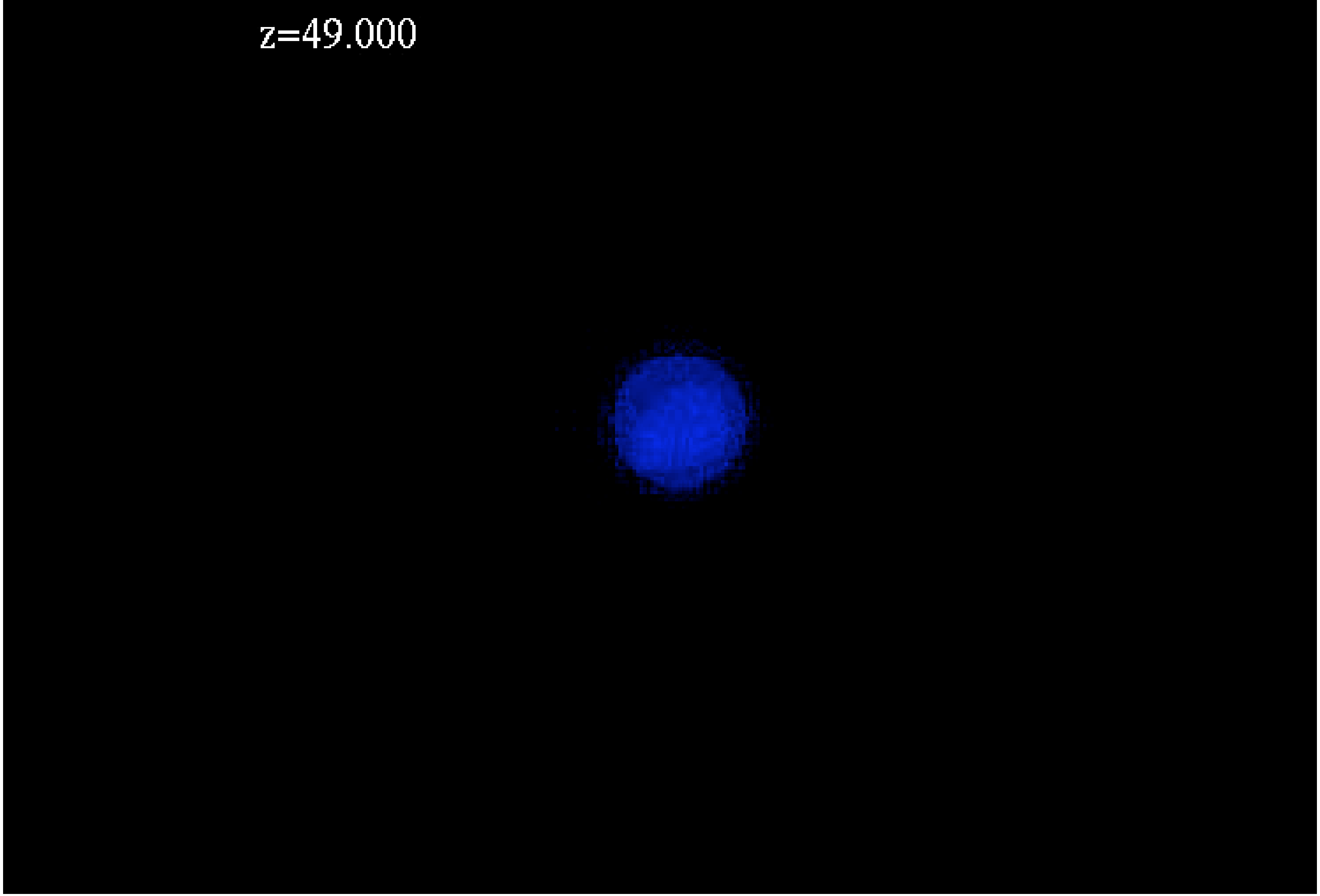
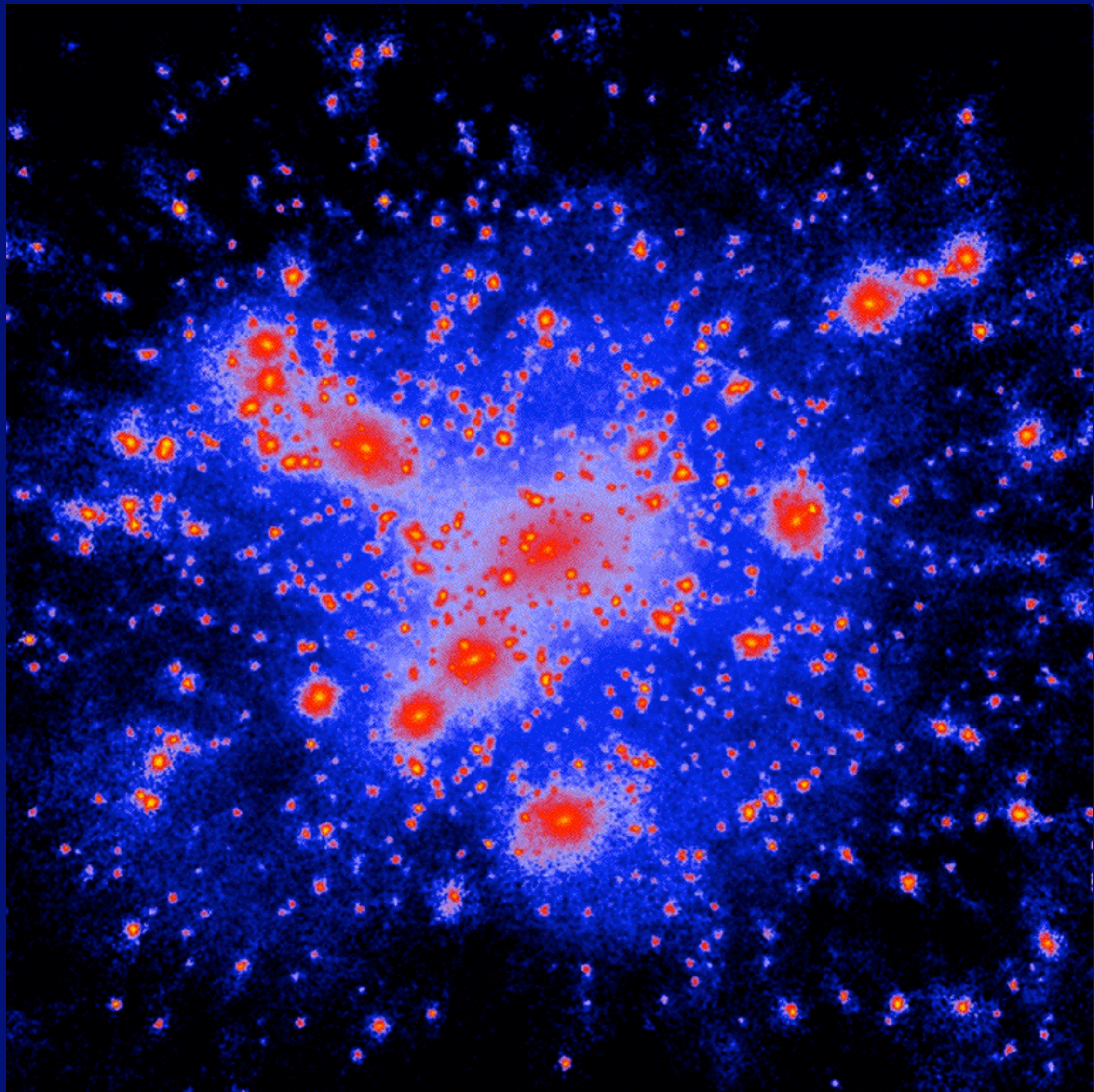
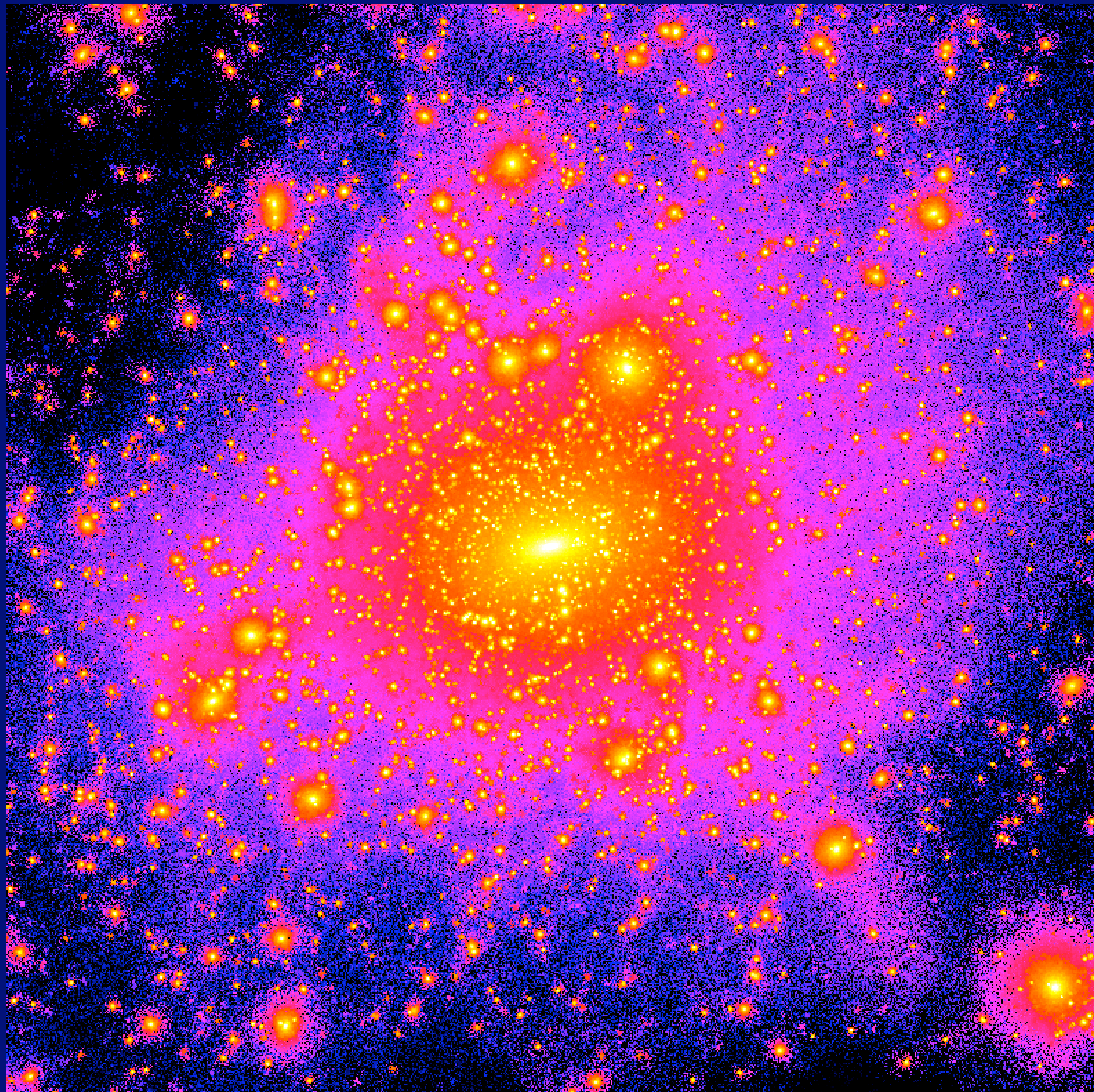
Lecture

