

Lecture

Structure of Dark-Matter Halos

Universal Halo profile

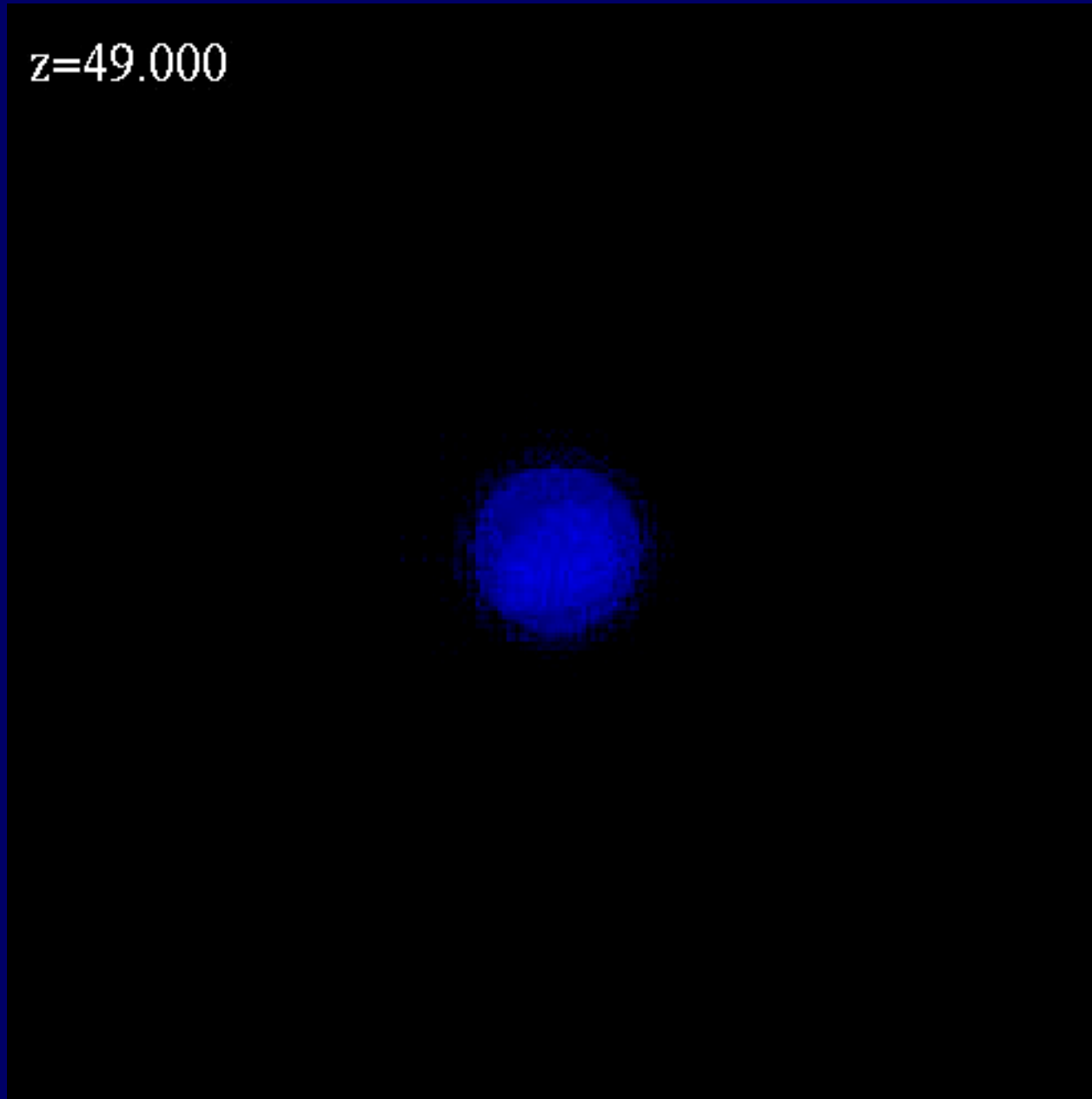
The cusp/core problem

Dynamical friction

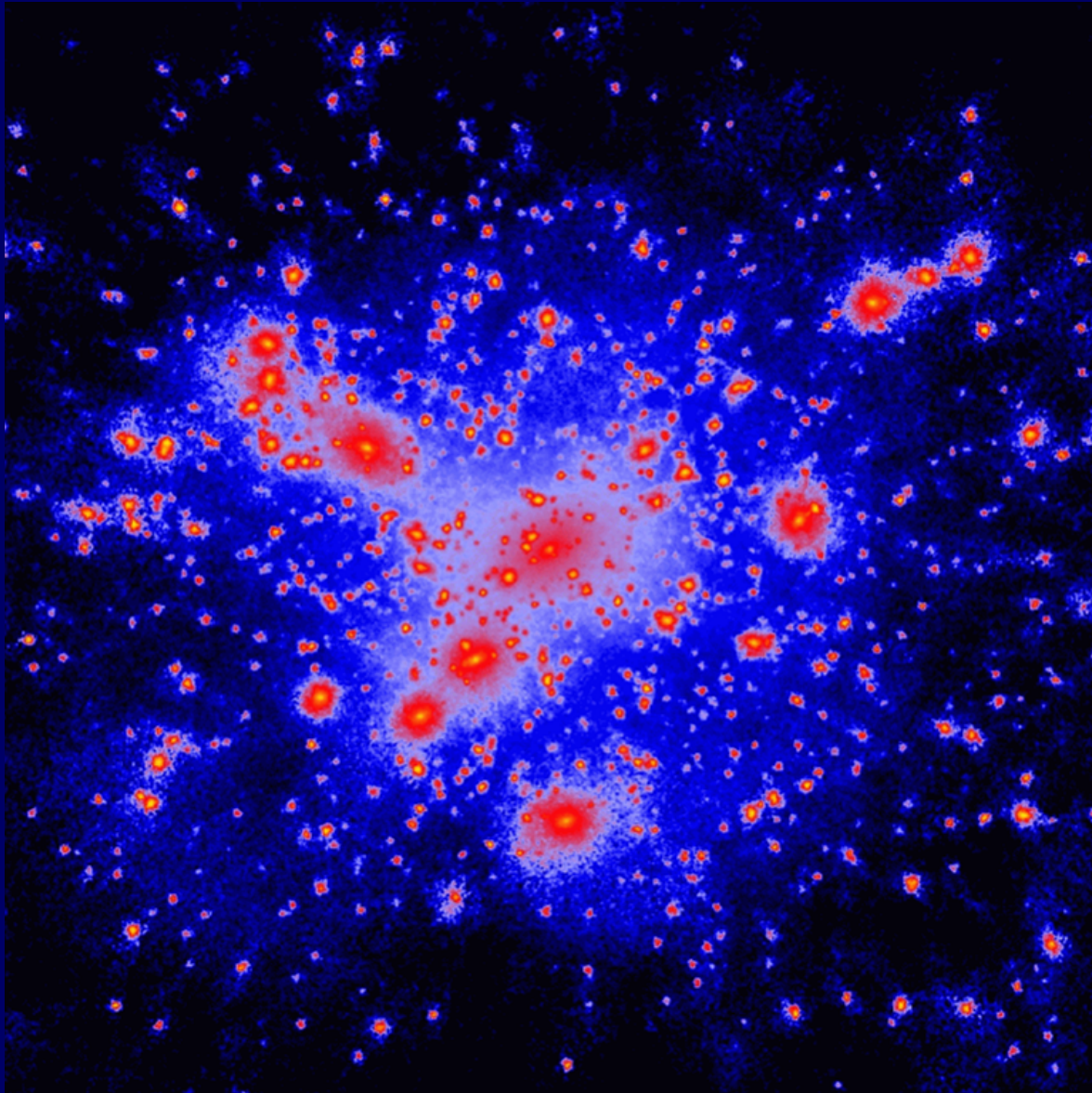
Tidal effects

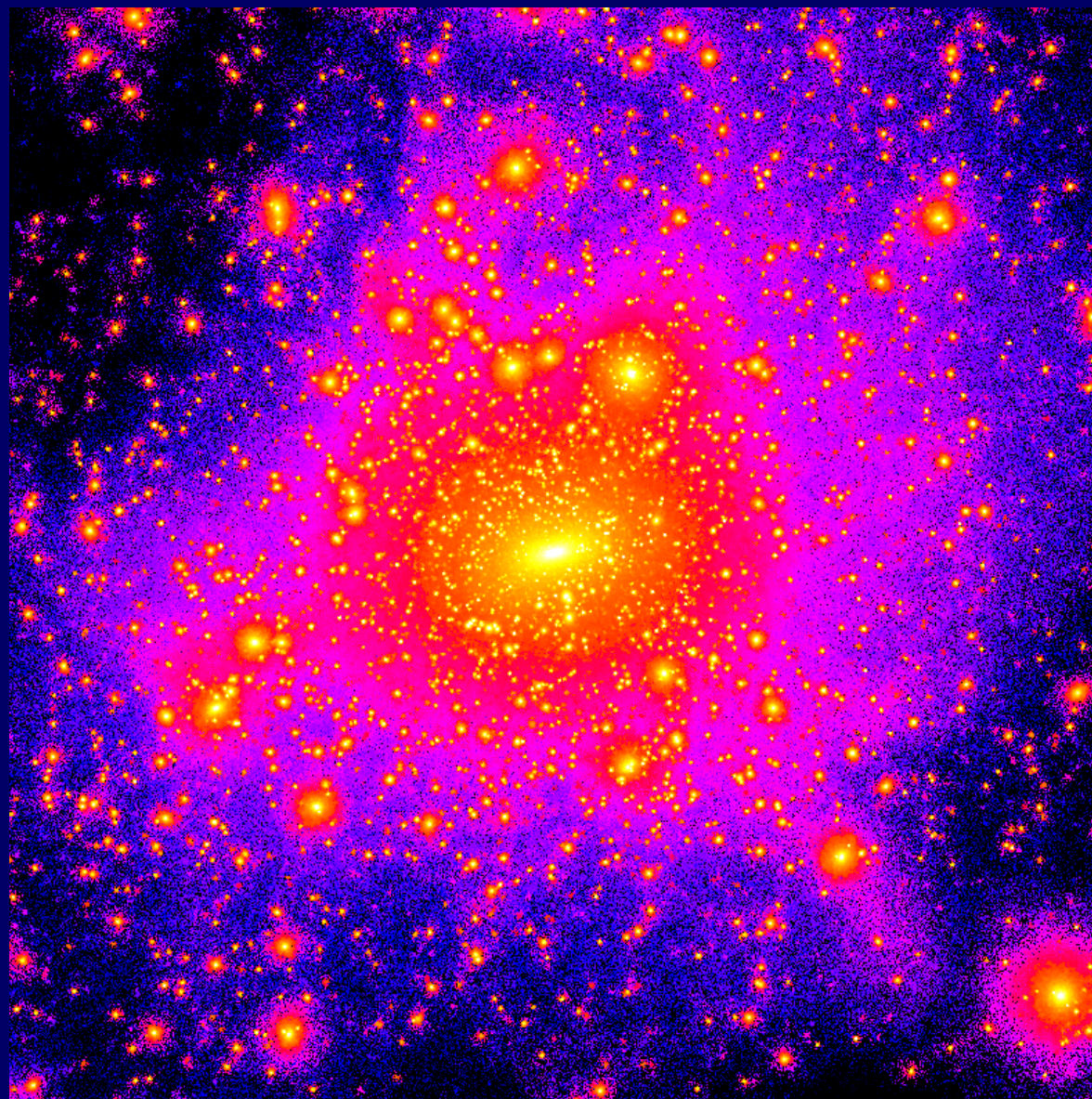
Origin of the cusp in hierarchical clustering

N-body simulation of Halo Formation

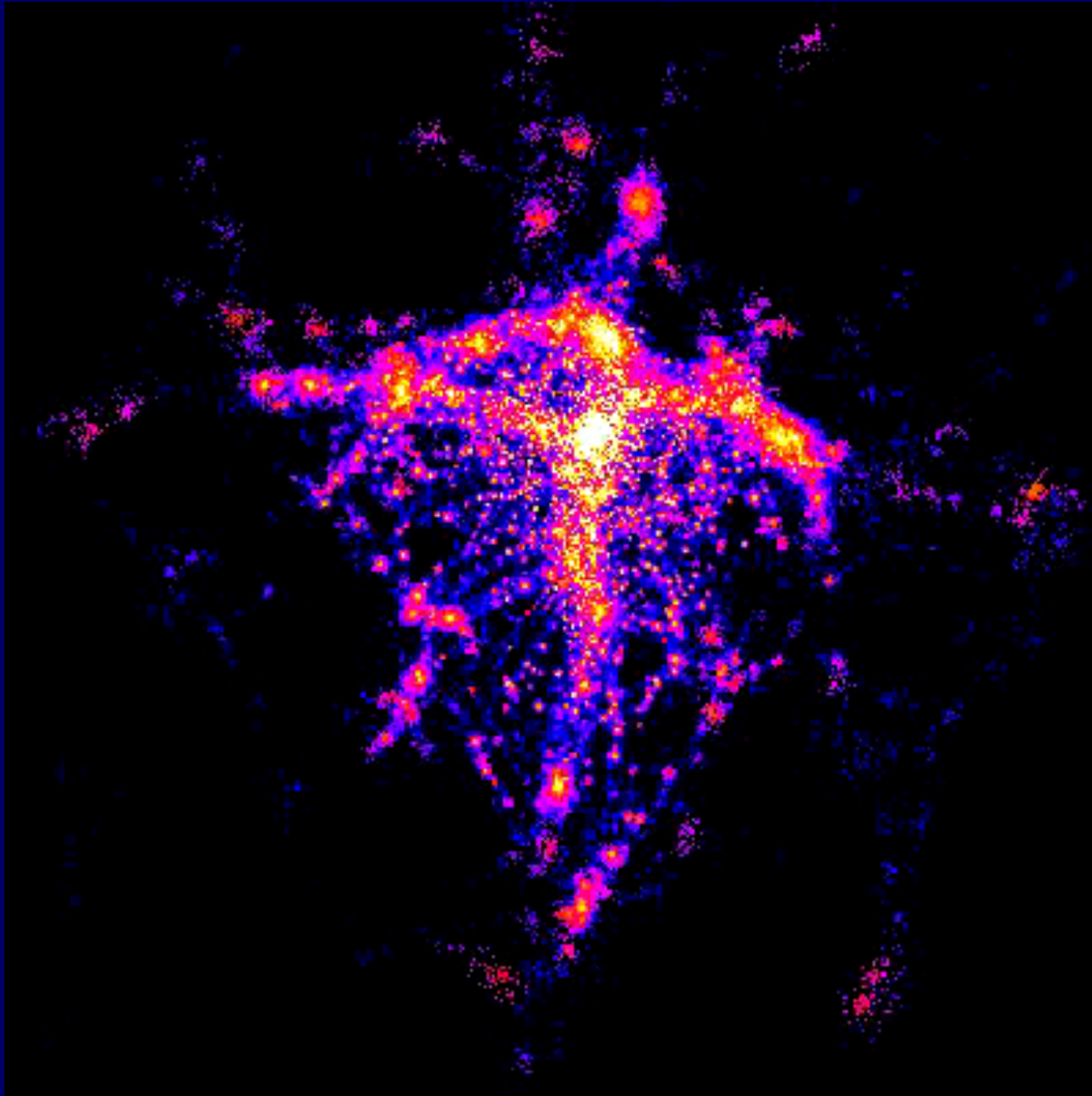


N-body simulation of Halo Formation





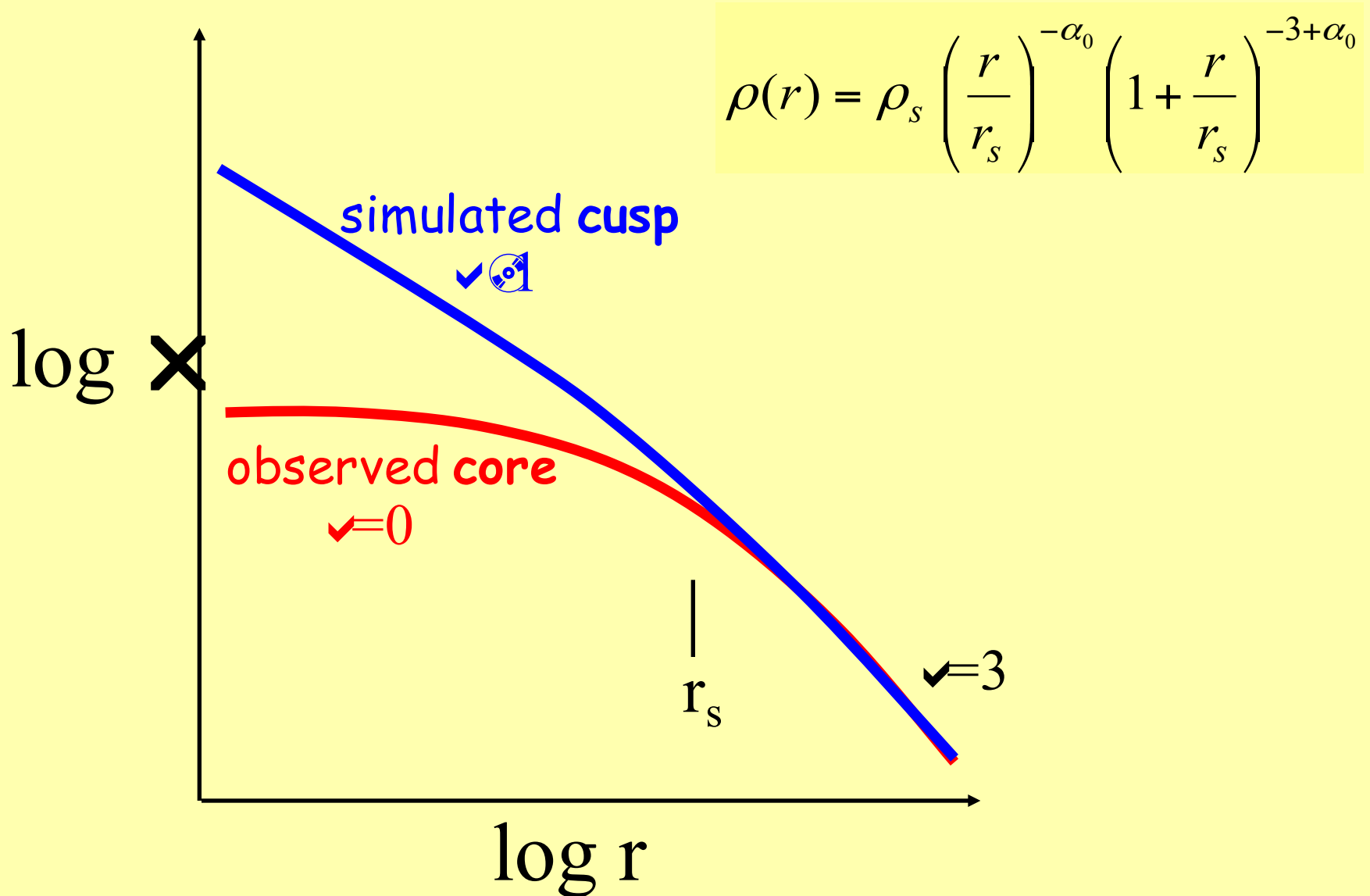
Dark Halo (Moore)



CDM halos (simulations)

- Density profiles are **universal**
shape independent of mass and cosmology.
- Density profiles are **cuspy**
density increases inward down to the innermost resolved radius. Asymptotic power-law near the center?
- Halos are **clumpy**
~10% of the mass is in self-bound clumps ---
the surviving cores of accreted satellites.

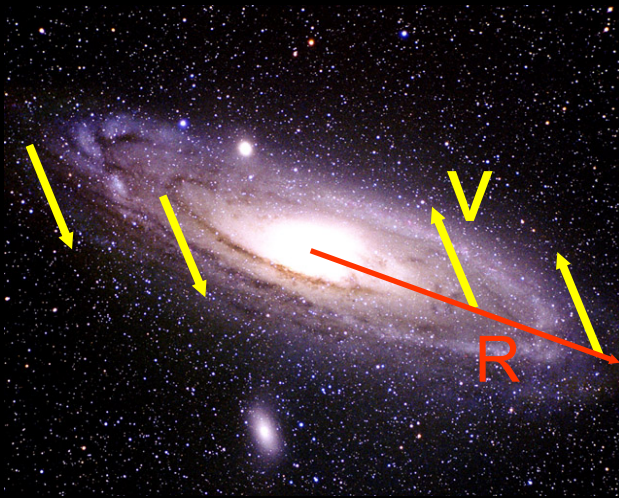
The dark-halo cusp/core problem



Universal Profile

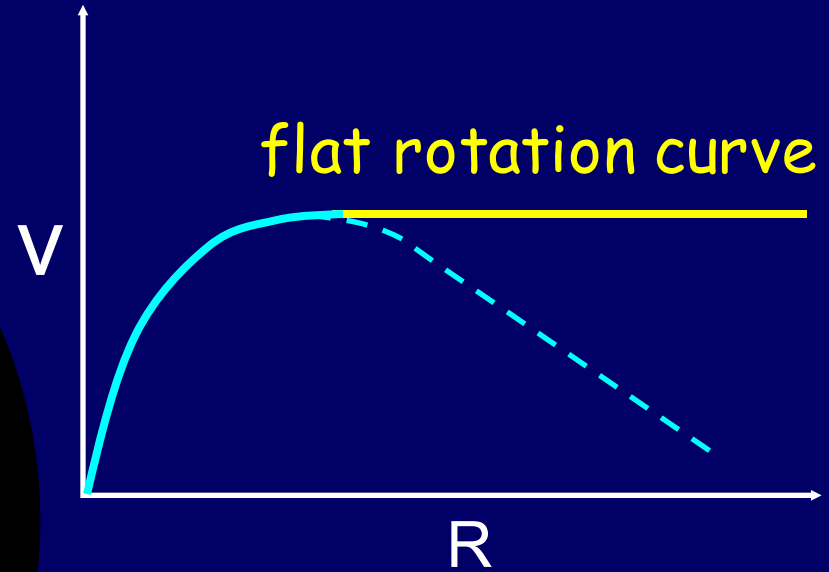
Dark Halos

dark halo

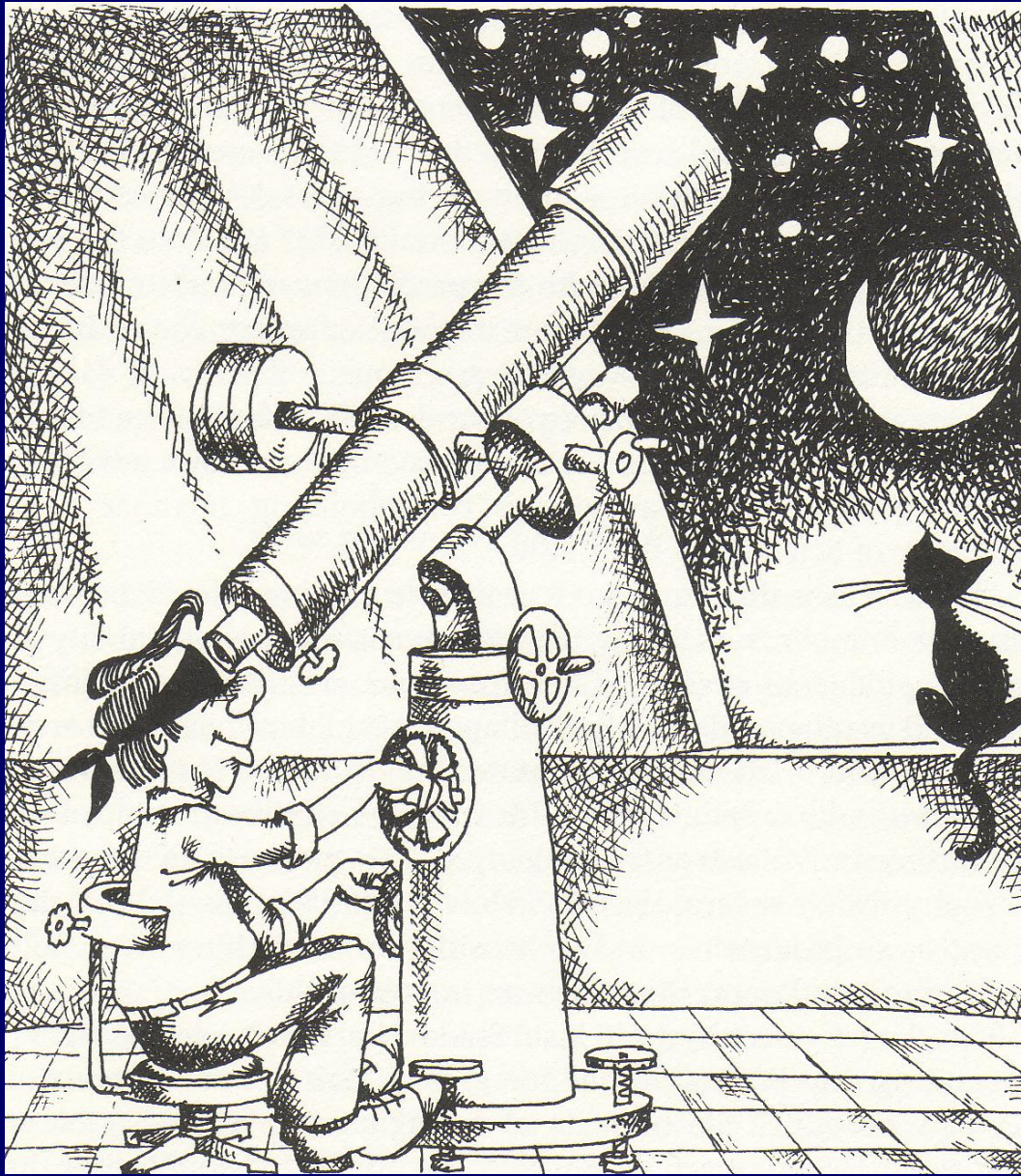


3,000 light years

30,000 ly



$$V^2 = \frac{GM(R)}{R}$$
$$\rightarrow M(R) \propto R$$



Isothermal Sphere

Hydrostatic equilibrium:

$$\frac{GM(r)\rho(r)}{r^2} = -\frac{dP}{dr} = \frac{\alpha\sigma^2\rho(r)}{r}$$

$$\rho(r) = \rho_0 r^{-\alpha} \rightarrow M(r) = \frac{4\pi}{(3-\alpha)} \rho_0 r^{(3-\alpha)}$$

$$P = nkT = \rho \frac{kT}{m} = \rho(r)\sigma^2 \rightarrow \frac{dP}{dr} = -\frac{\alpha\rho(r)\sigma^2}{r}$$

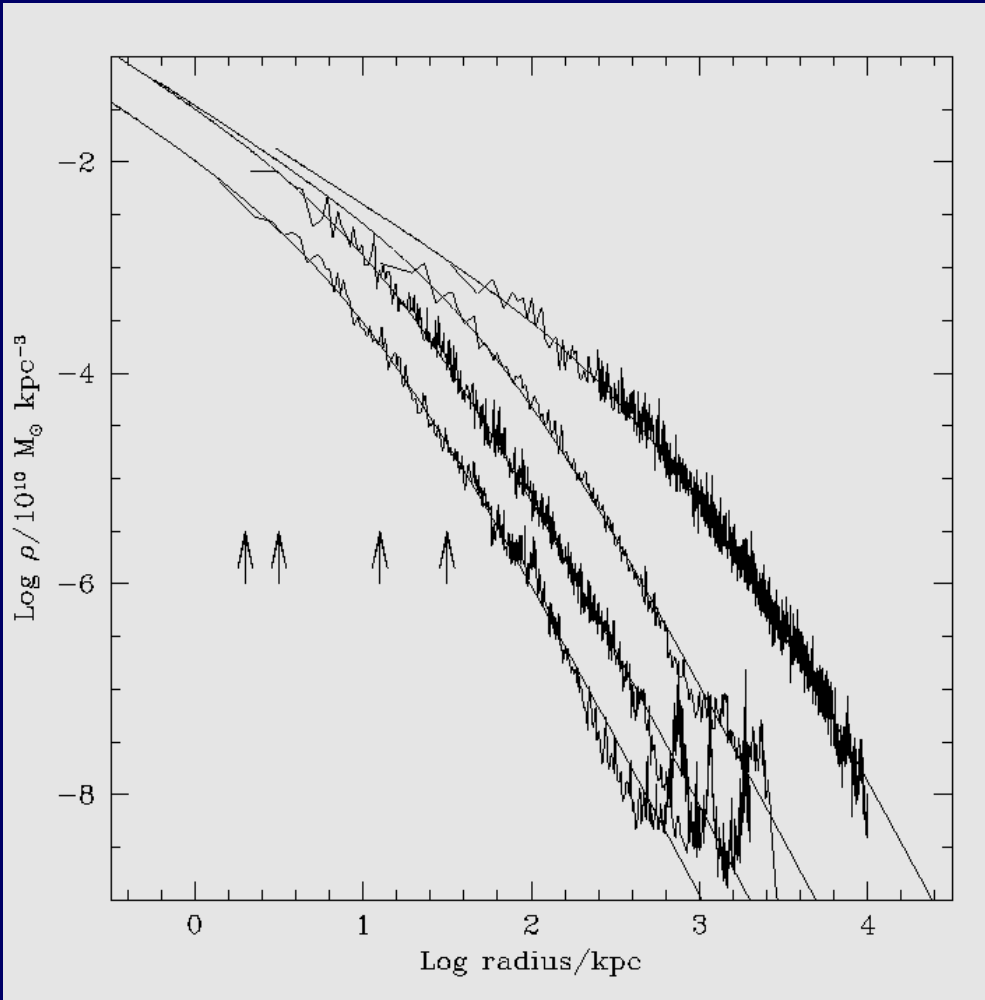
isothermal

$$\rightarrow M(r) = \frac{2\sigma^2}{G} r \rightarrow \rho(r) = \frac{\sigma^2}{2\pi G} r^{-2}$$

$$V^2(r) = \frac{GM(r)}{r} = 2\sigma^2$$

Universal Mass Profile of CDM Halos

Density



Radius

Mass profile general shapes are independent of halo mass & cosmological parameters

Density profiles differ from power law

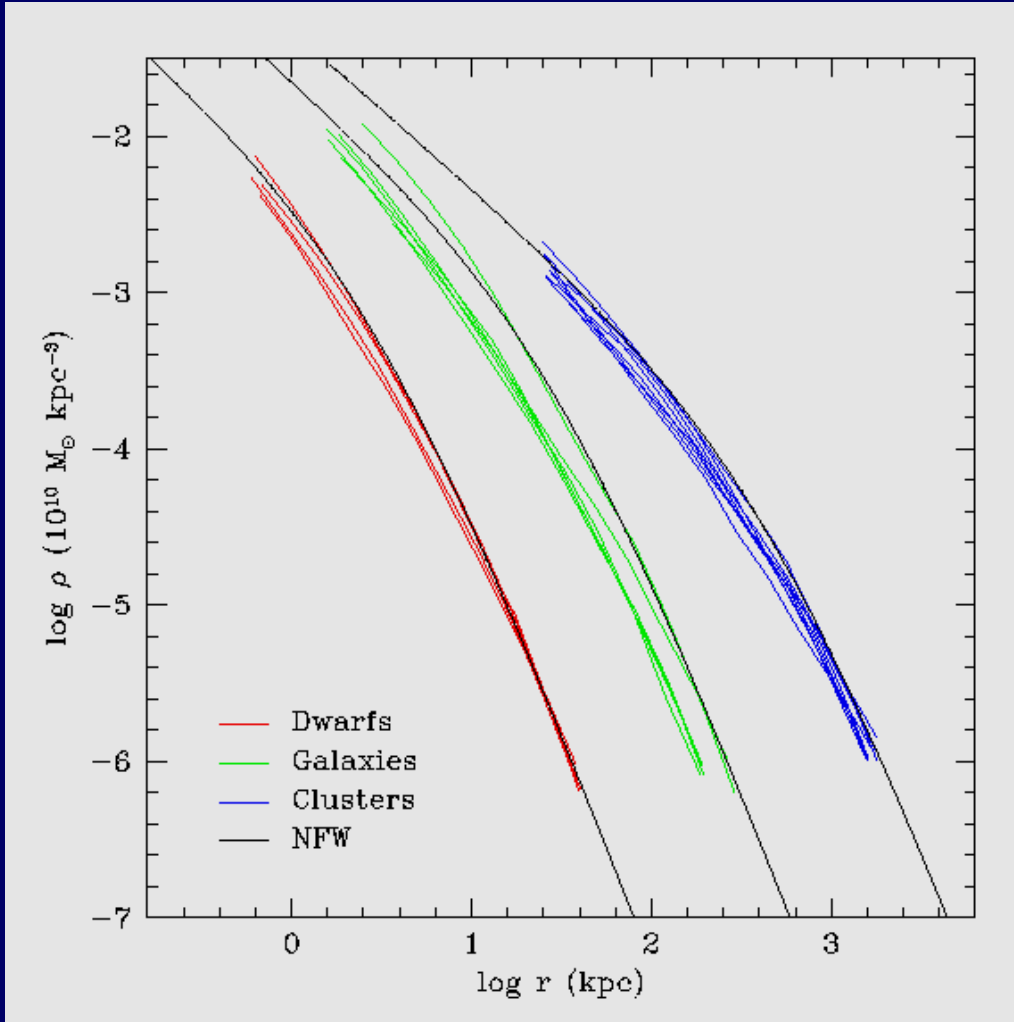
The profile is shallower than isothermal near the center

But no obvious flat-density core near the center

A cusp; some controversy about inner slope

New results for Λ CDM halos

Density



Radius

Simulations span ~ 6 decades in M_{vir} , from dwarf galaxies ($V_c \sim 50$ km/s) to galaxy clusters ($V_c \sim 1000$ km/s)

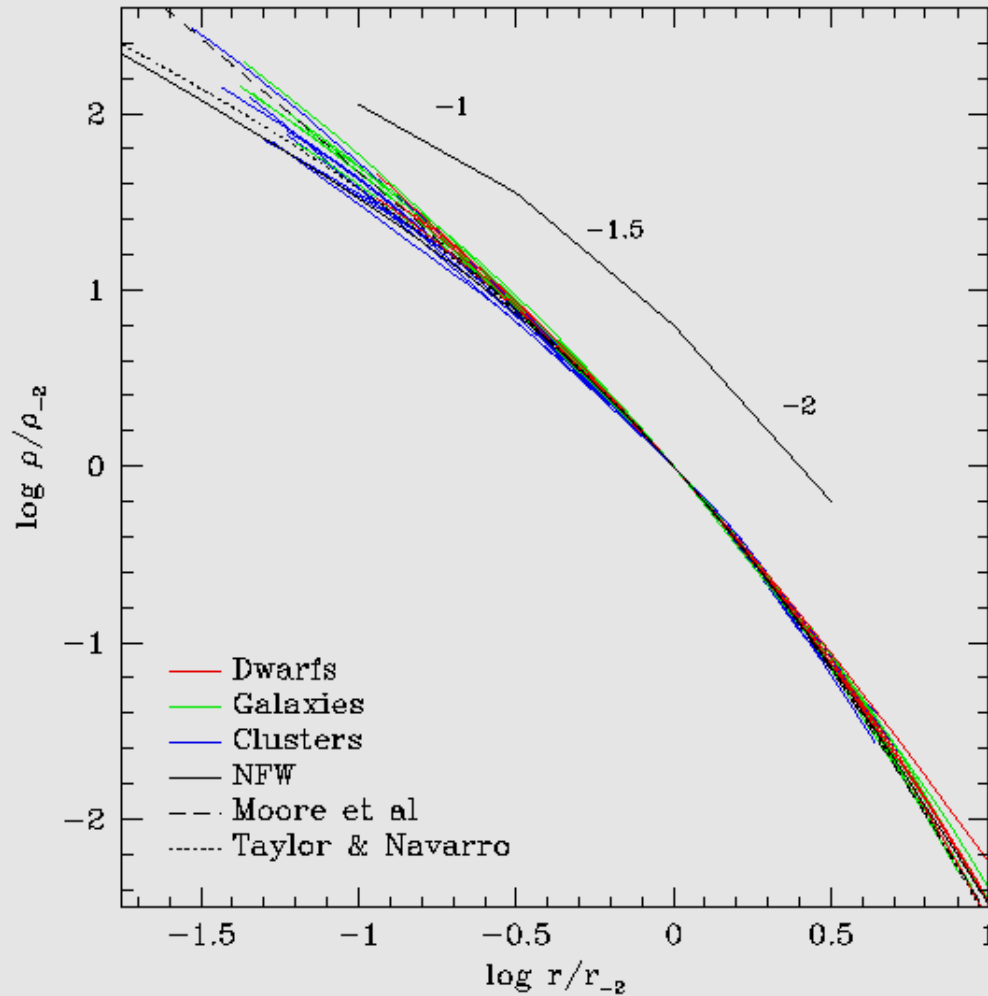
\sim million particles within R_{vir}

Controlled numerical effects via convergence studies

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Recent results for Λ CDM halos

Scaled Density



Properly scaled, all halos look alike: CDM halo structure appears to be "universal"

Scaled Radius

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Universal Profile: NFW

$$\rho(r) = \frac{\rho_s}{x(1+x)^2} \quad x \equiv \frac{r}{r_s} \quad \text{for } 0.01R_{vir} < r < R_{vir}$$

generalized
cusp:

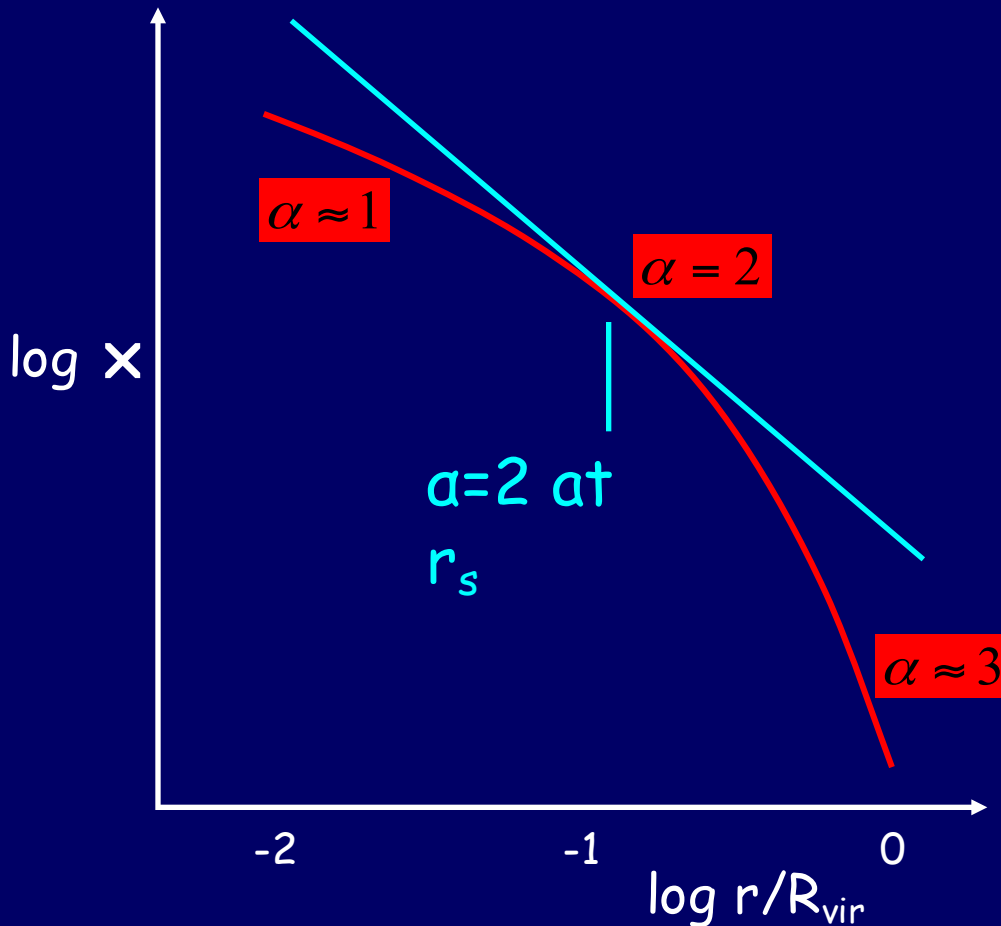
$$\rho(r) = \frac{\rho_s}{x^{\alpha_0} (1+x)^{3-\alpha_0}}$$

slope: $\alpha(r) = -\frac{d \ln \rho}{d \ln r}$

two parameters:

$$M_{vir} \quad C \equiv \frac{R_{vir}}{r_s} \sim 10$$

Ellipsoidal shape: $a_3/a_1 \sim 0.5$



Navarro, Frenk & White 95, 96, 97
 Cole & Lacey 96
 Moore et al. 98
 Ghinga et al. 00
 Klypin et al. 01
 Power et al. 02
 Navarro, Hayashi et al. 03, 04
 Stoehr et al. 04, 05

Halo Concentration vs Mass and History

Self-similar Toy model (Bullock et al. 2001):

Define a_c as the time when typically a constant fraction f of M is collapsing:

$$M_*(a_c) \equiv fM \quad (1)$$

Define a characteristic halo density:

$$\tilde{\rho}_s \equiv \frac{M}{(4\pi/3)r_s^3} = 3\rho_s \left(\ln(1+C) - \frac{C}{1+C} \right) \quad \text{for NFW}$$

Assume additional contraction of inner halo by a constant factor k :

$$\tilde{\rho}_s = k^3 \Delta(a) \rho_u(a_c) = k^3 \Delta(a) \rho_u(a) \frac{a^3}{a_c^3}$$

$$C \equiv \frac{R_{vir}}{r_s} \longrightarrow C(\mu, a) = k \frac{a}{a_c} \quad (2)$$

EdS
 $P_k \propto k^n$

$$\sigma \propto M^{-\alpha} \rightarrow M_* \propto a^{1/\alpha} \rightarrow^1 \frac{a_c}{a_0} = (\mu f)^\alpha$$

$$\mu \equiv M(a) / M_*(a)$$

$$C(\mu, a) = k(f\mu)^{-\alpha}$$

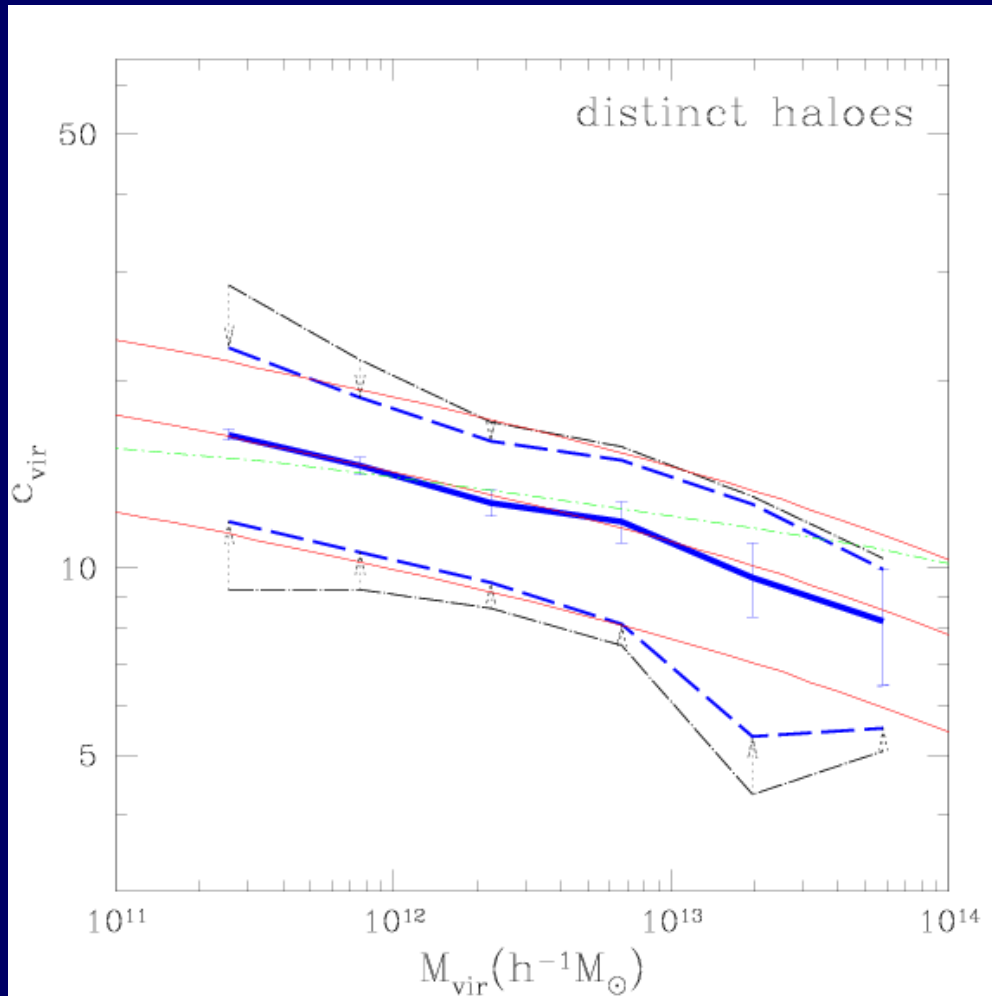
Determine parameters from simulations:

$$f \sim 0.01 \quad k \approx 4 \quad \alpha \approx 0.13$$

Excellent fit!

