$F(M,a) = \int_{\delta_c}^{\infty} d\delta\, [2\pi\sigma^2(M,a)]^{-1/2} \exp[-\delta^2 / 2\sigma^2(M,a)]$$

$$= (2\pi)^{-1/2} \int_{\delta_c/\sigma(M,a)}^{\infty} dx\, \exp(-x^2/2)$$

$$\nu_c \equiv \frac{\delta_c}{D(a)\,\sigma_0(M)}$$



PS ansaz: F is the mass fraction in halos >M (at a)

derivative of F with respect to M:

$$n(M,a)dM = \left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M} \nu_c \frac{d\ln\sigma_0}{d\ln M} \exp(-\nu_c^{\,2}/2) \frac{dM}{M}$$

Mo & White 2002

Press Schechter Formalism cont.

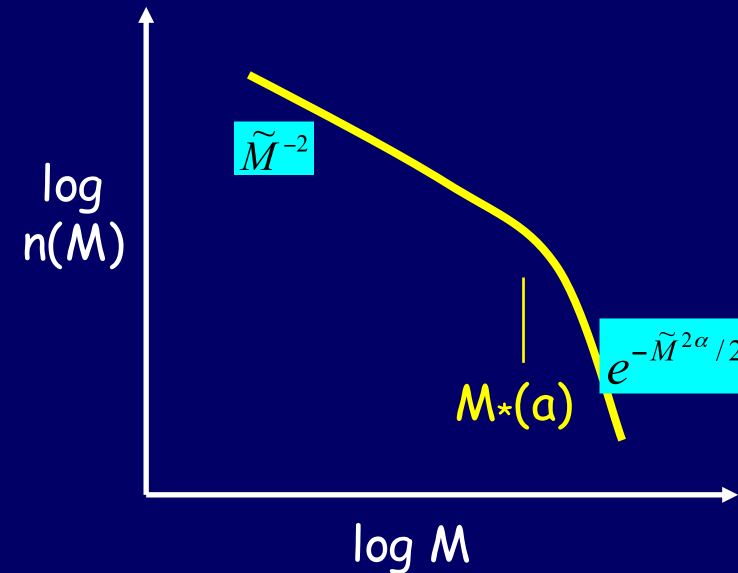
$$n(M,a)dM = \left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M} \nu_c \frac{d\ln\sigma_0}{d\ln M} \exp(-\nu_c{}^2/2) \frac{dM}{M}$$

Example: $P_k \propto k^n \rightarrow \sigma_0(M) \propto M^{-\alpha} \rightarrow \nu_c = (M/M_*)^{\alpha}$

$\alpha = (3+n)/6$ $\quad \frac{d\ln\sigma_0}{d\ln M} = \alpha$

$$n(M) \propto \alpha\, \tilde{M}^{\alpha-2} \exp(-\tilde{M}^{2\alpha}/2) \qquad \tilde{M} \equiv M/M_*$$

self-similar evolution, scaled with M*



$$\nu_c \equiv \frac{\delta_c}{D(a)\,\sigma_0(M)}$$

time $\quad P_k$

$M_*(a)$ defined by $\sigma(M_*, a) \equiv \delta_c$

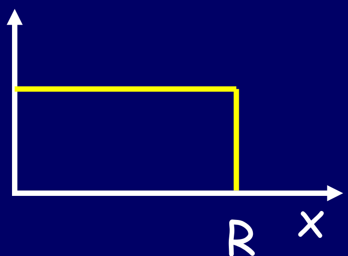
approximate

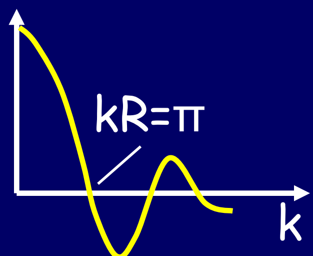
$$M_*(a) = M_{*0} D(a)^{1/\alpha} \sim 10^{13} M_{\circ}\, a^5$$