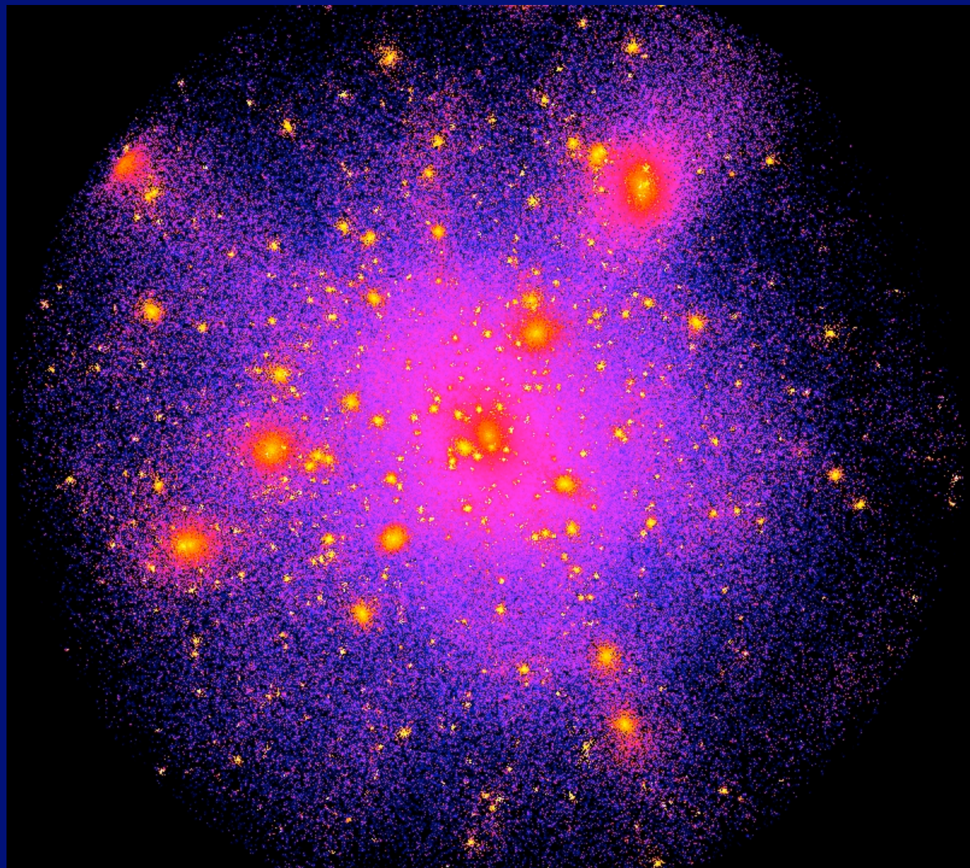
Sub-structure of Dark-Matter Halos

N-body simulation of Halo Formation



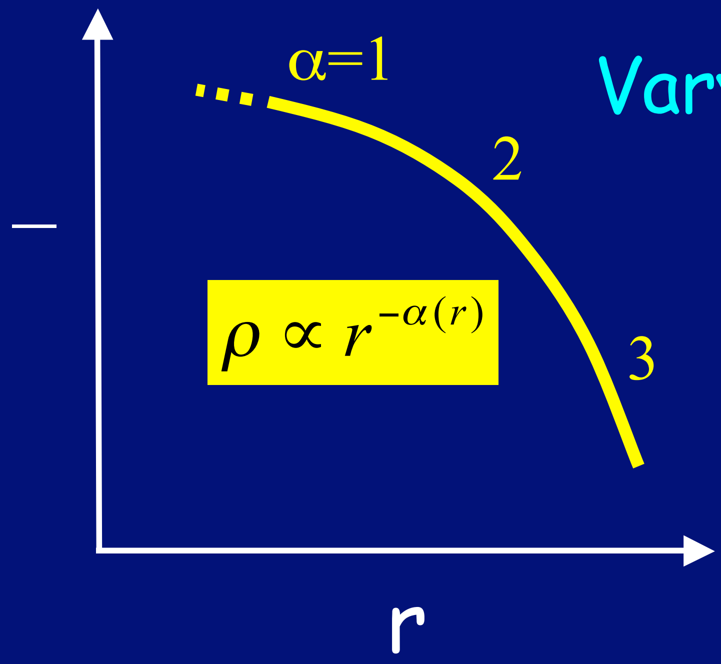






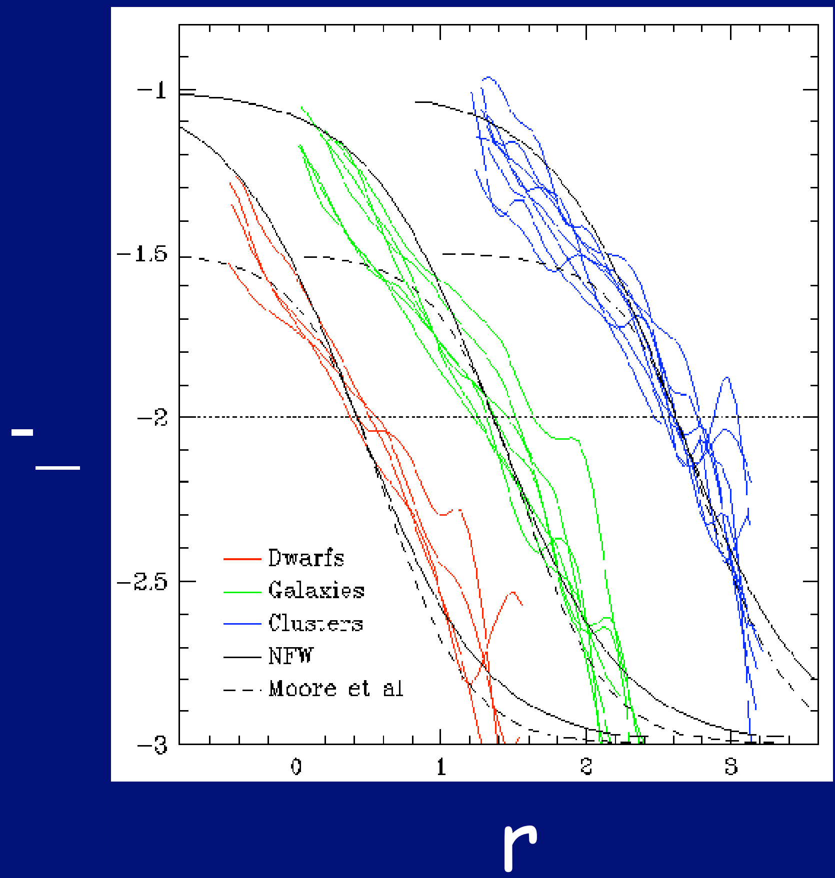
Stoehr

Origin of Halo Density Profile?



Varying slope

$$\alpha(r) \equiv -\frac{d \ln \rho(r)}{d \ln r}$$



Hayashi, Navarro et al.

Navarro & Taylor:

$$f_{NT} = \frac{\rho(r)}{\sigma_v^{\ 3}(r)} \propto r^{-1.875}$$

Phase-Space Density

$$f(\vec{x},\vec{v})$$

$$\rho(\vec{x}) = \int d\vec{v}\ f(\vec{x},\vec{v})$$

Vlasov eq.

$$\partial_t f + \vec{v}\cdot\vec{\nabla}_x f - \vec{\nabla}_x\phi\cdot\nabla_v f = 0$$

Poisson eq.

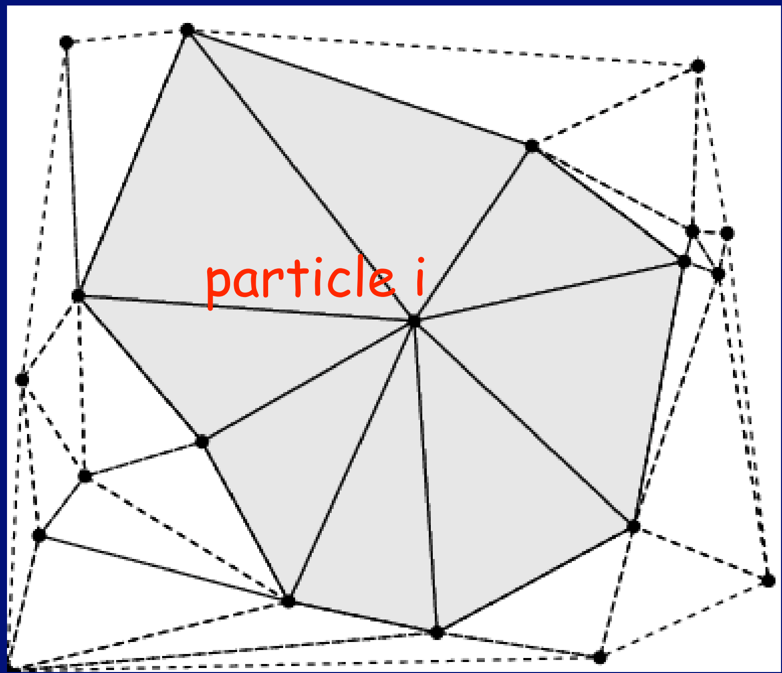
$$\phi(\vec{x}) = -G\int d\vec{x}'\,d\vec{v}\,\frac{f(\vec{x}',\vec{v})}{|\vec{x}-\vec{x}'|}$$

Distribution function of f:

$$V(f = f_0) \equiv \int d\vec{x}d\vec{v}\ \delta_{Dirac}[f(\vec{x},\vec{v},t) - f_0]$$

V(f)df = volume of phase space occupied by f in the range (f,f+df)