$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

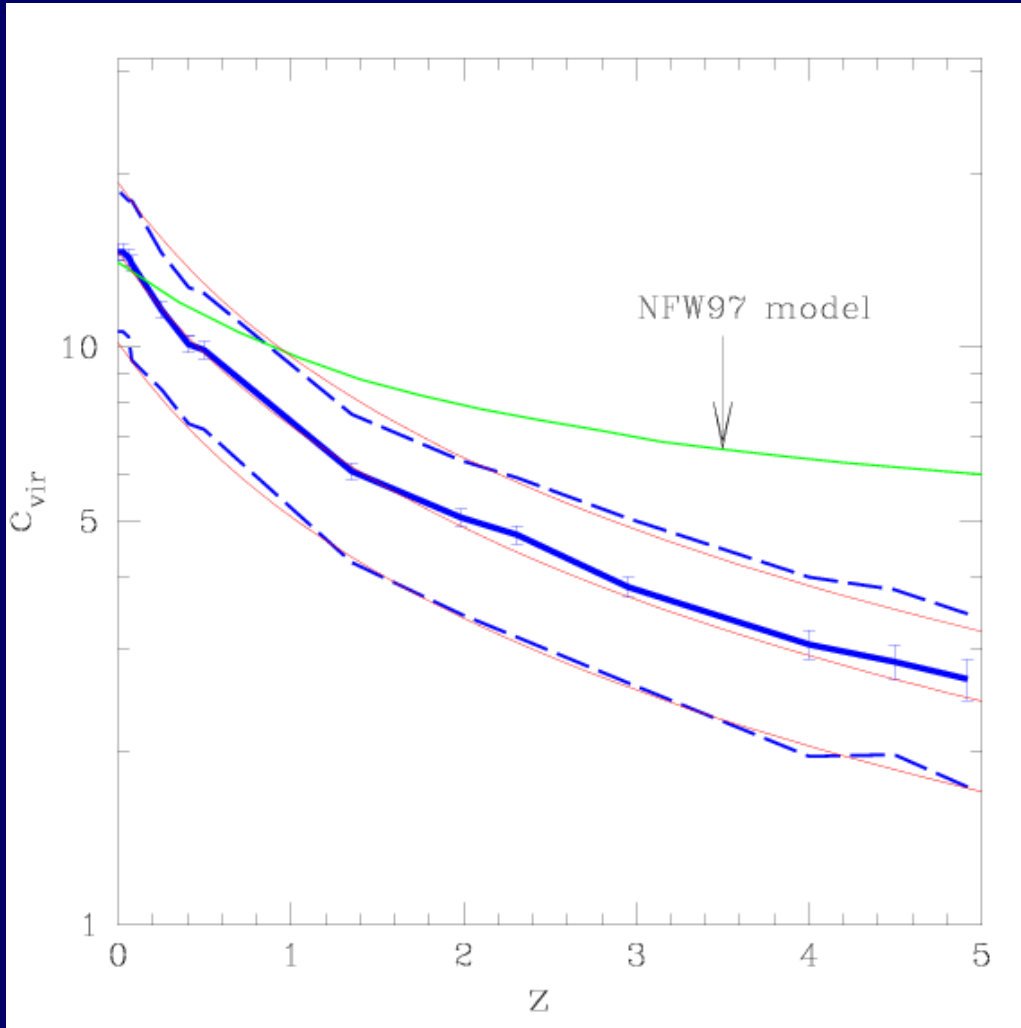
Concentration vs Mass



$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

Bullock et al. 2001

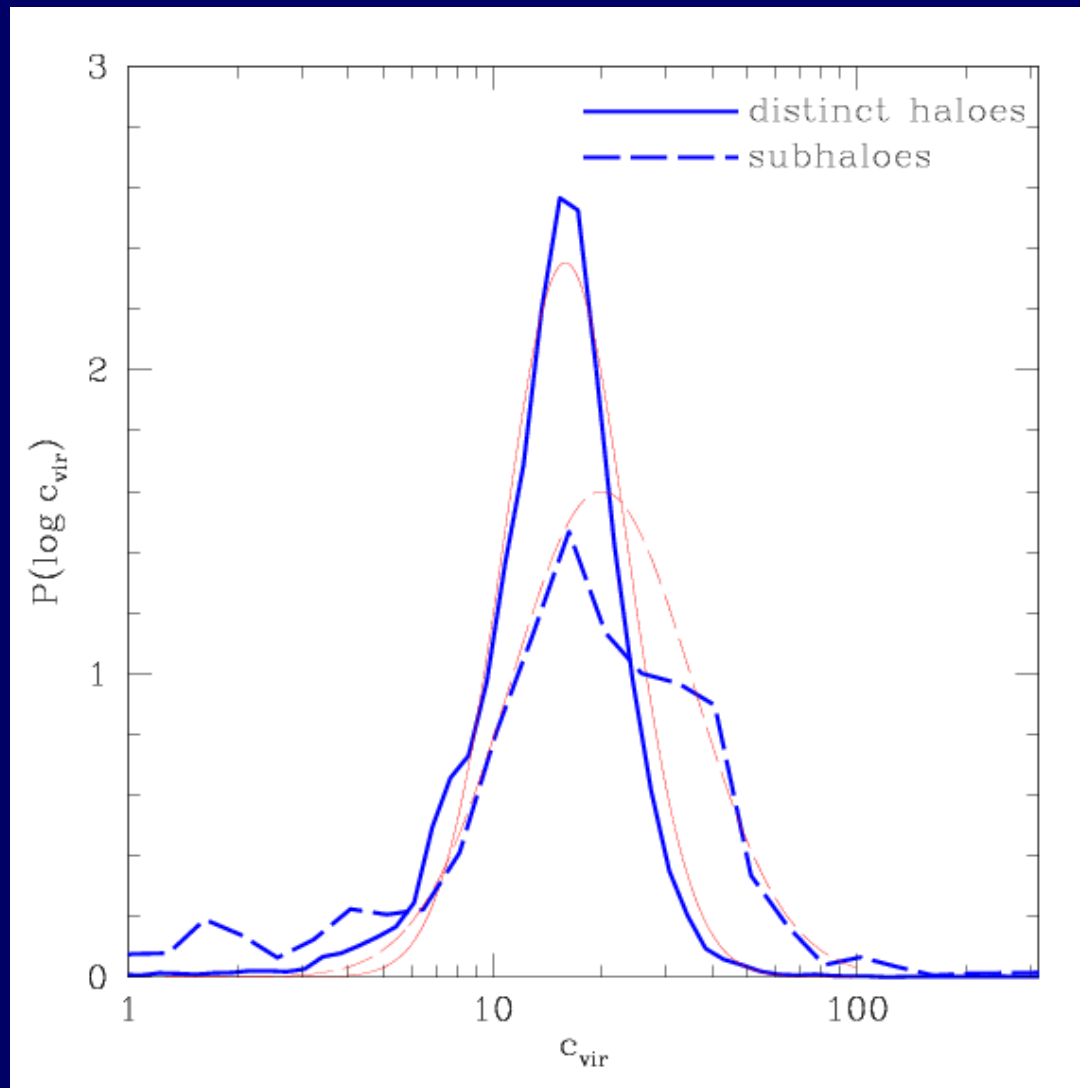
Concentration vs time, given mass



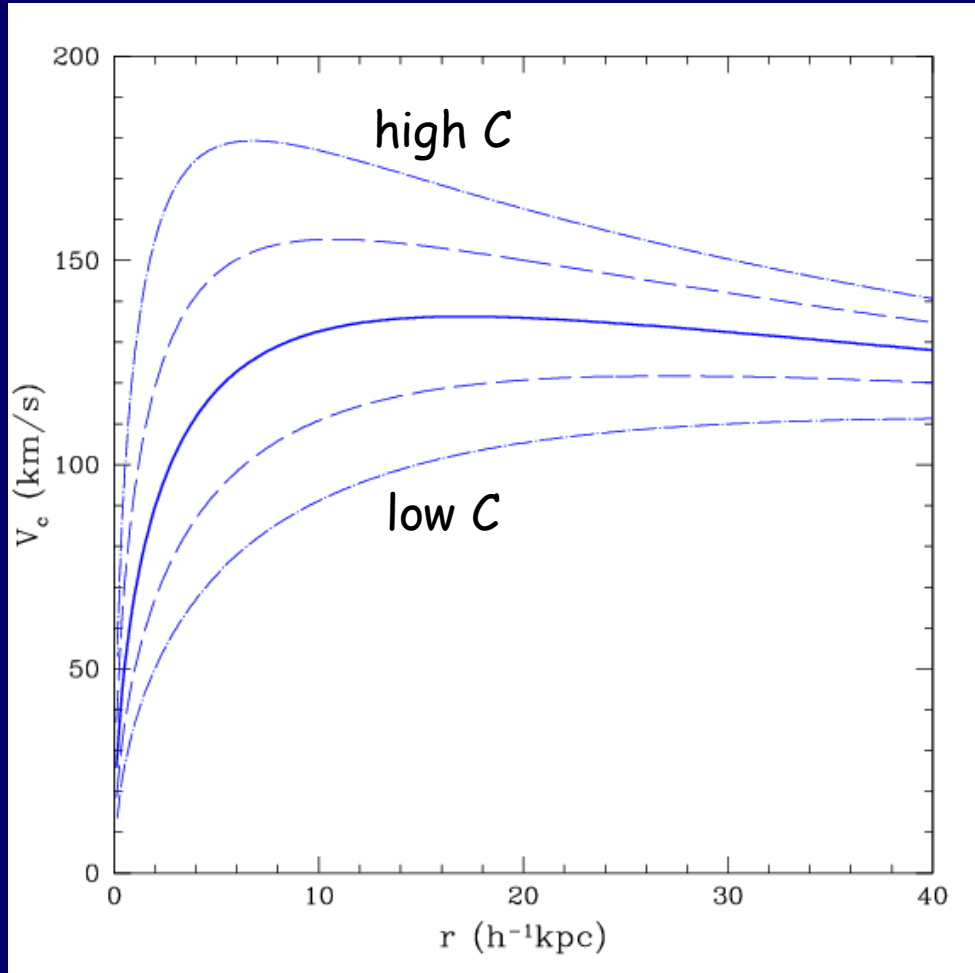
$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

Bullock et al. 2001

Distribution of C : log-normal



NFW Rotation Curve



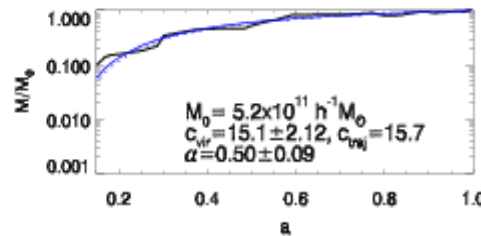
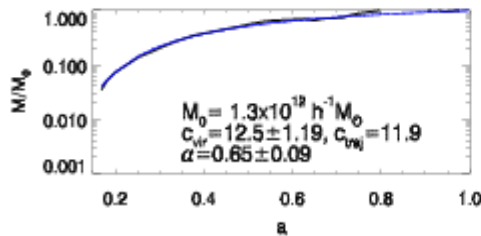
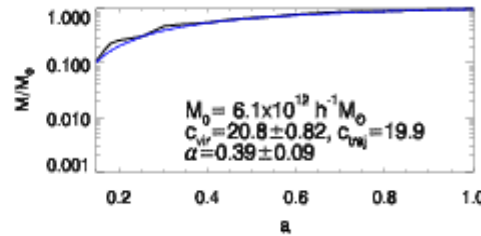
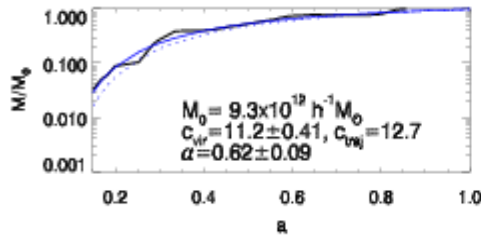
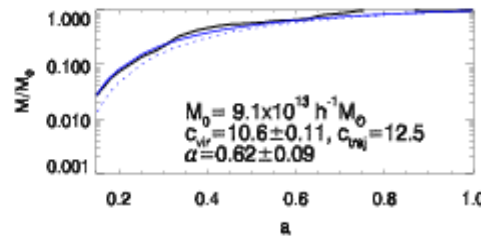
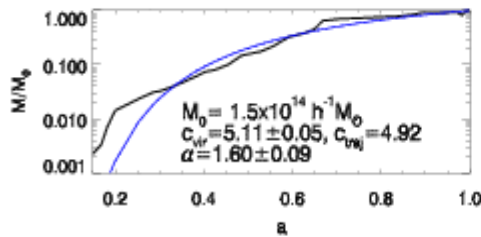
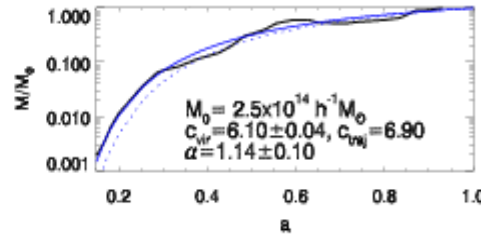
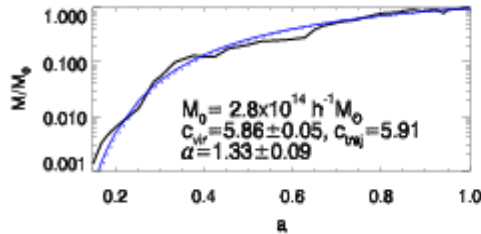
$$M = 4\pi\rho_s r_s^3 A(C) \quad A(C) \equiv \ln(1+C) - \frac{C}{1+C}$$

$$V^2(x) = V_{vir}^2 \frac{C}{A(C)} \frac{A(x)}{x}$$

$$r_{max} \approx 2.16r_s \quad \frac{V_{max}^2}{V_{vir}^2} \approx 0.216 \frac{C}{A(C)}$$

Mass Assembly History

Wechsler et al. 2002



$$M(a) \propto e^{-2a_c z}$$

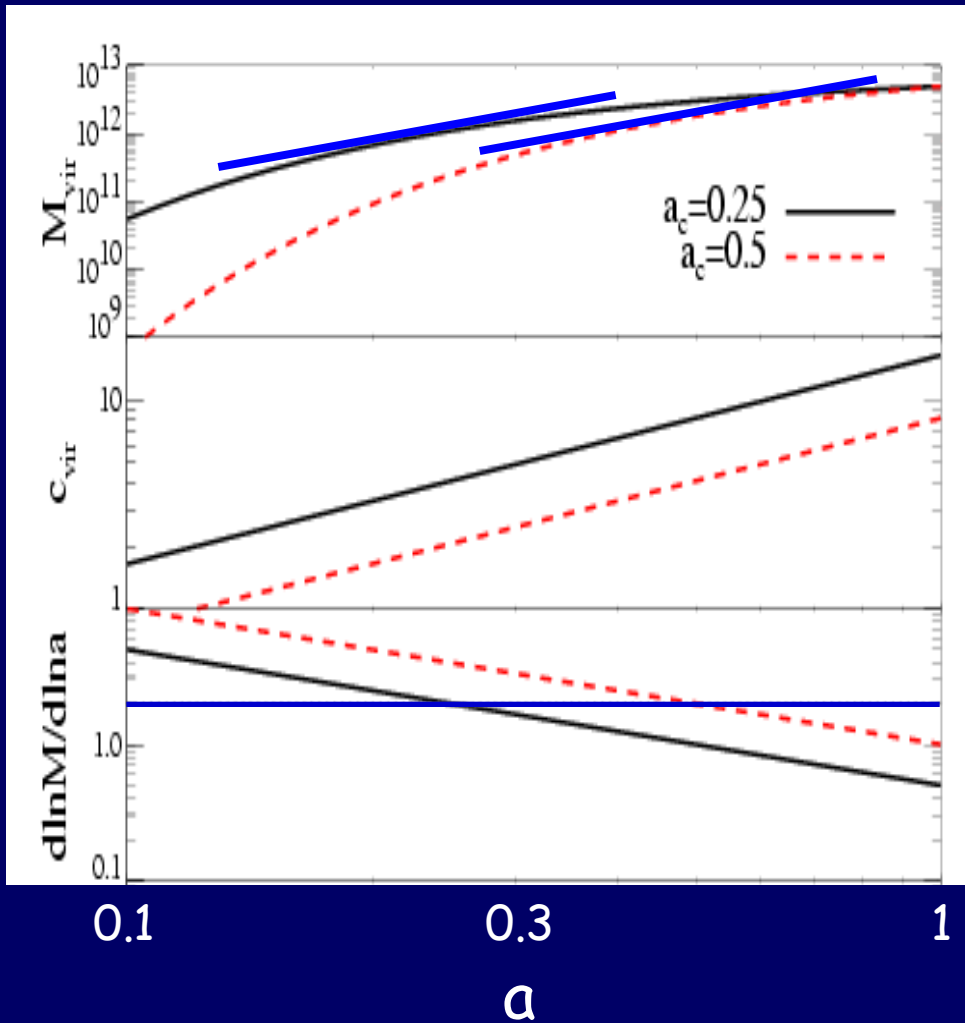
$$\frac{d \log M}{d \log a} = 2 \quad \text{defines } a_c$$

$$M = M_0 e^{-\alpha z}$$

$$\frac{\dot{M}}{M} = 0.04 \alpha (1+z)^{2.5} \text{ Gyr}^{-1}$$

Mass Assembly History

Wechsler et al. 2002

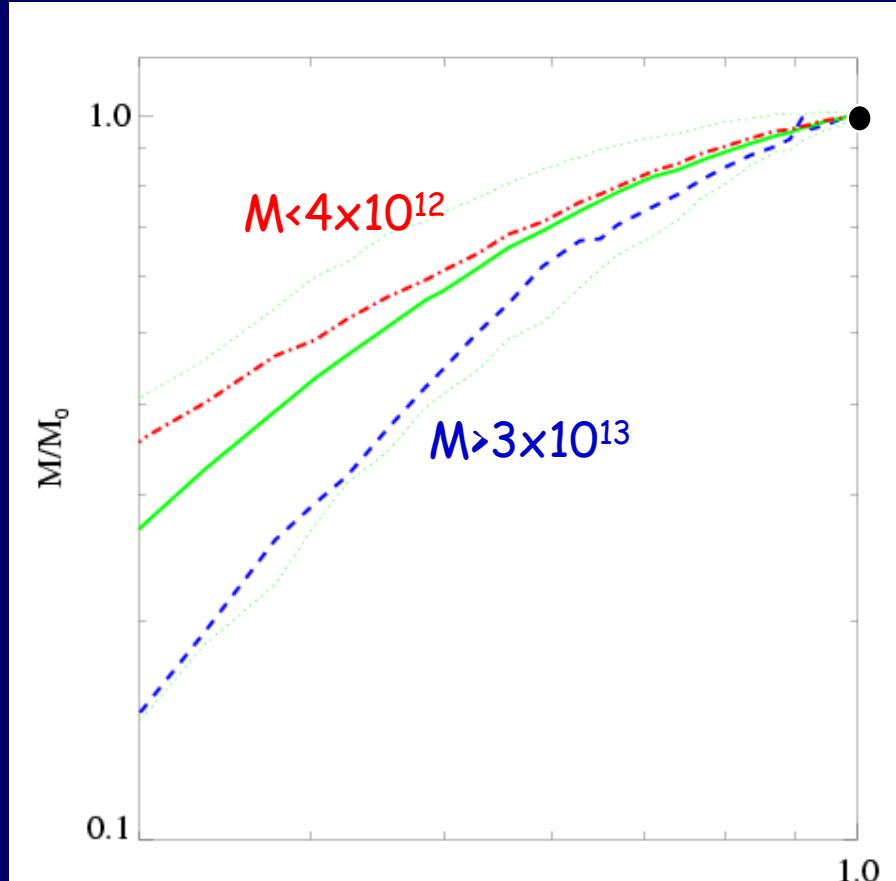


$$M(a) \propto e^{-2a_c z}$$

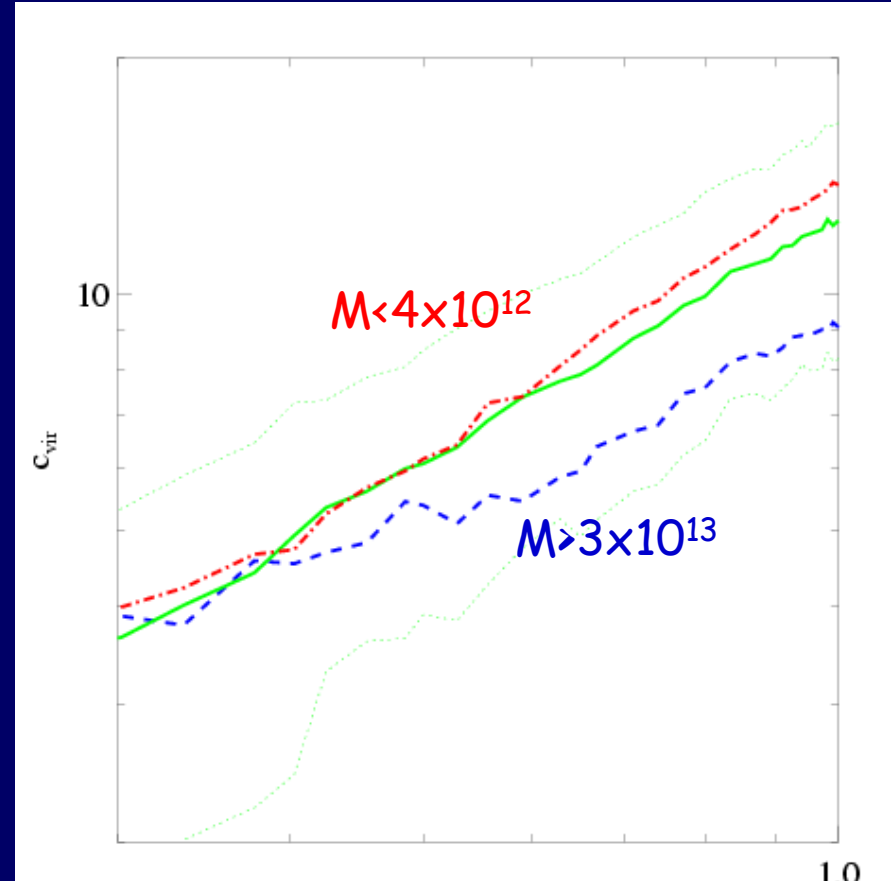
$$\frac{d \log M}{d \log a} = 2 \quad \text{defines } a_c$$

Mass dependence of History and Concentration

Wechsler et al. 2002



a

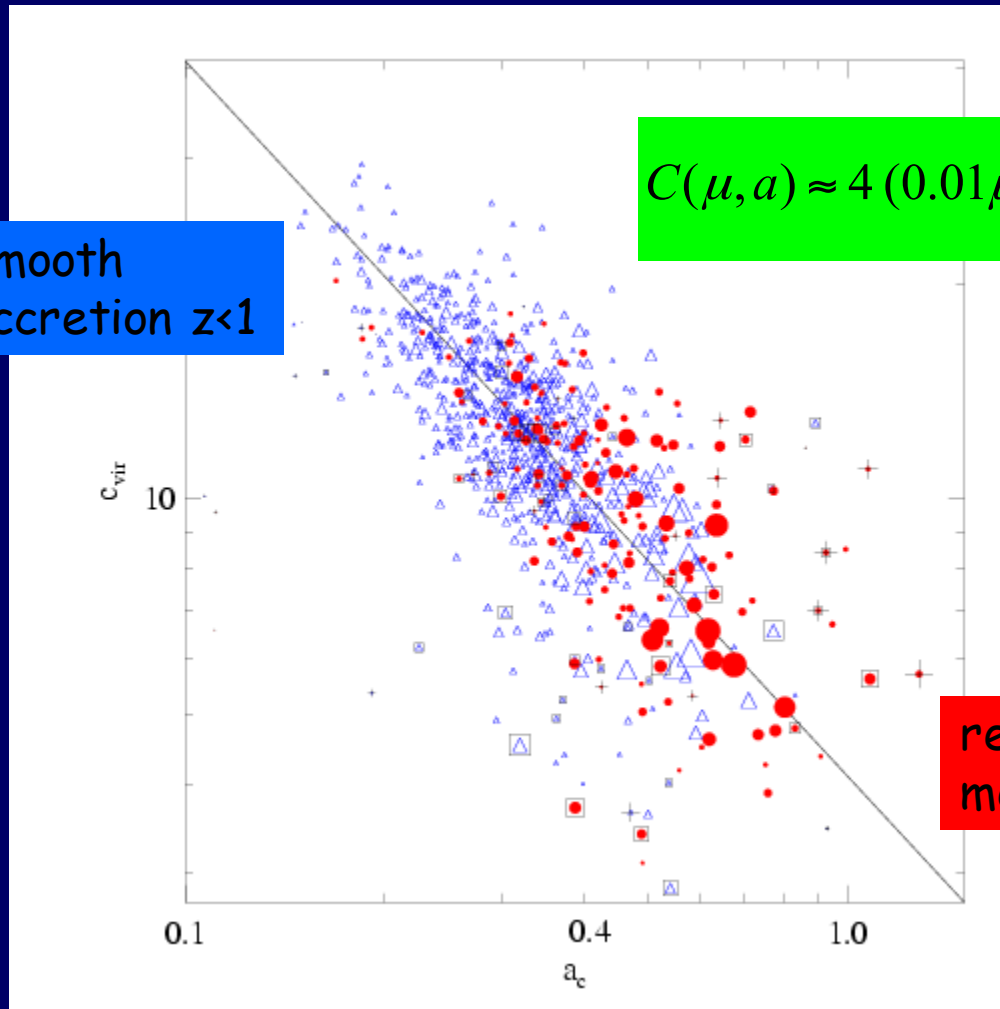


a

$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

Concentration vs History

Wechsler et al. 2002



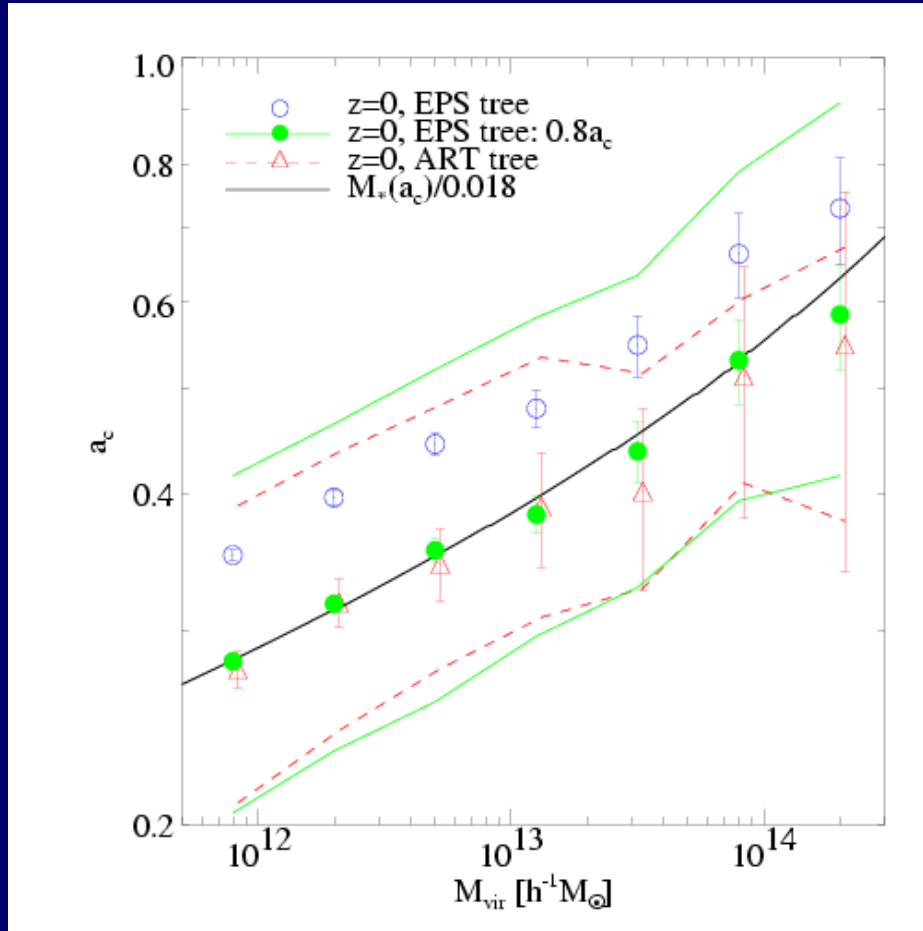
smooth
accretion $z < 1$

$$C(u, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

recent major
merger $z < 1$

History vs Mass

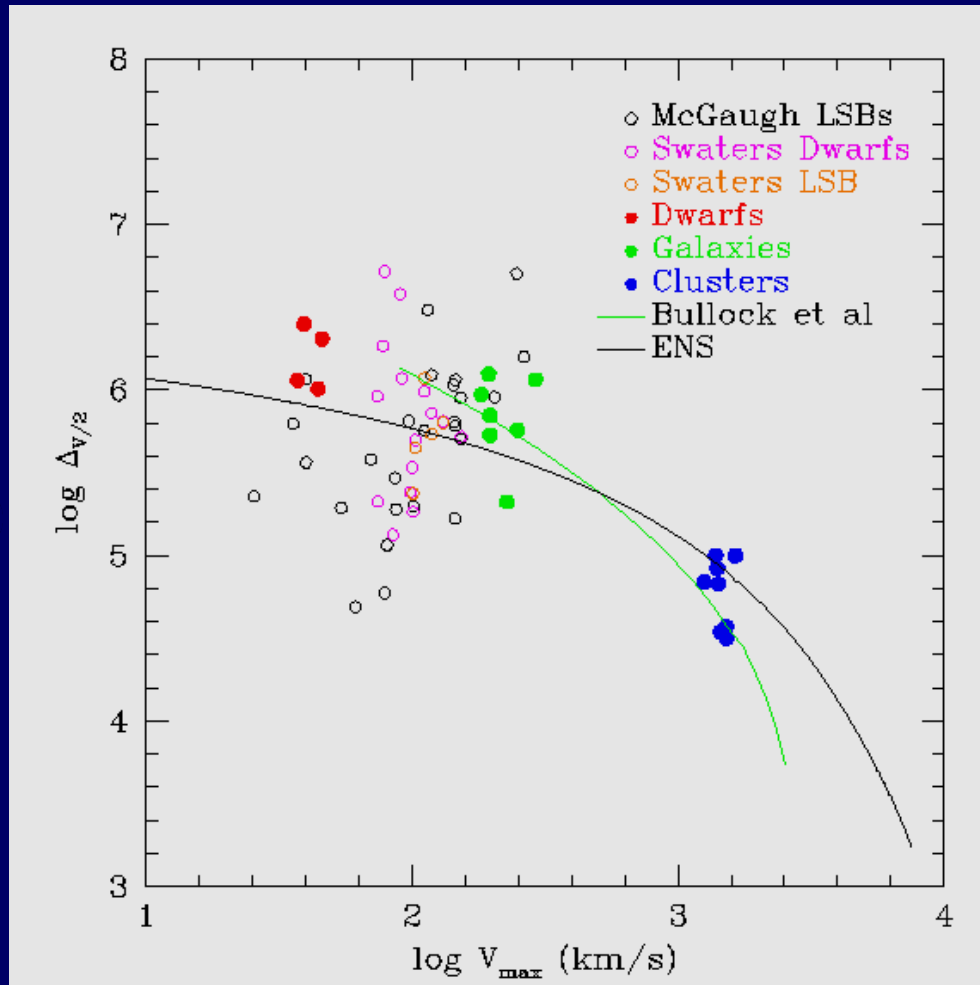
Wechsler et al. 2002



$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

Concentration of LSB galaxies and Λ CDM halos

Mean density contrast within $r(V_{\max}/2)$



Maximum Rotation Speed

The average intermediate-scale concentration and scatter of Λ CDM halos is roughly consistent with observations of LSB and dwarf galaxies

Average Assembly Rate into R_{vir} by EPS

Neistein, van den Bosch, Dekel 06; Birnboim, Dekel, Neistein 07, Neistein & Dekel 07, 08

Growth rate of main progenitor

Self-invariant time variable

$$\omega \equiv \frac{\delta_c}{D(t)} \longrightarrow \frac{dM}{d\omega} = \text{const.} \rightarrow \dot{M} \propto \dot{\omega}$$

In EdS regime ($z > 1$)

$$D(a) \propto a \propto t^{2/3} \rightarrow \dot{M} \propto a^{-5/2}$$

For a power-law power spectrum

$$\dot{M} \propto M^{1+\alpha}, \quad \alpha = (n+3)/6 \ll 1$$

Specific accretion rate

$$\frac{\dot{M}}{M} = s \text{ Gyr}^{-1} (1+z)^{2.5}$$

Mass growth

$$M = M_0 e^{-\alpha z}$$

$$\alpha = \frac{3}{2} s t_1 (1+z)^{-1} \approx \left(\frac{t}{t_1} \right)^{2/3} \quad t_1 \approx 17.5 \text{ Gyr}$$

For a general power spectrum

$$\frac{d \ln M}{d\omega} \approx - (2/\pi)^{1/2} \left(\sigma^2(M/2.2) - \sigma^2(M) \right)^{-1/2}$$

$$\left\langle \frac{\dot{M}}{M} \right\rangle_{\text{vir}} \approx 0.03 \text{ Gyr}^{-1} M_{12}^{0.15} (1+z)^{2.5}$$

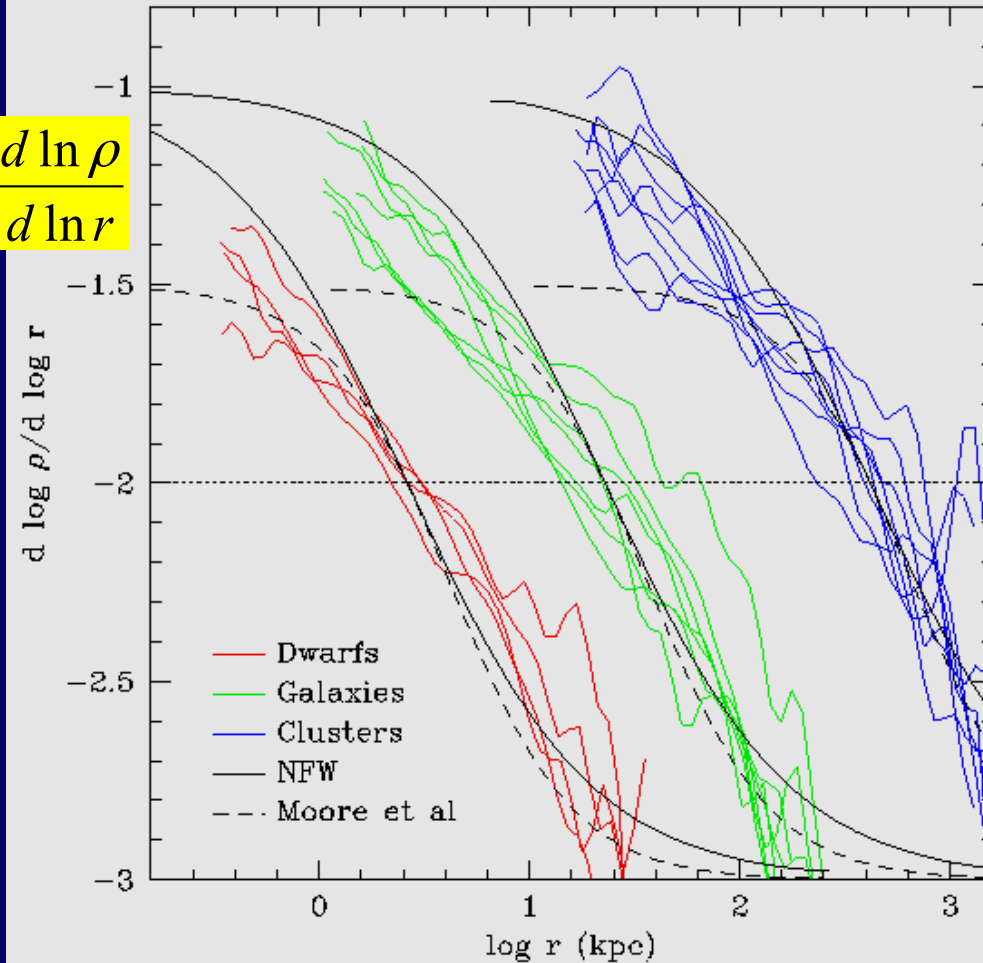
$$\left\langle \dot{M} \right\rangle_{\text{vir}} \approx 30 M_{\odot} \text{ yr}^{-1} M_{12}^{1.15} (1+z)^{2.5}$$

Simulated Cusp

Recent results for Λ CDM halos

$$-\alpha(r) = \frac{d \ln \rho}{d \ln r}$$

Logarithmic Slope



Radius

No obvious convergence to a power law: profiles get shallower all the way in.