$$\sigma^2(R) = (2\pi)^{-1} \int_0^{\infty} dk\, k^2\, P(k)\, \tilde{W}^2(kR)$$

Top Hat

$$W_R(x) = \Theta(x/R) \qquad \tilde{W}_R(k) = 3[\sin(kR) - kR\cos(kR)]/(kR)^3$$





in a flat universe

$$D(a) = a\, g(a)/g(1)$$

$$g(a) \approx \frac{5}{2}\Omega_m(a)\left(\Omega_m(a)^{4/7} - \Omega_\Lambda(a) + \frac{1+\Omega_m(a)/2}{1+\Omega_\Lambda(a)/70}\right)^{-1}$$

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{\Omega_\Lambda + \Omega_m a^{-3}}$$

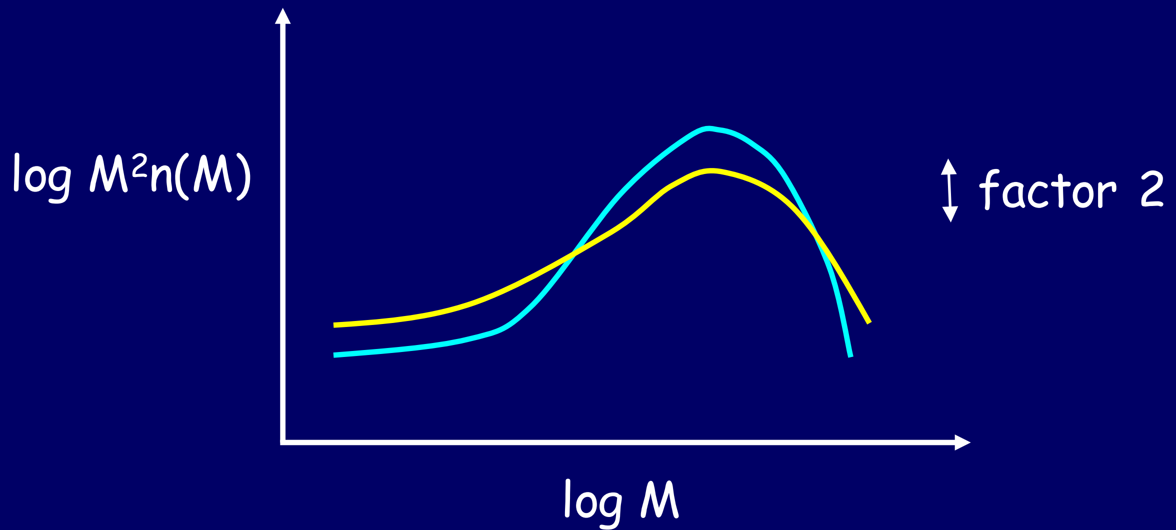
Press Schechter cont.

Better fit using ellipsoidal collapse (Sheth & Tormen 2002)