Measuring f(x,v) using an adaptive "grid"
Delaunay Tesselation

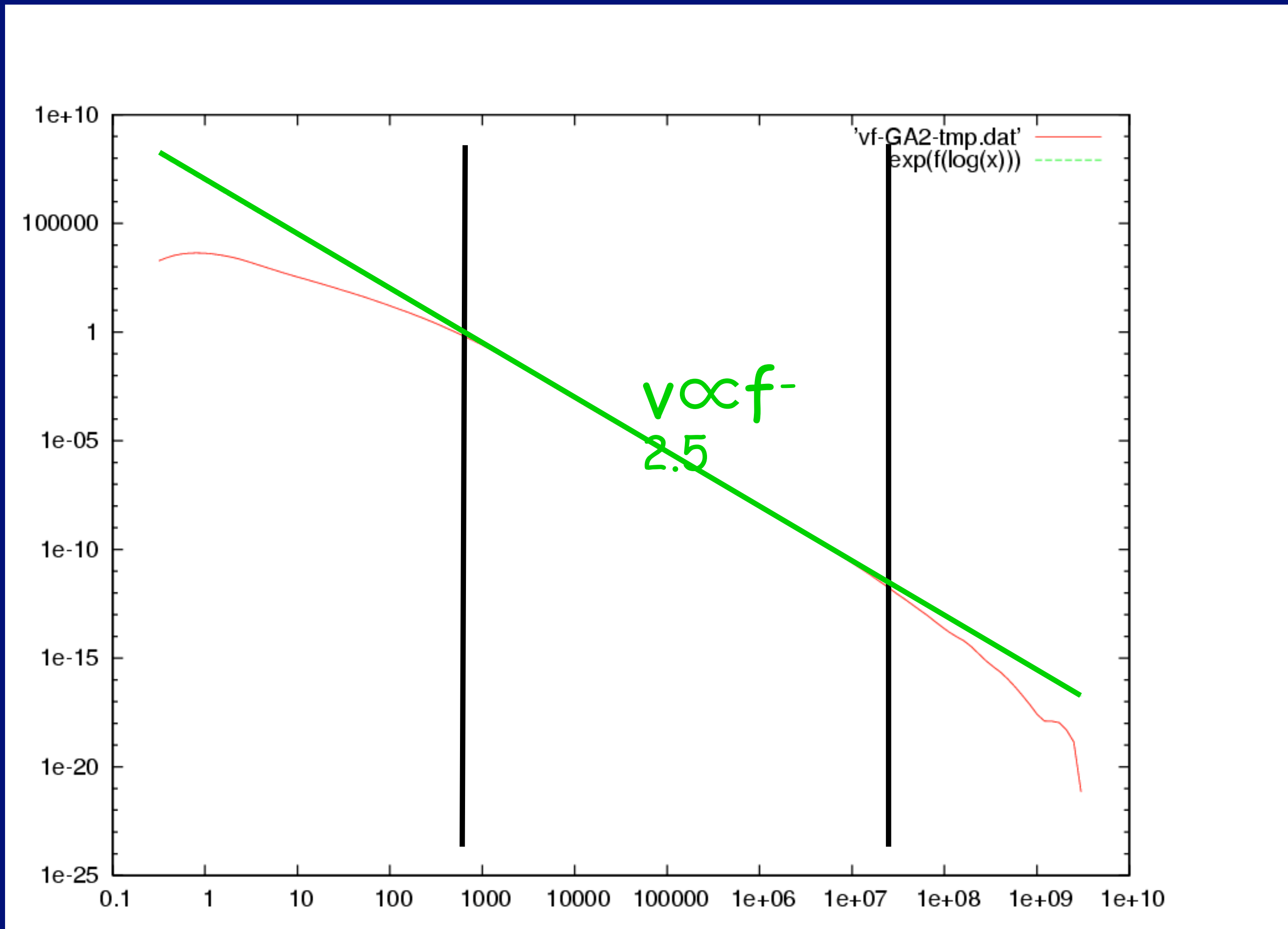


$$f_i = (d+1)\frac{m}{V_i}$$

Arad, Dekel & Klypin

PDF of Phase-Space Density

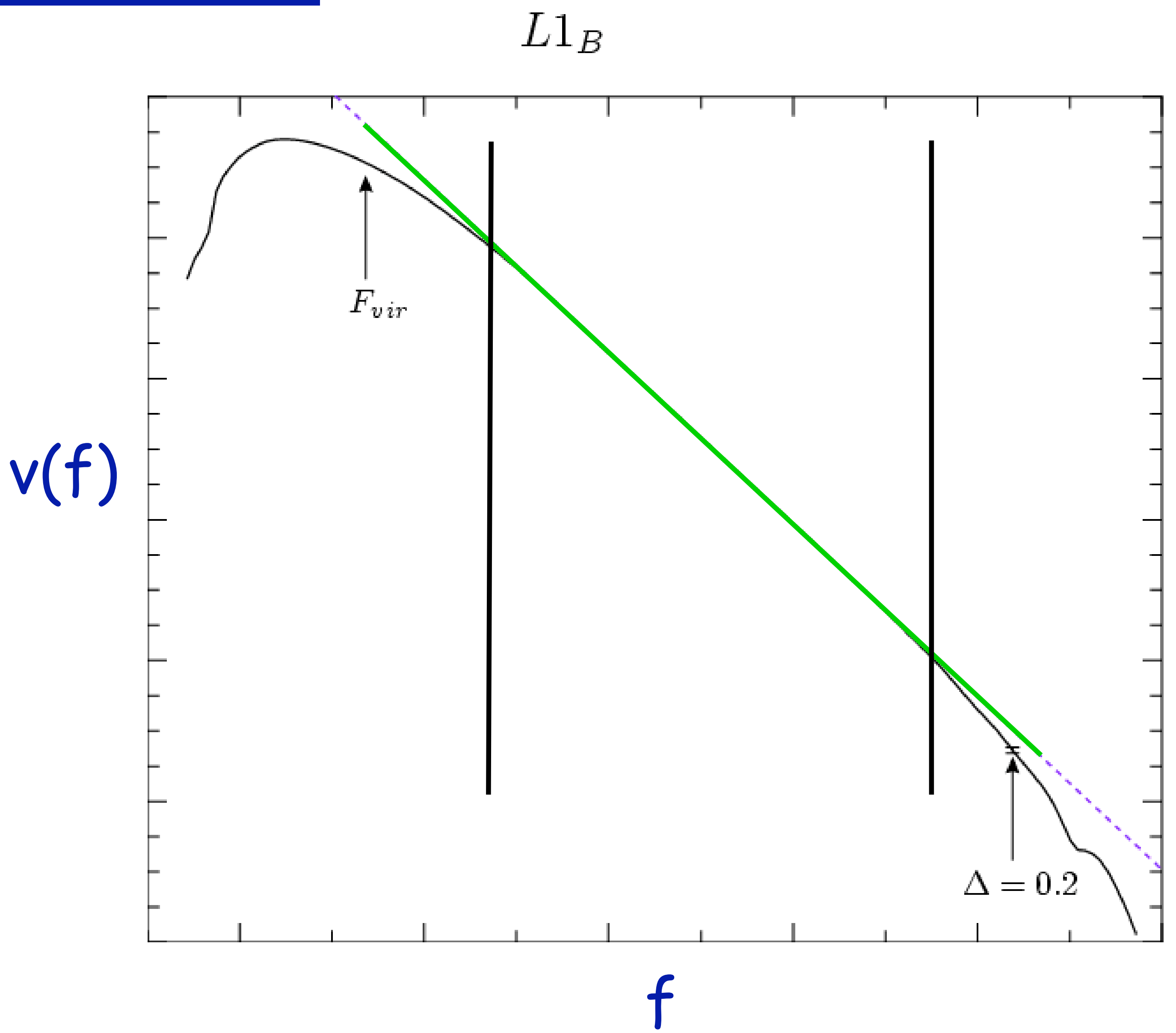
V(f)