Innermost slopes are shallower than -1.5

Improved profile:

$$\alpha_{\beta}(r) \equiv -\frac{d \ln \rho}{d \ln r} = 2 \left(\frac{r}{r_s} \right)^{\beta}$$

$$\ln \left(\frac{\rho_{\beta}}{\rho_s} \right) = -\frac{2}{\beta} \left[\left(\frac{r}{r_s} \right)^{\beta} - 1 \right]$$

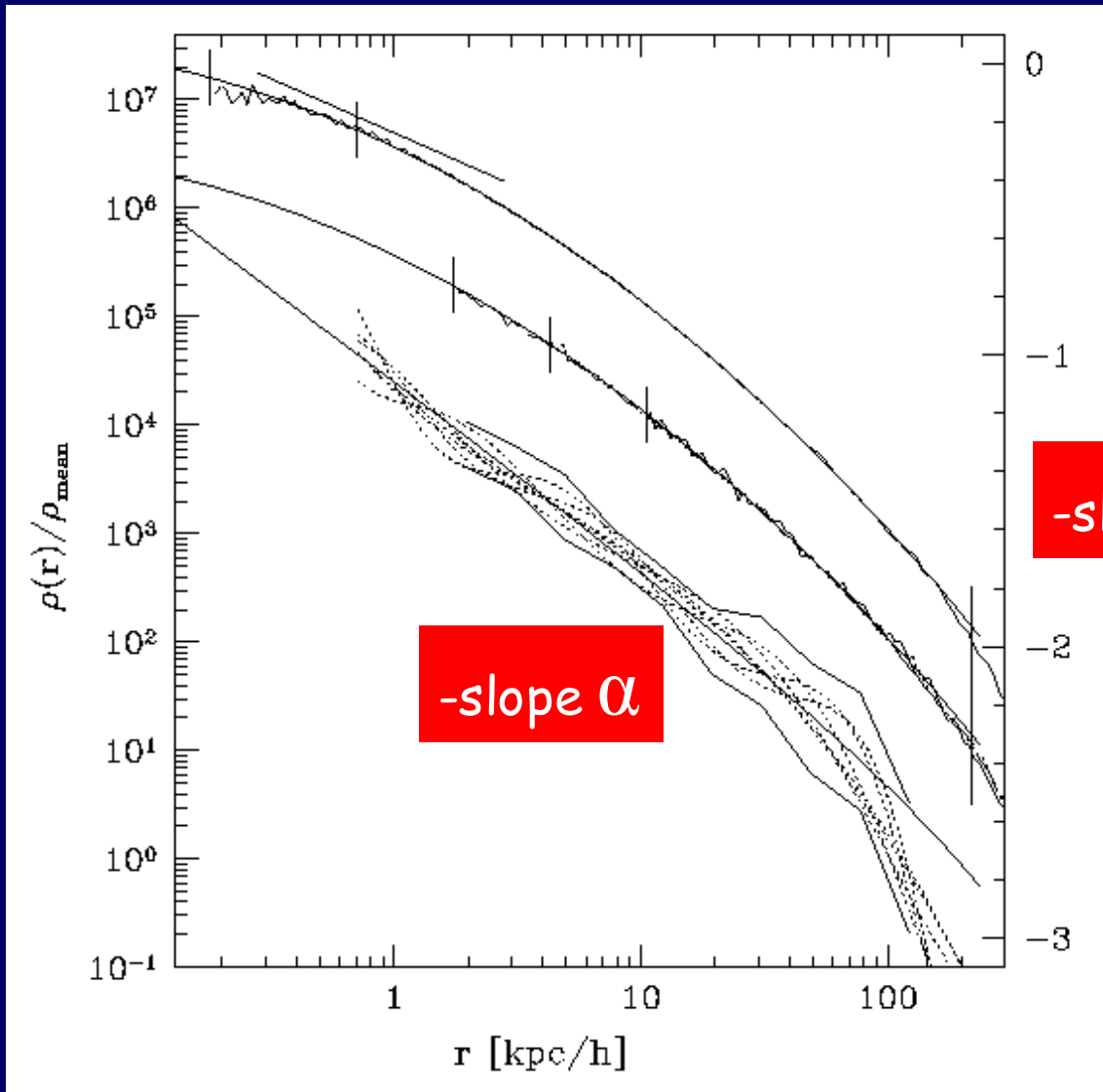
$$\beta \sim 0.1 - 0.2$$

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Improved Cusp Profiles

Stoehr et al. 2004

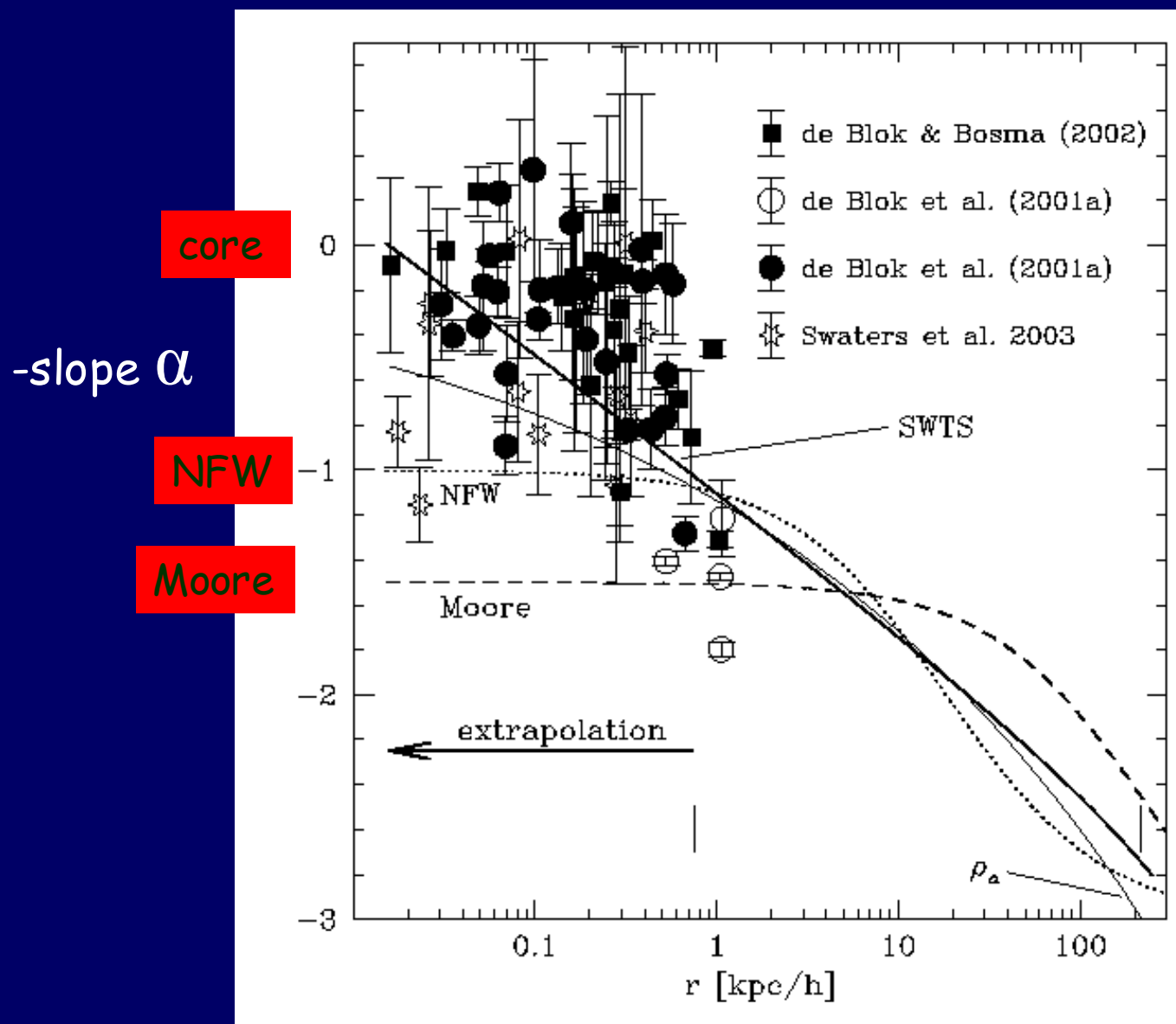
$$\log\left(\frac{V}{V_{\max}}\right) = -a \left[\log\left(\frac{r}{r_{\max}}\right) \right]^2$$



-slope α

-slope α

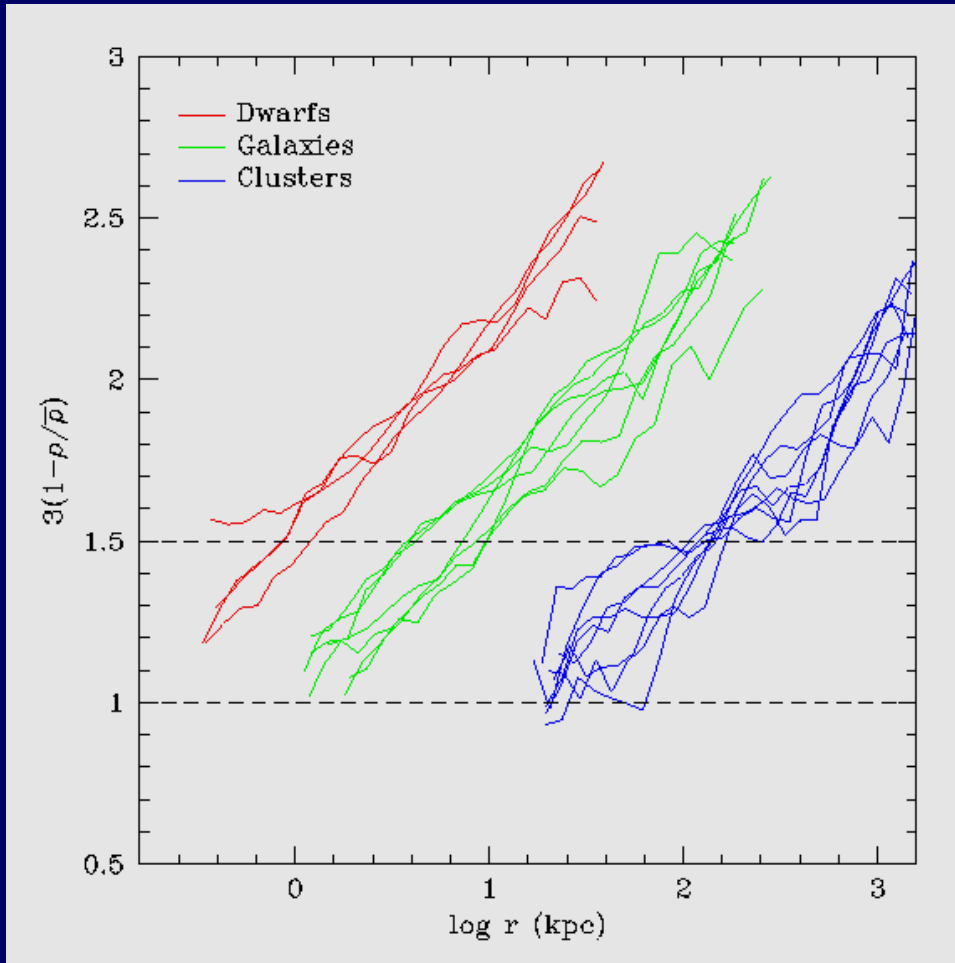
Improved Cusp Profiles: extrapolated to the inner cusp



Maximum Asymptotic Inner Slope

$$\rho = r^{-\alpha} \quad r < r_p \quad \rightarrow \quad \bar{\rho}(r) = \frac{1}{(4\pi/3)r^3} \int_0^r 4\pi r'^2 dr' \rho(r') = \frac{3}{3-\alpha} r^{-\alpha}$$

$$\rightarrow \alpha = 3[1 - \rho(r) / \bar{\rho}(r)] \quad \text{upper limit for slope in } r < r_p$$



Radius

$M(r)$ is robustly measured in the simulations.

With the local density, it provides an upper limit to the inner asymptotic log slope

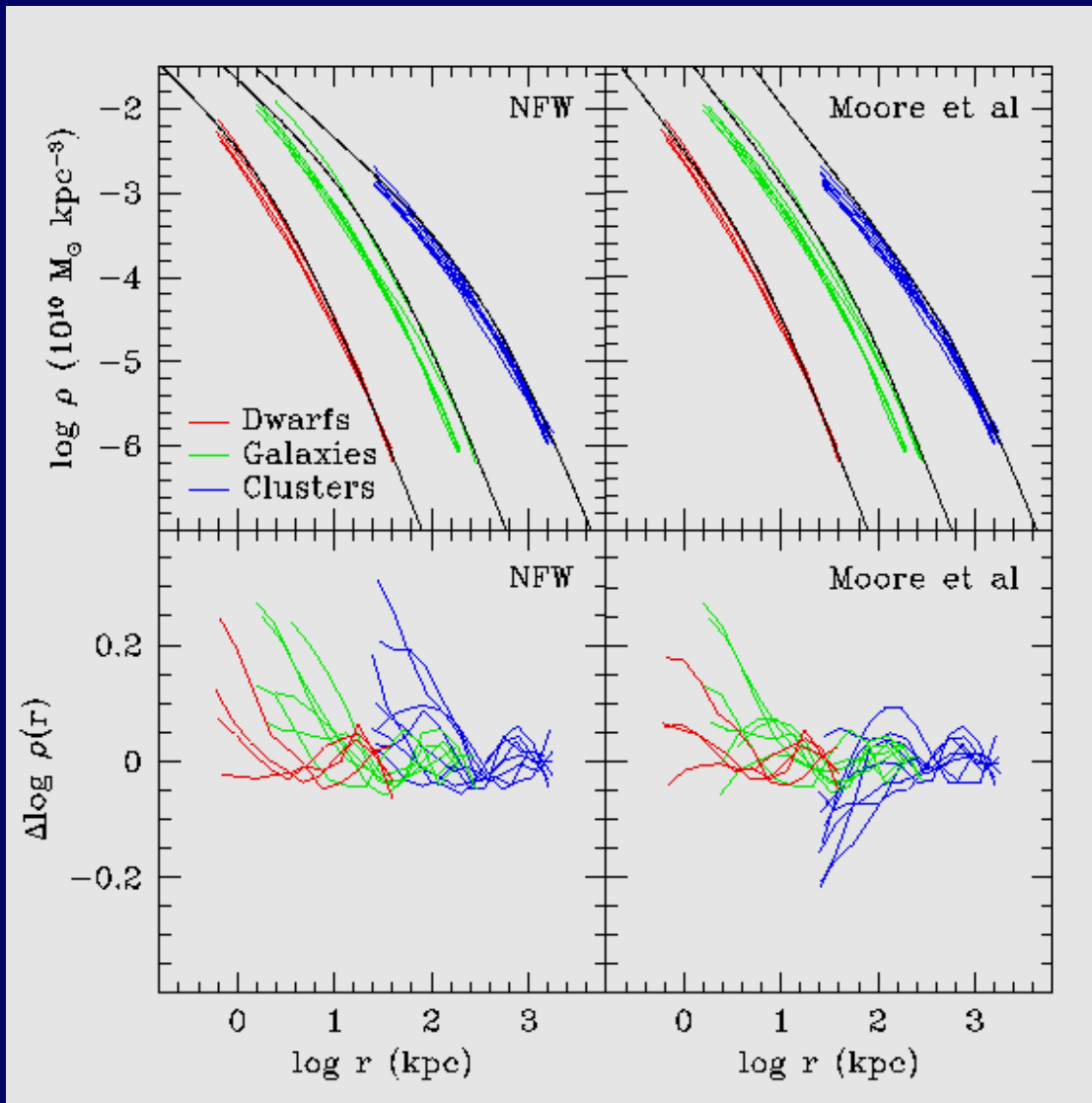
→ There is not enough mass in cusp to sustain a power-law as steep as $\propto r^{-1.5}$

Navarro, Hayashi, Frenk, Jenkins, White, Power, Springel, Quinn, Stadel

How good or bad are simple fits?

Density

residuals



Over the well resolved regions, both NFW and Moore functions exhibit comparable systematic deviations when fitted to simulated CDM halos.

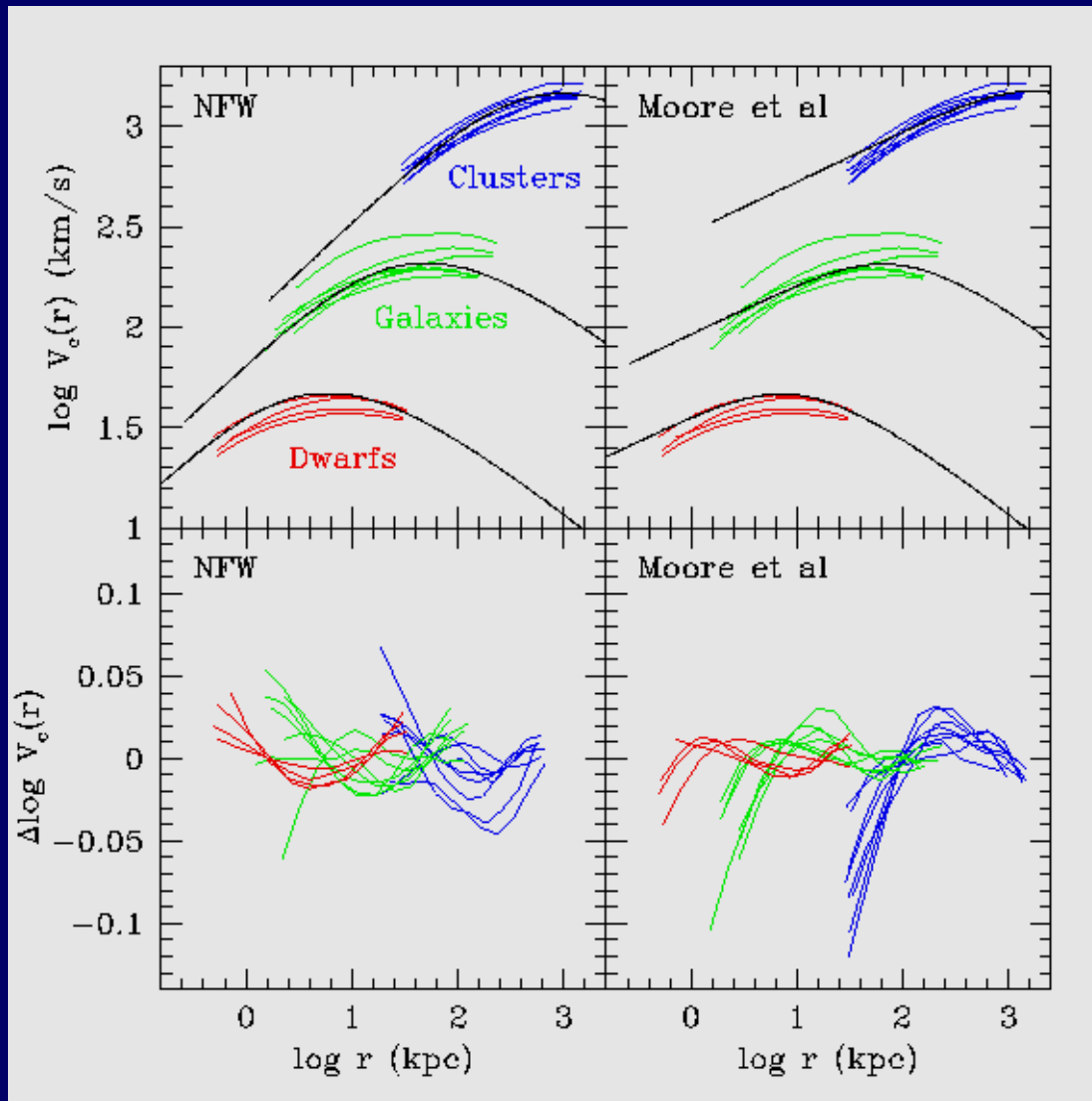
Radius

Navarro, Frenk, White,
Hayashi, Jenkins, Power,
Springel, Quinn, Stadel

How good or bad are simple fits?

Circular Velocity

residuals



Over the well resolved regions, both NFW and Moore functions exhibit comparable systematic deviations when fitted to simulated CDM halos.

Radius

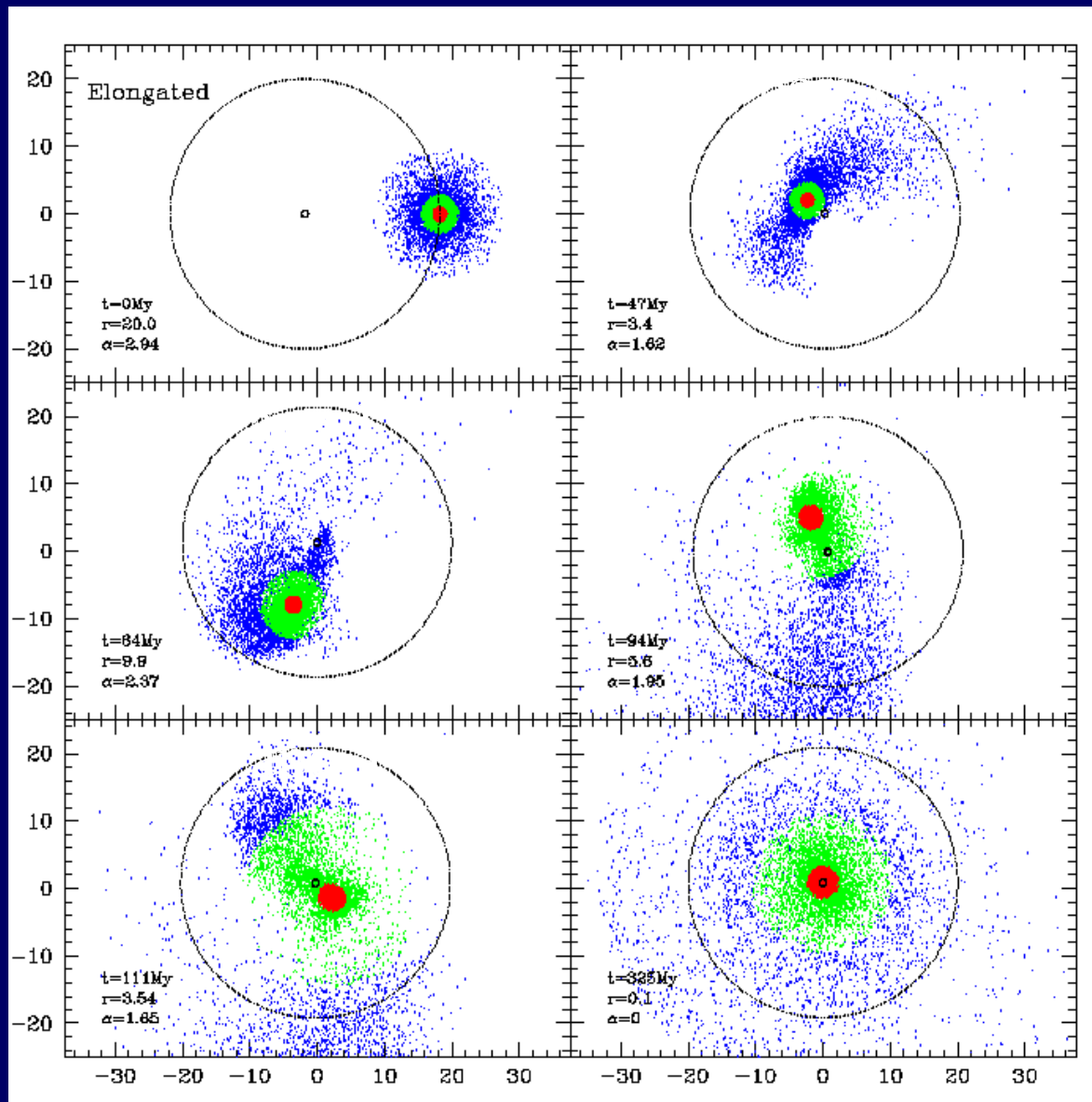
Navarro, Frenk, White,
Hayashi, Jenkins, Power,
Springel, Quinn, Stadel

Origin of the Halo inner Cusp?

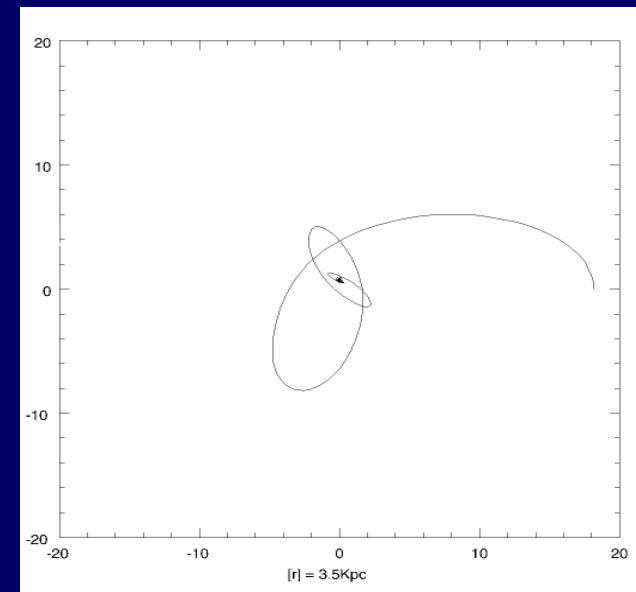
Dynamical Friction and Tidal Effects

Dekel, Arad, Devor, et al. 2003

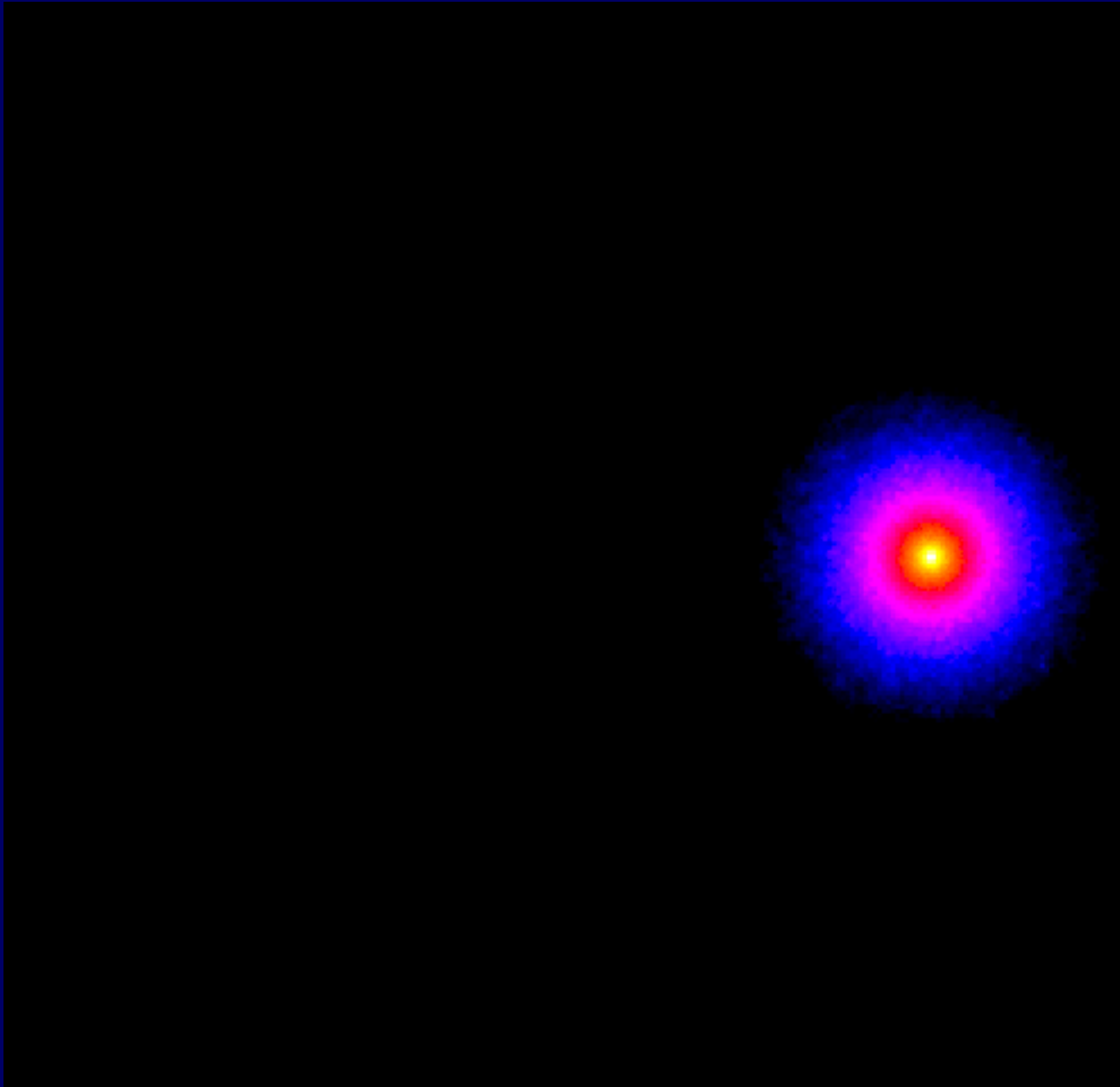
Halo Bulidup by Mergers



tidal stripping & dynamical friction



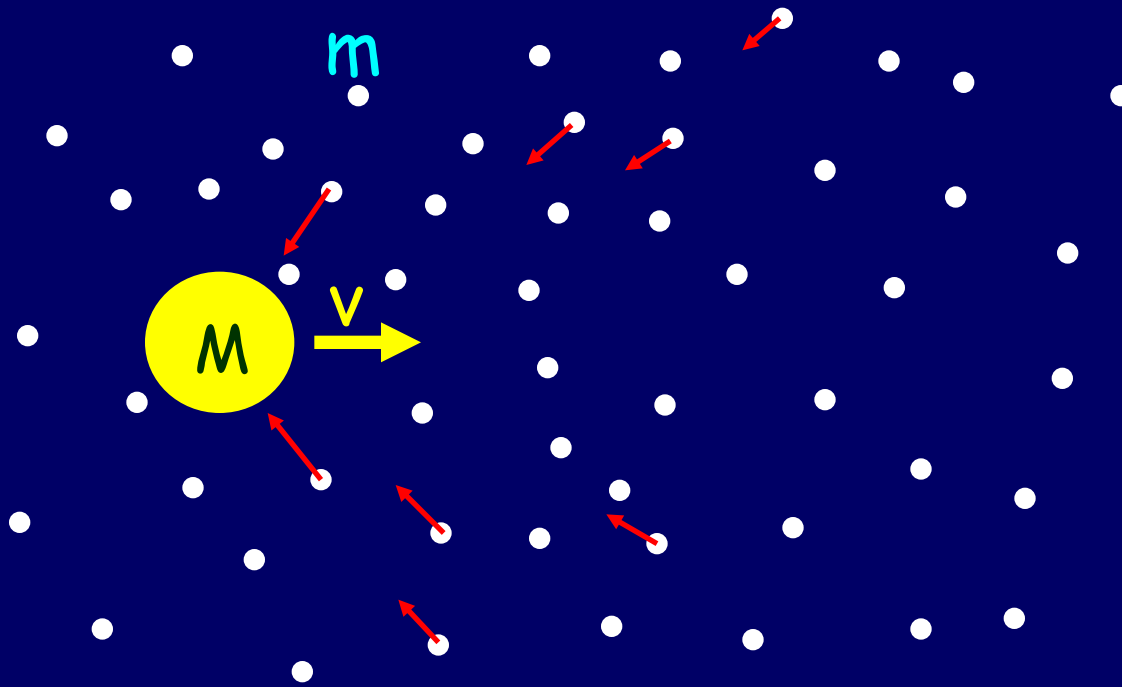
Dynamical Friction and Tidal stripping



Moore et al.

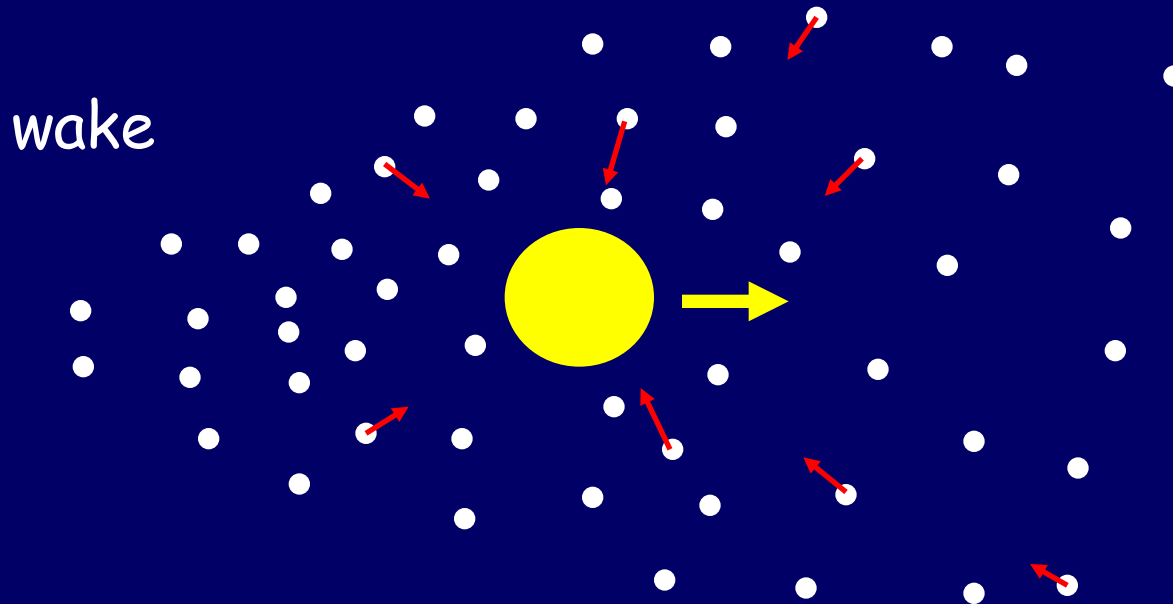
Dynamical Friction

Dynamical Friction

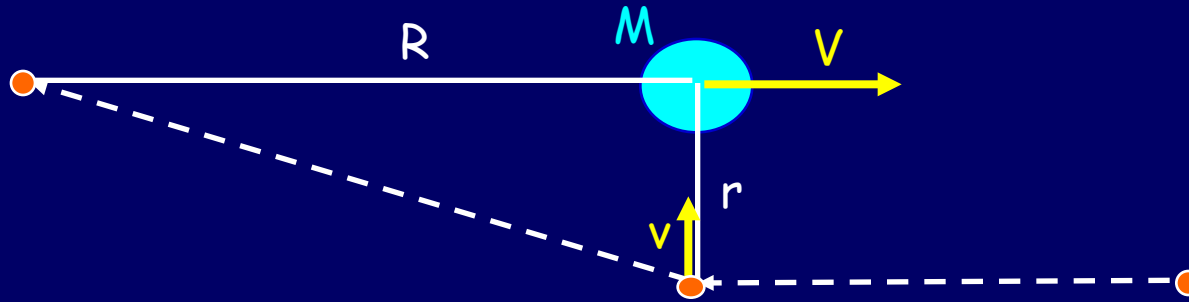


$$m \ll M$$

Dynamical Friction



Dynamical Friction



Impulse approximation

$$v \approx f \Delta t \approx \frac{GM}{r^2} \frac{r}{V} \rightarrow r \approx \frac{GM}{Vv}$$

$$\frac{r}{R} \approx \frac{v}{V}$$

$$f_{\text{DF}} \approx \frac{GM_{\text{wake}}}{R^2} \approx \frac{G\rho r^2 R}{R^2} \approx \frac{G^2 \rho M}{V^2}$$

Dynamical Friction

Chandrasekhar formula:

$$\frac{d\vec{v}}{dt} = -4\pi G^2 \ln \Lambda \rho(< v) M_{sat} \frac{\vec{v}}{v^3} \left[\text{erf}(X) - \frac{2X}{\pi^{1/2}} e^{-X^2} \right] \quad m \ll M_{sat}$$

$$X \equiv \frac{v}{\sqrt{2}\sigma}$$

$$\text{erf}(X) \equiv \frac{2}{\sqrt{\pi}} \int_0^X e^{-t^2} dt$$

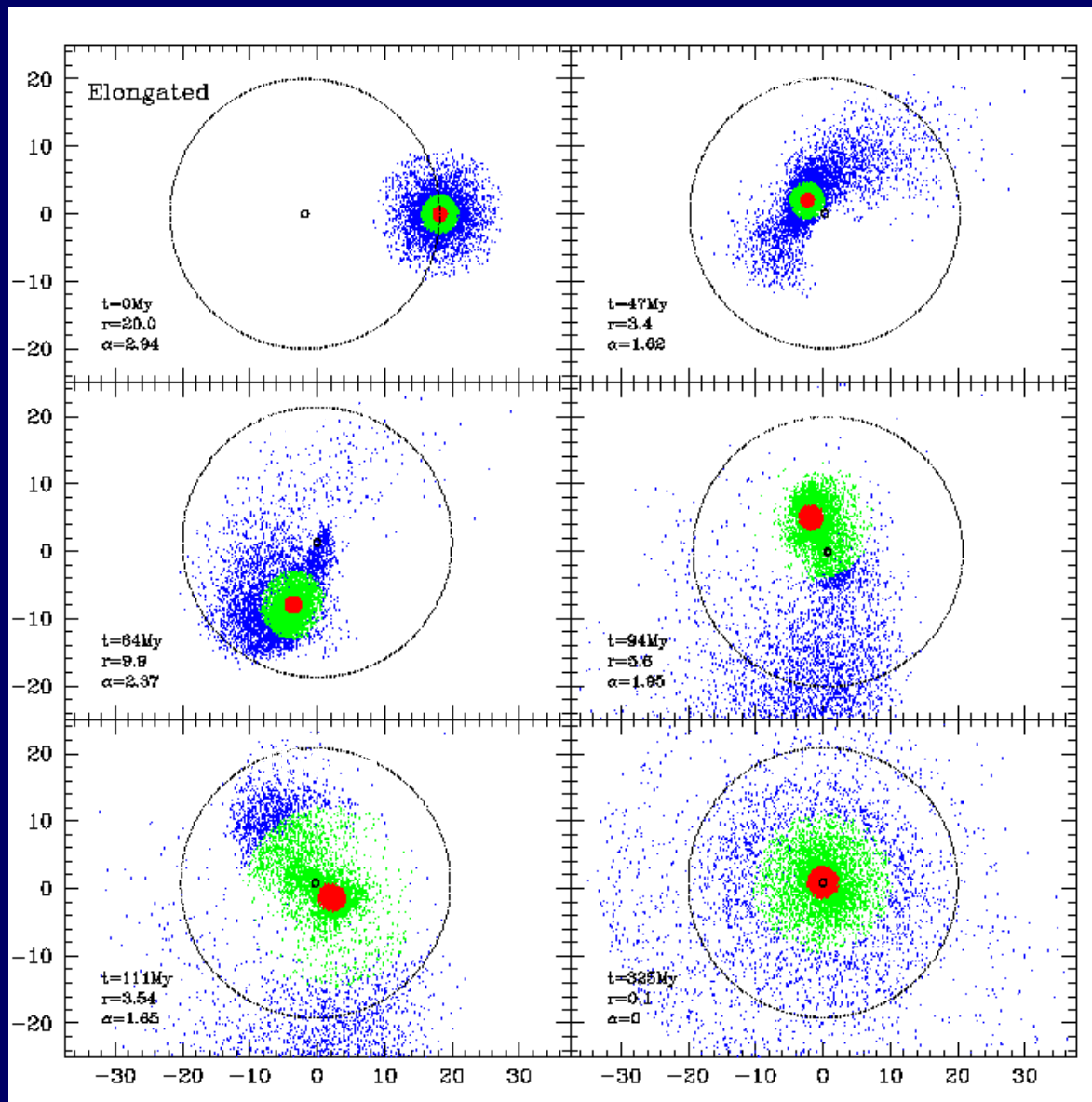
Coulomb logarithm:

$$\Lambda = \frac{b_{max} v_0^2}{GM_{sat}} \approx \frac{M_{halo}}{M_{sat}}$$

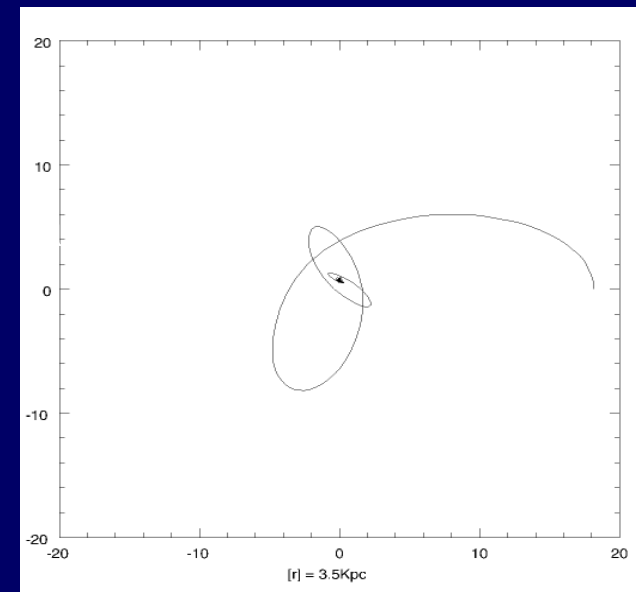
drag proportional to ρ but independent of m

acceleration propto M (because wake density propto M)

Halo Bulidup by Mergers



tidal stripping & dynamical friction



Tidal Effects



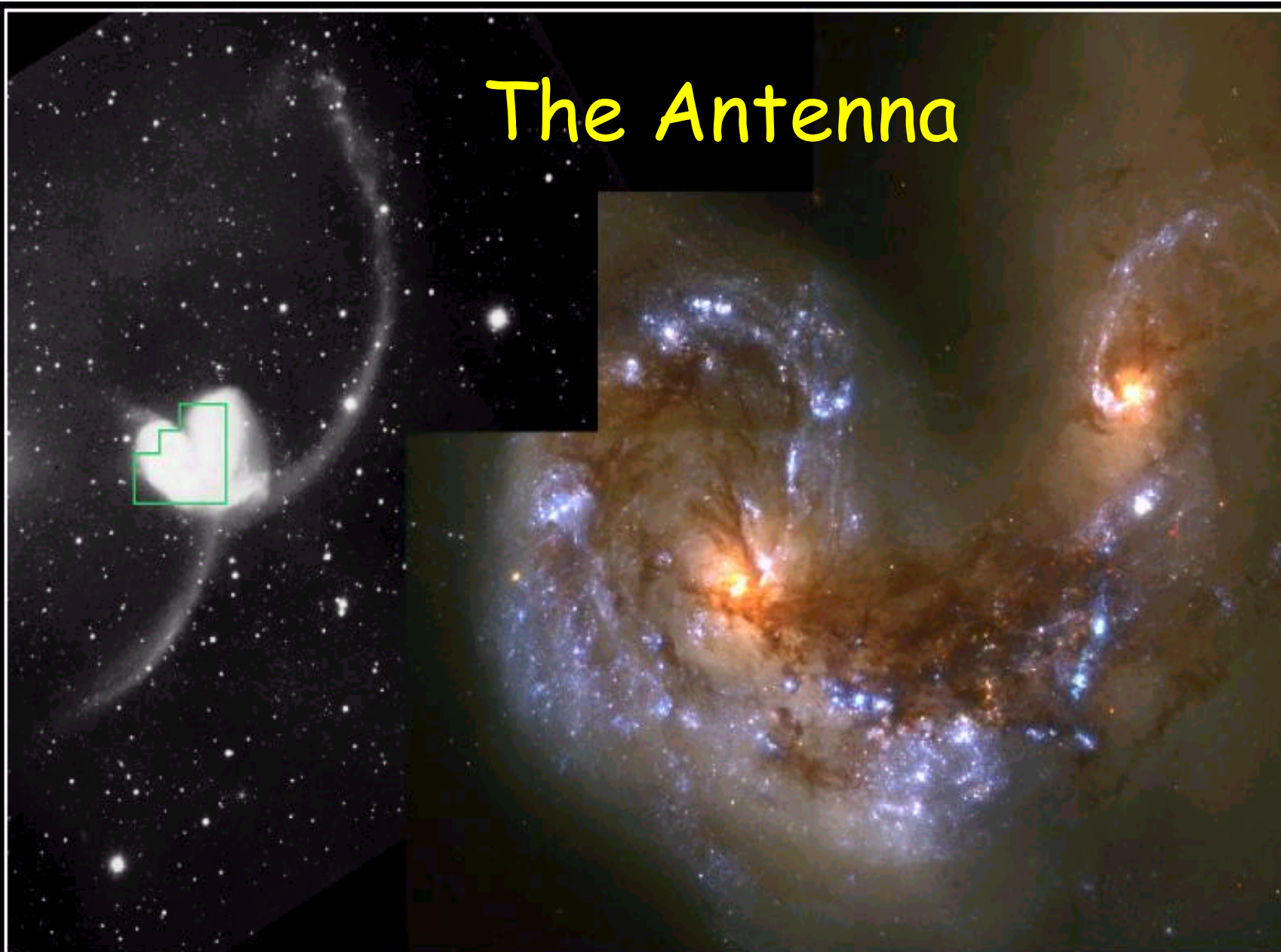
12-hour period

Tidal interaction & Merger



The Mice • Interacting Galaxies NGC 4676
Hubble Space Telescope • Advanced Camera for Surveys

The Antenna

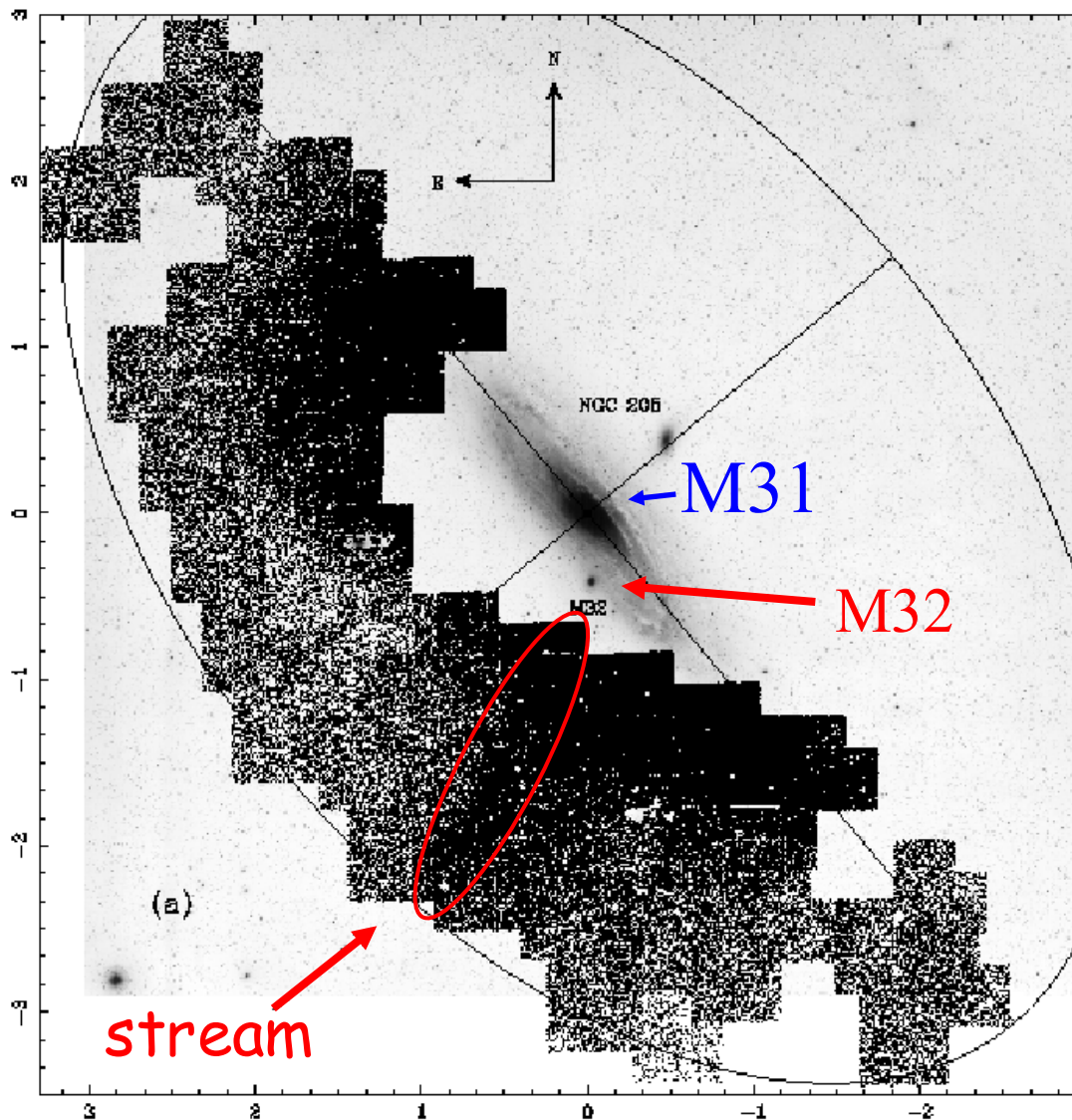


Colliding Galaxies NGC 4038 and NGC 4039

HST • WFPC2

PRC97-34a • ST ScI OPO • October 21, 1997 • B, Whitmore (ST ScI) and NASA

Tidal stripping of a satellite?