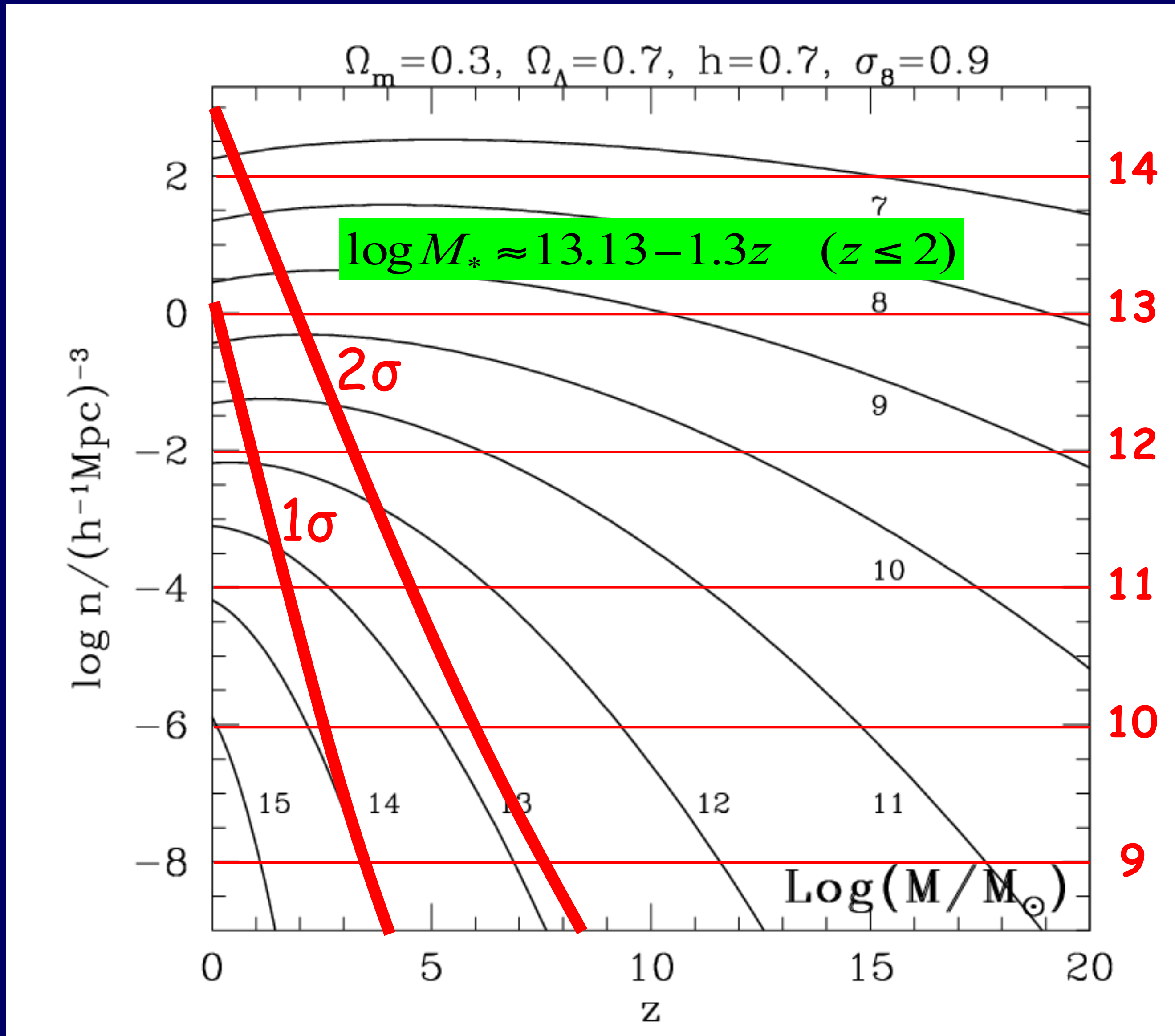
$$F(>M,a) \approx 0.4(1+0.4/\nu^{0.4})\ \mathrm{erfc}(0.85\nu/2^{1/2})$$