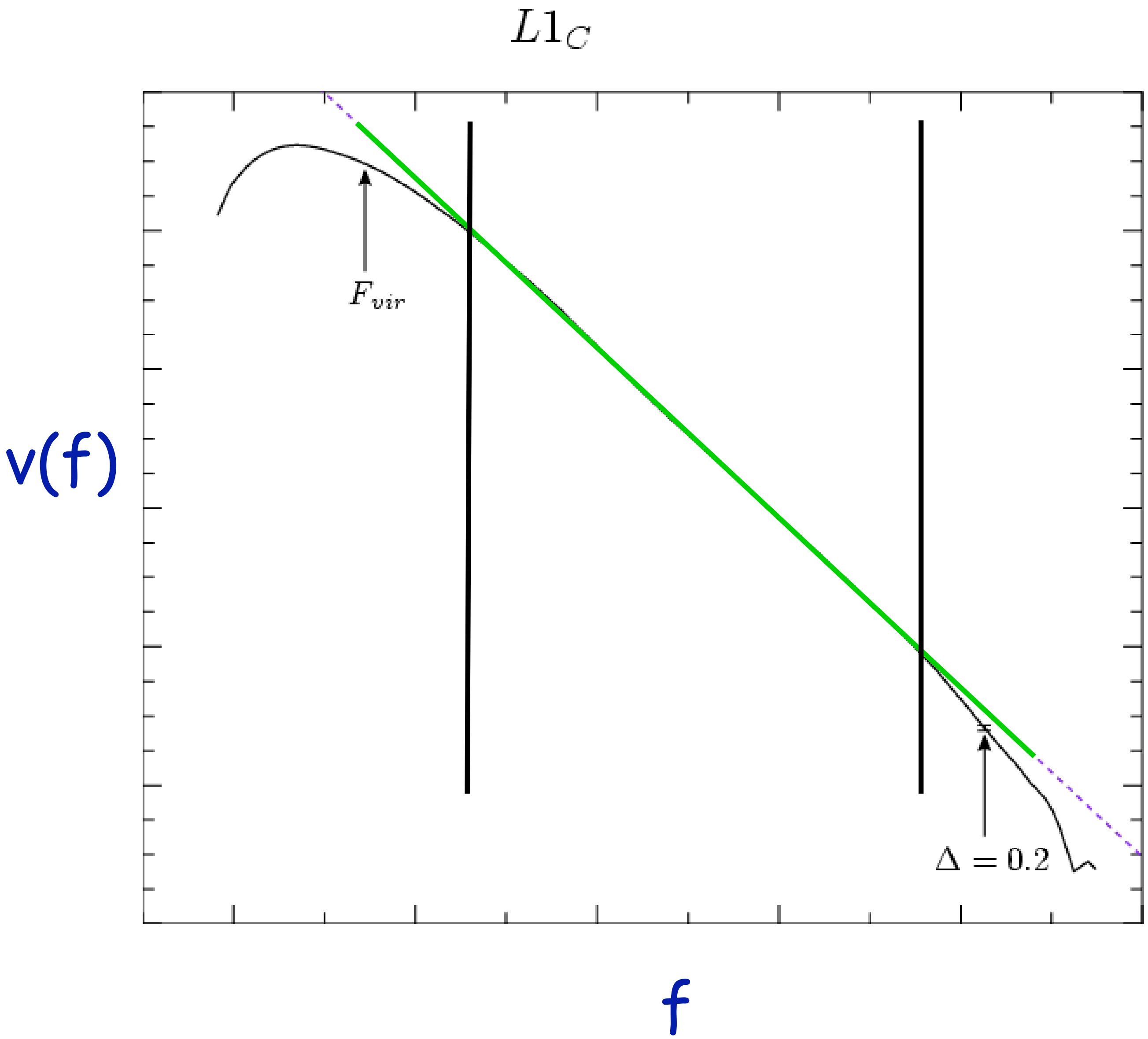
Arad, Dekel & Klypin

f

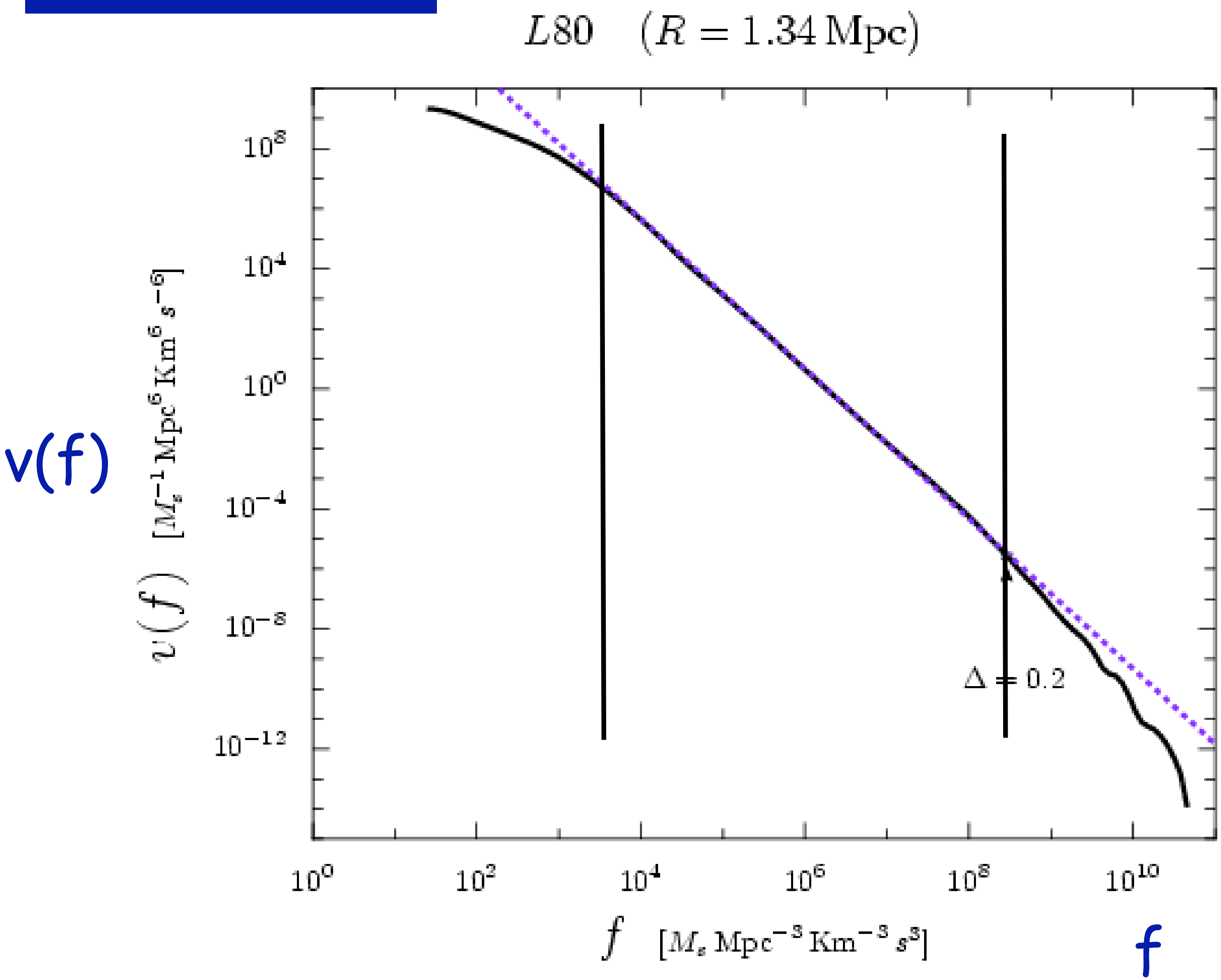
Dwarf



Dwarf

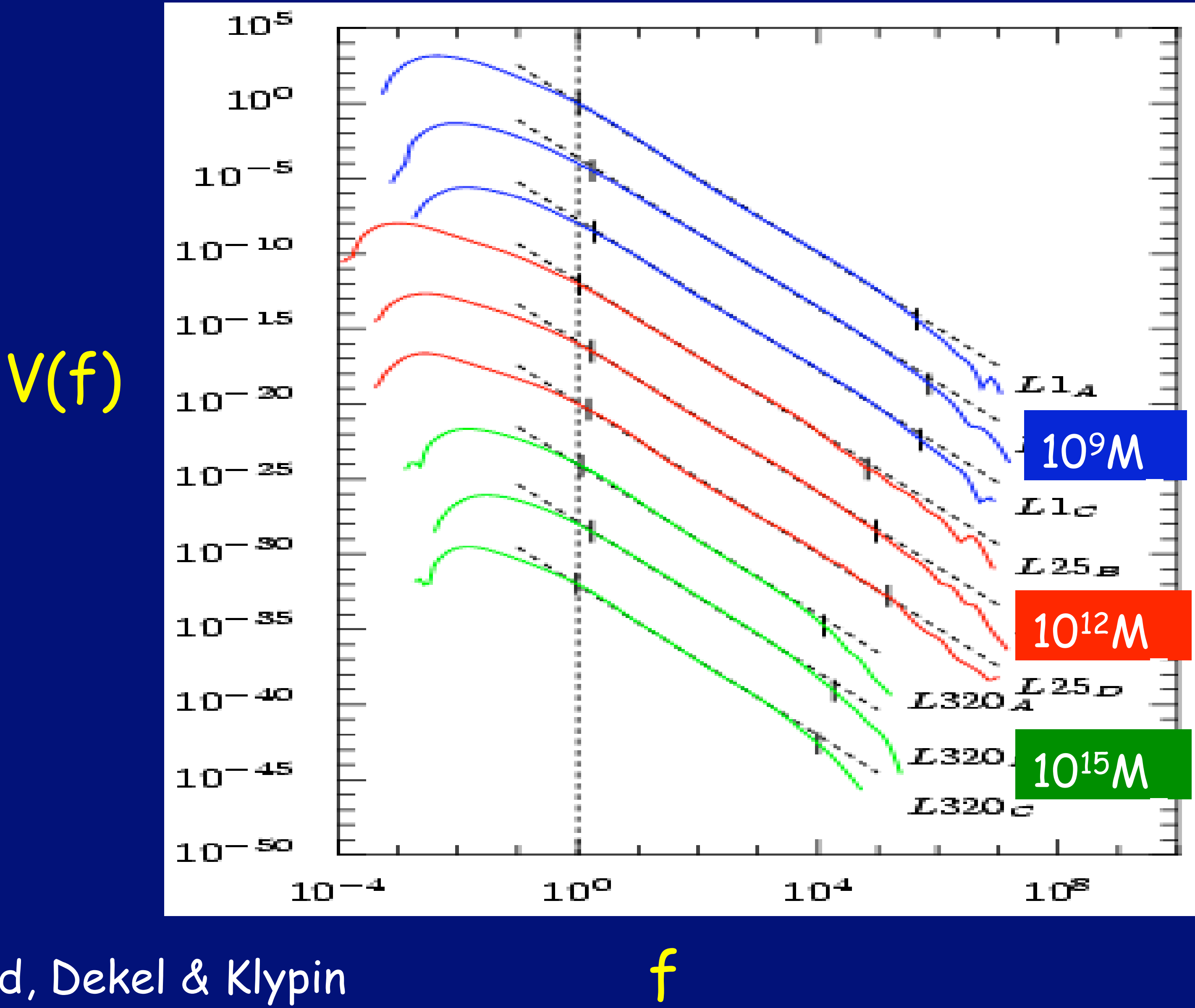


Cluster



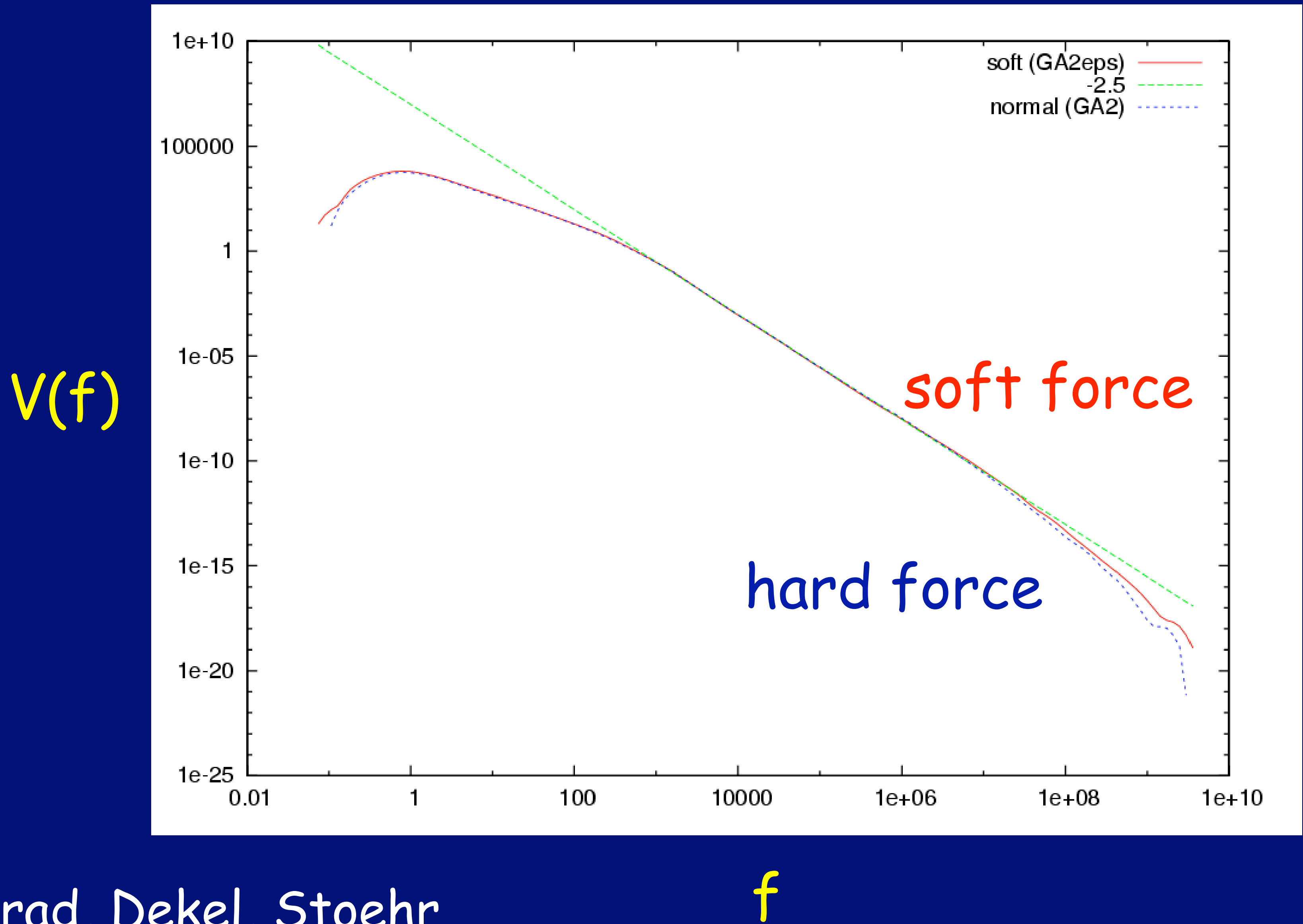
L80 (R = 1.34 Mpc)
v(f)
$v(f)$ $[M_s^{-1}\,\mathrm{Mpc}^6\,\mathrm{Km}^6\,s^{-6}]$
10^8
10^4
10^0
10^{-4}
10^{-8}
10^{-12}
$\Delta = 0.2$
10^0
10^2
10^4
10^6
10^8
10^{10}
f $[M_s\,\mathrm{Mpc}^{-3}\,\mathrm{Km}^{-3}\,s^3]$
f

PDF of Phase-Space Density



Arad, Dekel & Klypin

Not 2-body relaxation



Arad, Dekel, Stoehr

V(f) related to _(r)?