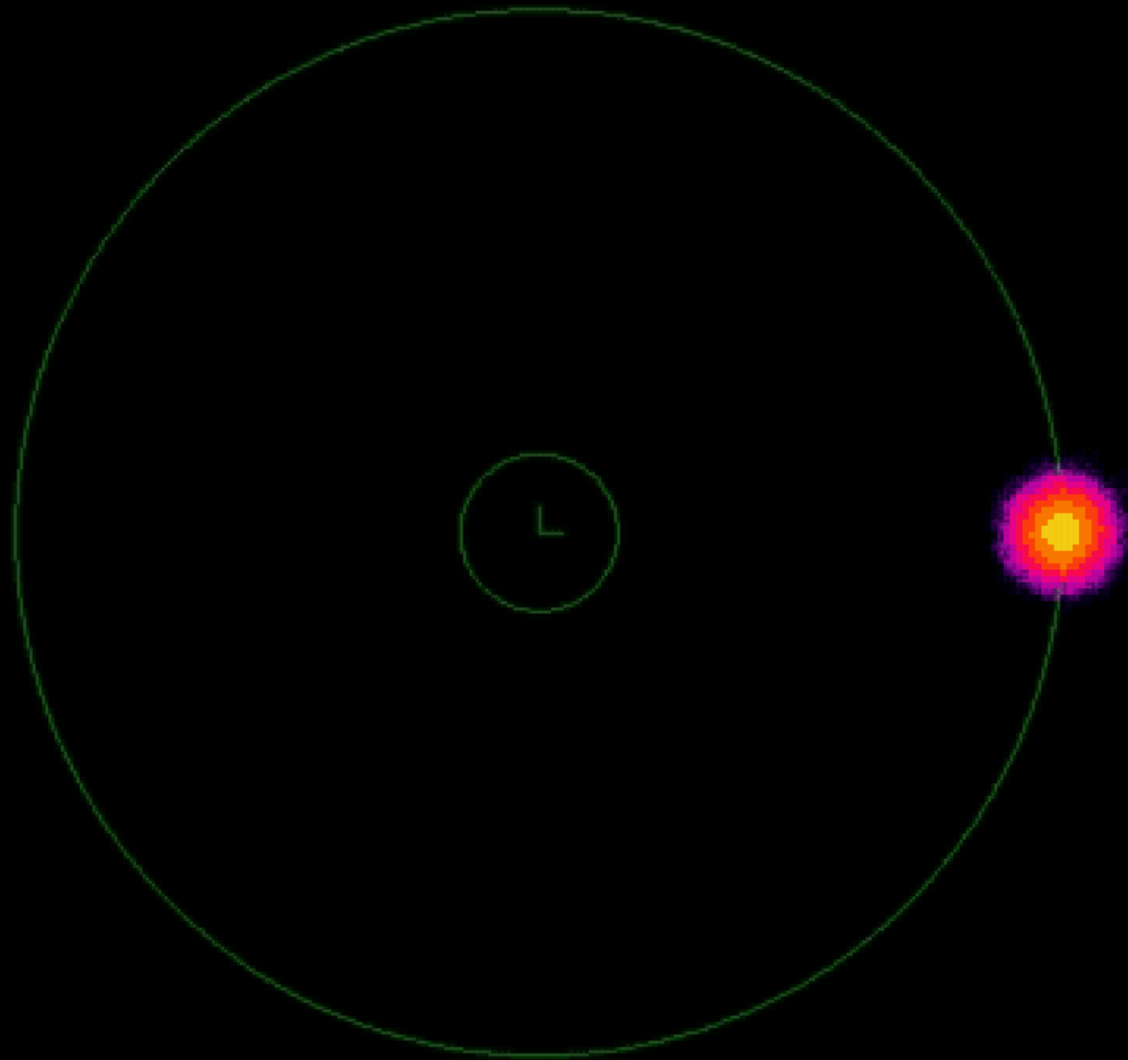


Ibata et al. 2001

The tidal disruption of an NFW Satellite halo

Eric Hayashi & Julio Navarro

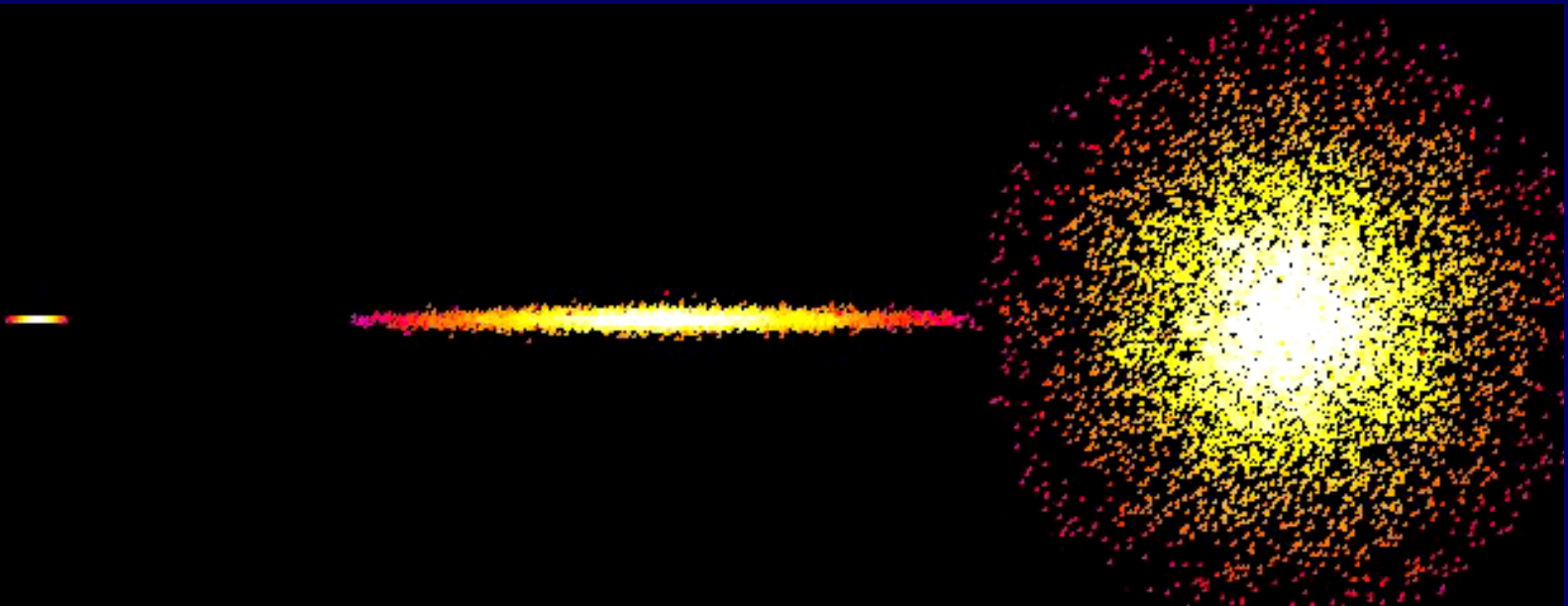
University of Victoria



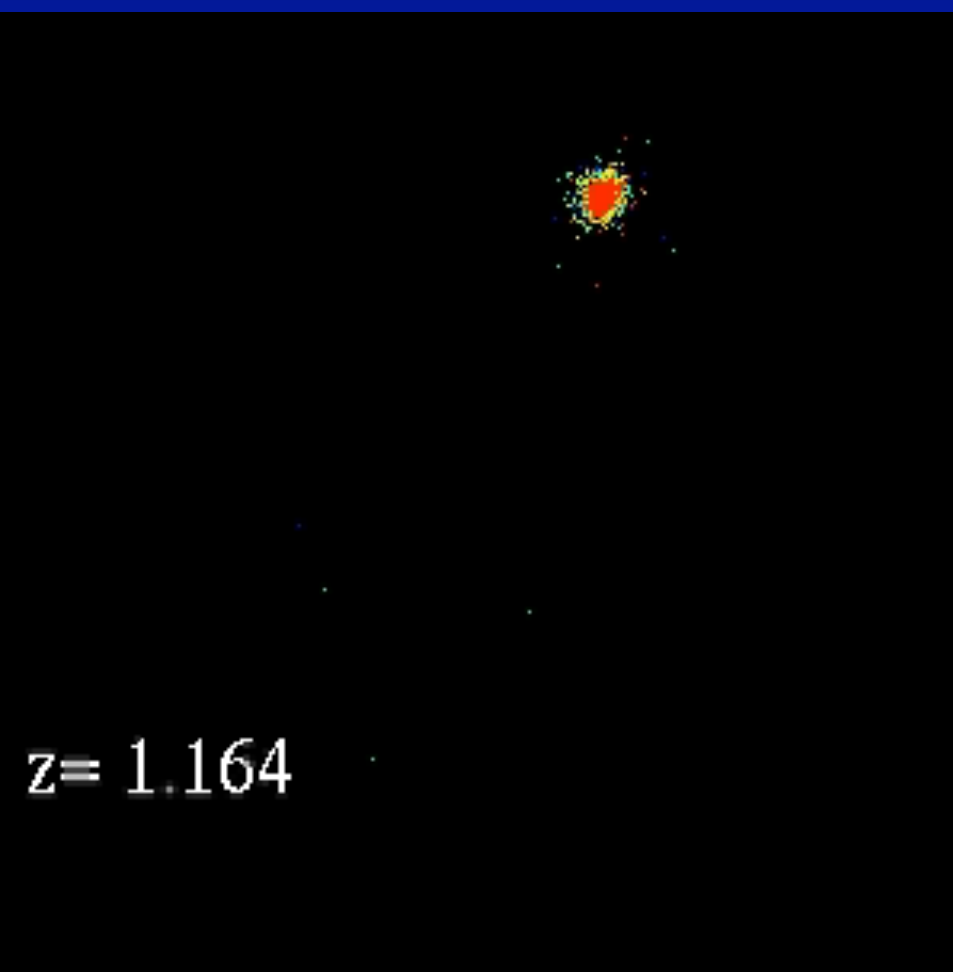
time = 0.00 t_{orb}

Harrassment of a satellite

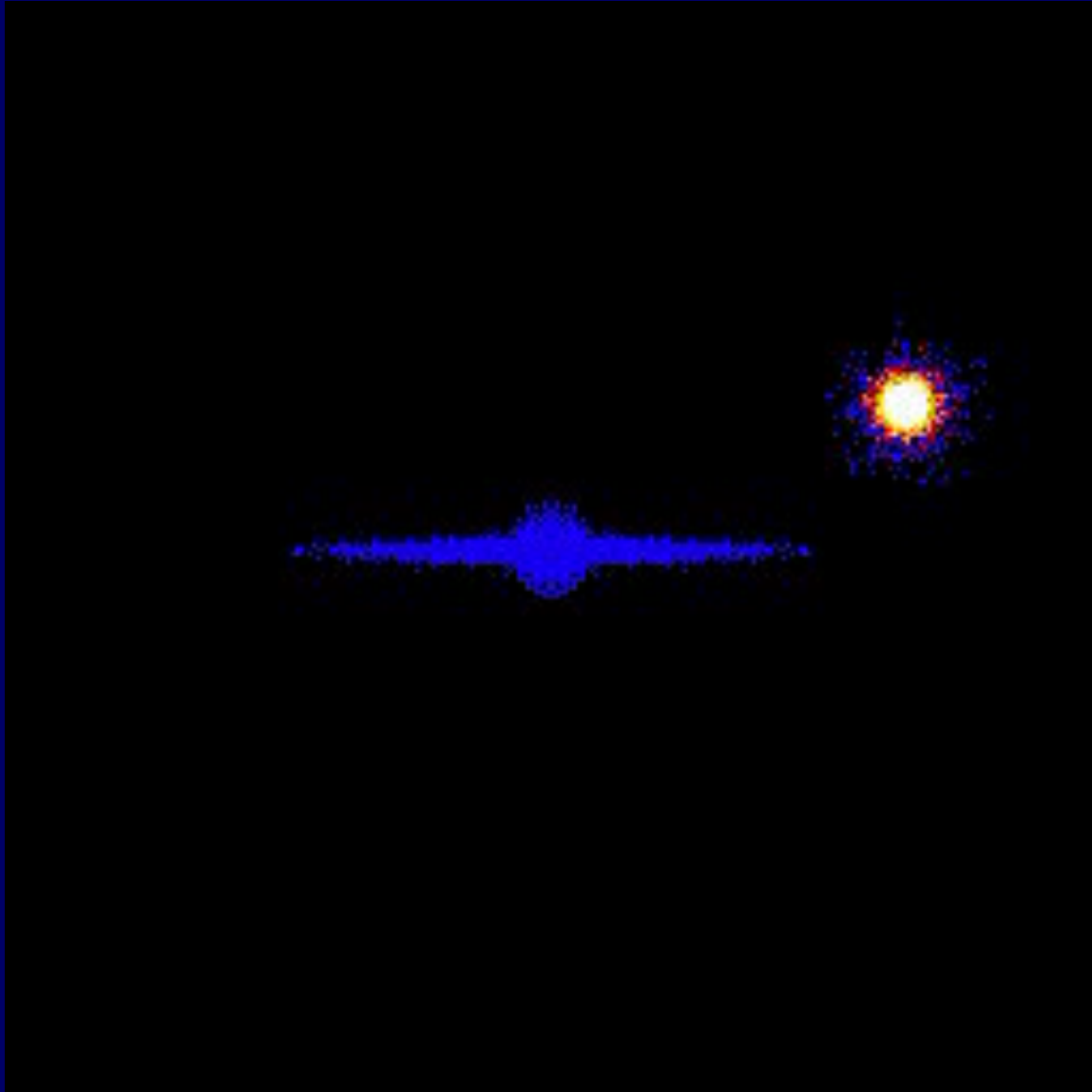
$z=0.500$



Moore et al.

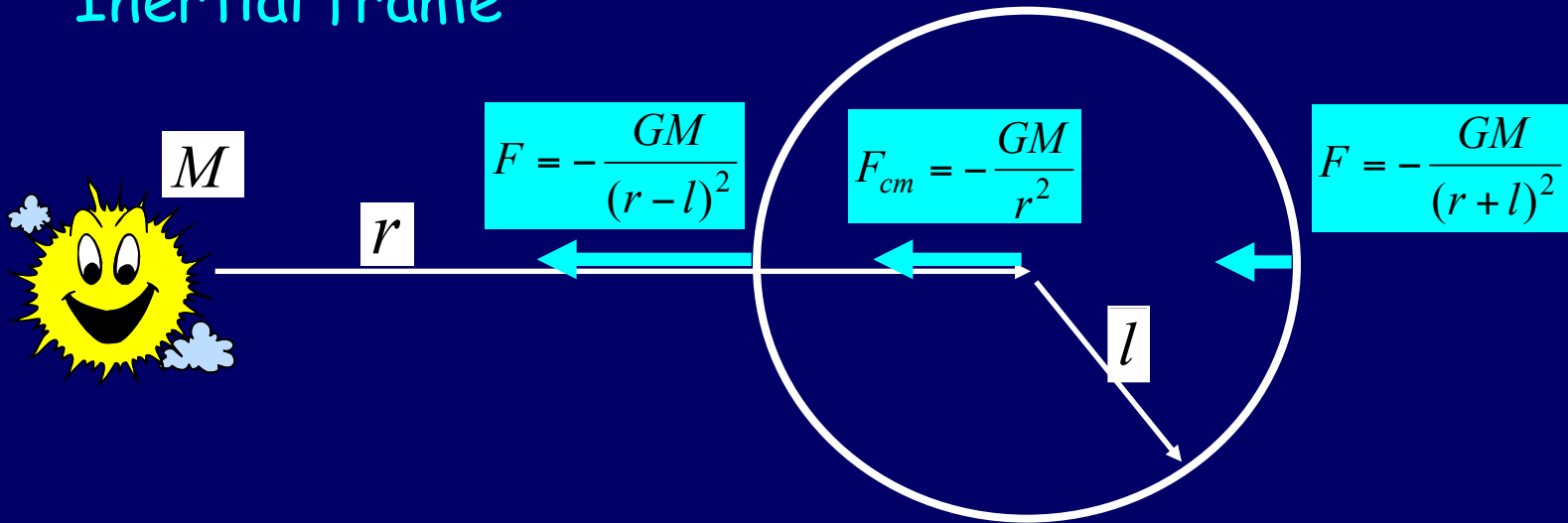


Sagittarius Dwarf

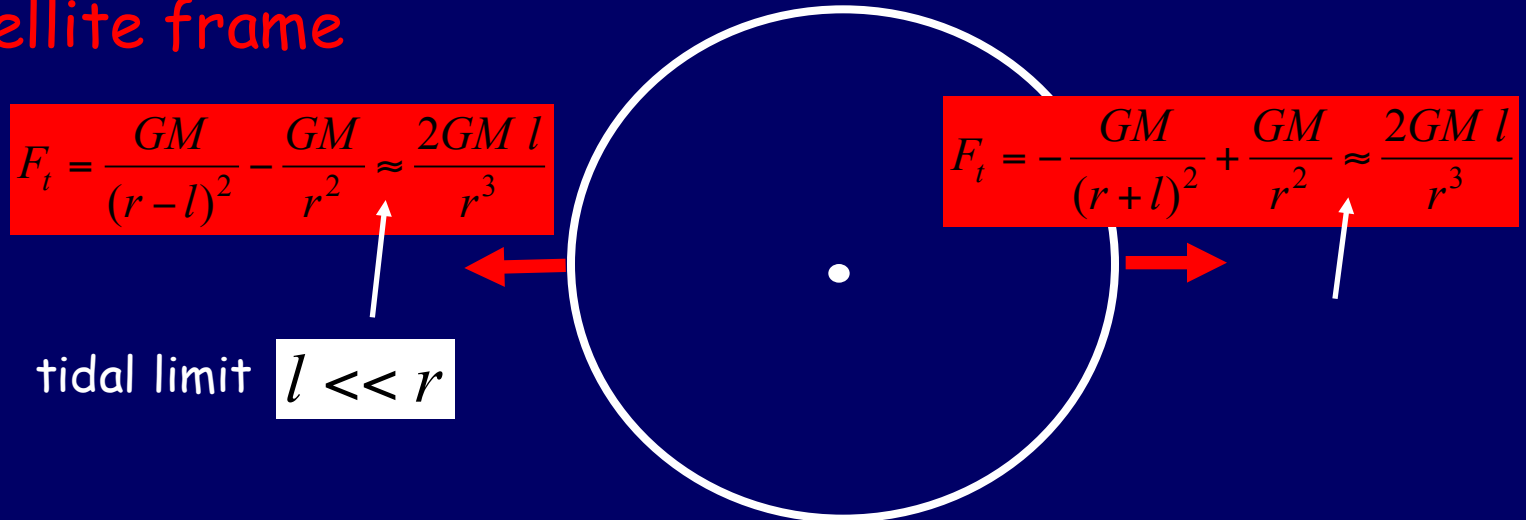


Tidal Force by a Point Mass

Inertial frame



Satellite frame



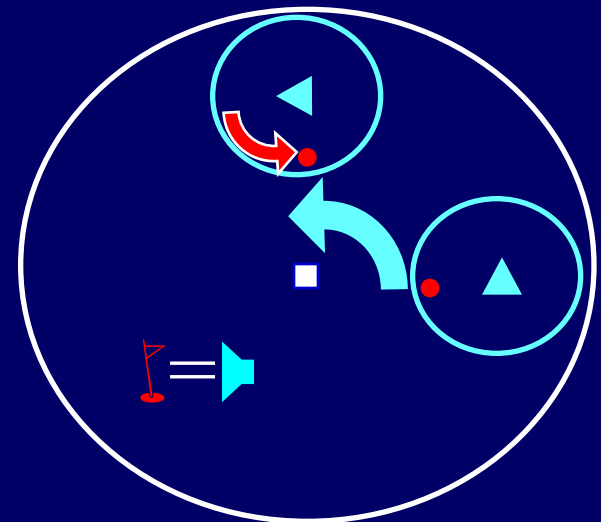
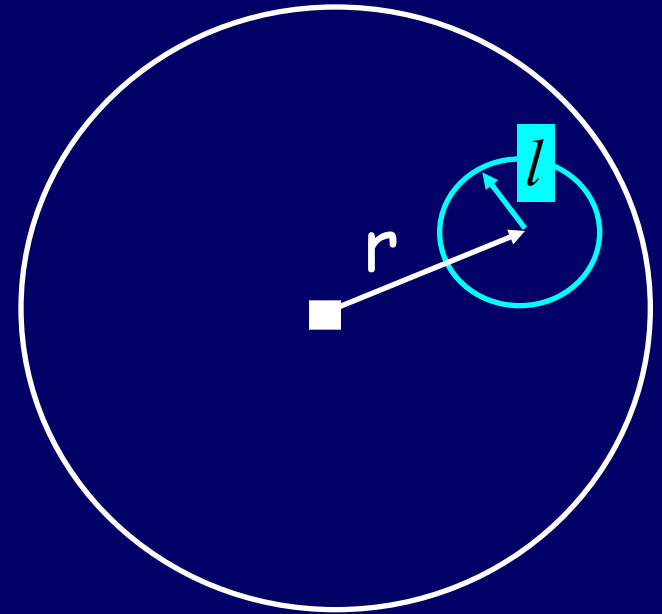
Tidal Radius of a Satellite

self-gravity force $\frac{Gm(l_t)}{l_t^2} = \frac{2GM(r)l_t}{r^3}$ tidal force

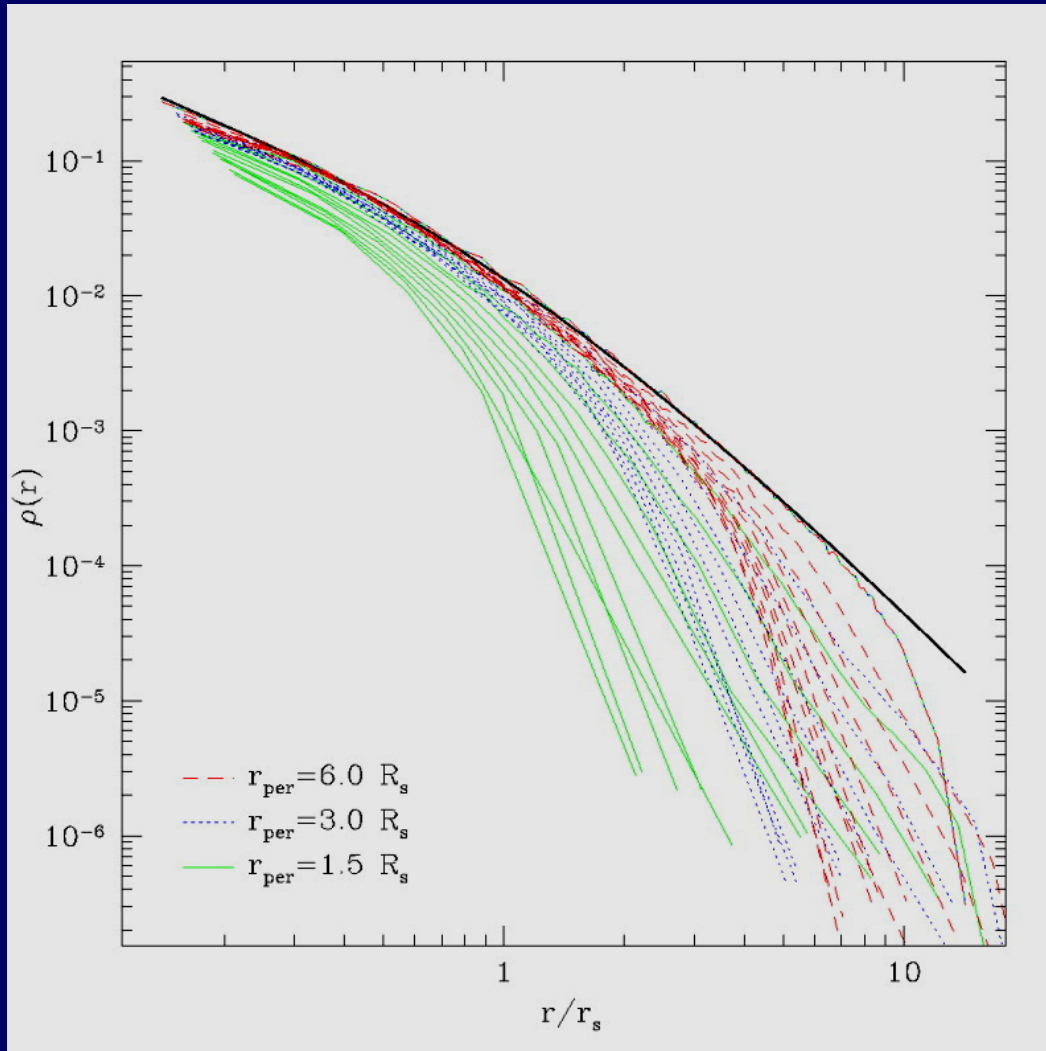
→ $\bar{\rho}_{sat}(l_t) \sim \frac{m(l_t)}{l_t^3} \sim \frac{M(r)}{r^3} \sim \bar{\rho}_{halo}(r)$

$t \propto \frac{R}{V} \propto \frac{R}{(GM/R)^{1/2}} \propto \left(\frac{R^3}{M}\right)^{1/2} \propto \rho^{-1/2}$

→ $t_{sat}(l_t) \sim t_{halo}(r)$
resonance



Density Profiles of stripped NFW halos



Profiles of sub-halos
Stoehr et al 2004:

$$\log\left(\frac{V}{V_{\text{max}}}\right) = -a \left[\log\left(\frac{r}{r_{\text{max}}}\right) \right]^2$$

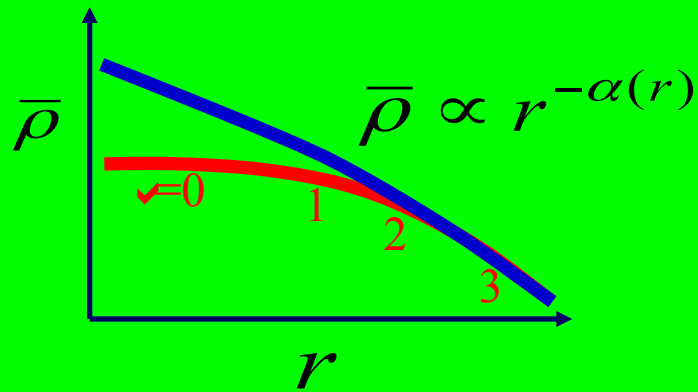
$$a \approx 0.45 \Leftrightarrow \beta \approx 0.7$$

Origin of a cusp: tidal effects in mergers

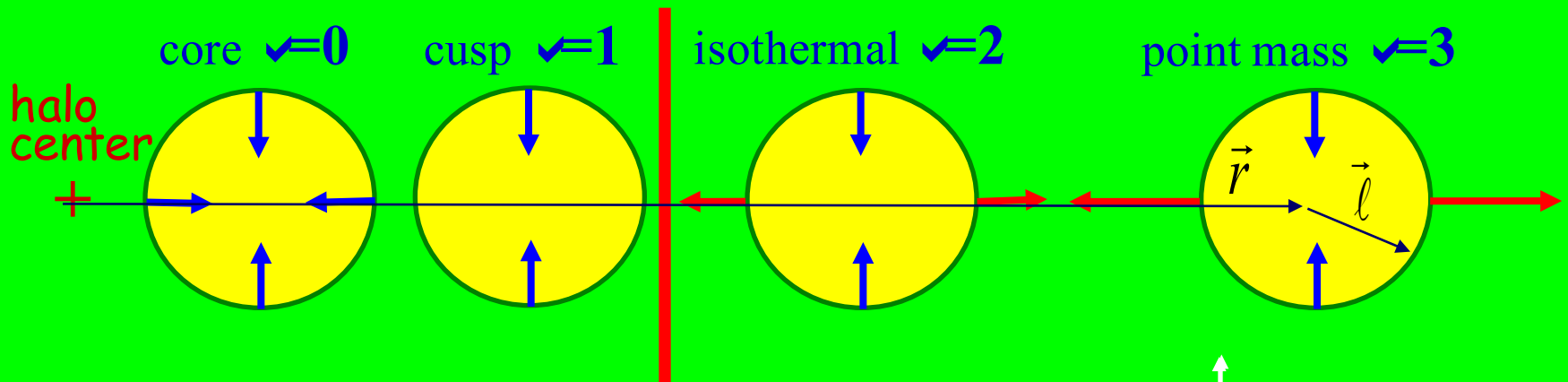
Dekel, Devor, Arad et al.

- a. If satellites settle in halo core 🤖
steepening to a cusp ✓📡
- b. Mass-transfer recipe 🤖
convergence to a universal slope ↗1
- c. Flat-density core? Only if satellites are
puffed up, e.g. by gas blowout

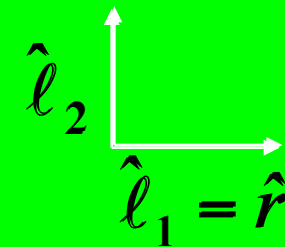
Tidal force on a satellite



$$\alpha(r) \equiv -\frac{d \ln \bar{\rho}(r)}{d \ln r}$$

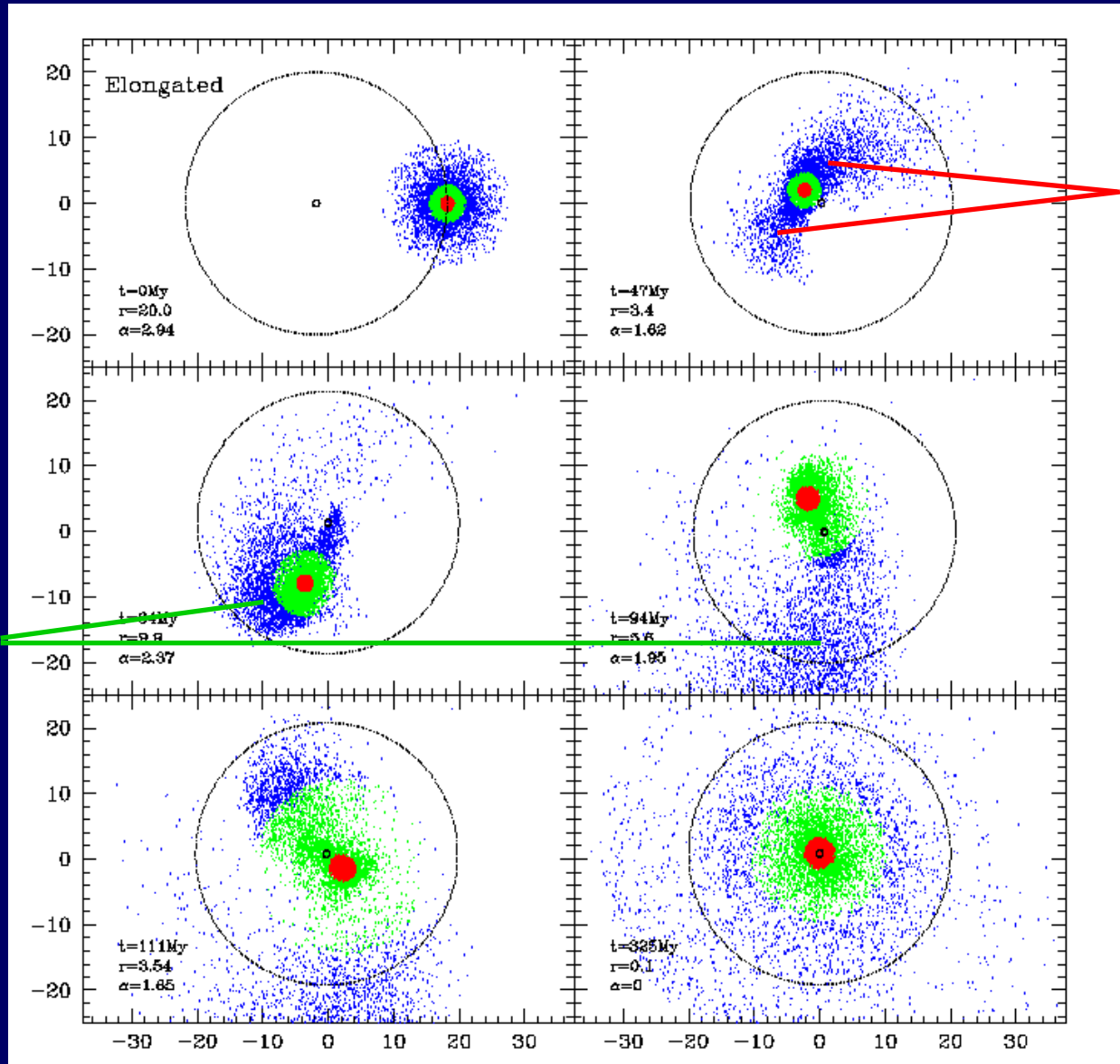


$$\vec{F}_{\text{tidal}} \propto \frac{GM(r)}{r^3} \left(\underline{[\alpha(r) - 1]} \vec{\ell}_1 - \vec{\ell}_2 - \vec{\ell}_3 \right)$$