$$1\sigma, 2\sigma, 3\sigma \quad 22\%, 4.7\%, 0.54\%$$

Comparison of PS to N-body simulations



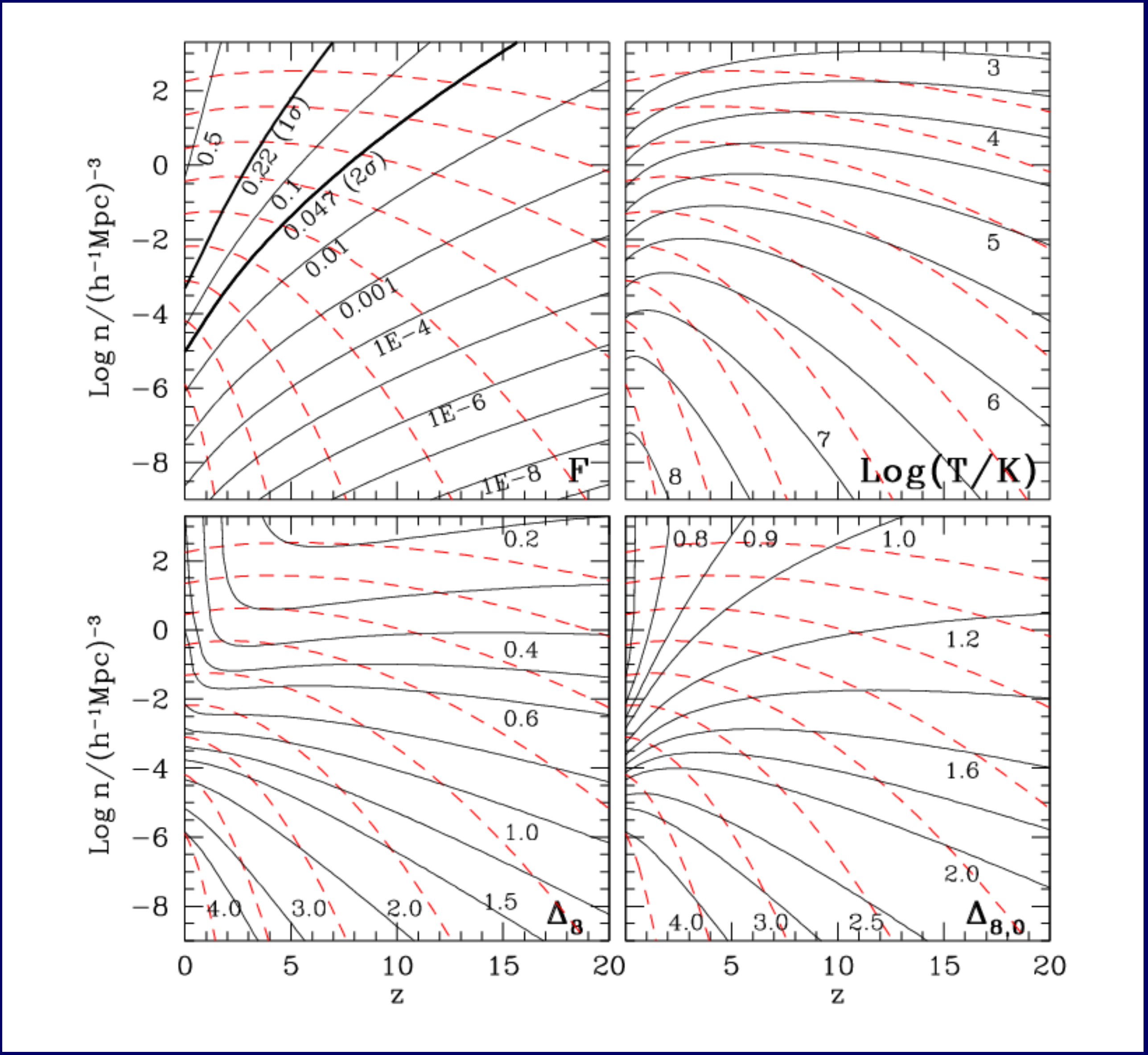
Press-Schechter in ΛCDM



log $M/M_\odot$

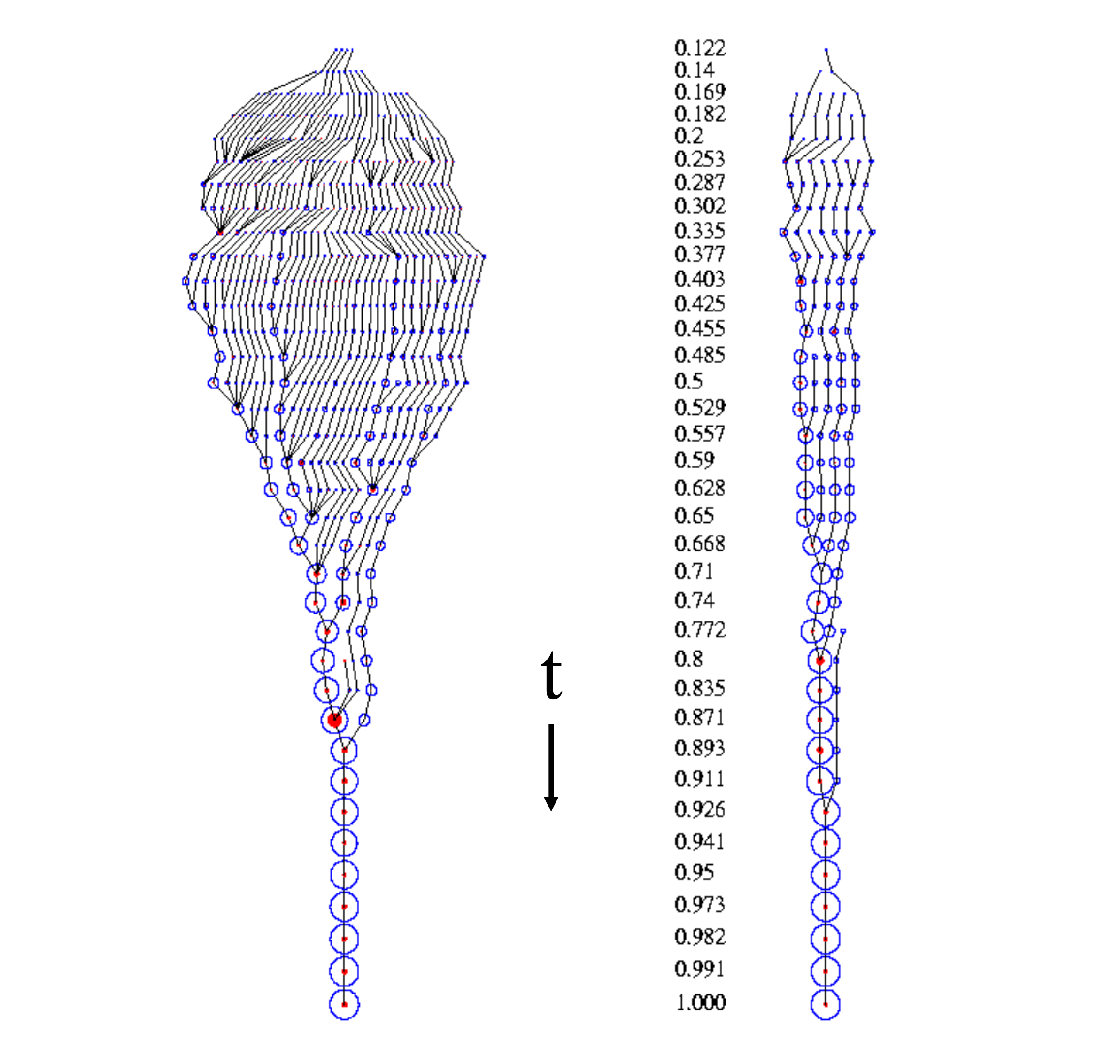
Mo & White 2002

Press-Schechter



Mo & White 2002

Merger Tree



0.122
0.14
0.169
0.182
0.2
0.253
0.287
0.302
0.335
0.377
0.403
0.425
0.455
0.485
0.5
0.529
0.557
0.59
0.628
0.65
0.668
0.71
0.74
0.772
0.8
0.835
0.871
0.893
0.911
0.926
0.941
0.95
0.973
0.982
0.991
1.000
t

Mass versus Light Distribution