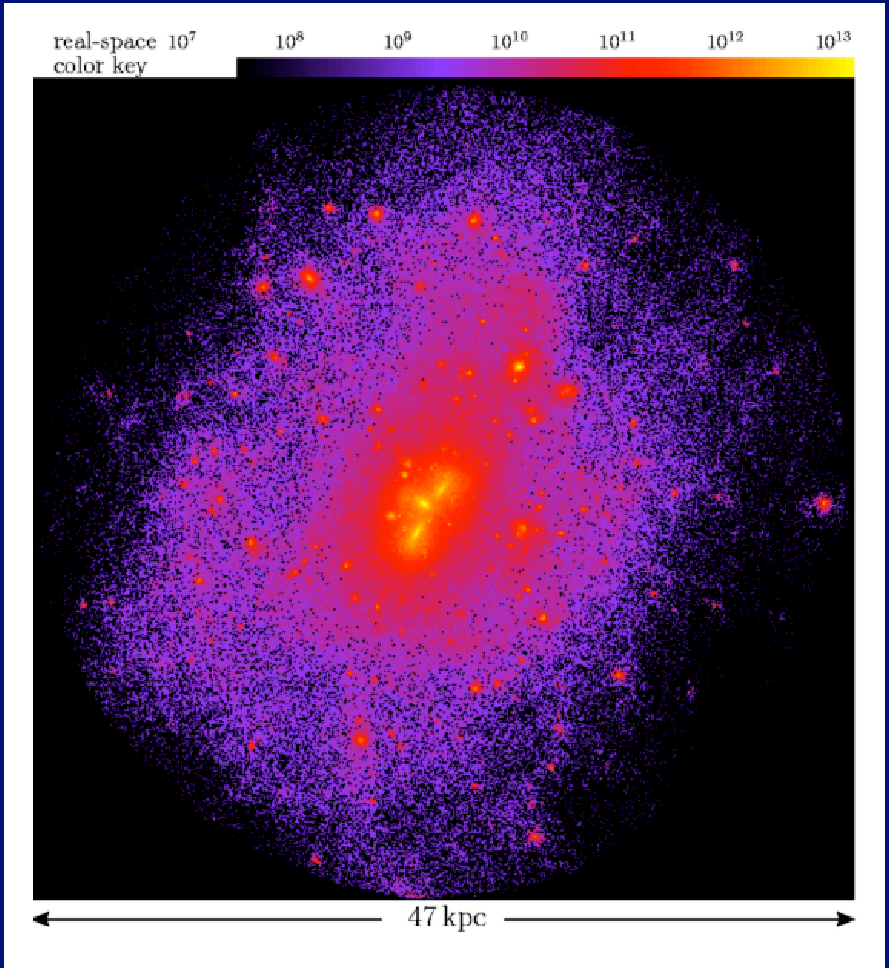
if $f(\vec{x},\vec{v}) = f(E)$ (crossed out) e.g., spherical & isotropic

$$\rho(r) \propto r^{-\alpha}, \quad V(f) \propto f^{-\beta}, \quad \beta = \frac{18-4\alpha}{6-\alpha}$$

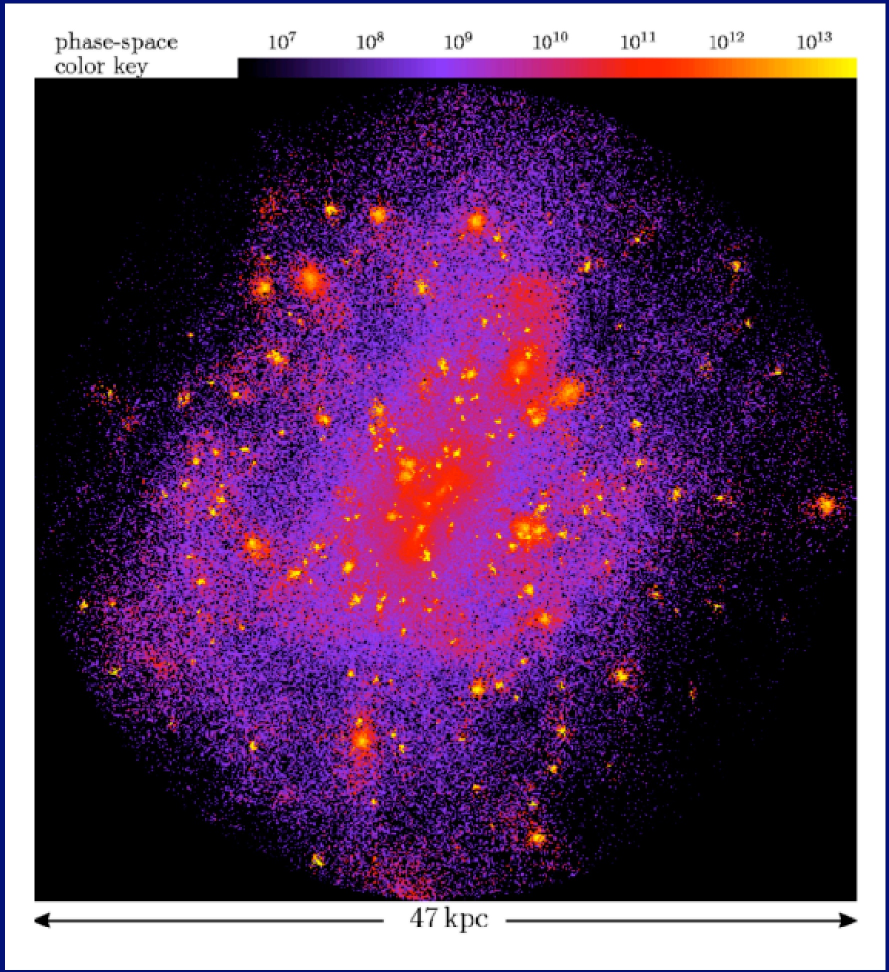
$$\alpha = 3 \Leftrightarrow \beta = 2$$
$$\alpha = 2 \Leftrightarrow \beta = 2.5$$
$$\alpha = 1 \Leftrightarrow \beta = 2.8$$
$$\alpha = 0 \Leftrightarrow \beta = 3$$