no mass transfer where $\sqrt{<1}$

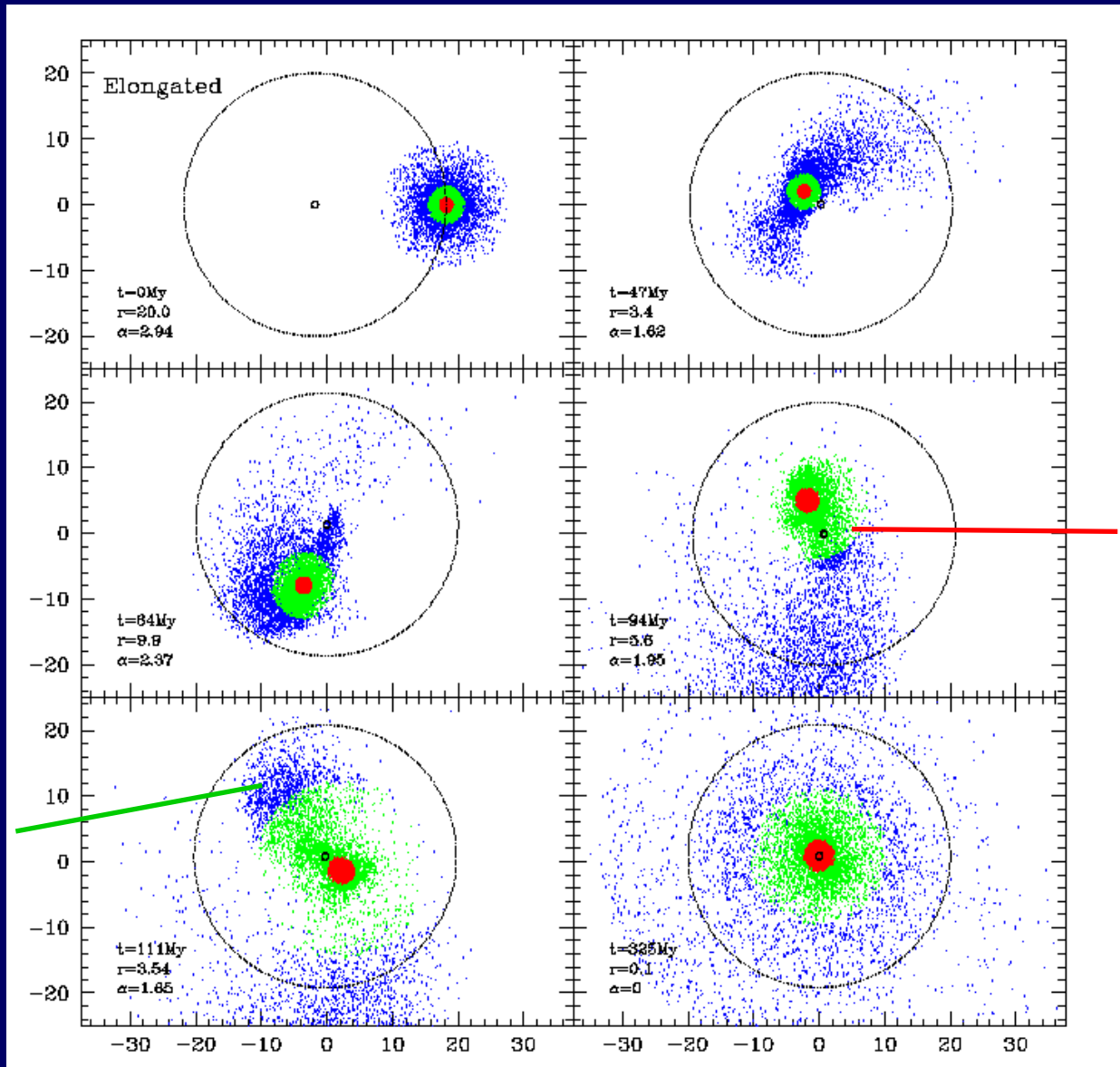
Impulsive stripping and deposit



pericenter stripping

apocenter deposit

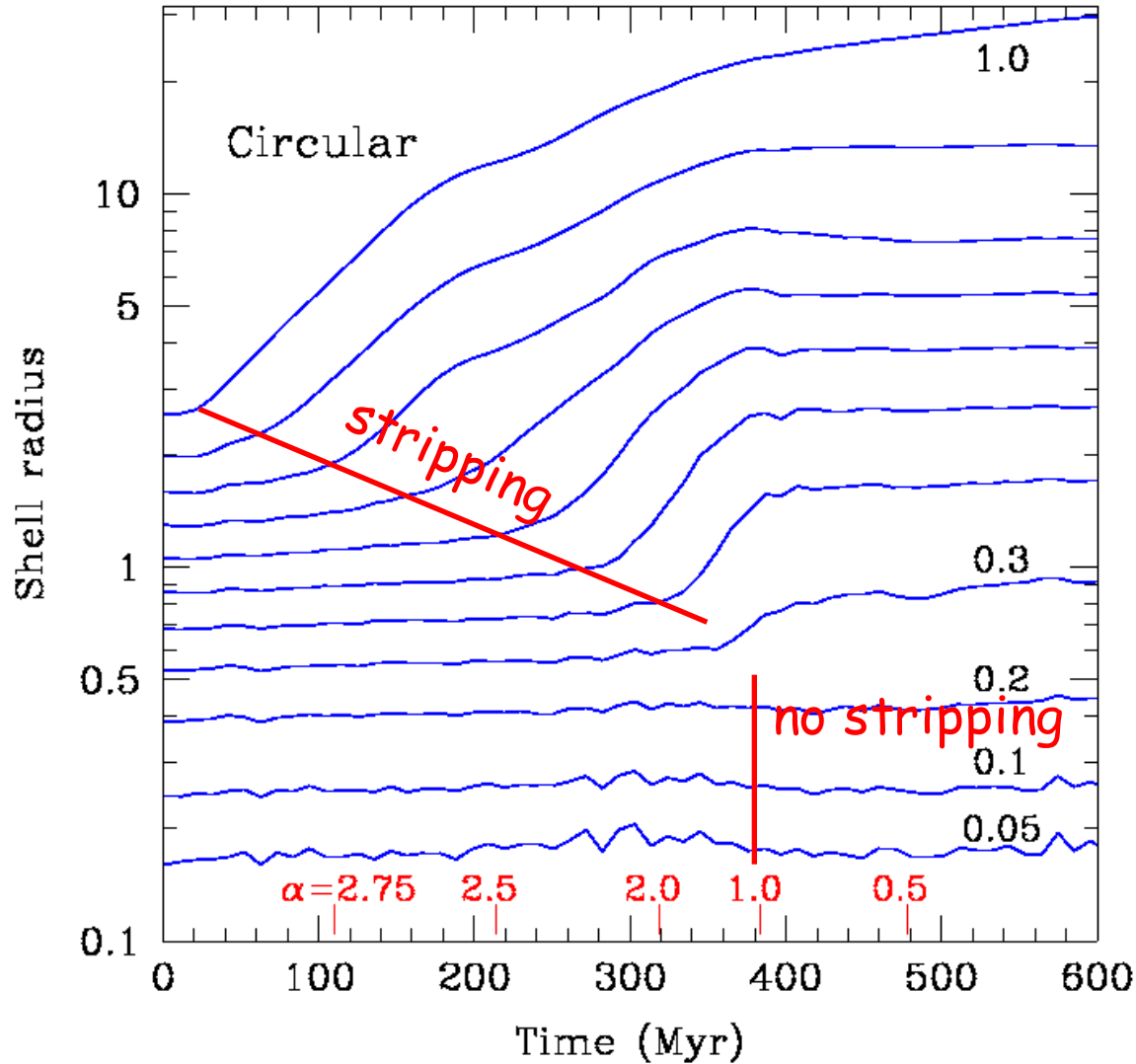
Impulsive stripping and deposit



pericenter stripping

apocenter deposit

Adiabatic evolution of satellite profile



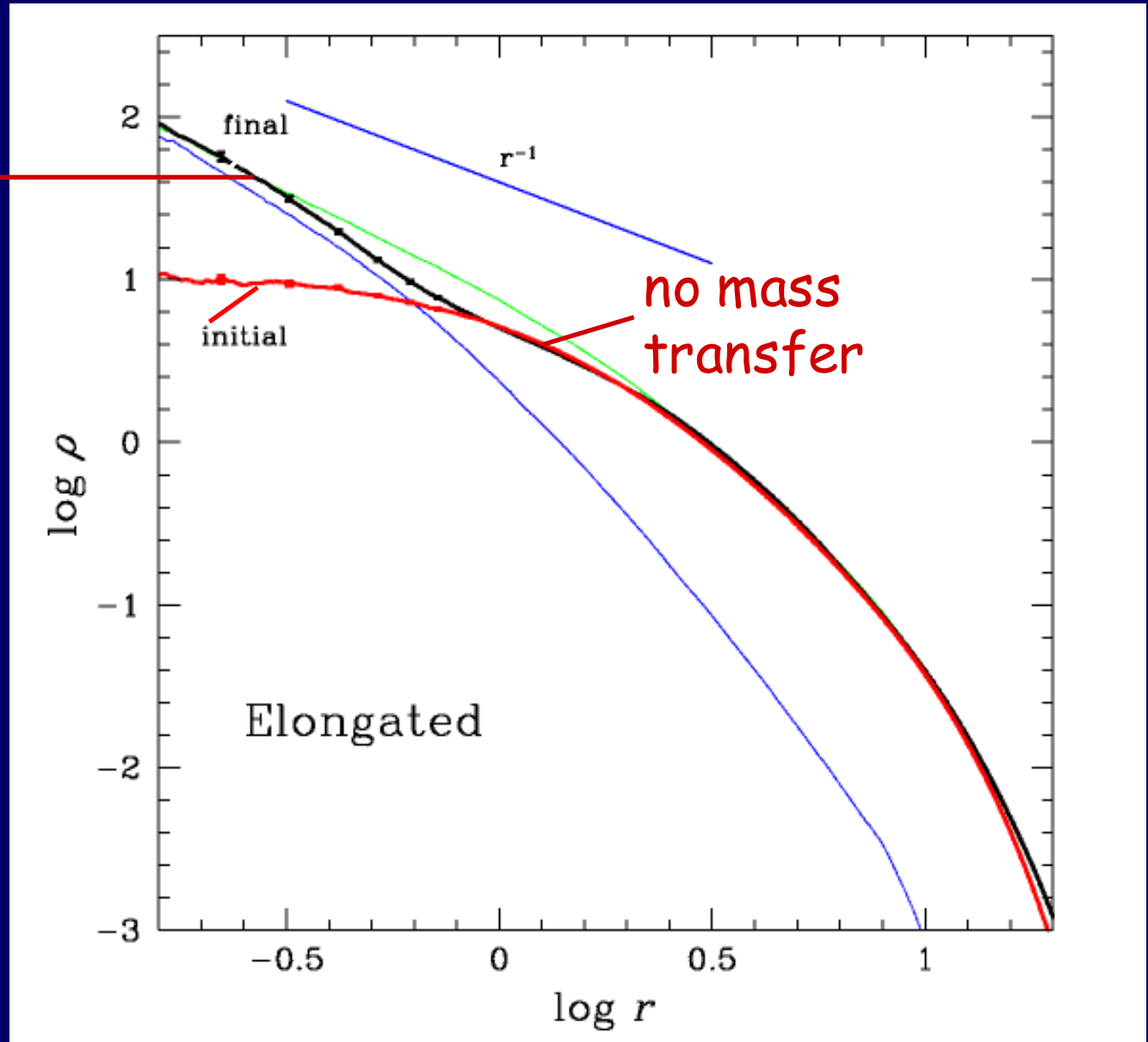
tidal
compression in
halo core

Merger of a compact satellite

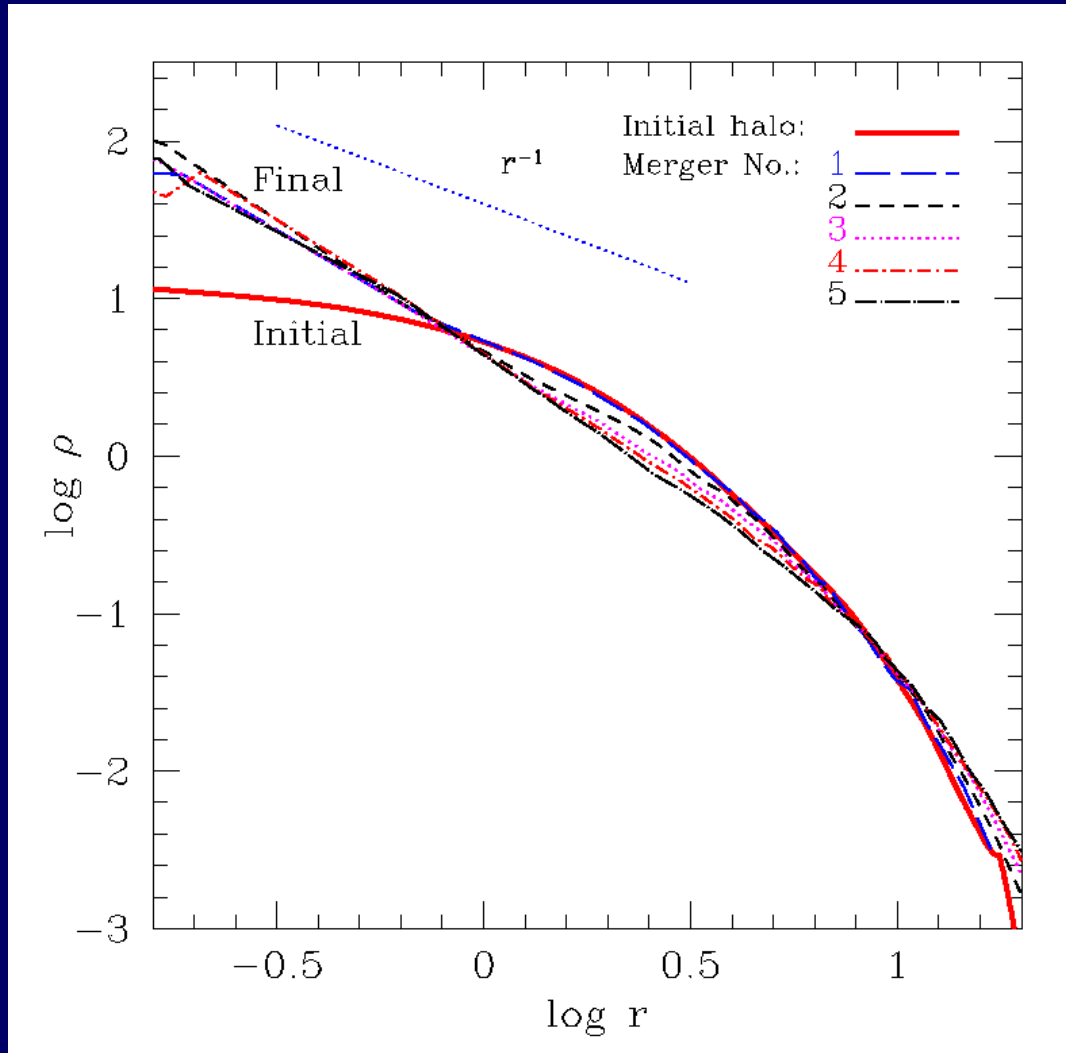
satellite
decays intact
to halo center

N-body
simulation

Dekel, Devor &
Hetzroni 03



Tandem mergers with compact satellites

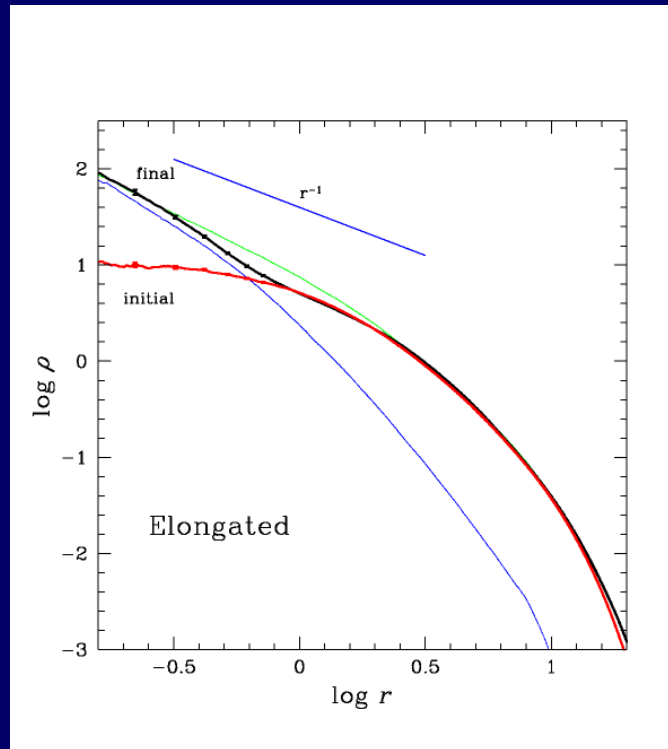


→ The cusp is stable!

Result:

No mass transfer in core 🤔

rapid steepening to a cusp of ✓🚀

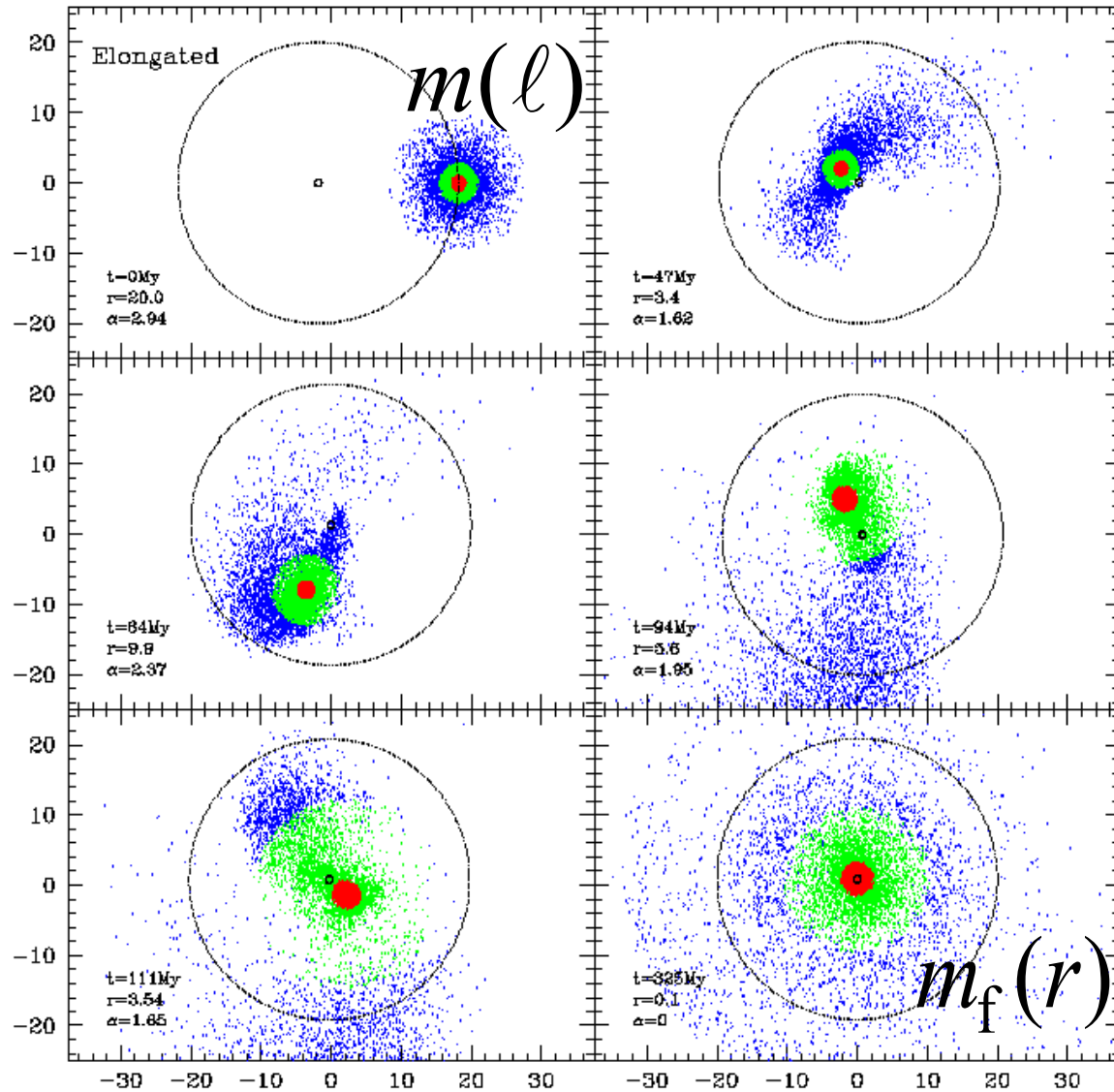


Tidal mass-transfer recipe at $\checkmark 1$

final initial satellite profile

$$m_f(r) = m(\ell) \rightarrow \ell(r)$$

Deposit radius



Tidal mass-transfer recipe at $\checkmark > 1$

final initial satellite profile

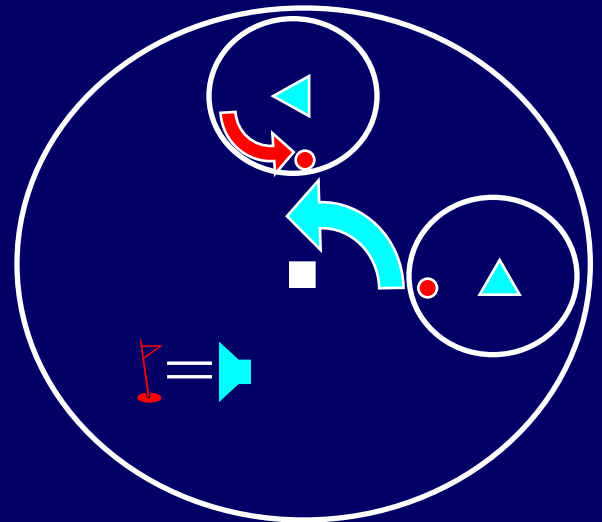
$$m_f(r) = m(\ell) \rightarrow \ell(r)$$

halo

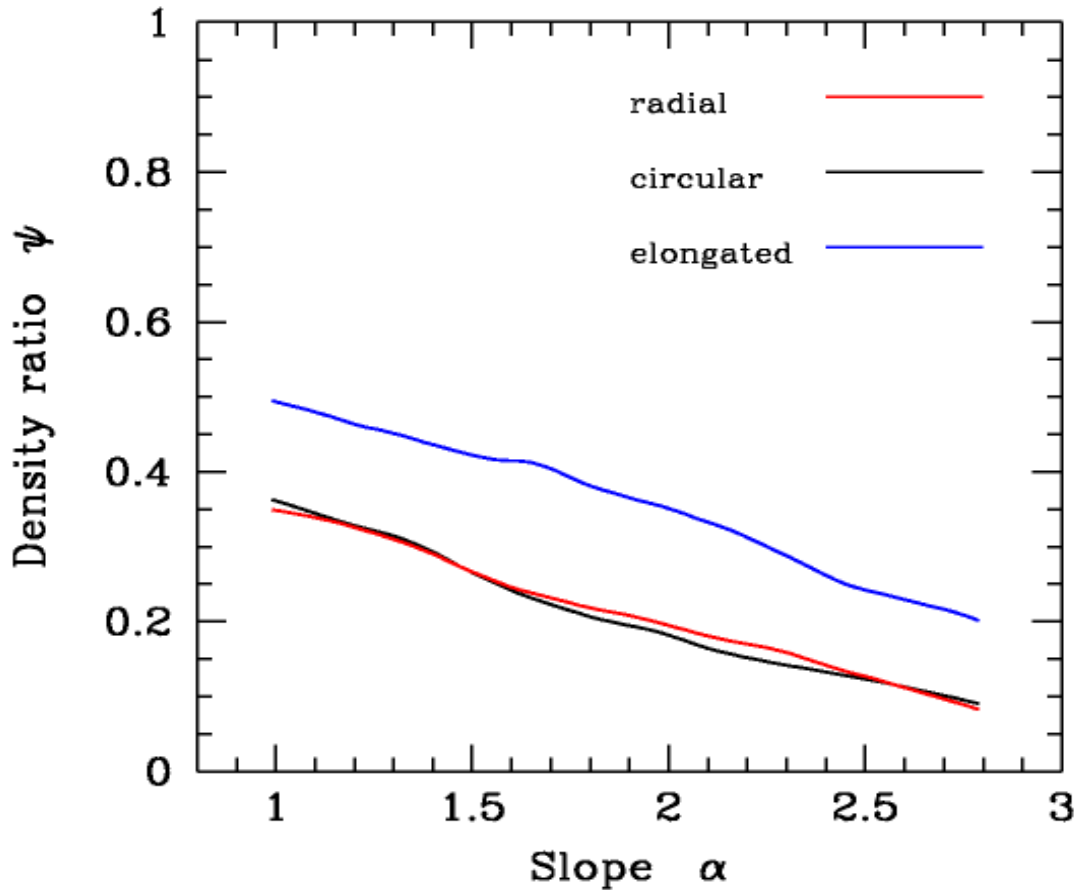
$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)]$$

initial satellite

=1? resonance



Tidal mass-transfer recipe at $\checkmark 1$



final initial sat. profile

$$m_f(r) = m(\ell) \rightarrow \ell(r)$$

halo

$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)] \approx \frac{1}{2\alpha}$$

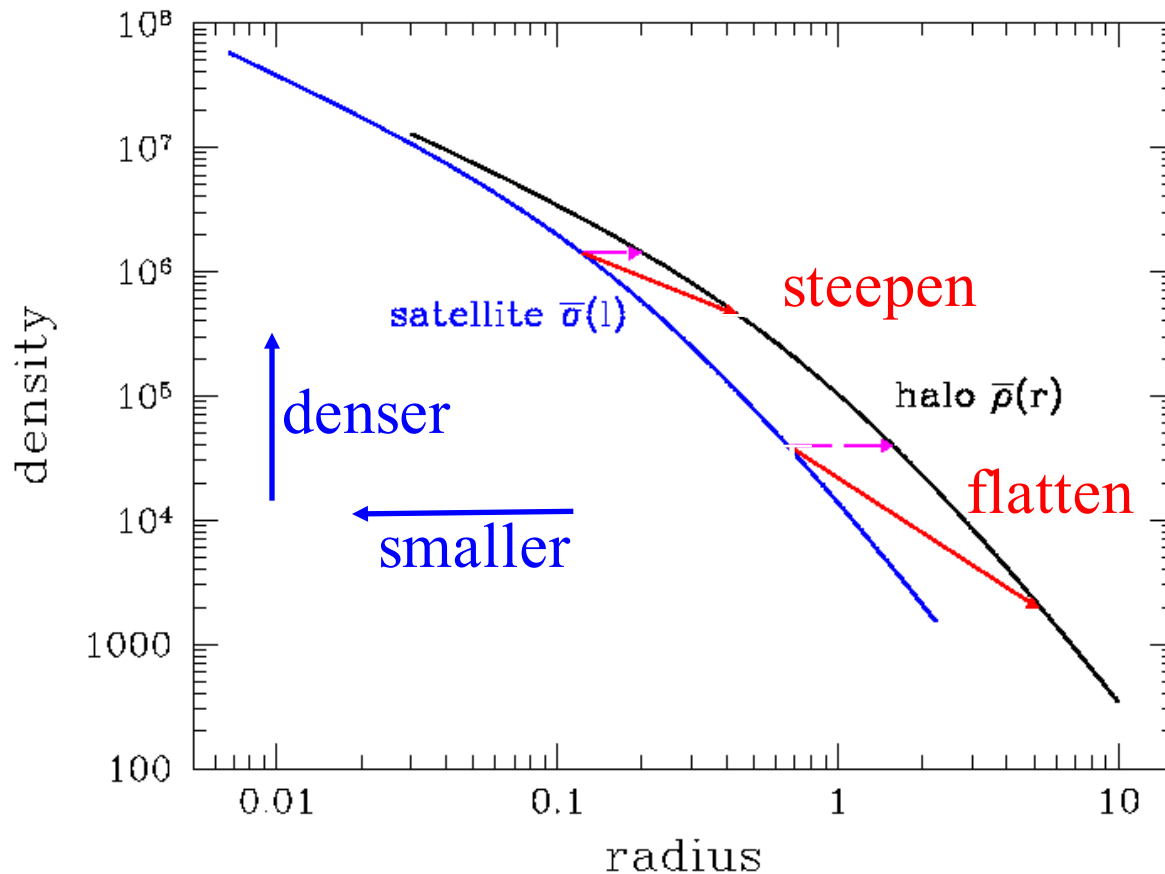
satellite

☺ stripping efficiency grows with \checkmark

Steepening / flattening

homologous halo and satellite

scaling: $\rho_s \propto m^{-(3+n)/2}$ $r_s \propto m^{(5+n)/6}$



$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)]$$
$$\psi \approx \frac{1}{2\alpha}$$

Adding satellite to halo profile

$$\bar{\rho}_{\text{new}}(r) = \bar{\rho}_{\text{old}}(r) + \bar{\sigma}(\ell) \frac{\ell^3}{r^3}$$



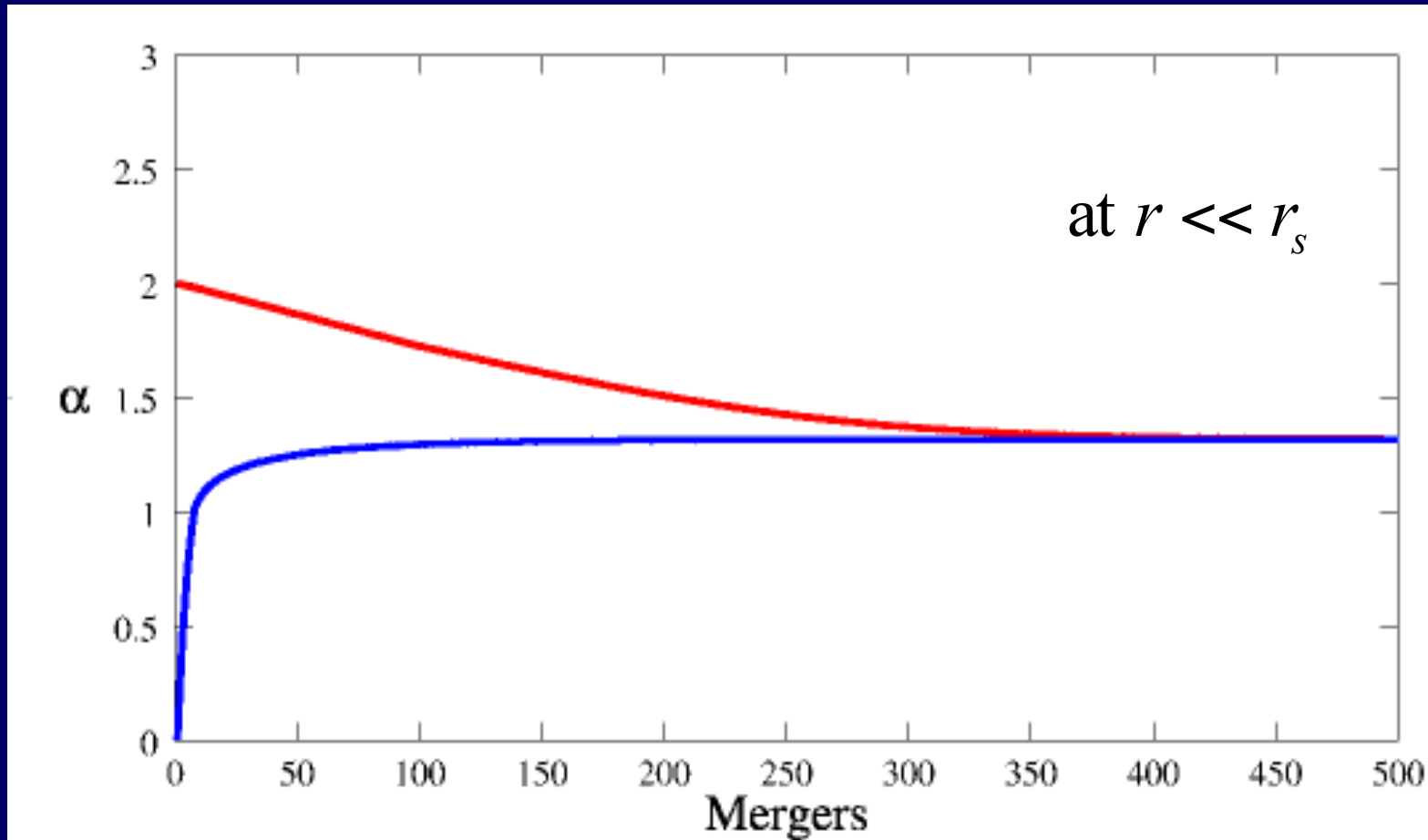
$$\Delta\alpha(r) \propto -\frac{d}{dr} \left[\frac{\bar{\sigma}(\ell) \ell^3}{\bar{\rho}(r) r^3} \right]$$

linear perturbation analysis 



asymptotic

Convergence to an asymptotic slope



Dekel, Arad, Devor, Birnboim 03

Summary: Cusp

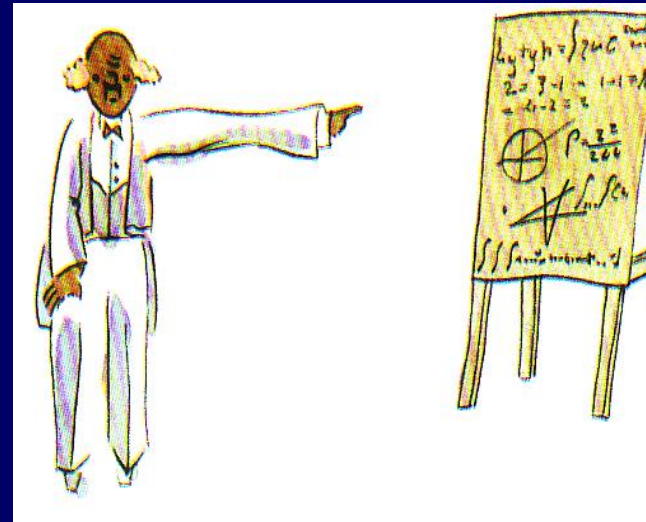
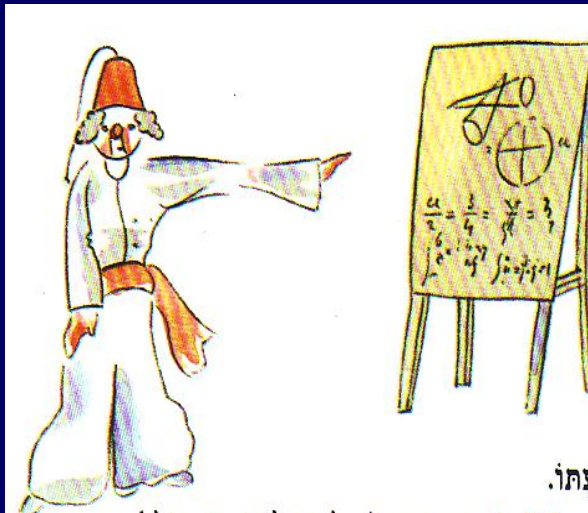
Dark-matter halos in CDM naturally form **cusps** due to merging compact satellites

Observed Core

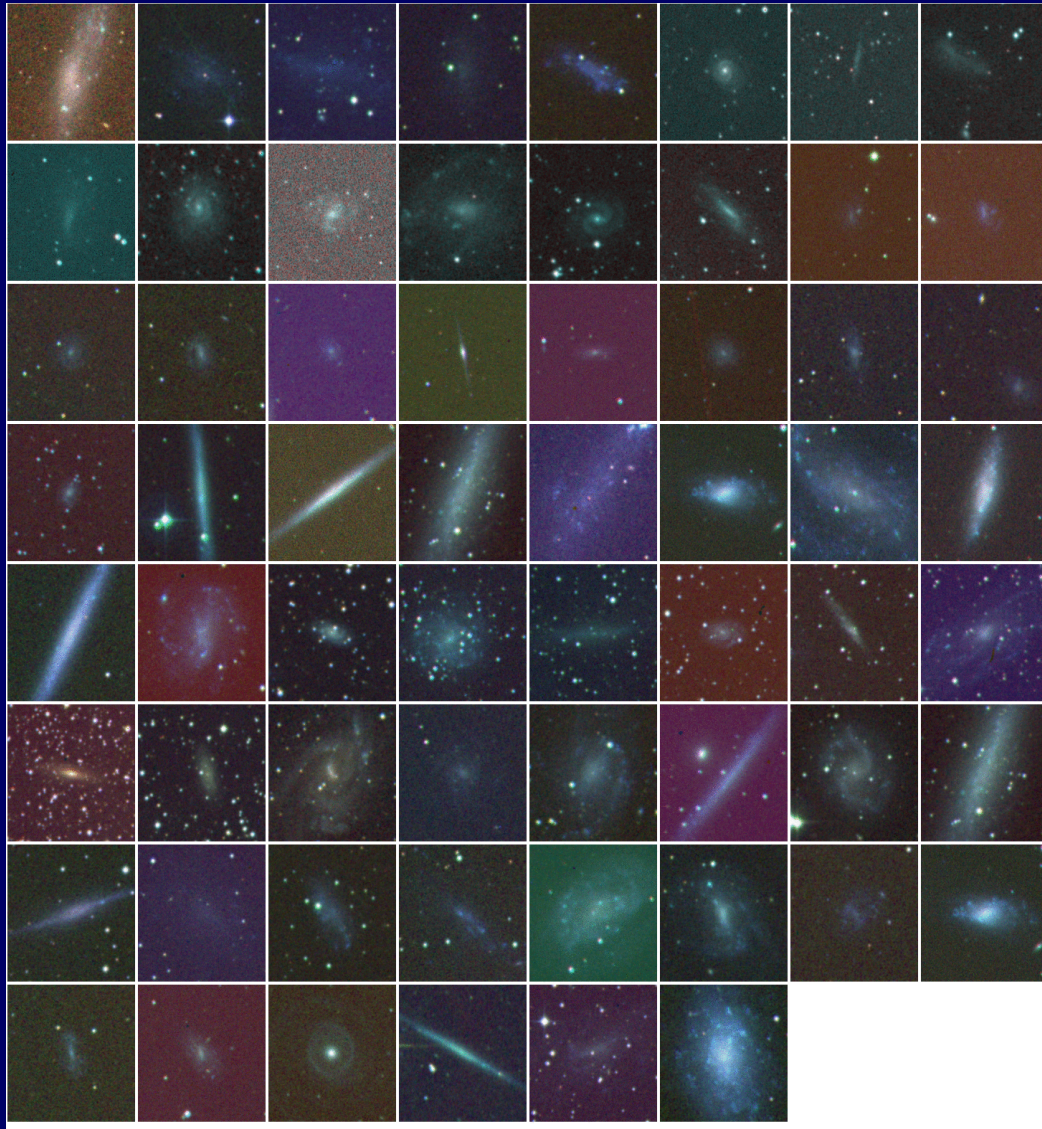
האסטרונום התורכי (הנסיך הקטן)



15



Low Surface Brightness Galaxies



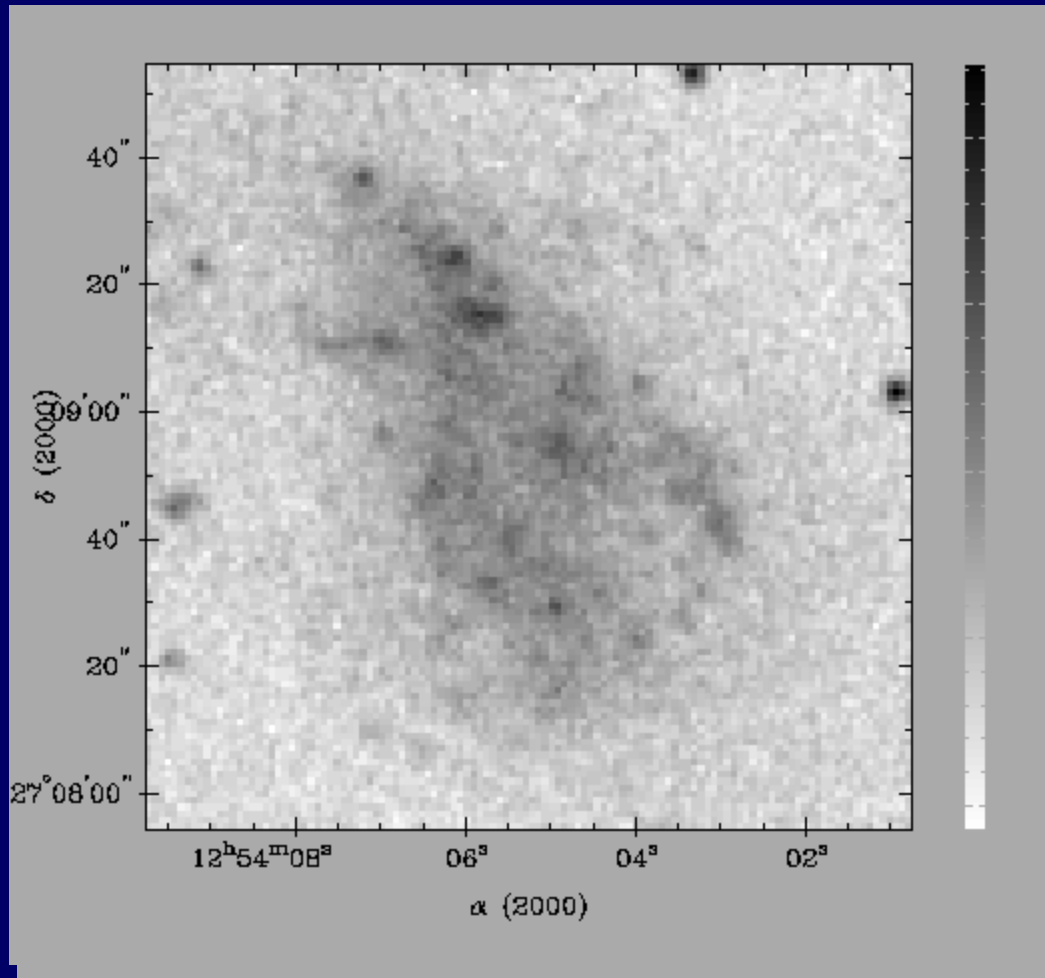
Compare simulated $V_c(r)$ with rotation curves of dark-matter dominated LSB galaxies

Observations:

de Blok et al (2001) (B01),
de Blok & Bosma (2002) (B02),
and Swaters et al (2003) (S03)

Peak velocities range from 25
km/s to 270 km/s

These measurements are hard!