Halo Phase-Space Density

Real Density

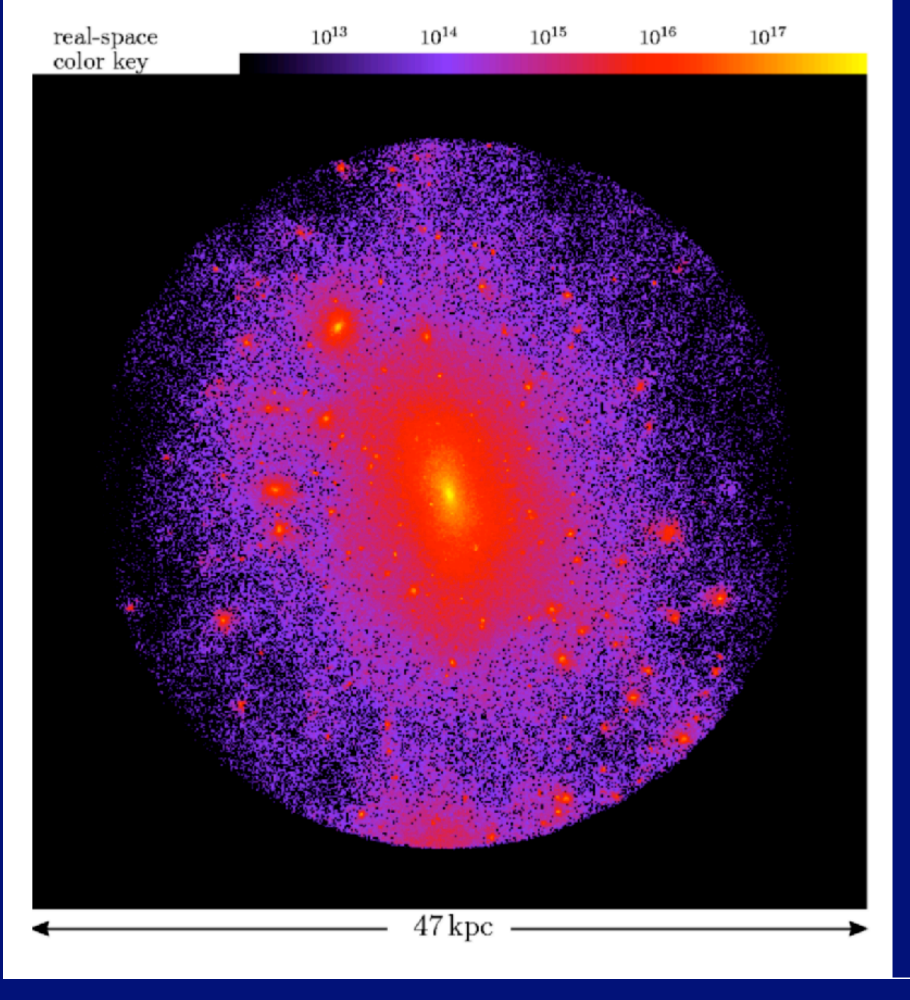


Phase-Space Density

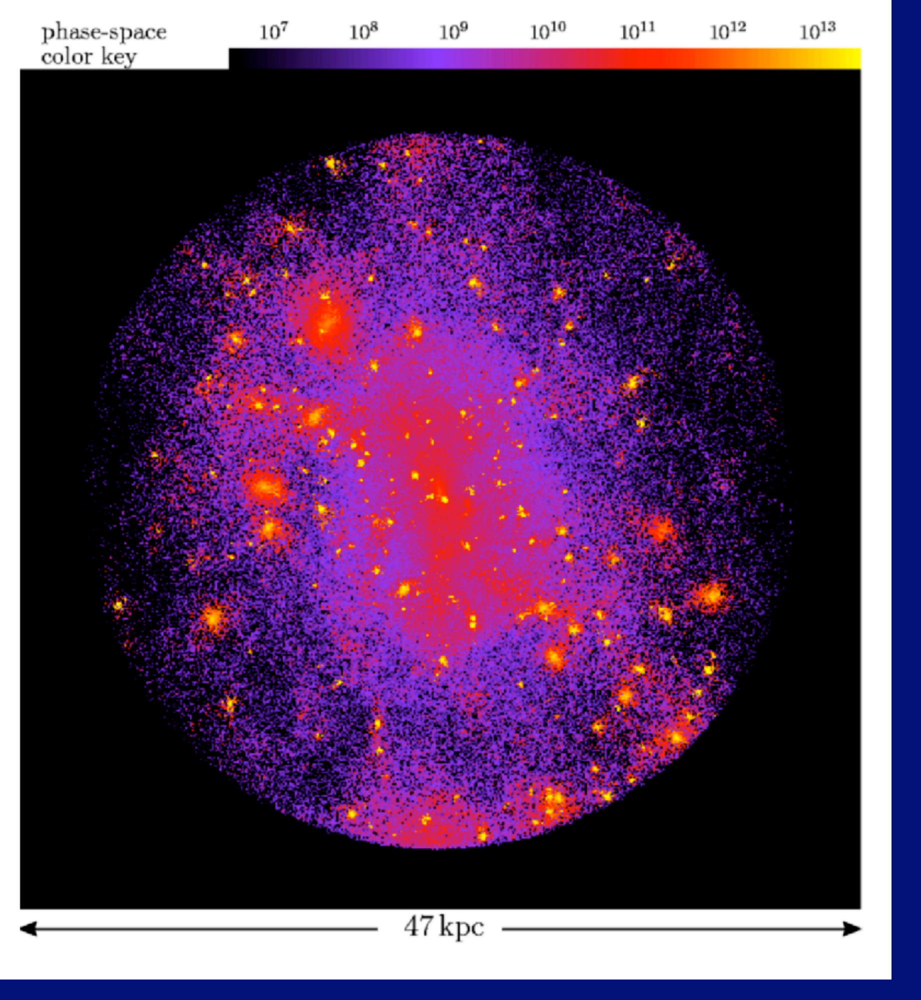


Halo Phase-Space Density

Real Density



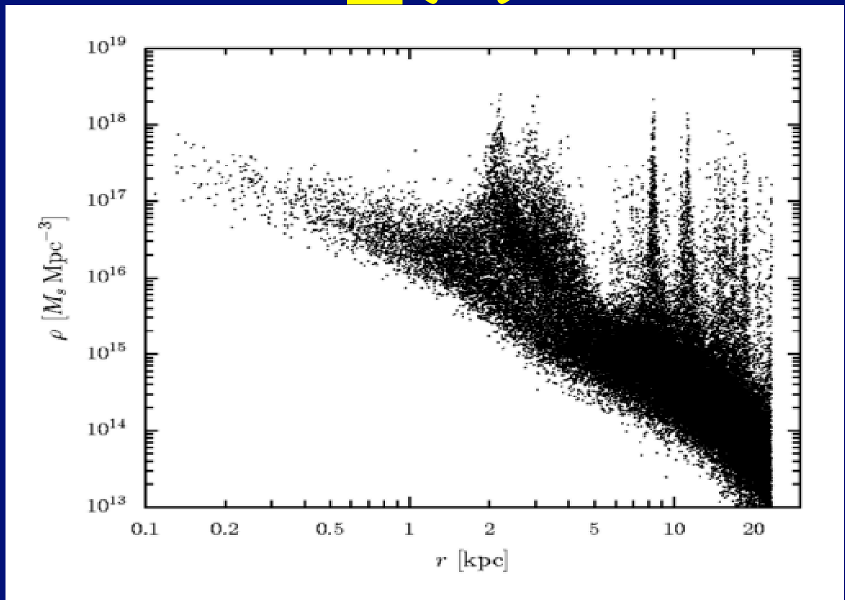
Phase-Space Density

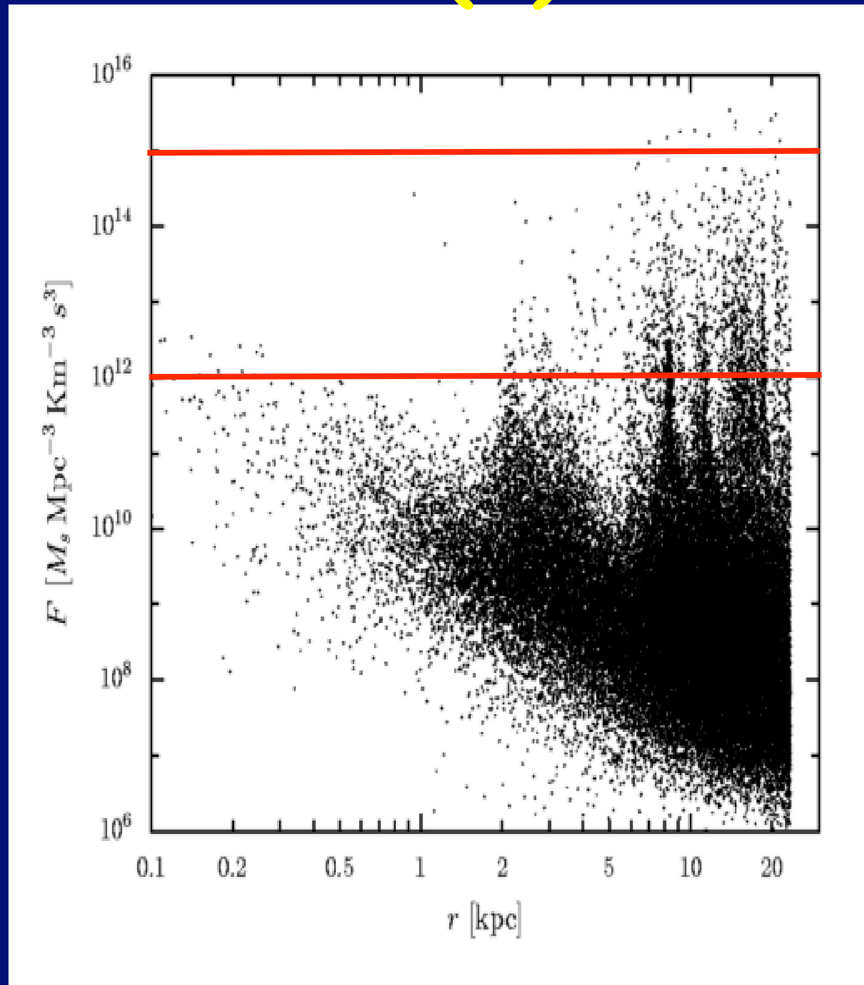


Profiles in Real Space and Phase Space

f(r)

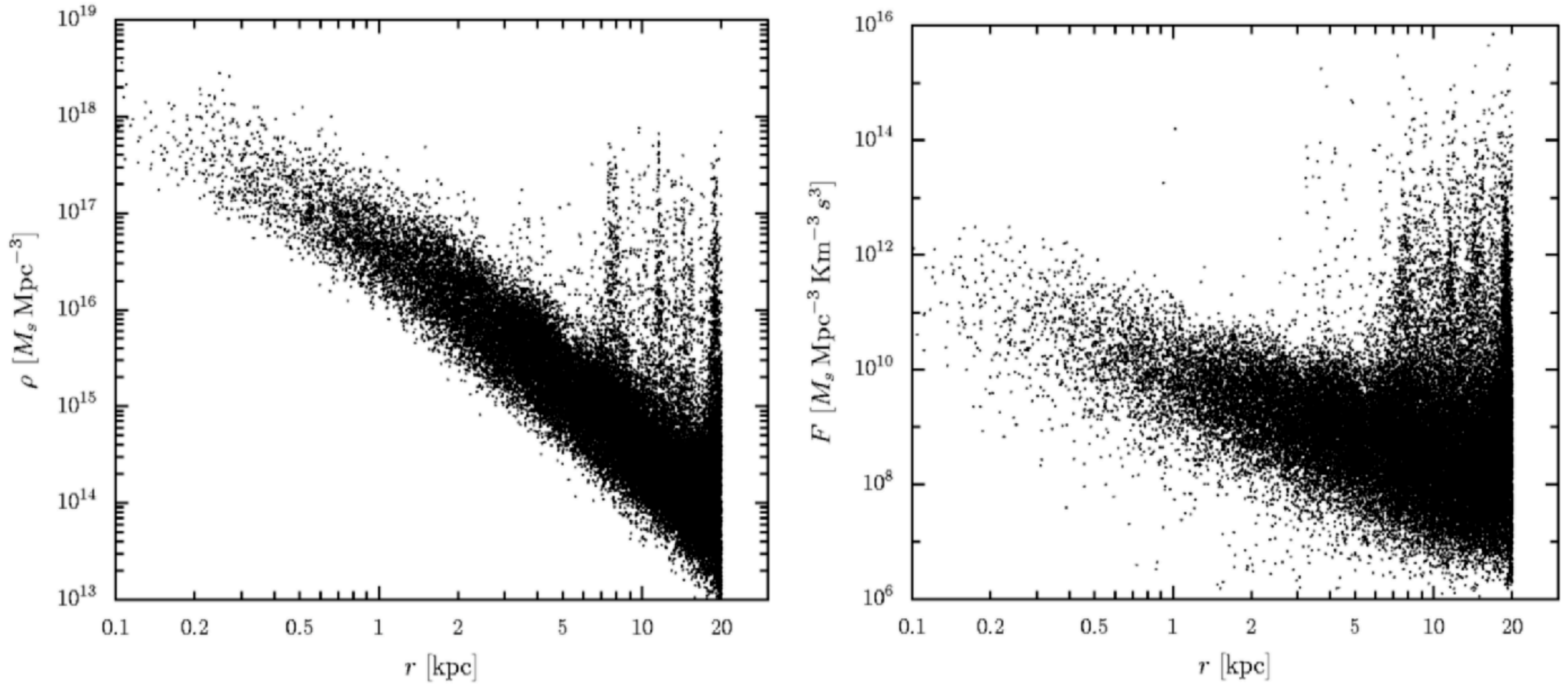
_(r)



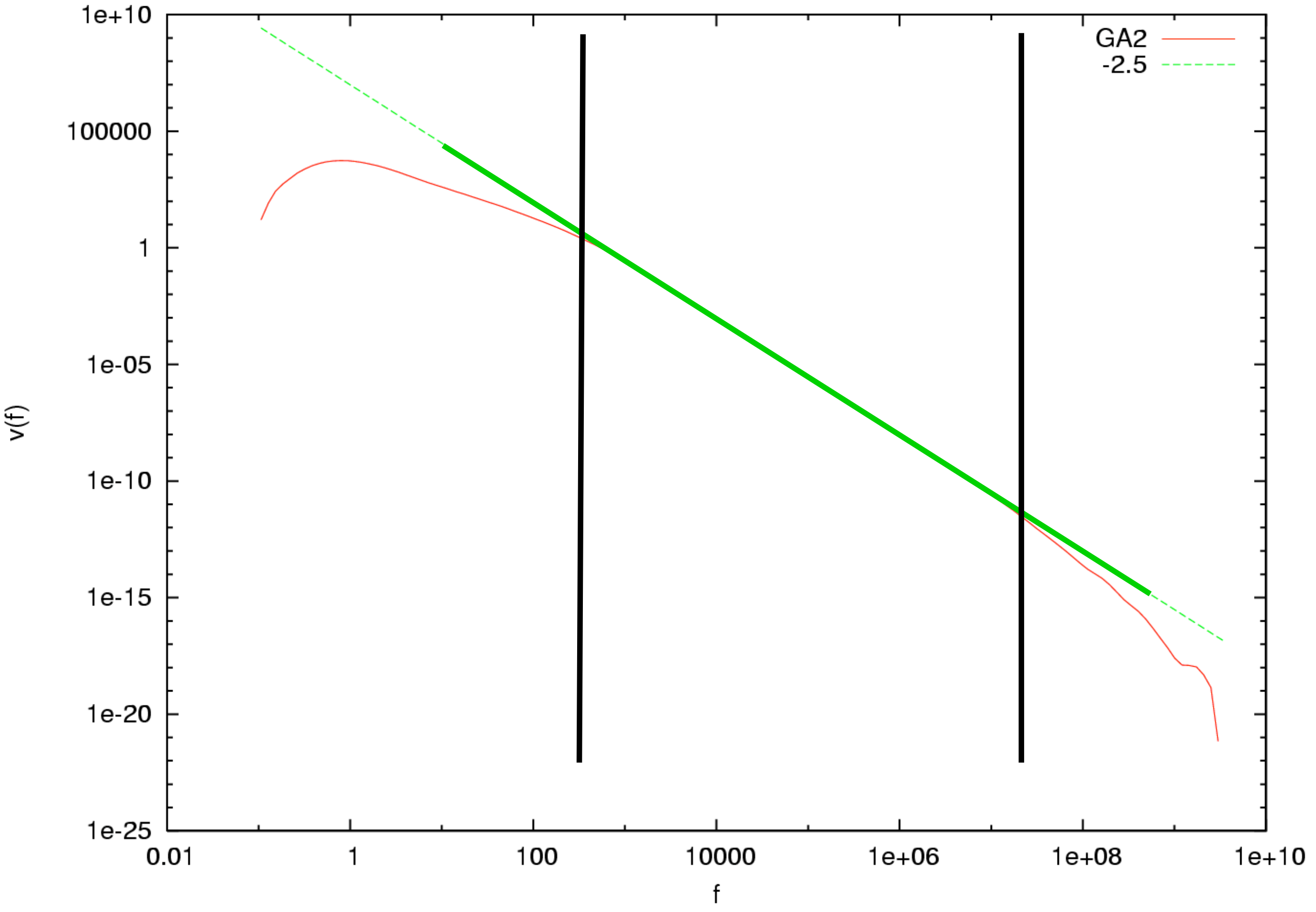


radius

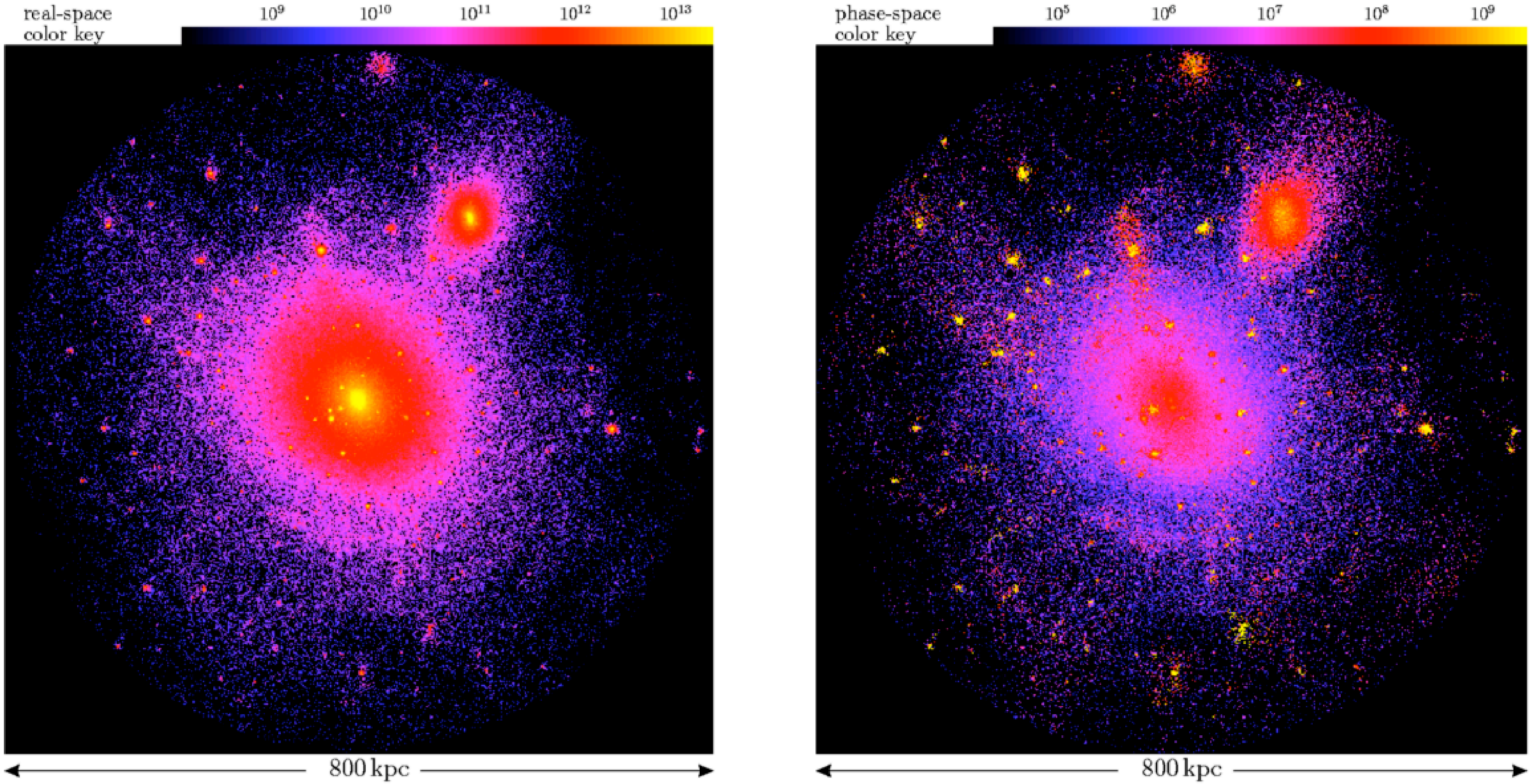
radius

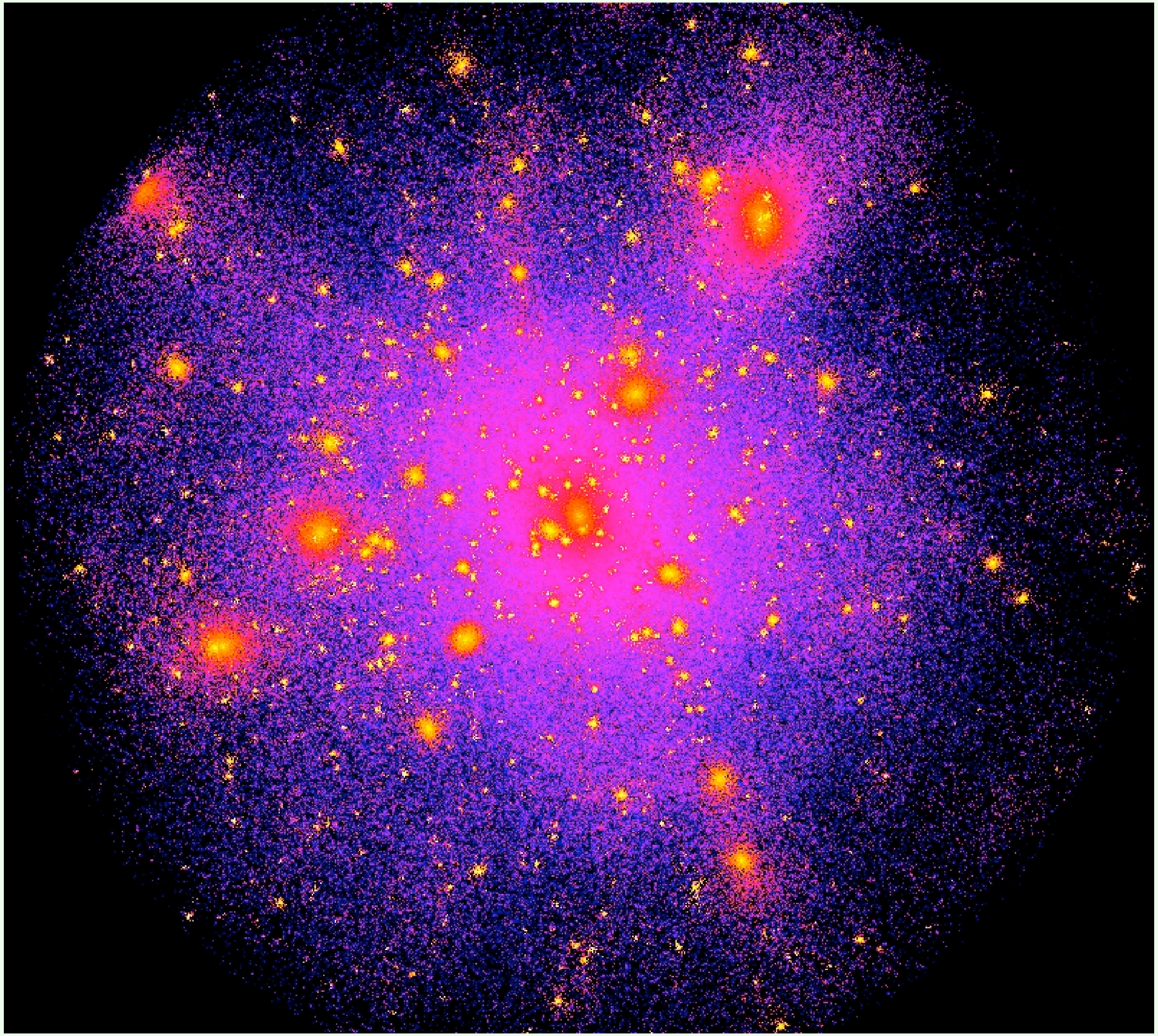


$\rho\ [M_s\,\mathrm{Mpc}^{-3}]$
r [kpc]
$F\ [M_s\,\mathrm{Mpc}^{-3}\,\mathrm{Km}^{-3}\,s^3]$
r [kpc]

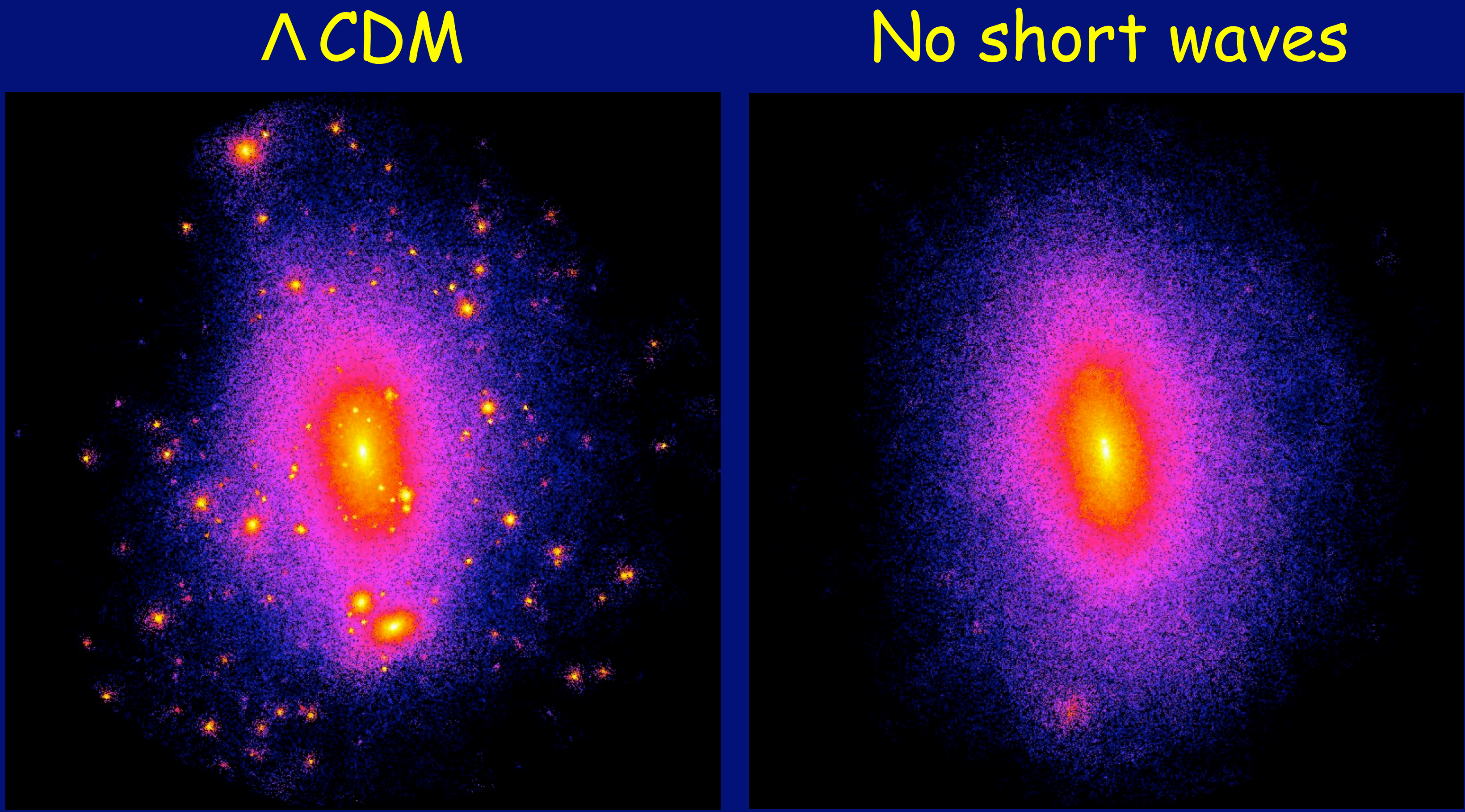


1e+10
100000
1
1e-05
1e-10
1e-15
1e-20
1e-25
v(f)
0.01
1
100
10000
1e+06
1e+08
1e+10
f
GA2
-2.5


real-space
color key
10^9
10^{10}
10^{11}
10^{12}
10^{13}
800 kpc
phase-space
color key
10^5
10^6
10^7
10^8
10^9
800 kpc



Is $v(f) \propto f^{-2.5}$ determined by substructure?