DDO154 (a dwarf LSB)

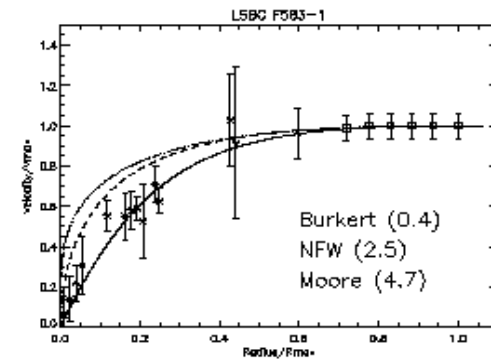
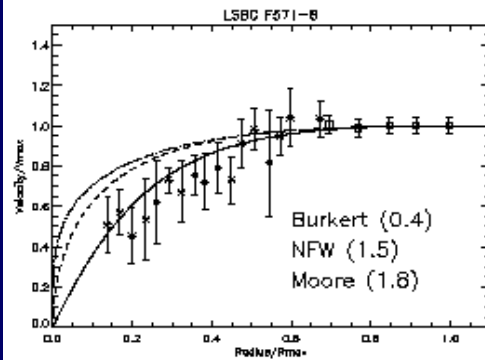
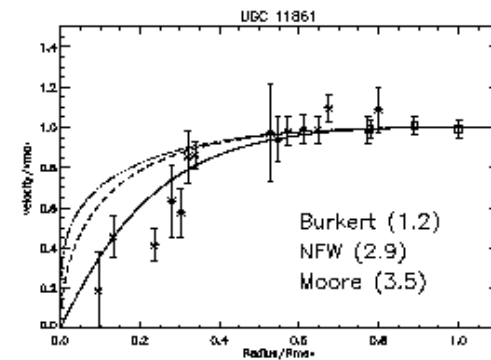
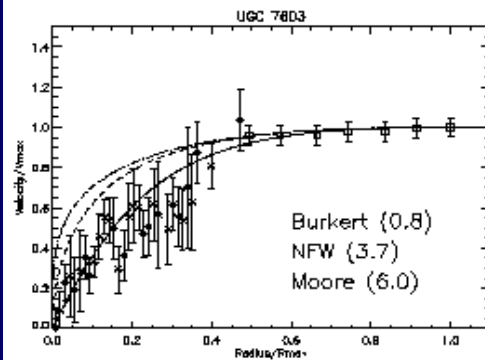
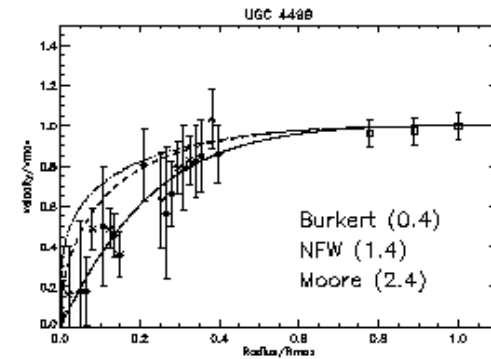
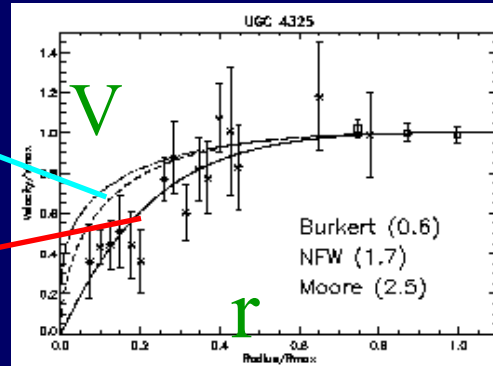
Observed cores vs. simulated cusps

cusps

core $\alpha=0$

$$V^2 = \frac{GM(r)}{r}$$

$$\rightarrow V \propto r^{1-\alpha/2}$$

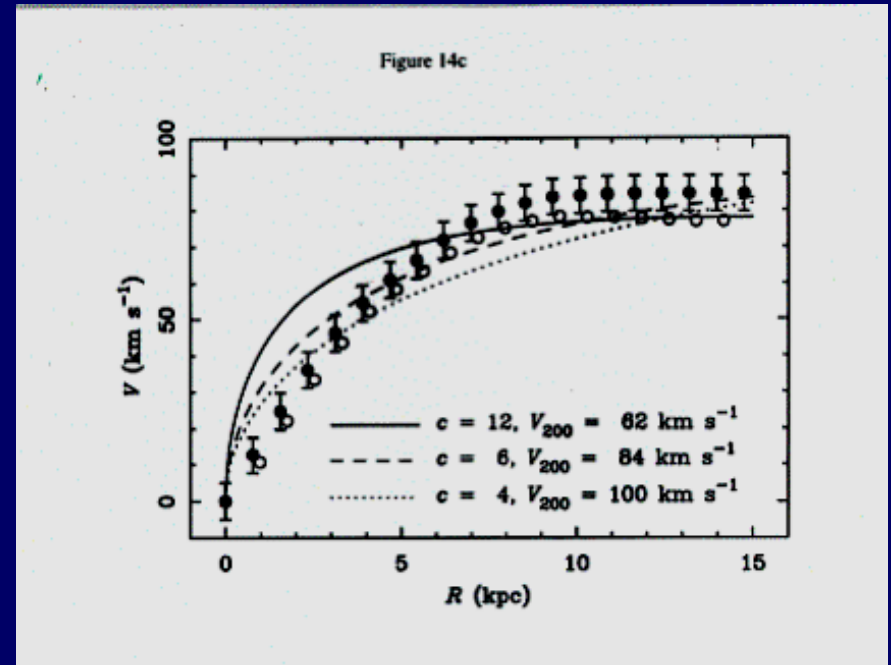


LSB rotation curves and CDM halos

Two problems:

The **shape** of LSB galaxy rotation curves is inconsistent with the circular velocity curves of CDM halos.

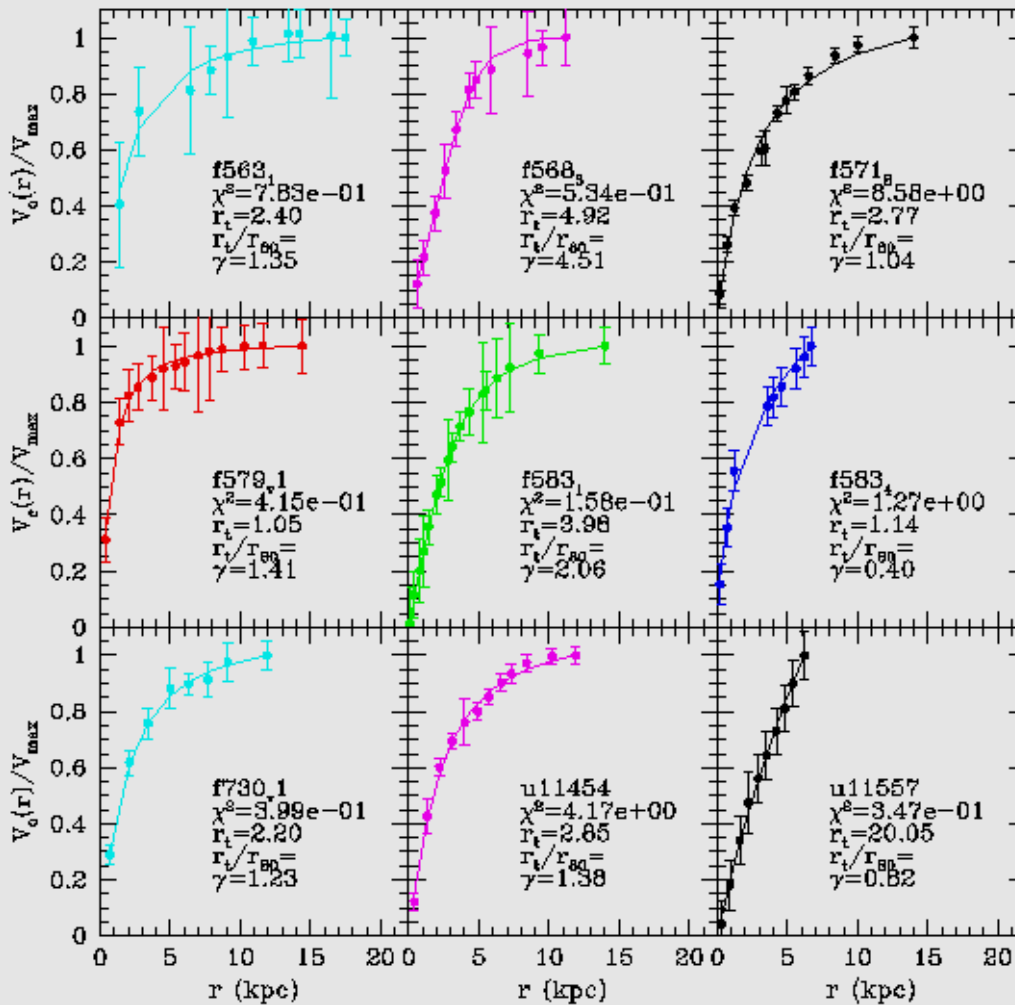
The **concentration** of dark matter halos is inconsistent with rotation curve data: there is too much dark matter in the inner regions of LSB galaxies.



McGaugh & de Blok 1998
see also Moore 1994
Flores & Primack 1994

LSB rotation curves (McGaugh et al sample)

Rotation Speed



Radius

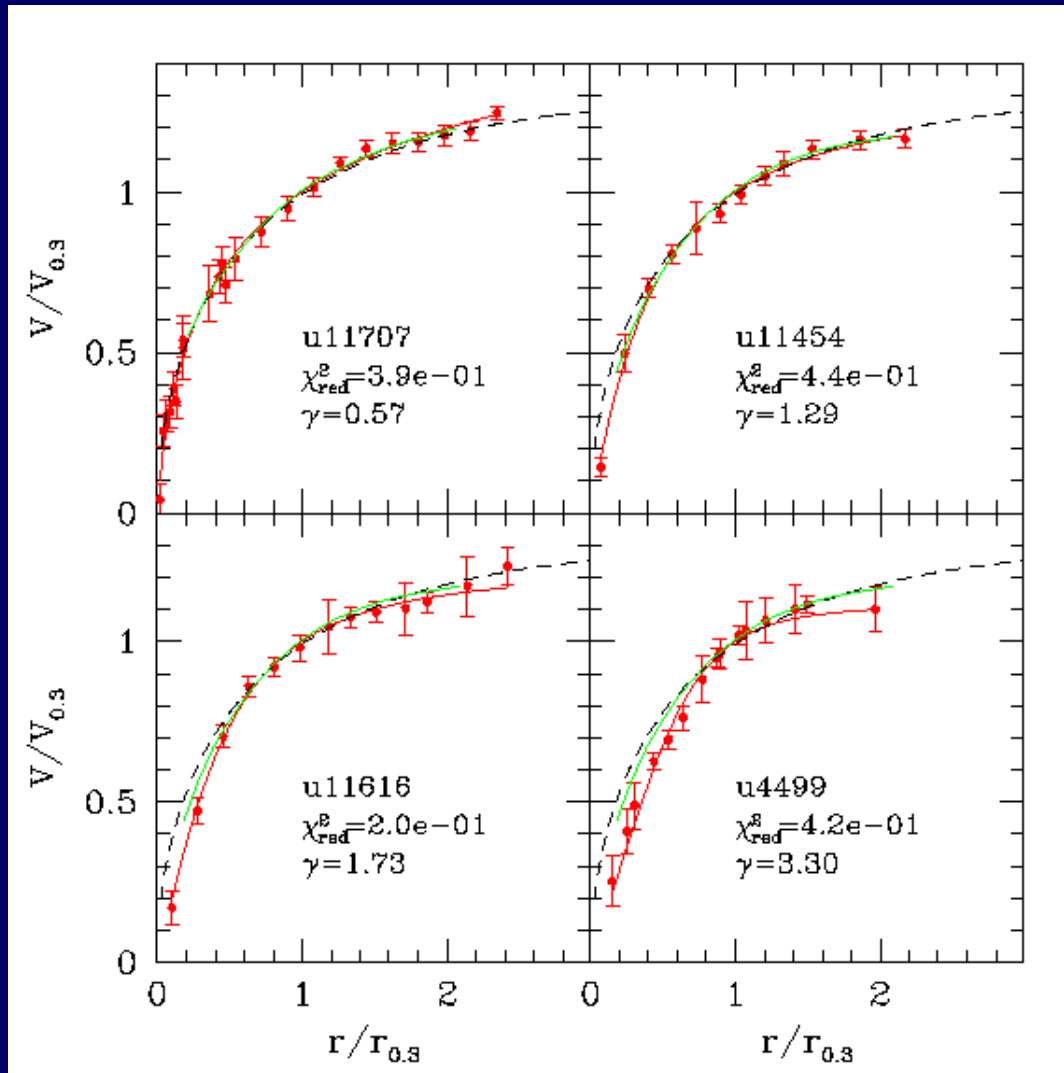
The shape of $V(r)$ varies from galaxy to galaxy

A fitting function:
 $V(r) = V_0 (1 + (r/r_t)^\gamma)^{-1/\gamma}$

The parameter γ is a good indicator of the shape of the rotation curve, the rate of change from rising to flat.

Scaled LSB rotation curves: a representative sample

Rotation Speed



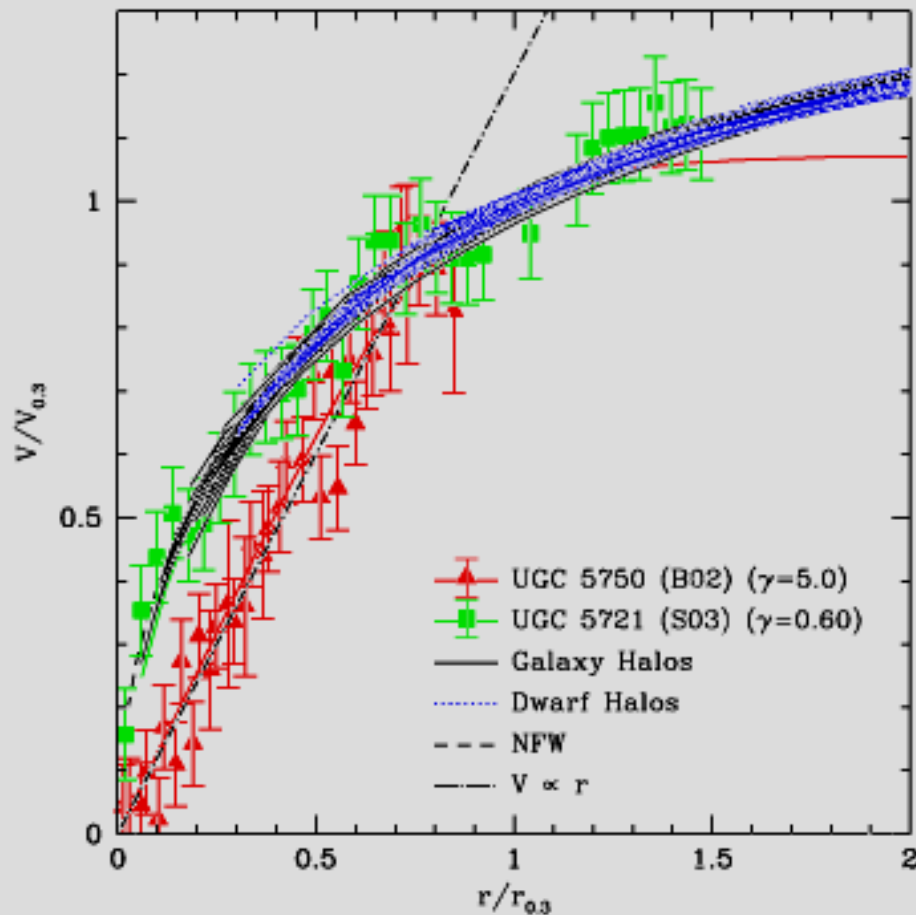
Radius

75% of LSB have $0.5 < \chi^2$
(~CDM halos)

25% have $\chi^2 > 2$
(in conflict with CDM halos)

Scaled LSB rotation curves

Rotation Speed

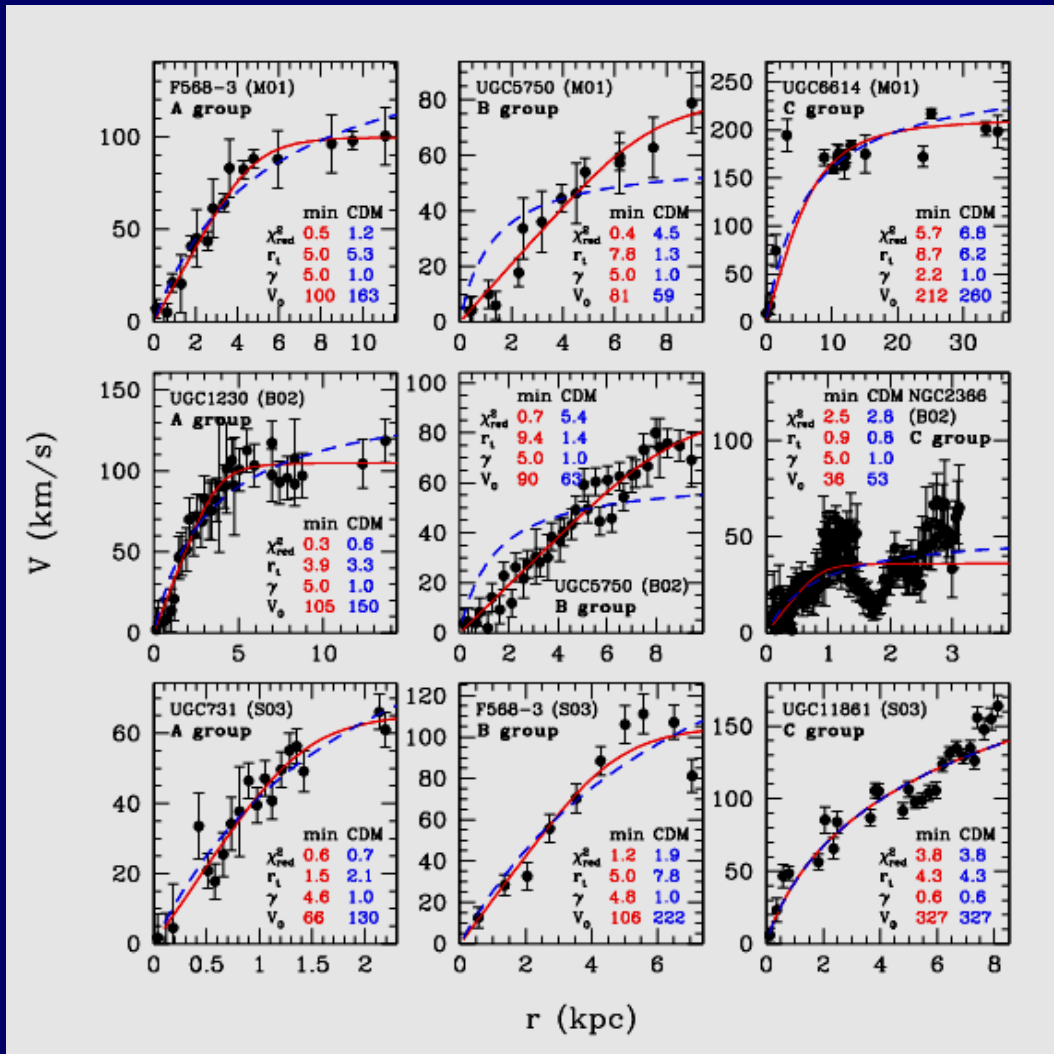


Radius

75% of LSB have $0.5 < \gamma < 2$
(~CDM halos)

25% have $\gamma > 2$
(in conflict with CDM halos)

Rotation Curves Inconsistent with CDM Halos



Three categories of rotation curves:

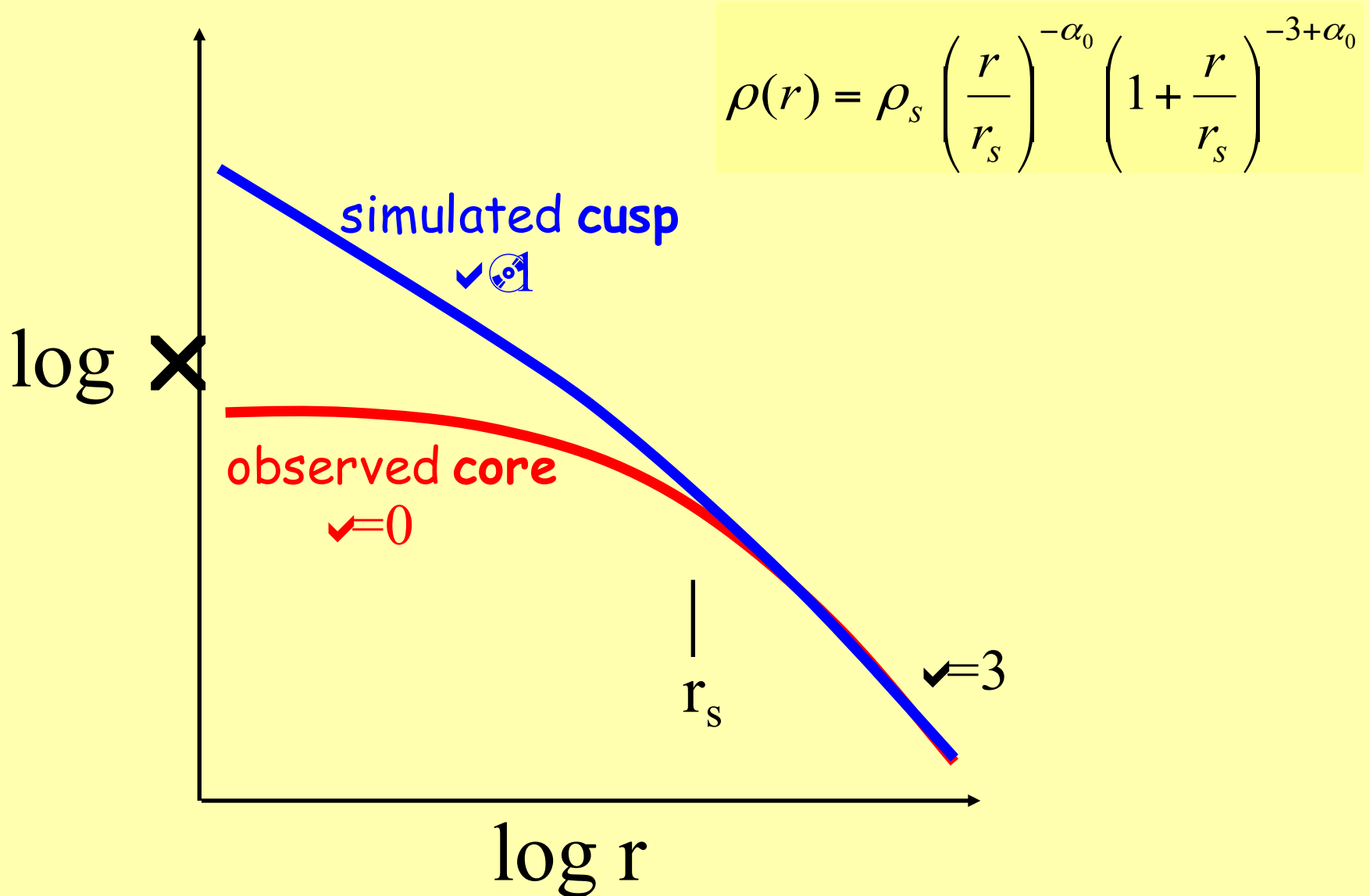
A) Well fit by V_g with LCDM compatible parameters (70%)

B) Poorly fit by V_g with LCDM-compatible parameters (10%)

C) Poorly fit by V_g with any parameters (20%)

Only 10% of LSB rotation curves are robustly inconsistent with LCDM halo structure

The dark-halo cusp/core problem

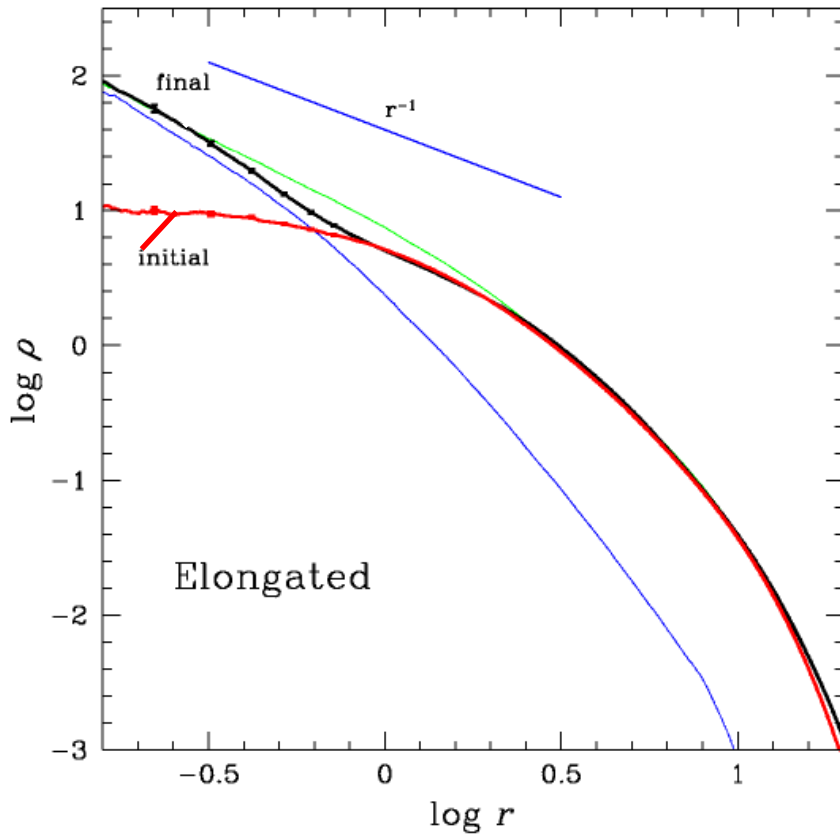


How to make and maintain a core?

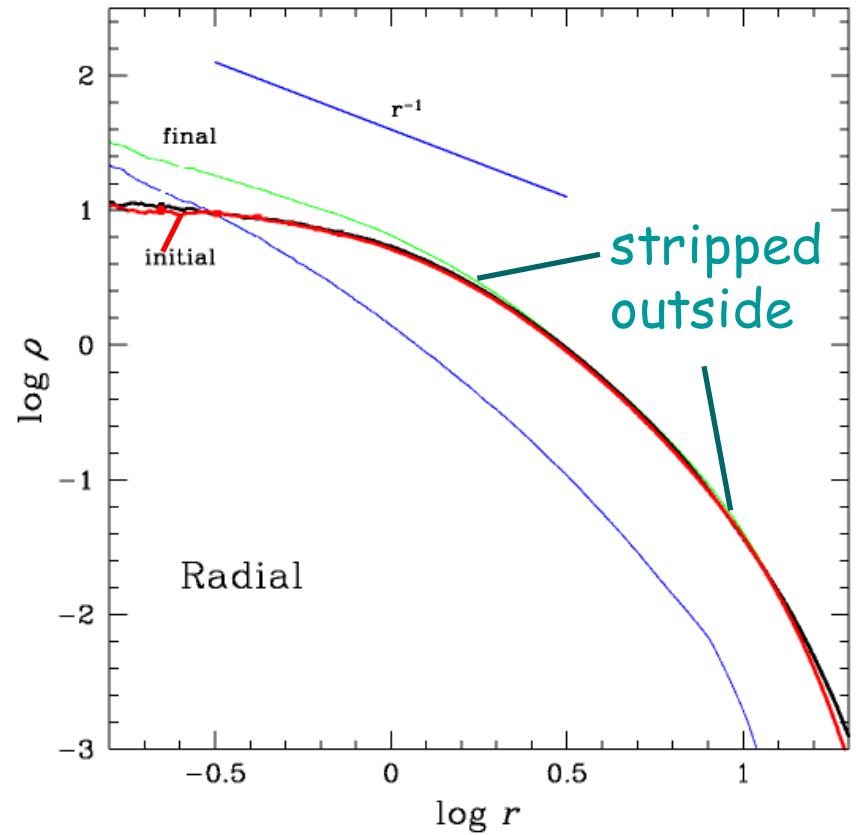
must suppress satellite
mergers with the halo core!

Compact vs. puffy satellite

compact



puffy 1/3 density



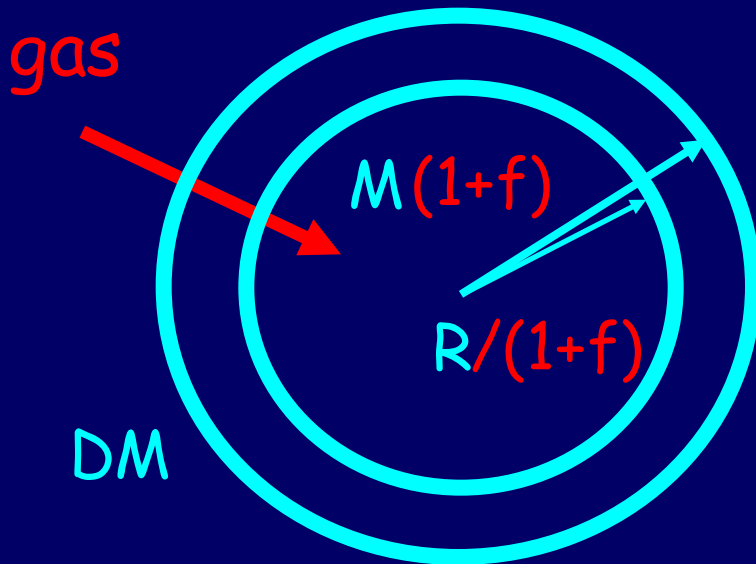
Adiabatic Contraction

Periodic motion under a slowly varying potential

Adiabatic invariant:

$$I \approx \int_0^T v^2 dt \approx v^2 T$$

$$t_{\text{dyn}} \sim \frac{R}{V} \sim \frac{R}{(GM/R)^{1/2}} \sim (GM/R^3)^{-1/2} \sim (G\rho)^{-1/2}$$



$$I \approx \frac{GM}{R} \left(\frac{M}{R^3} \right)^{-1/2} \propto (MR)^{1/2}$$

$$R \propto M^{-1}$$

Adiabatic Contraction

Periodic motion under a slowly varying potential

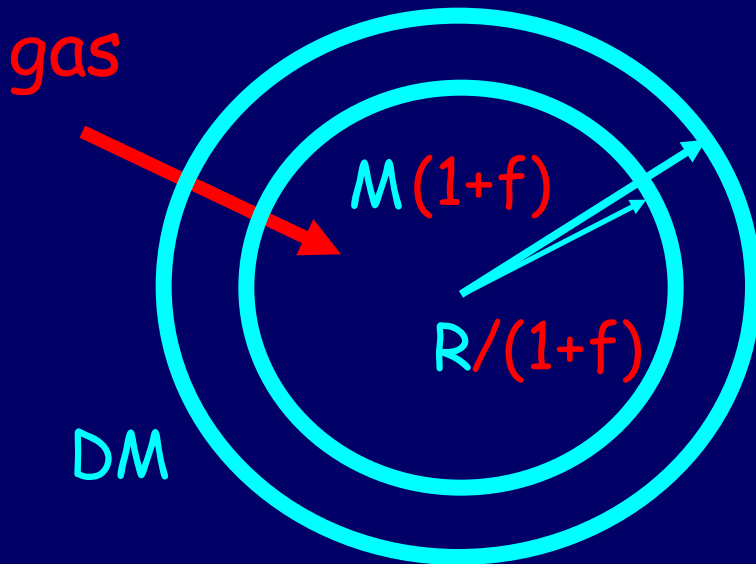
Adiabatic invariant:

$$I \approx \int_0^T v^2 dt$$

e.g. circular motions:

$$I = V^2 T \approx VR = j \quad \text{angular momentum}$$

$$V^2 = \frac{GM}{R} \rightarrow I \approx (MR)^{1/2}$$



$$R \propto M^{-1}$$

Feedback



Instant Blowout

$$E_{\text{before}} = -\frac{GM^2}{R} + \frac{1}{2}MV^2$$

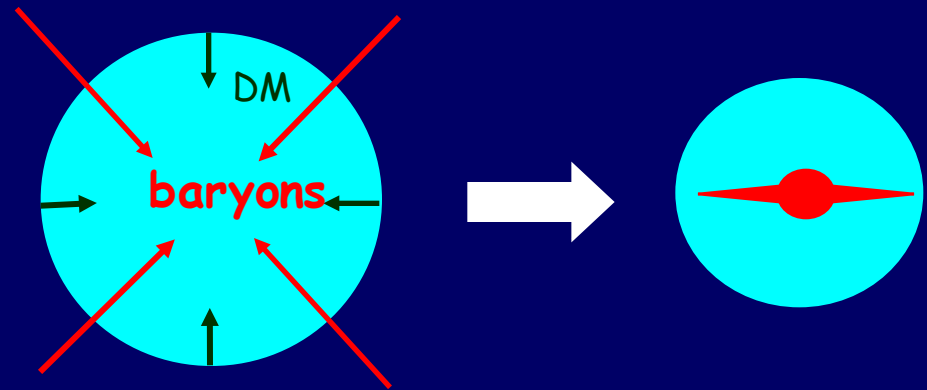
Lose $M/2$ while V^2 is unchanged:

$$E_{\text{after}} = -\frac{G(M/2)^2}{R} + \frac{1}{2}(M/2)V^2 = 0$$

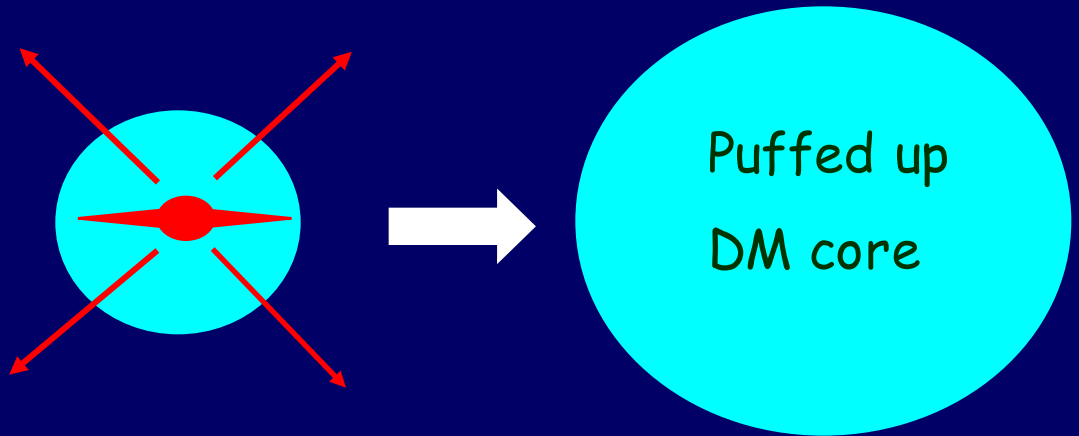
unbound!

DM-halo reaction to blowout

Adiabatic contraction:



Instant blowout:
by supernova feedback



only 1/6 in density (Gnedin & Zhao 02)
not enough in big galaxies?

Enough in satellites?

Toy Model

Dekel, Dutton, Ishai +

A shell of DM at r encompassing mass M in **virial** equilibrium

A mass m falls into (or ejected out of) the center **instantly**

Step 1: **U changes while $T = \text{const}$** . $E = U + T$ changes. Out of virial eq.

Step 2: U and T relax to a **virial** equilibrium while **$E = U + T$ is conserved**
The radius of the shell encompassing mass M contracts (or expands)

Alternatively, adiabatic contraction and instant expansion,
with same qualitative results

Shell Approximation

$$E = U + T \quad U = -\frac{GM}{r} \quad T = -\frac{1}{2} \frac{GM}{r}$$

Instant inflow or outflow

$$\frac{r_f}{r} = \frac{(1+f)}{(1+2f)}$$

$$f = \frac{m}{M}$$

Instant outflow after instant inflow

$$f_{out} = \beta f_{in}$$

$$f_{out} < 0.5$$

$$\frac{r_f}{r} = \frac{(1 - \beta f_{in})}{(1 - 2\beta f_{in})(1 + f_{in})}$$

$$\frac{M_f}{M} = \frac{1 - \beta f_{in}}{1 - f_{in}}$$

$$f \ll 1: \quad \frac{r_f}{r} \approx 1 - (1 - \beta)f + (2\beta^2 - \beta + 1)f^2$$

$$f \ll 1 \text{ and } \beta = 1: \quad \frac{r_f}{r} \approx 1 + 2f^2$$

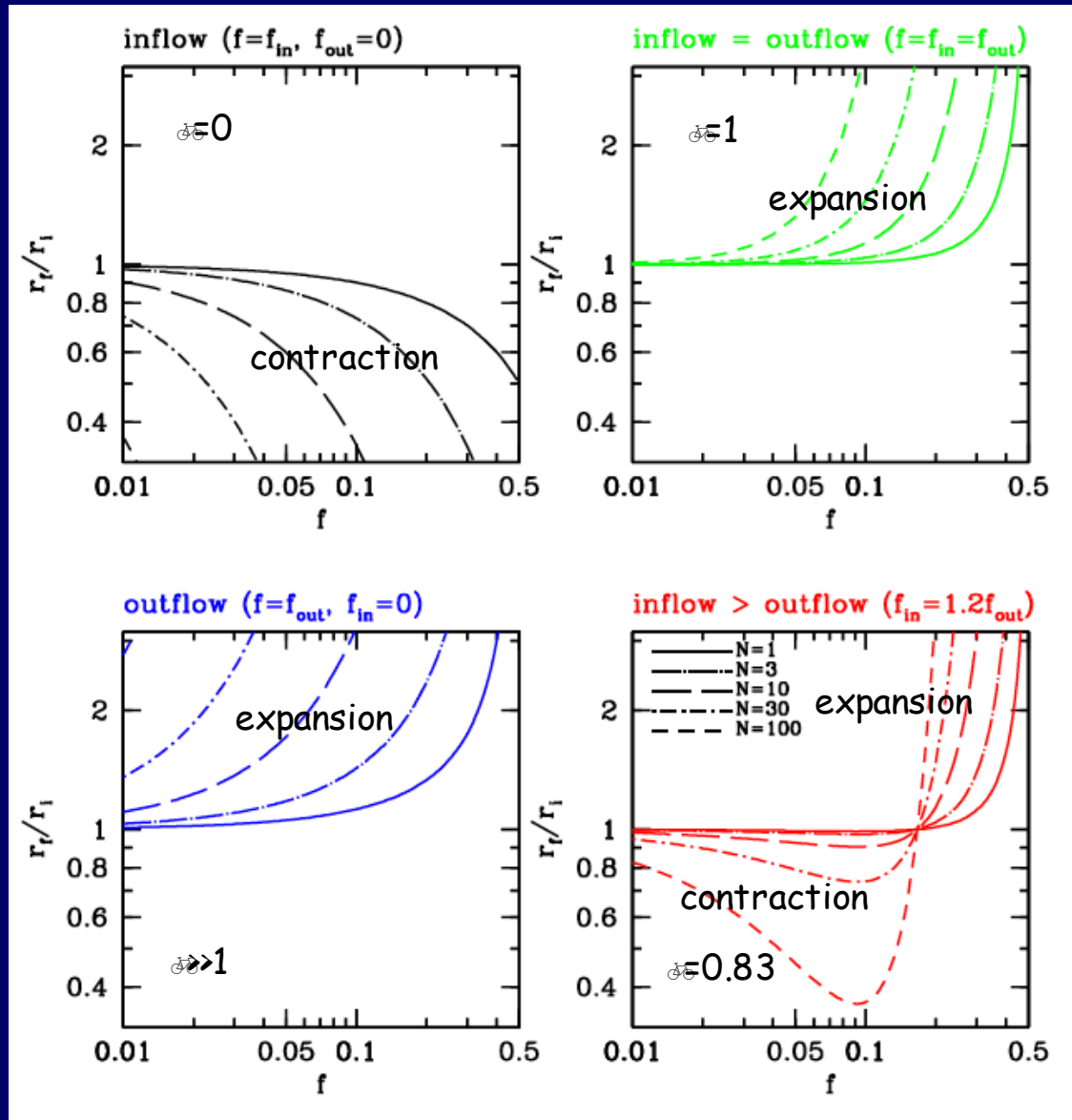
Net expansion if $\rightarrow (1+2f)^{-1}$, large and f
 Net contraction if $\leftarrow (1+2f)^{-1}$, small or f

Instant outflow after adiabatic inflow

Net expansion if $\rightarrow (1+f)^{-1}$

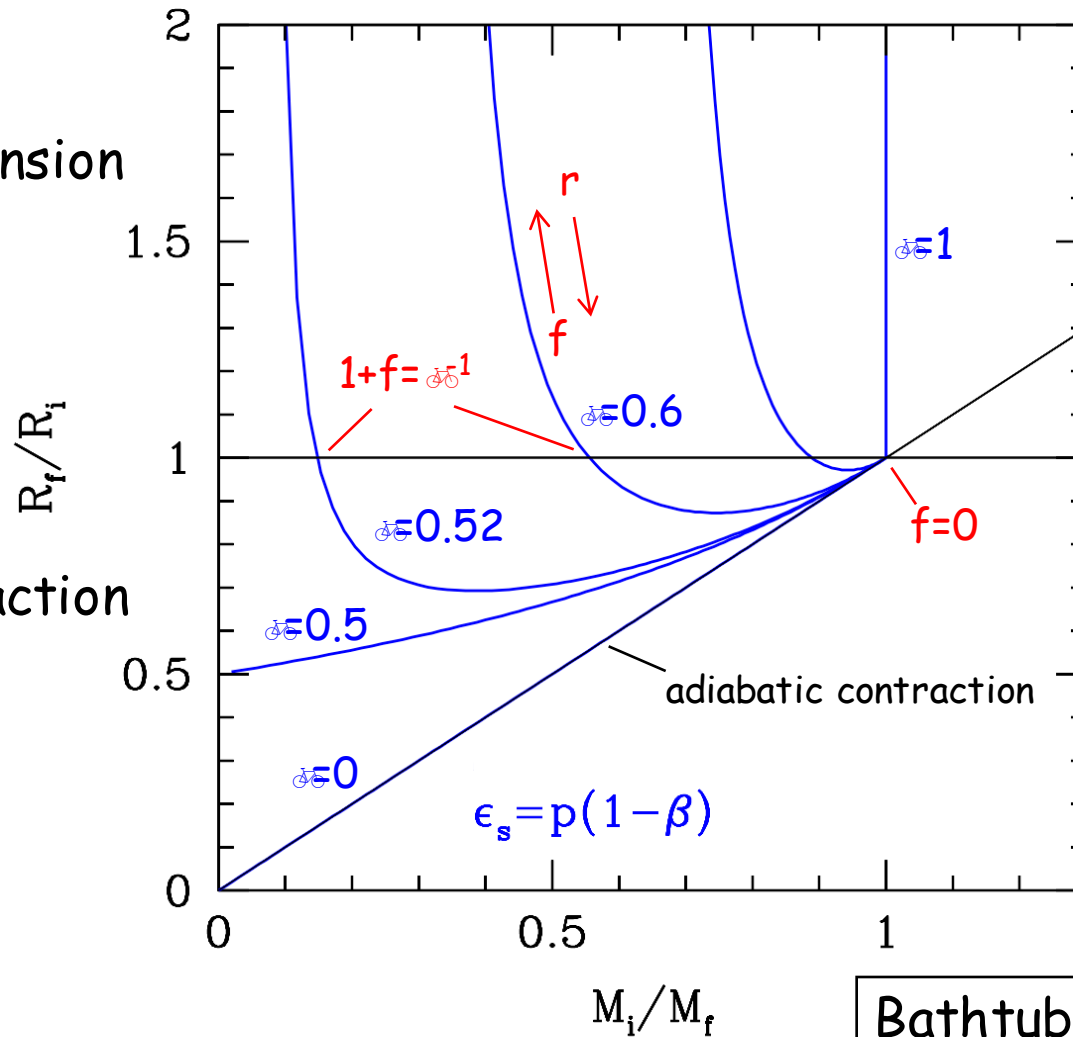
$$\frac{r_f}{r} = \frac{(1 - \beta f_{in})(1 - f_{in})}{(1 - 2\beta f_{in})} \approx 1 + f^2$$

Model: Halo Response dependence on



Toy-Model: One Episode

Expansion

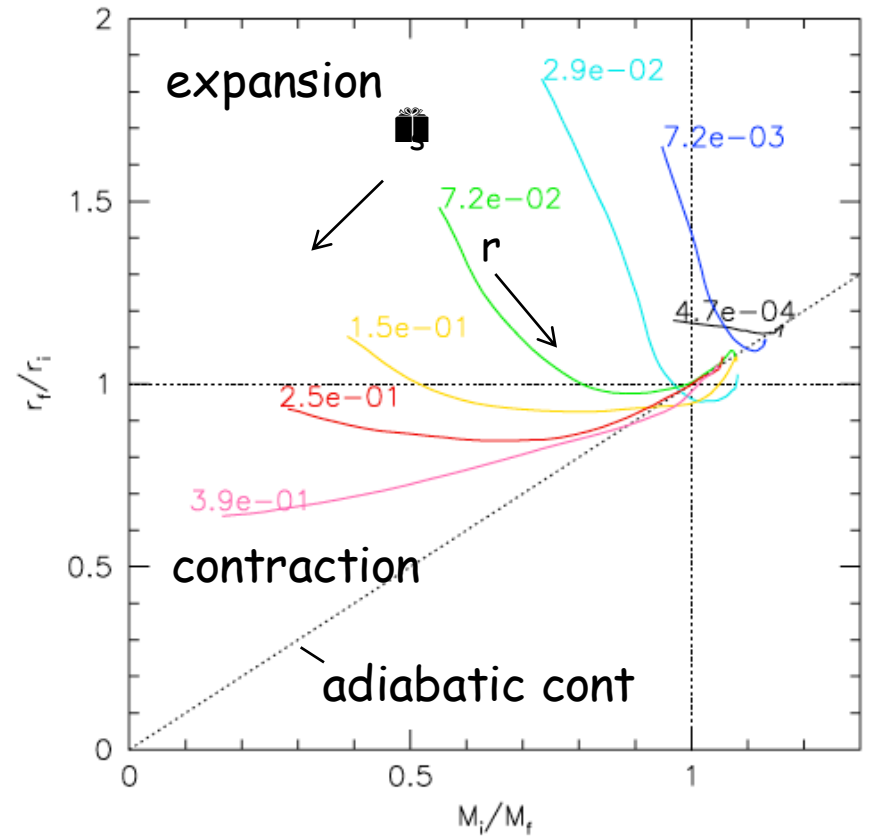
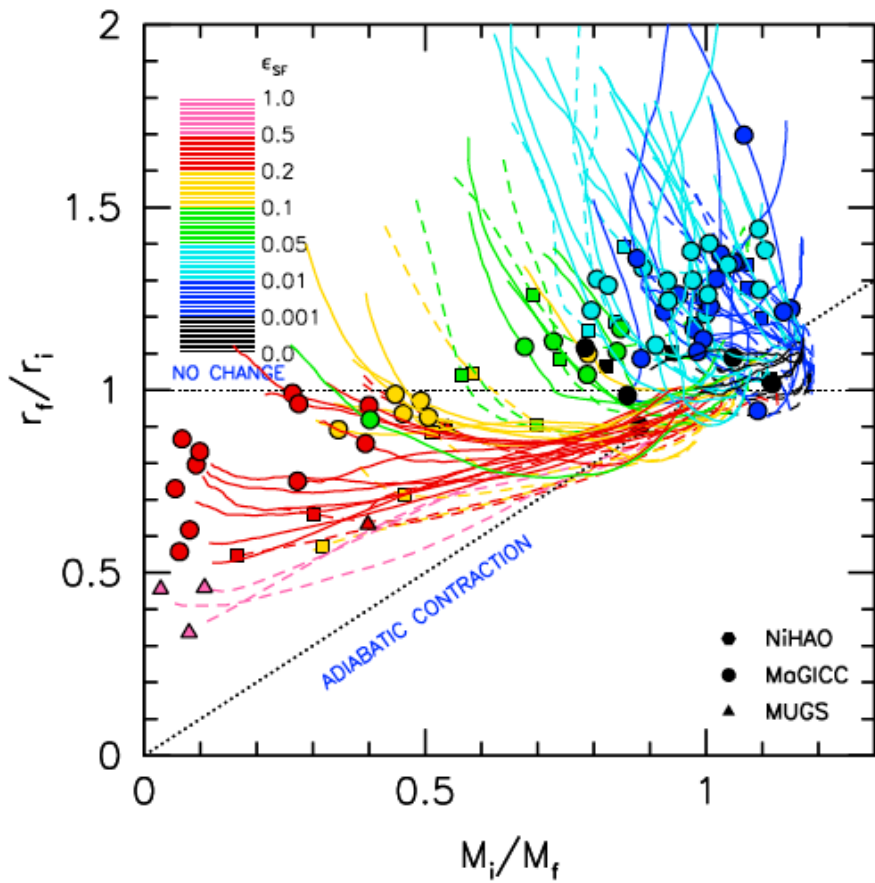


Contraction

Bathtub Model:

$$\epsilon_s = M_{\text{star}} / (f_b M_{\text{halo}}) = p(1 - \epsilon_s)$$

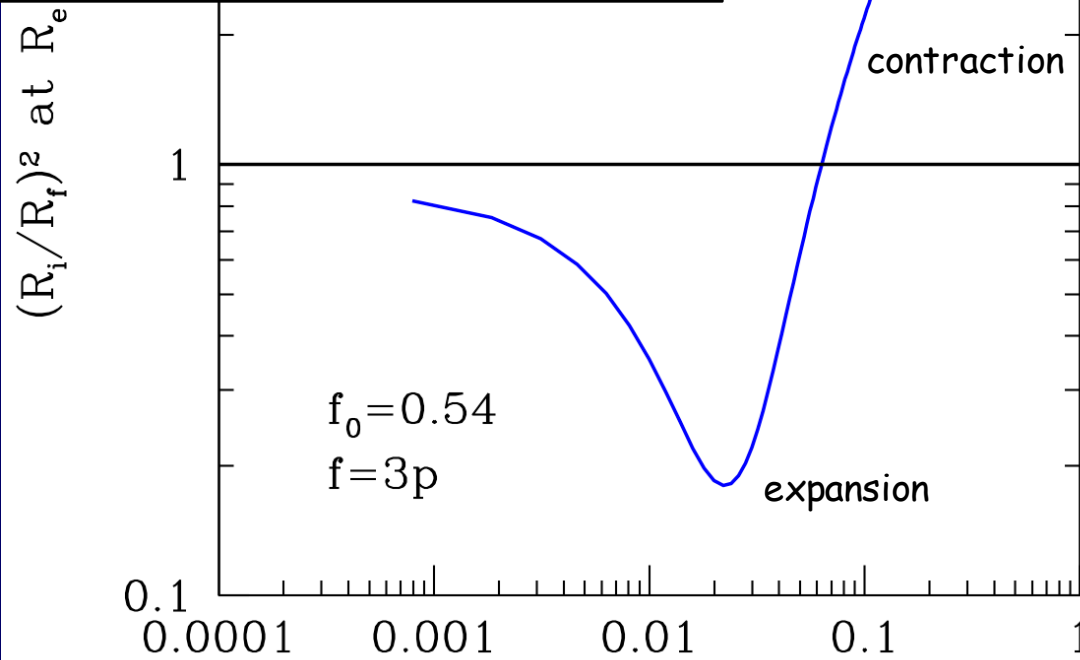
Inner Halo Response: Contraction/Expansion



Different Behavior at Low and High Mass

$p \sim 1$ at high M
 (high ☐ , low 🚲 , weak fdbk)
 recycling + inflow at $V_v > 100$ km/s

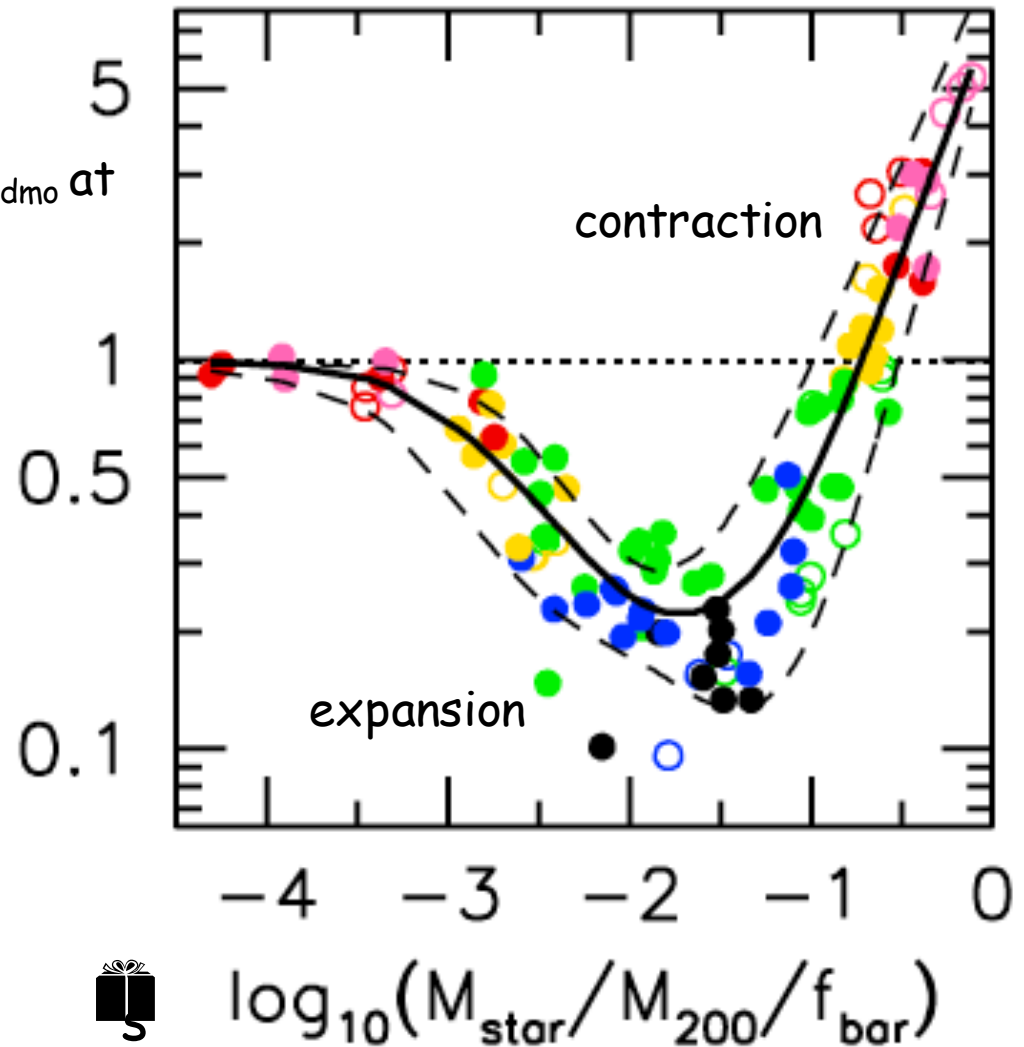
P & f drop at low M
 (low ☐ , $\text{🚲} \rightarrow 1$, strong fdbk)
 ejection & halt inflow at $V_v \ll 100$



$$\text{☐} = M_{\text{star}} / (f_b M_{\text{halo}}) = p(1 - \text{🚲})$$

Response at $0.01R_{\text{vir}}$: Contraction/Expansion

$M_{\text{hydro}}/M_{\text{dmo}}$ at
 $0.01R_{\text{vir}}$



Color (red to black)
 $= R_{\text{star}}/R_{\text{halo}}$



5

Multiple Episodes

$$\left(\frac{r_f}{r}\right)_N = \left(\frac{r_f}{r}\right)^N \approx 1 + 2Nf^2 \quad (f \ll 1)$$

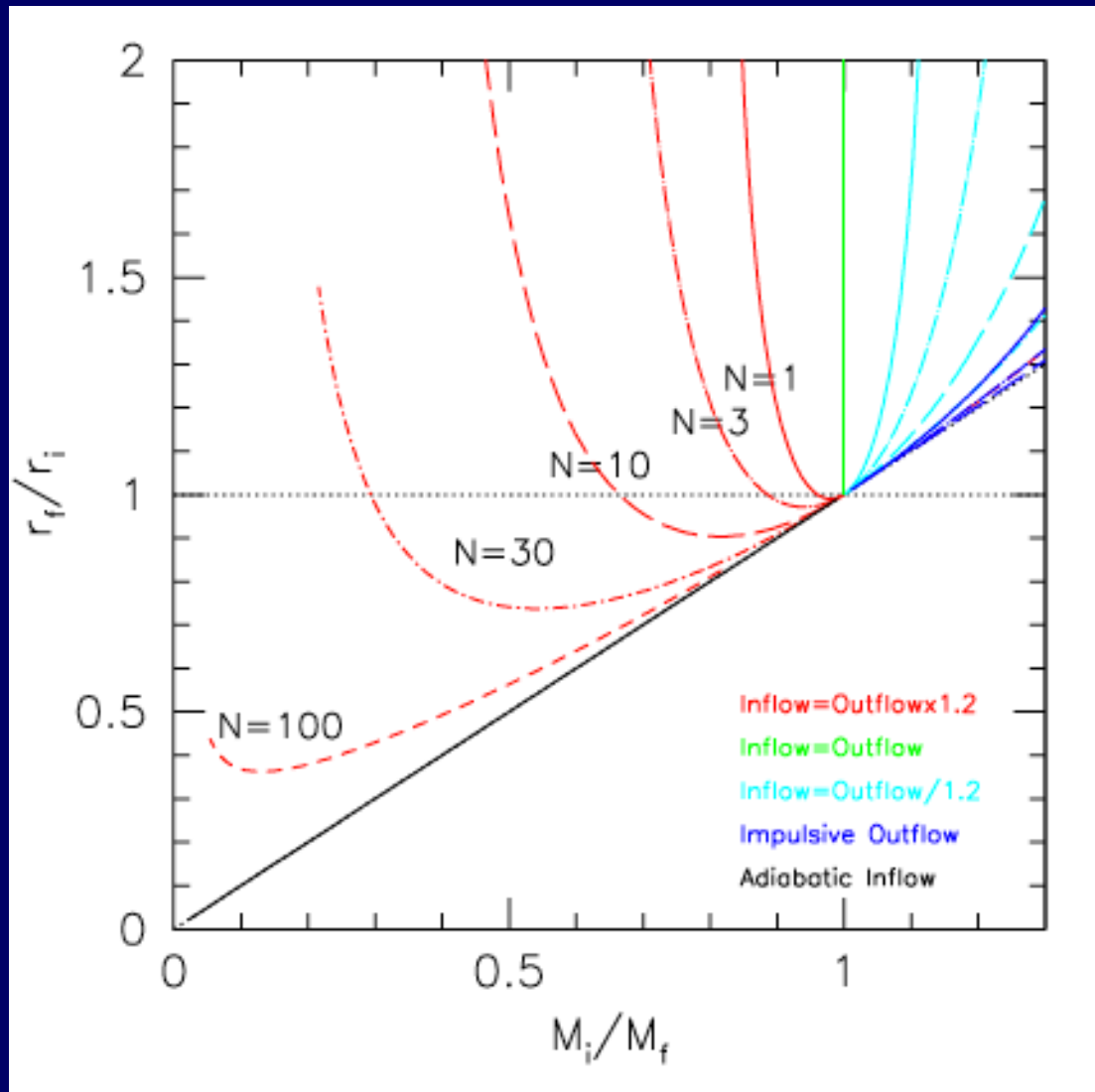
1. **Cosmological accretion:** per episode $f = f_{\text{tot}}/N$
($f_{\text{tot}}/N < 1/2$ to remain bound)

→ maximum effect at $N=1$

2. **Recycling:** per episode $f = f_0$

→ Stronger effect with many episodes

Model: Multiple Episodes



A Whole Virialized Halo

$$E(r) = U(r) + T(r) \quad U(r) \neq -\frac{GM(r)}{r} \quad T = -\frac{1}{2} \frac{GM(r)}{r}$$

The virial relation is the same,
but the potential should include the response of outer shells

$$U(r) = U_{\text{int}}(r) + U_{\text{ext}}(r) = -\frac{GM(r)}{r} - \int_r^{R_v} \frac{4\pi r'^2 \rho(r')}{r'} dr'$$

Instant inflow or outflow, then virialization while conserving mass:

$$U_i(r_i) + \frac{1}{2} \frac{[M_i(r_i) - 2m]}{r_i} = U_f(r_f) + \frac{1}{2} \frac{[M_f(r_f) - m]}{r_f}$$

$$M_f(r_f) = M_i(r_i)$$

Use a parametric functional form for $M(r)$ and $U(r)$
Apply at many radii and determine the best-fit parameters

Analytic Profile for Dark-Matter Halos

Dekel, Ishai 16

eNFW

$$\rho(r) = \frac{\rho_s}{x^\alpha (1+x)^{3-\alpha}} \quad x = \frac{r}{r_s} \quad r_s = \frac{R_v}{c}$$

parameters $\bar{\rho}_v, c, \alpha$

No analytic $M(r)$ or $U(r)$

New Profile

$$\bar{\rho}(r) = \frac{\bar{\rho}_s}{x^\alpha (1+x)^{3-\alpha}} \quad x = \frac{r}{r_s} \quad r_s = \frac{R_v}{c}$$

$$\bar{\rho}_s = \bar{\rho}_v c^\alpha (1+c)^{3-\alpha}$$

Trivial integration

$$M(r) = \mu M_v \frac{x^{3-\alpha}}{(1+x)^{3-\alpha}}$$

$$V^2(r) = c\mu V_v^2 \frac{x^{2-\alpha}}{(1+x)^{3-\alpha}}$$

$$F(r) = c^2 \mu F_v \frac{x^{1-\alpha}}{(1+x)^{3-\alpha}}$$

The coefficients are functions of the parameters

Derivative

$$\rho(r) = \frac{1}{4\pi r^2} \frac{dM}{dr} = \rho_s \frac{1}{x^\alpha (1+x)^{4-\alpha}}$$

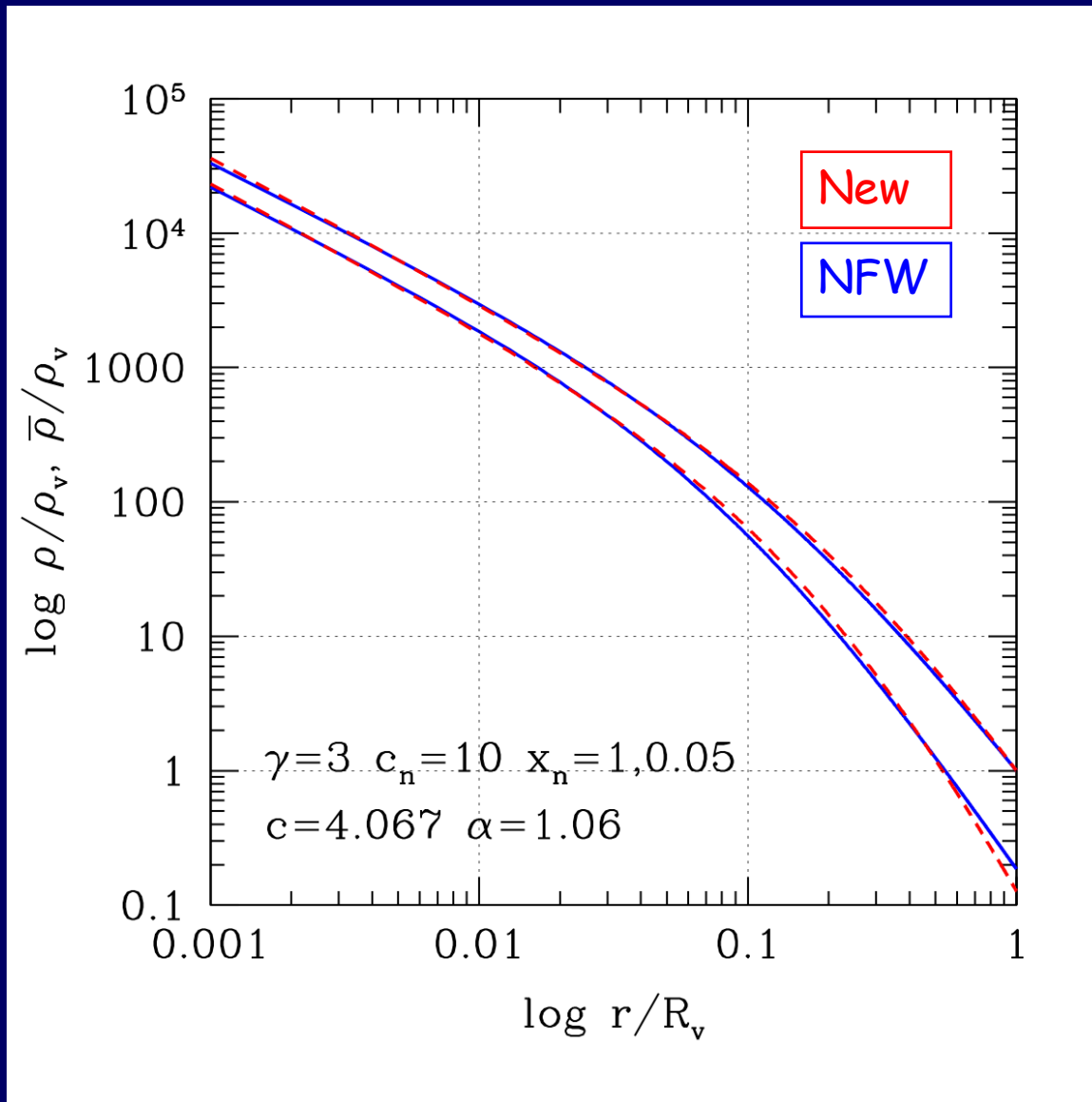
steep outer slope is compensated by c

Potential

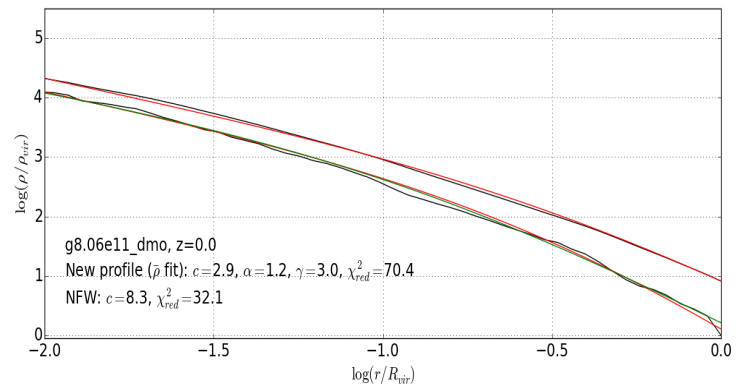
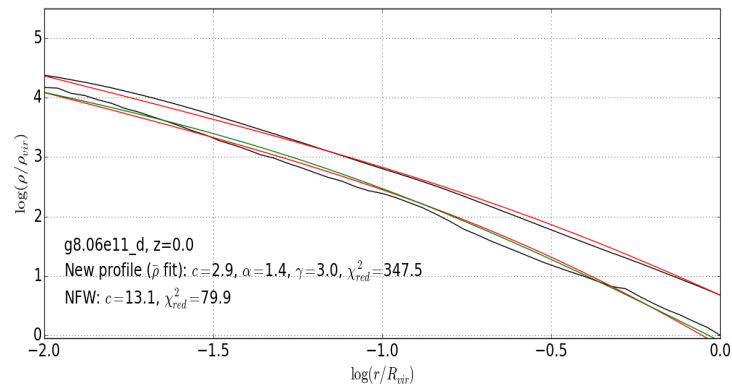
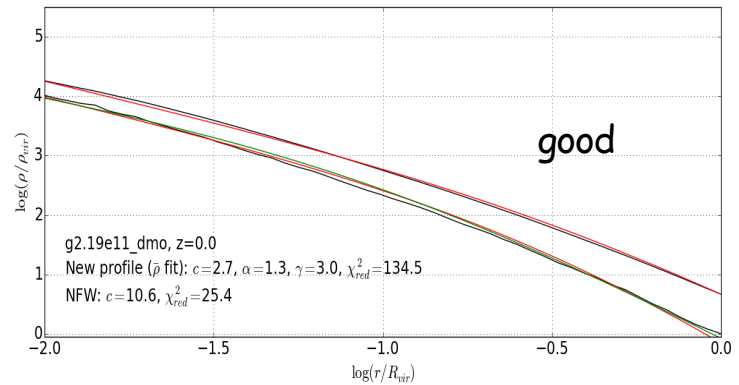
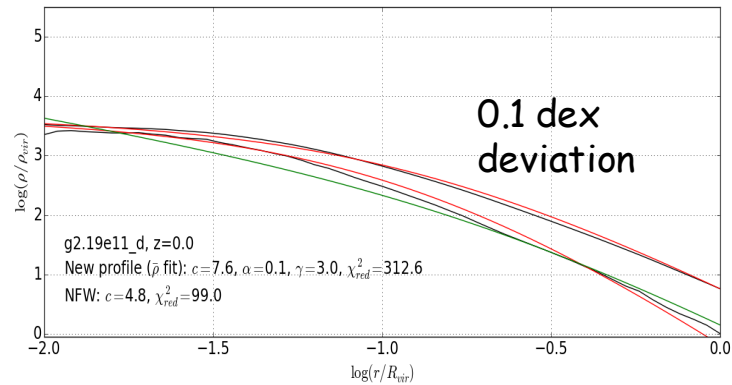
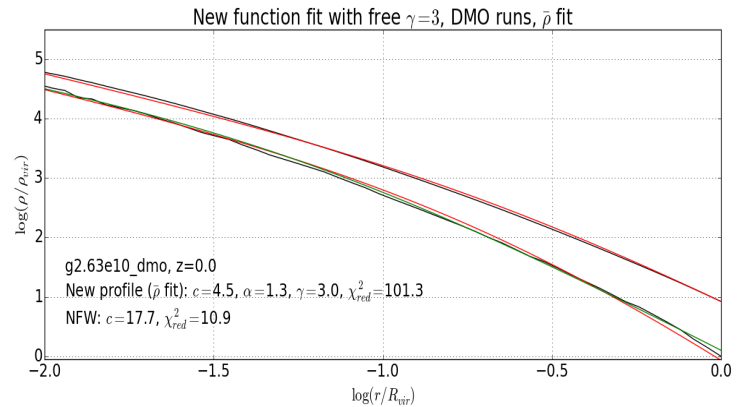
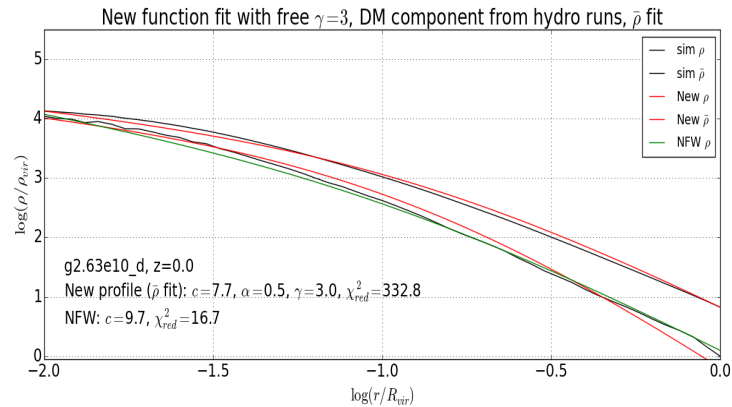
$$U(r) = -\int_r^\infty \frac{M(r')}{r'^2} dr' = \omega V_v^2 \left[\left(\frac{x}{1+x} \right)^{2-\alpha} - \left(\frac{c}{1+c} \right)^{2-\alpha} \right] - V_v^2$$

Analytic $\rho(r)$, $M(r)$ and $U(r)$, good fit to simulations

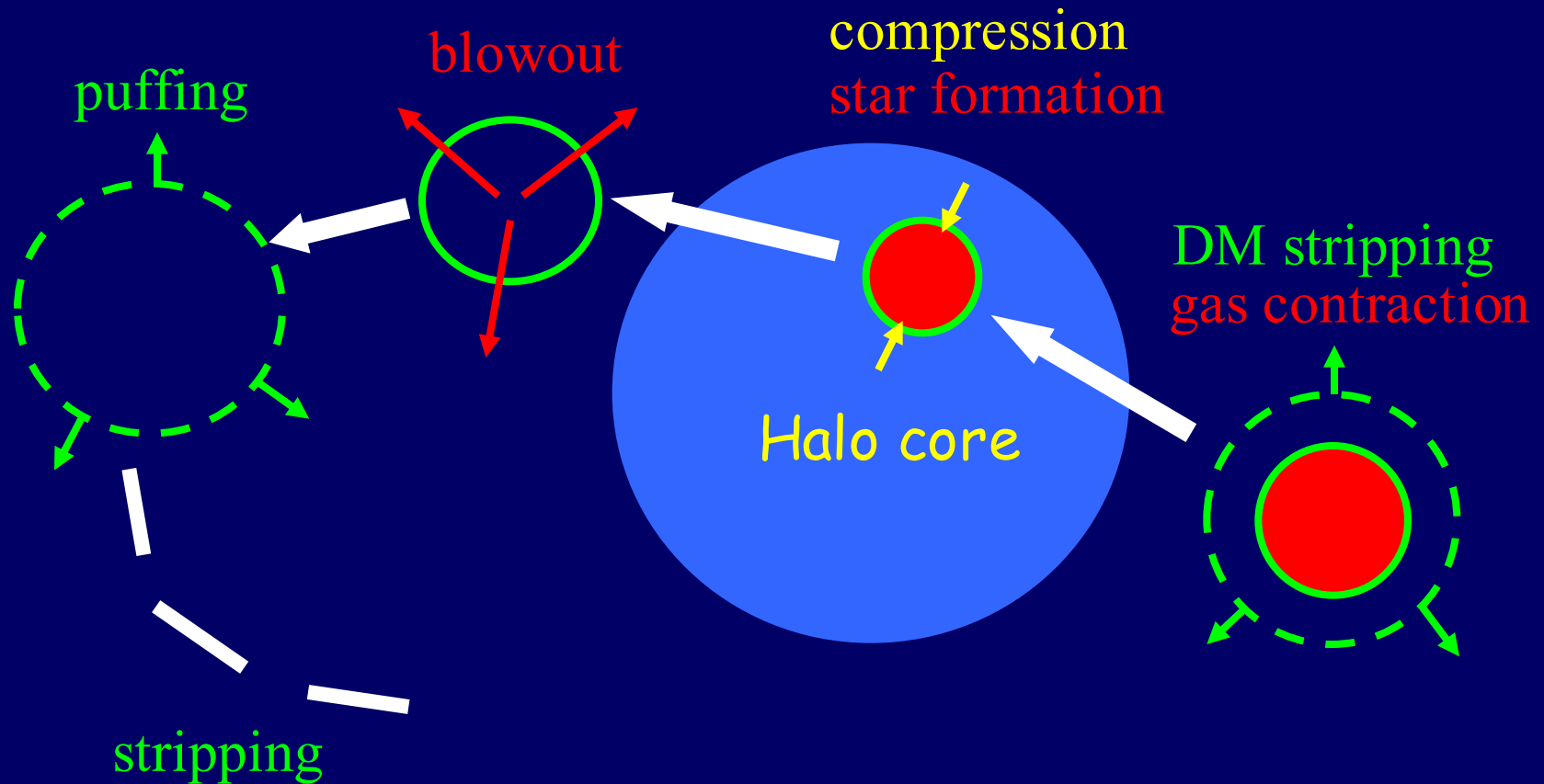
A Better Match to NFW with $\gamma=3$ ✓ 1



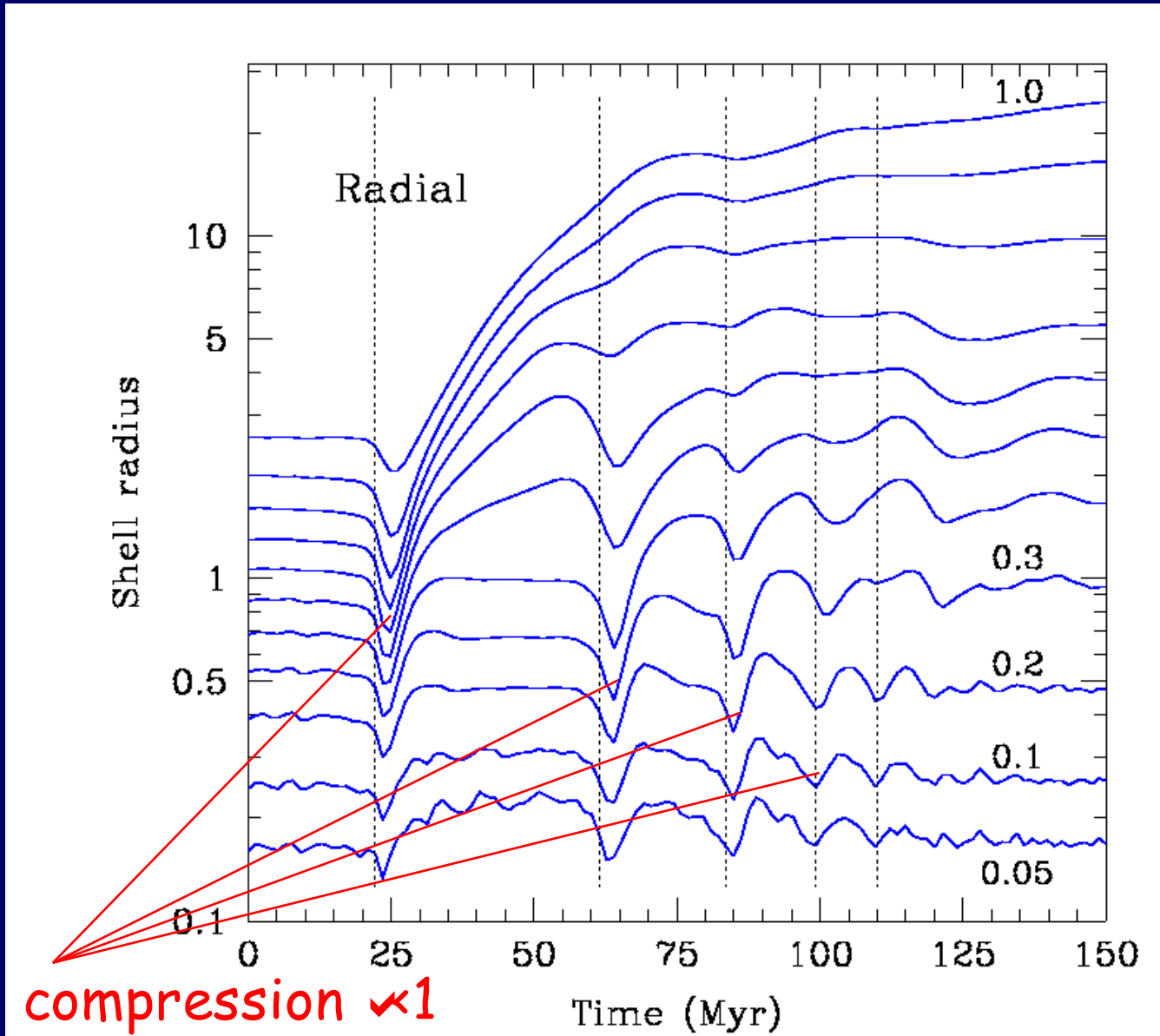
Fit $\rho \propto r^{-3}$ Profile to Simulated Halos ($z=0$)



Satellite disruption by stimulated feedback




Compression in core



Summary: Core

Feedback may lead to a core
by puffing small satellites

Caveats

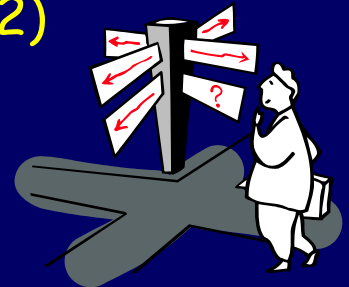
 Cusps (though flatter) form also in simulations where satellites are suppressed

 Cores detected in big galaxies and clusters (?)

→ Puffing-up of satellite halos is **necessary** for cores, but perhaps **not sufficient**

Other scenarios for core formation

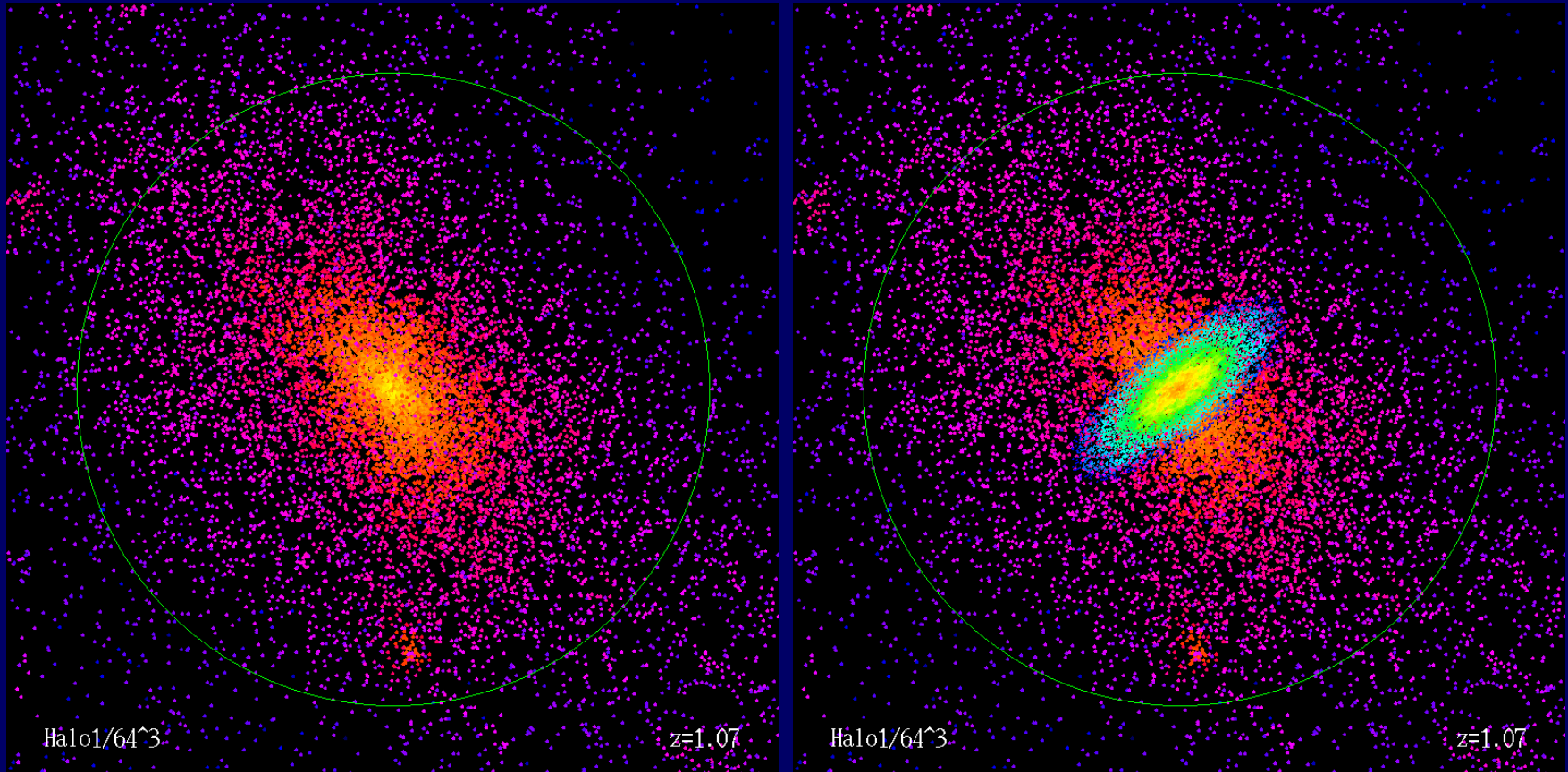
- Warm dark matter, Interacting dark matter
☹️ suppress satellites
- Disruption of satellites by a massive black hole
(Merritt & Cruz 01)
- Angular-momentum transfer from a big bar
to the halo core (Weinberg & Katz 02)
- Delicate resonant tidal reaction of halo-core
orbits
if the system is noise-less (Katz & Weinberg 02)
- Heating of the cusp by merging clouds
(El-Zant, Shlosman & Hoffman 02)



Origin of Core: Disk in Triaxial Halo

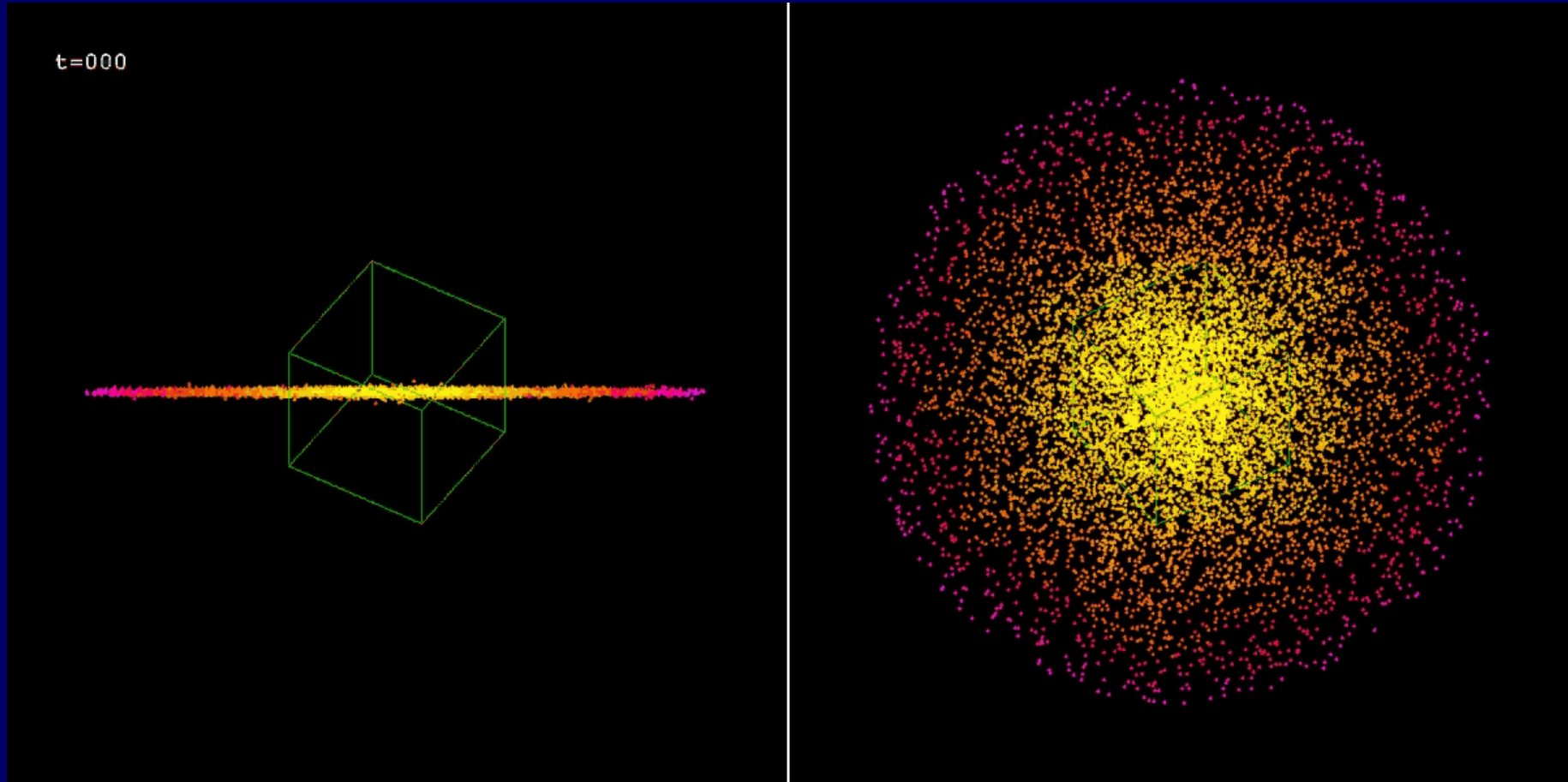
Disk Rotation curve is NOT $V^2=GM(r)/r$
Hayashi, Navarro et al.

Disks in realistic dark matter halos



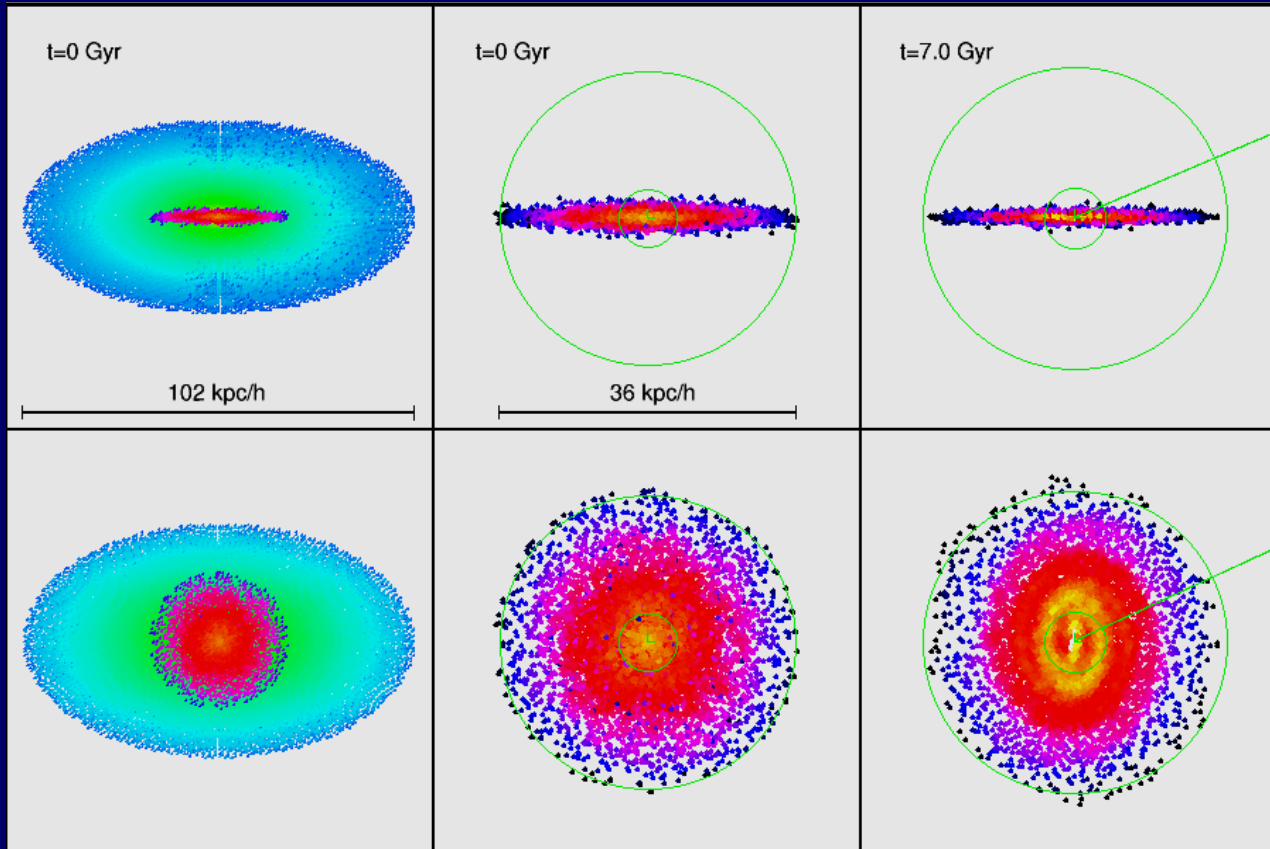
Massless isothermal gaseous disk in the non-spherical DM halo potential tracks the closed orbits within this potential

Disks in realistic dark matter halos



Massless isothermal gaseous disk in the DM halo potential

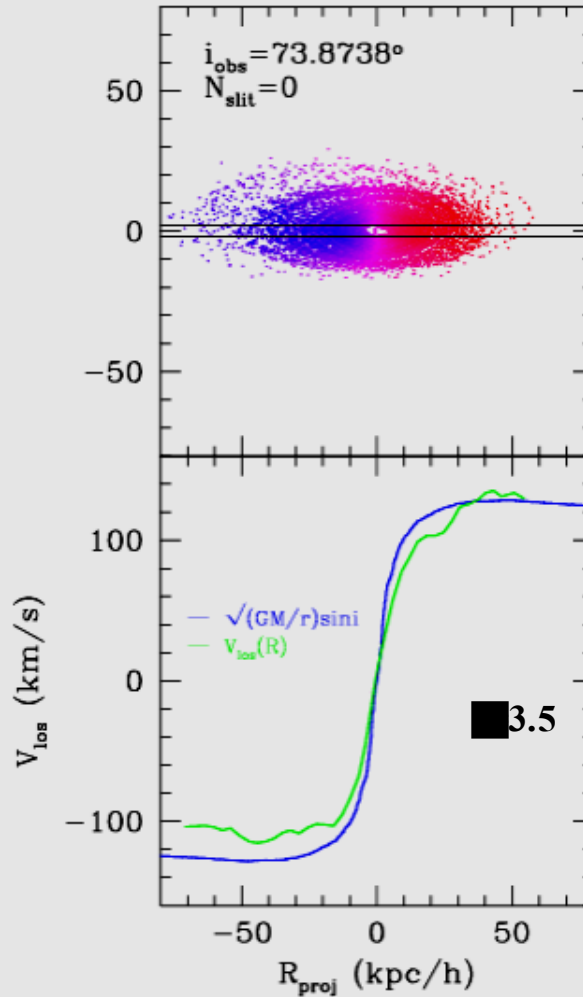
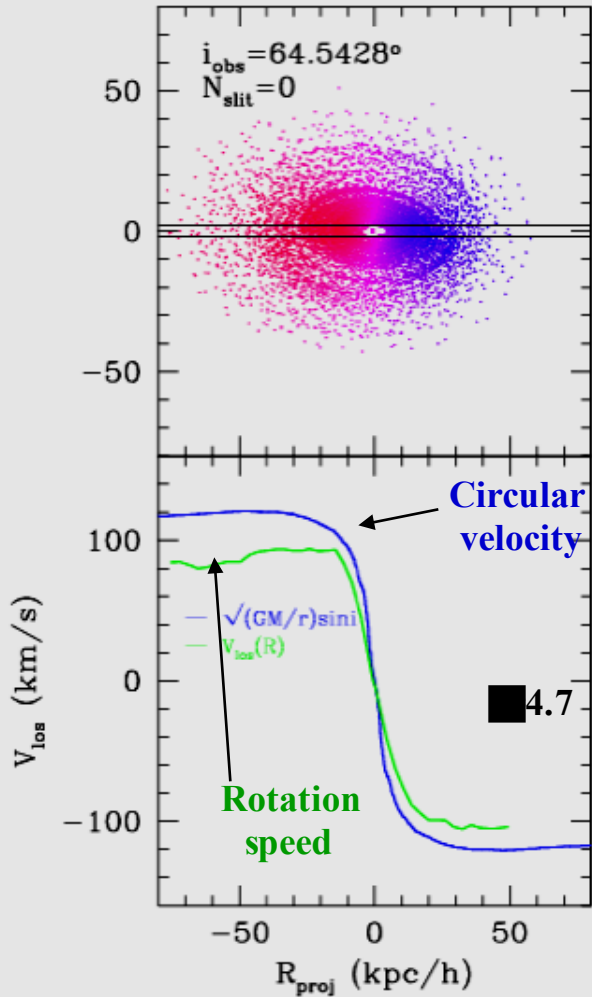
Dynamics of a Gaseous Disk



Closed orbits in triaxial potentials are not circular, and not limited to a plane.

High γ ?

Disks in triaxial dark matter halos



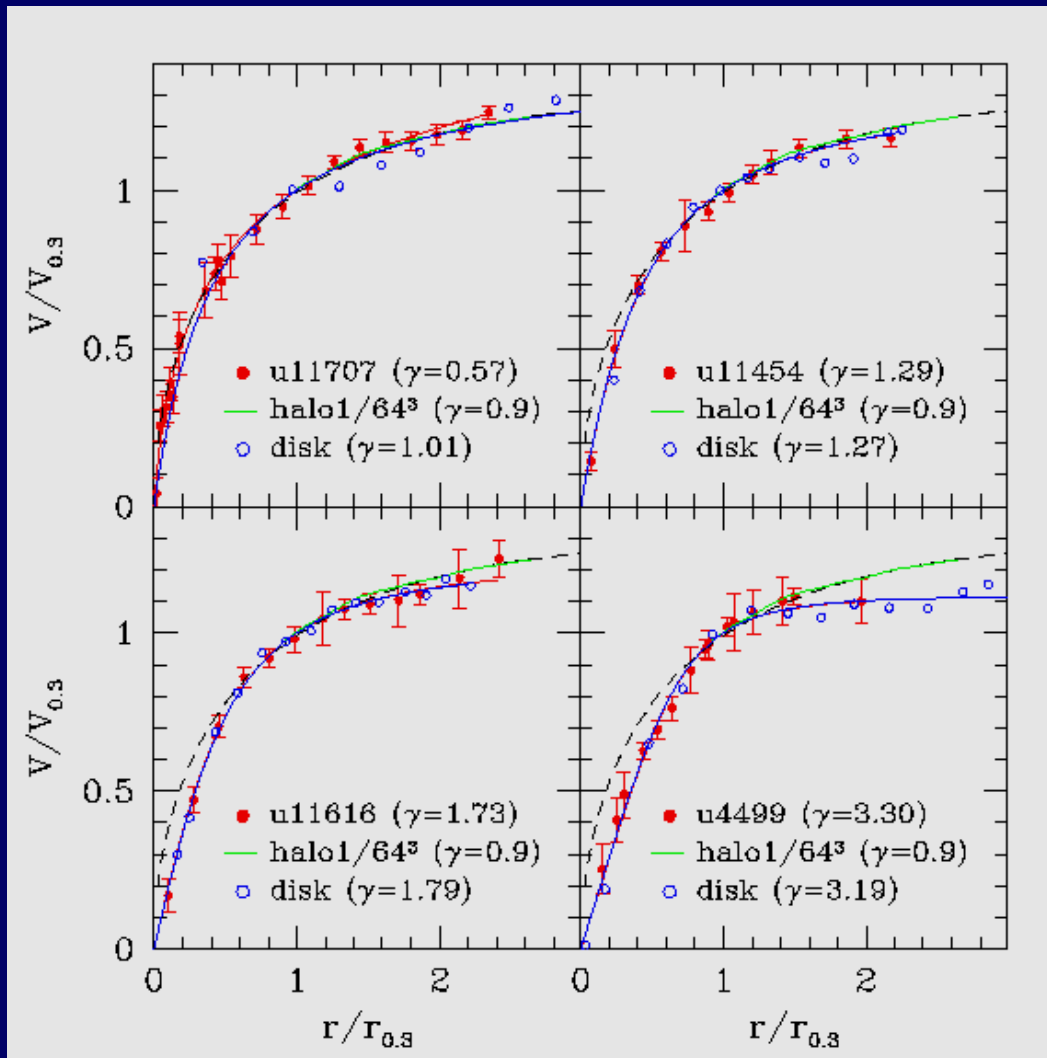
Inferred rotation speeds may differ significantly from actual circular velocity.

Inclination: 50 degrees

67 degrees

Scaled Rotation Curves: disk in CDM halo vs LSBs

Scaled Rotation Speed



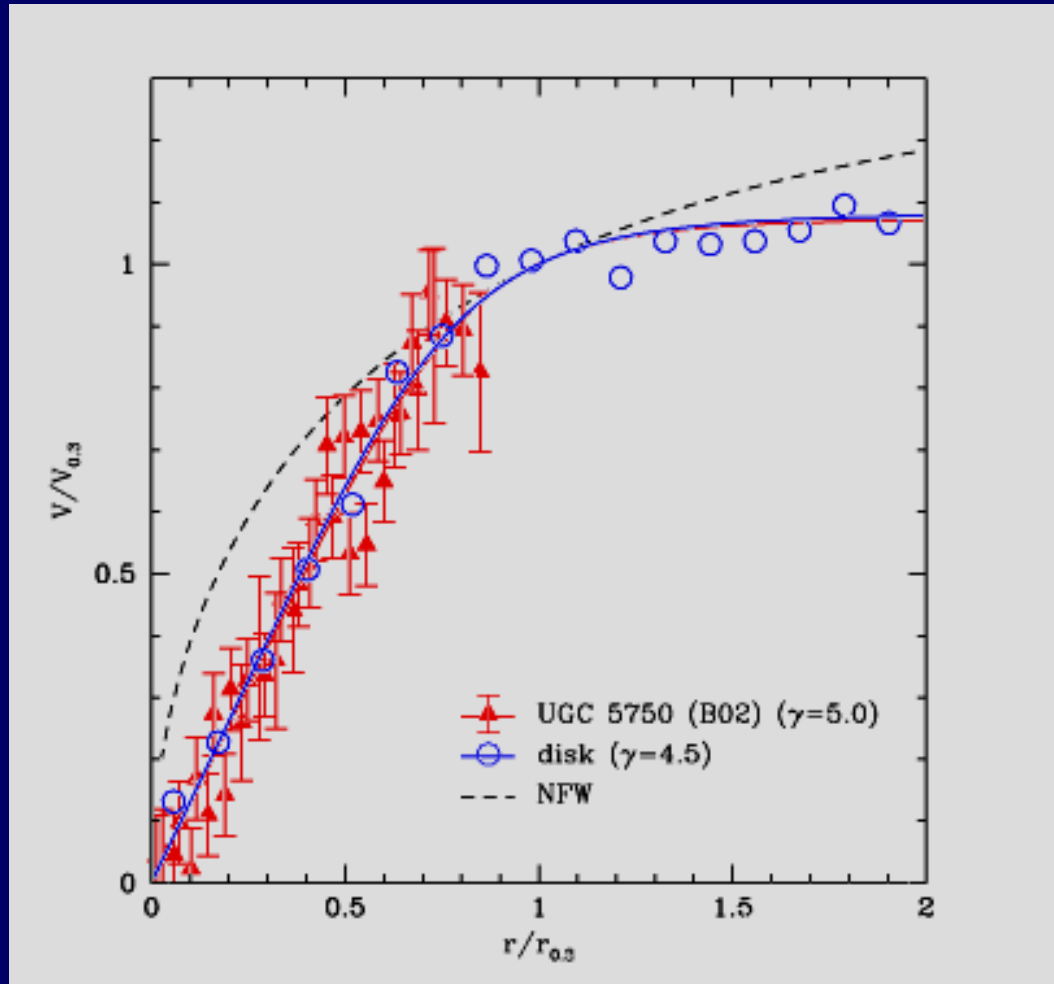
Scaled radius

All LSB rotation curve shapes may be accounted for by various projections of a disk in a single CDM halo

Hayashi et al 2003

Scaled LSB rotation curves: a representative sample

Rotation Speed



Radius

LSB rotation curve shapes may be accounted for by various projections of a disk in a single CDM halo

Triaxiality in the halo potential may be enough to explain the "cusp-core" discrepancy.

Hayashi et al 2003

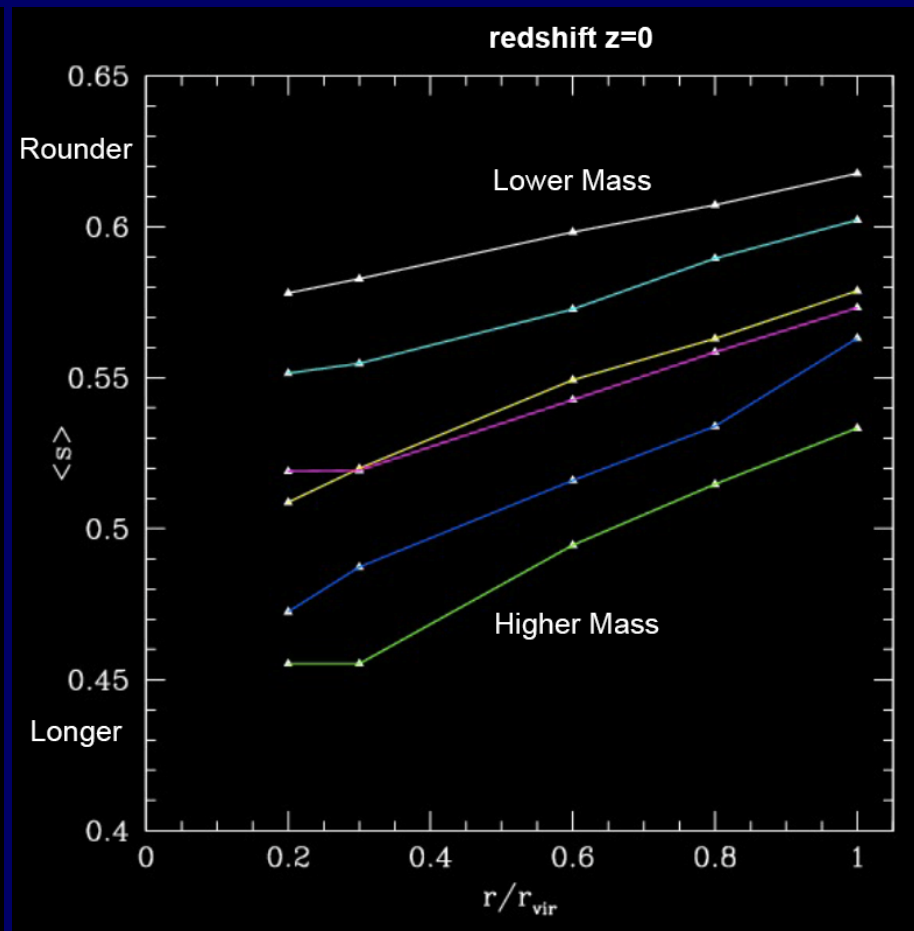
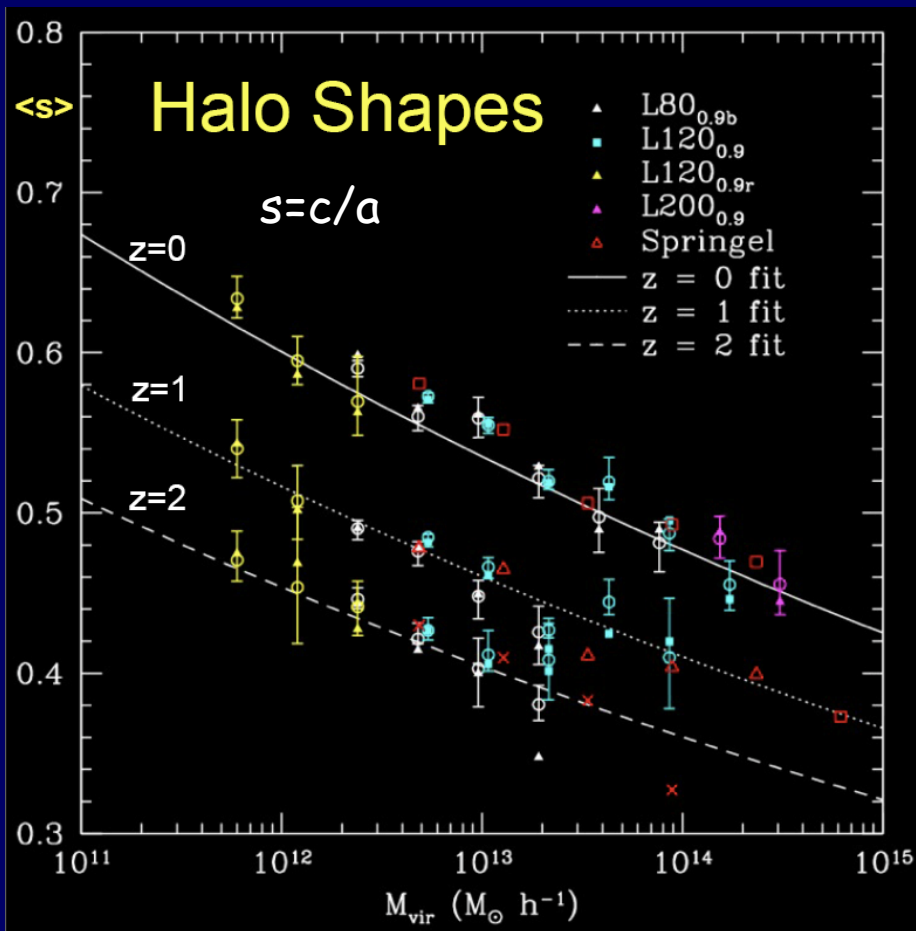
Halo Shape

Halo Shapes

Allgood et al 06

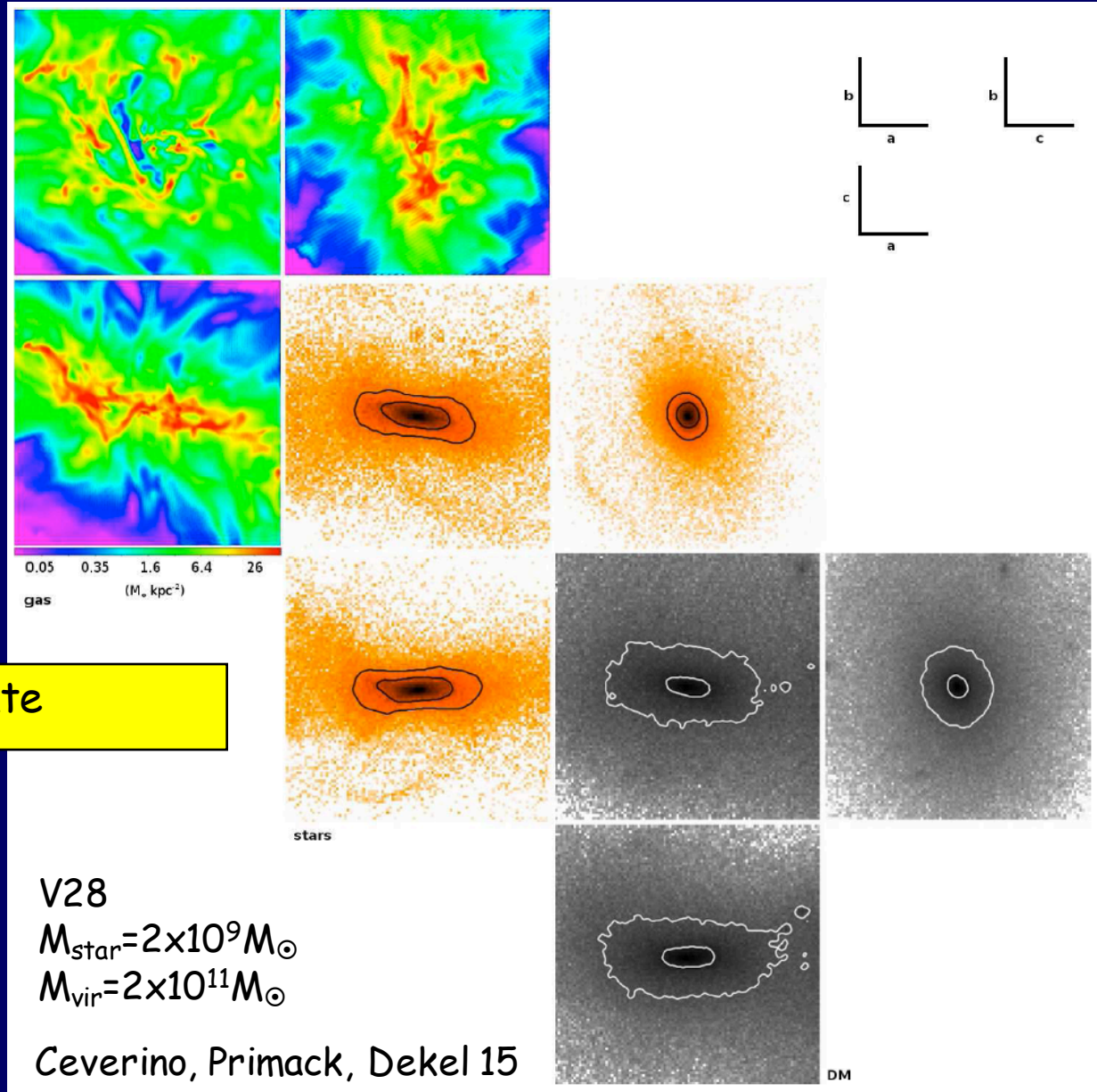
Halos are flatter at higher masses and higher redshifts

Halos are rounder at outer radii



A Prolate Low-Mass Galaxy at $z=2.2$

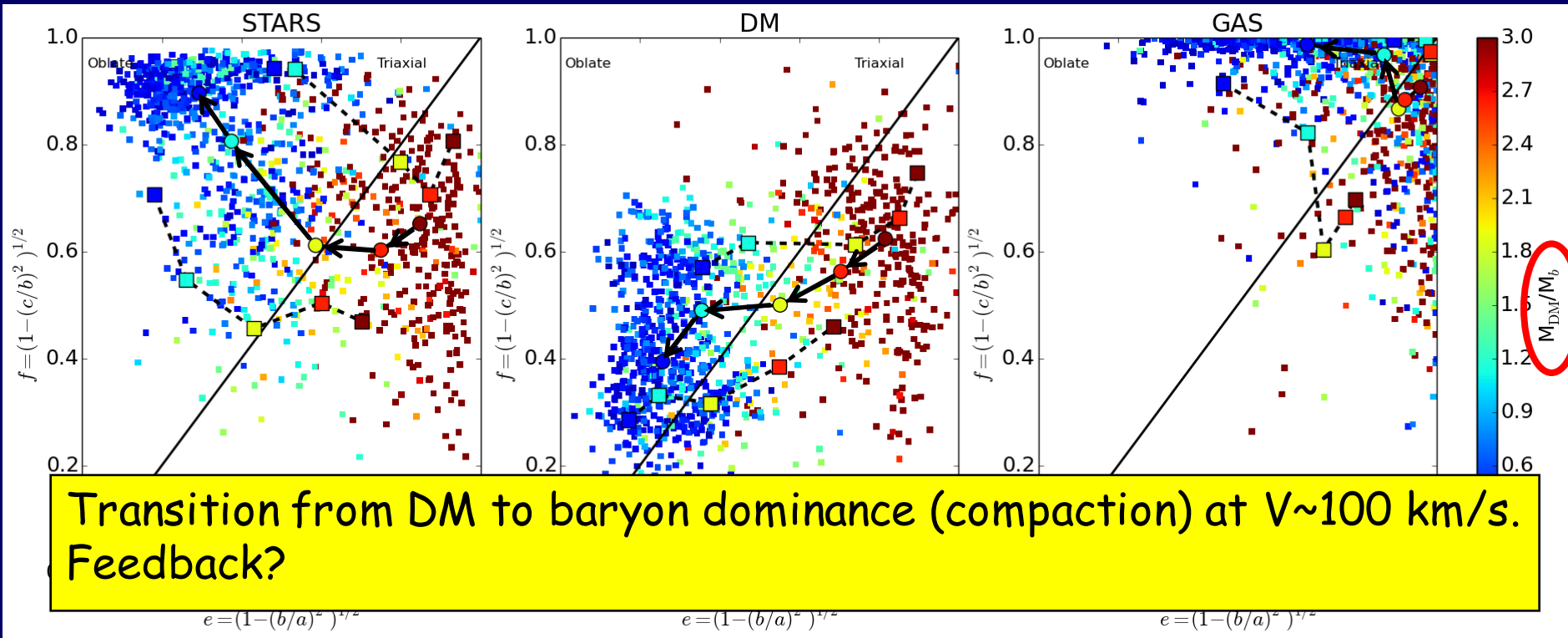
Gas: disk



Stars and DM: prolate

Consistent with
van der Wel+ 14
CANDELS

Evolution of Shape

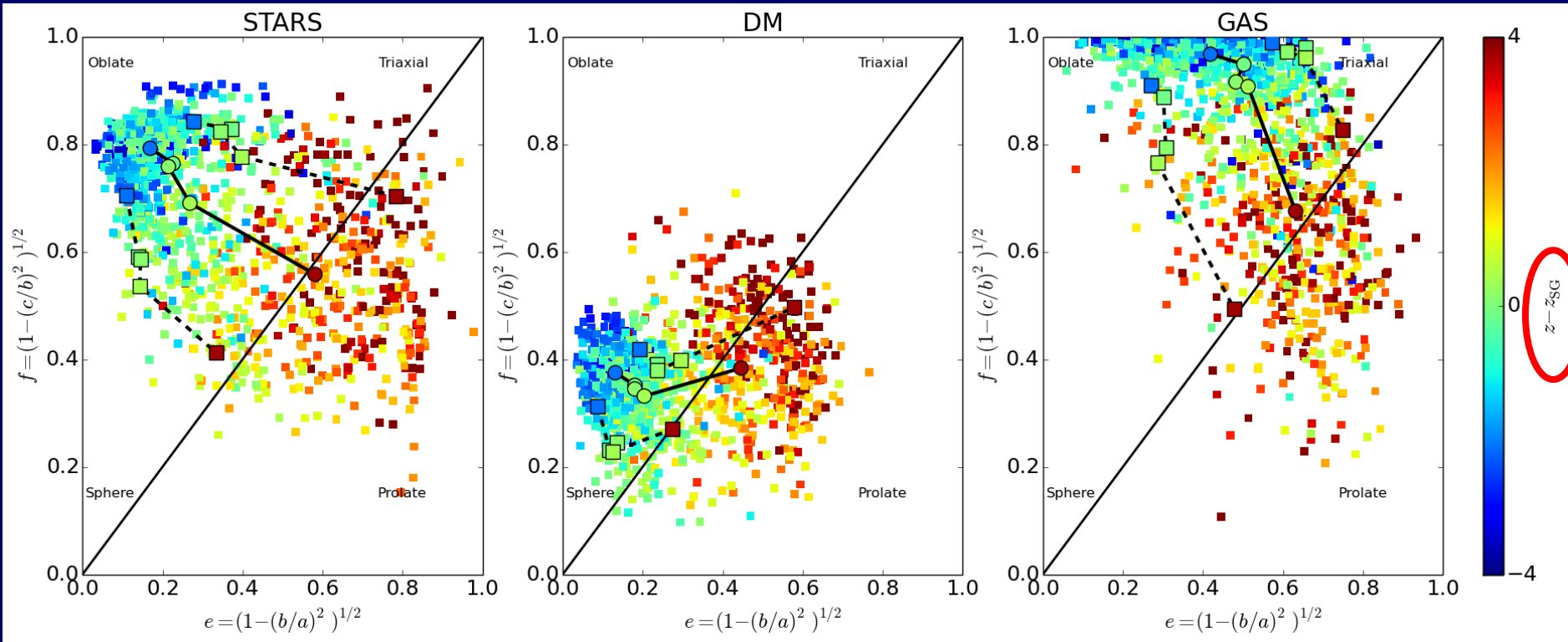


Pre-compaction: DM-dominated core, $M_* < 10^9 M_\odot$ $V < 100$ km/s \rightarrow outflows
 \rightarrow prolate (triaxial) DM & stellar system, anisotropic dispersion

Post-compaction: baryonic core, $M_* > 10^9 M_\odot$ $V > 100$ km/s - no outflow
 \rightarrow box orbits deflected \rightarrow oblate, rotation-dominated

Gas: triaxial \rightarrow disk

Evolution of Shape



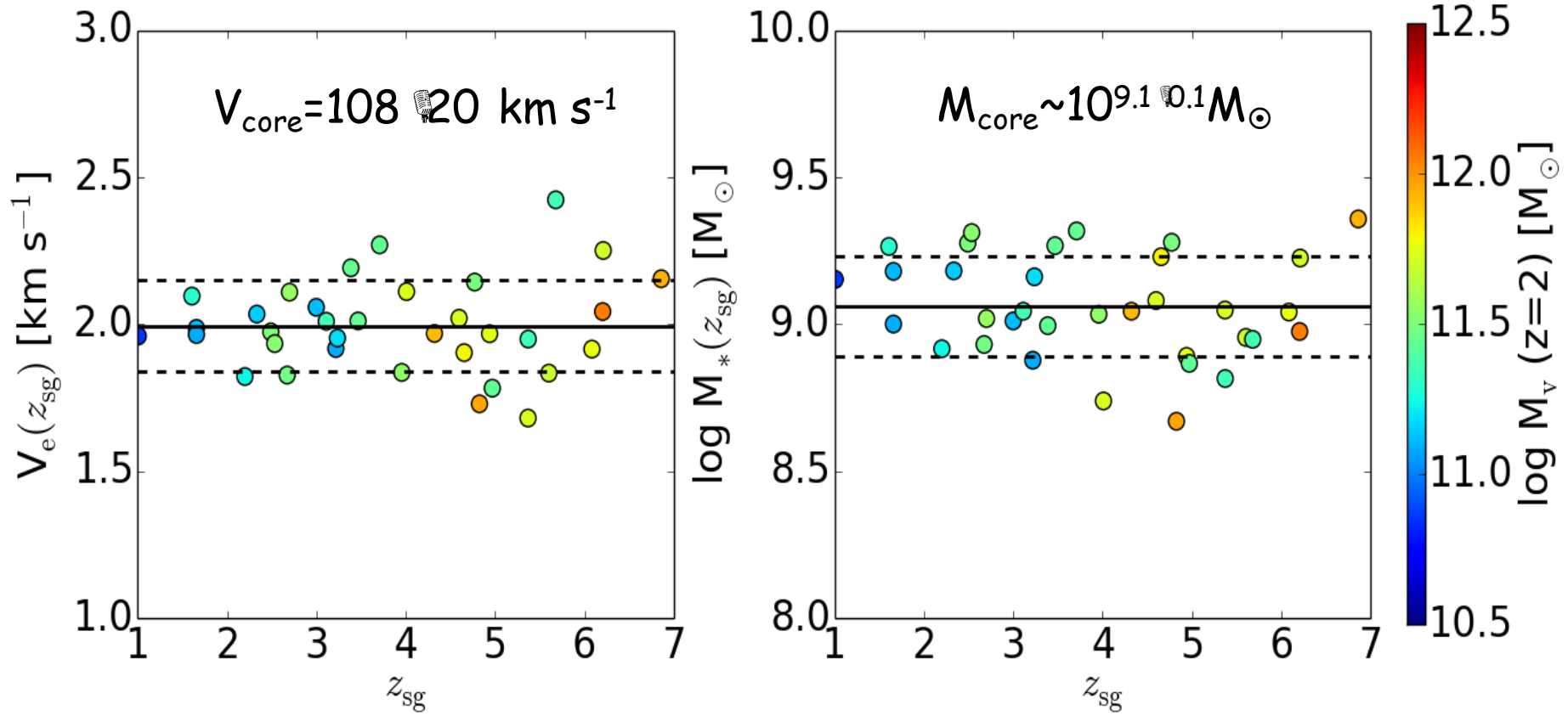
Stars and DM:

Pre-compaction: DM-dominated \leftrightarrow triaxial (prolate) \rightarrow more spherical

Post-compaction: stars self-gravitating \leftrightarrow oblate

Gas: triaxial \rightarrow oblate (disk)

Transition DM to Self-Gravity at a Critical M, V



A clue:
critical depth of potential well for SN-driven outflows (Dekel & Silk 86) ?

Compaction and Quenching by Elongated Halo

Halo core forms elongated (anisotropic velocities)
due to streaming within a dominant filament (including mergers)

Inflowing gas streams with AM form a disk ($V/\sigma \sim 3$)

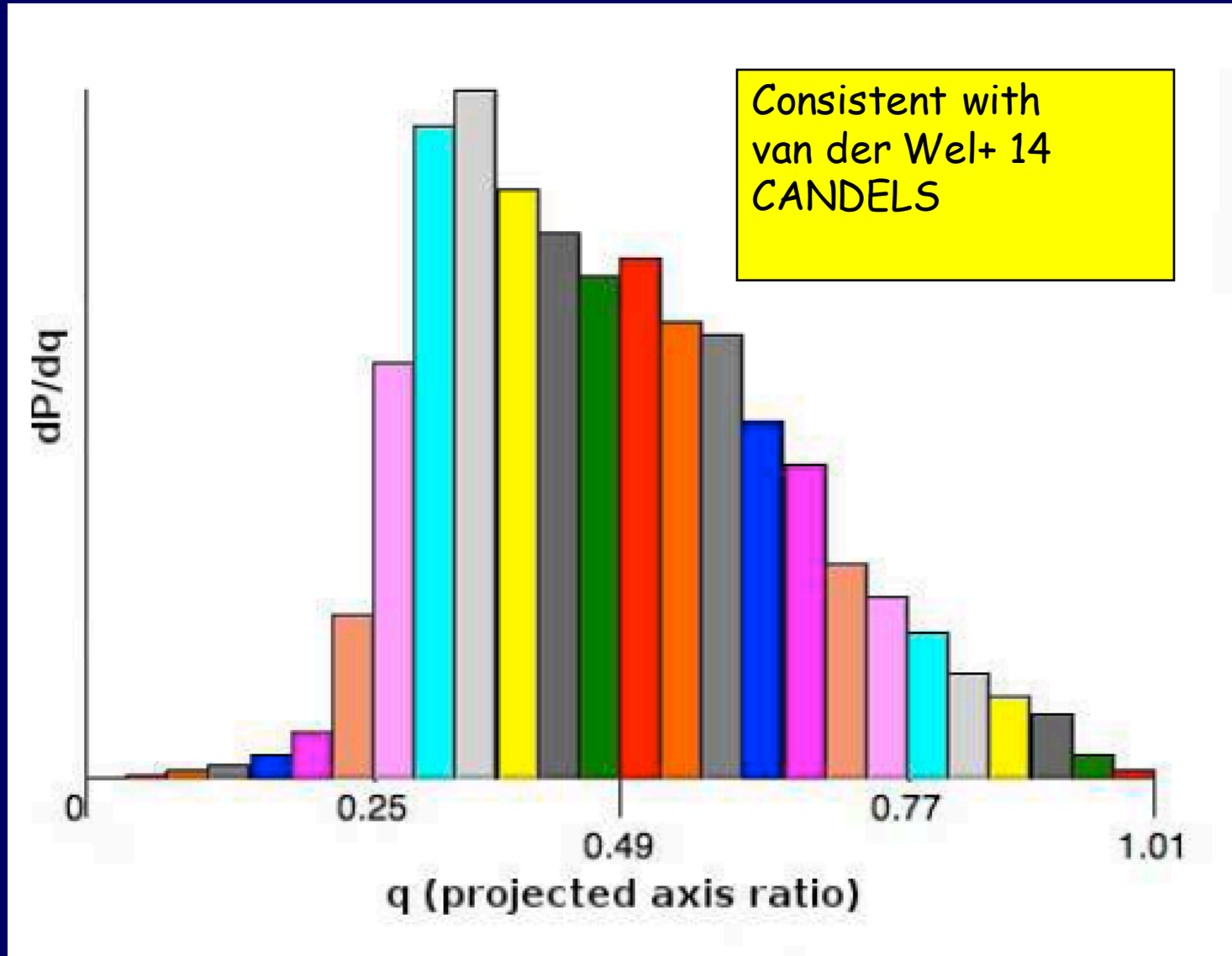
Local torques by the elongated halo cause AM loss and dissipation
→ gas inflow - compaction → high SFR, $V/\sigma \sim 1$

DM-dominated core: pre-compaction, $V < 100 \text{ km s}^{-1}$ → outflows
→ elongated stellar system following the elongated halo (tidal torques)

Self-gravitating stellar core: post-compaction, $V > 100 \text{ km s}^{-1}$, no outflow
→ halo and galaxy get rounder (by deflection)
→ stellar system becomes oblate, following the gas, reflecting rotation

→ no torques → no gas inflow → gas depletion and central quenching

Distribution of Projected Axial Ratio



Distribution of Projected Axial Ratio