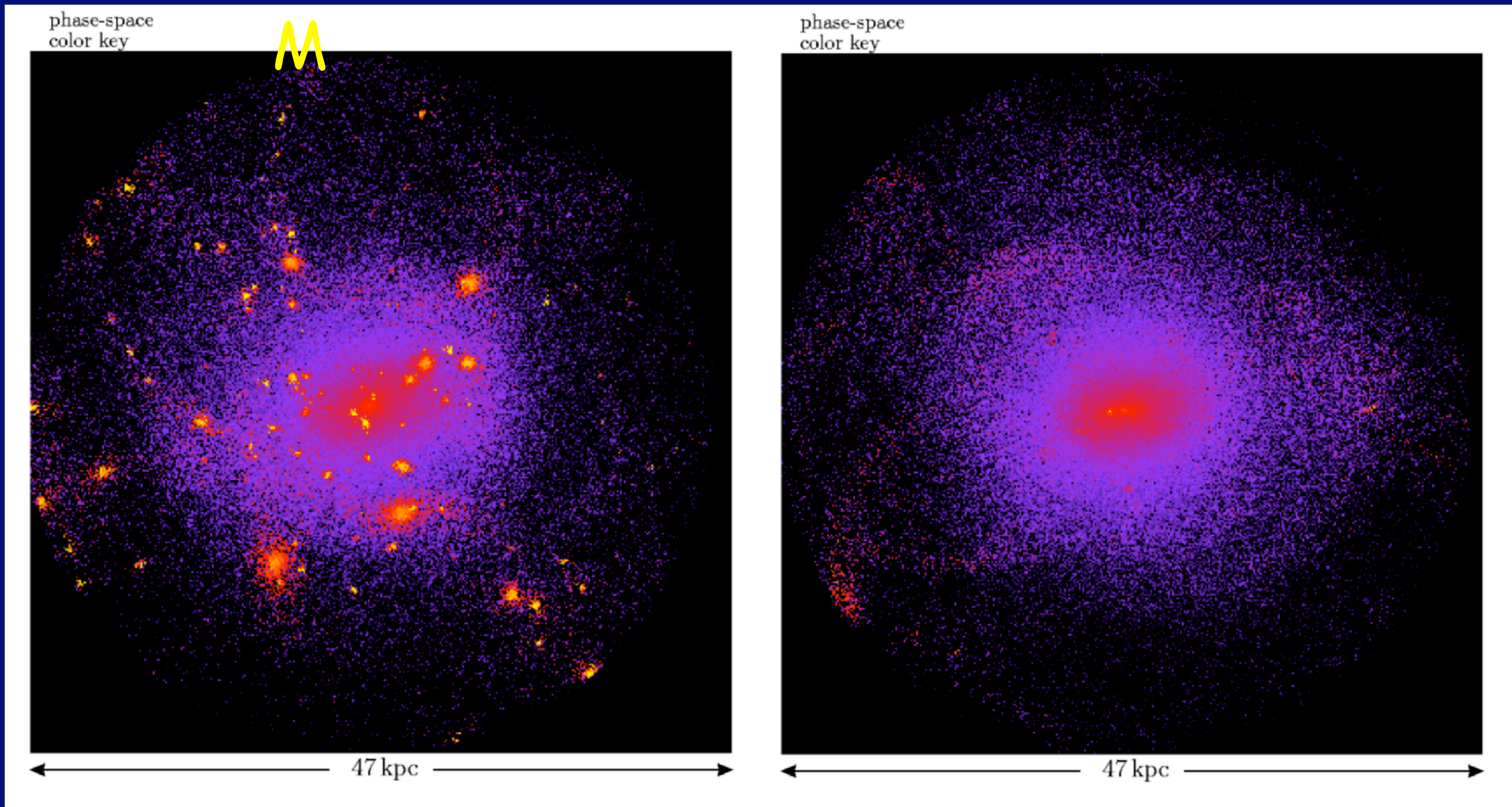


Real-Space Density

Moore et al.

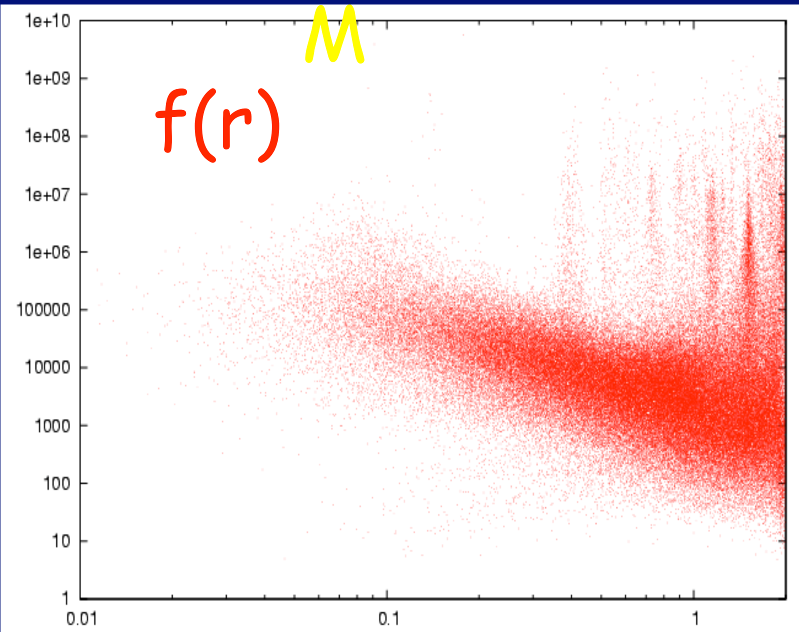
Phase-Space density

ΛCDM　　　　No short waves



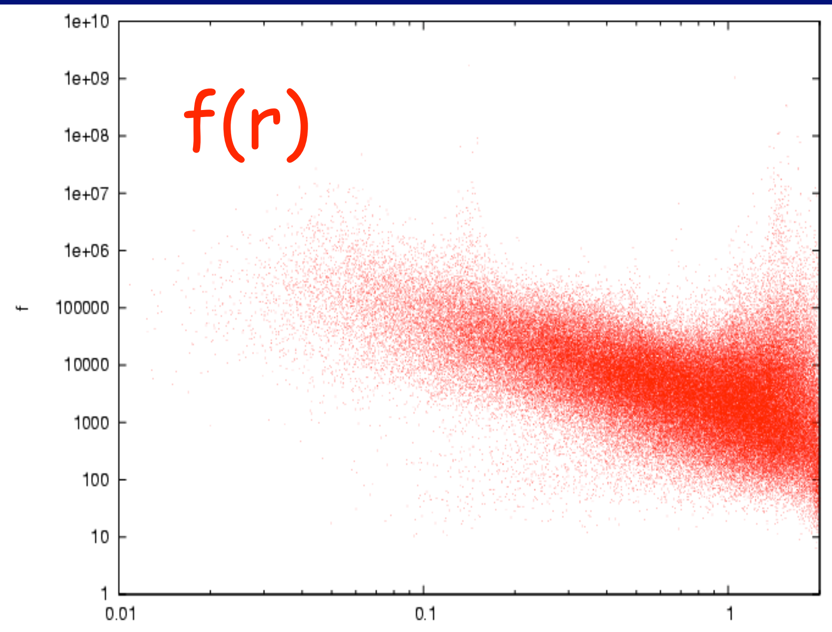
Phase-Space Density Profile

ΛCDM



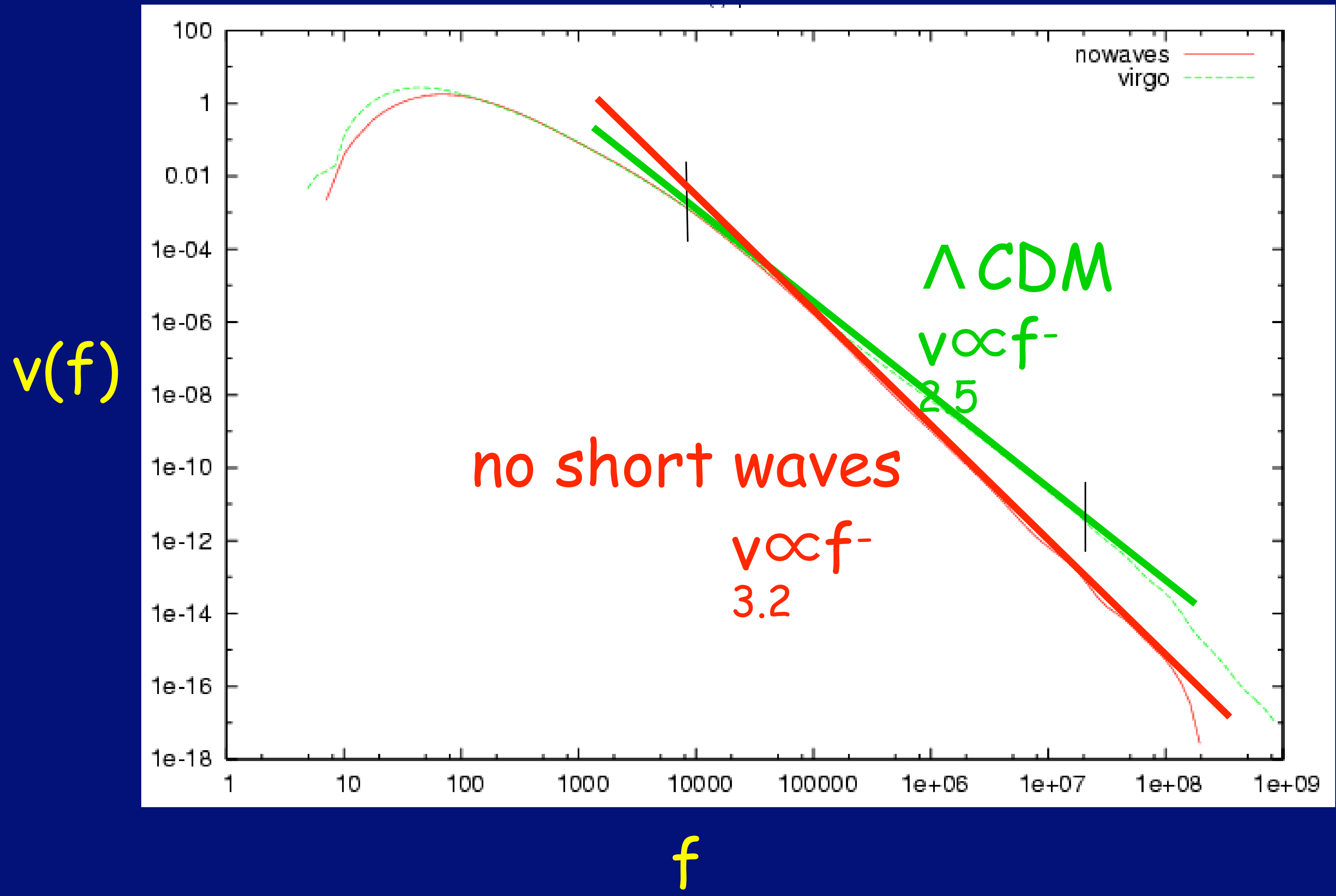
radius

No short waves

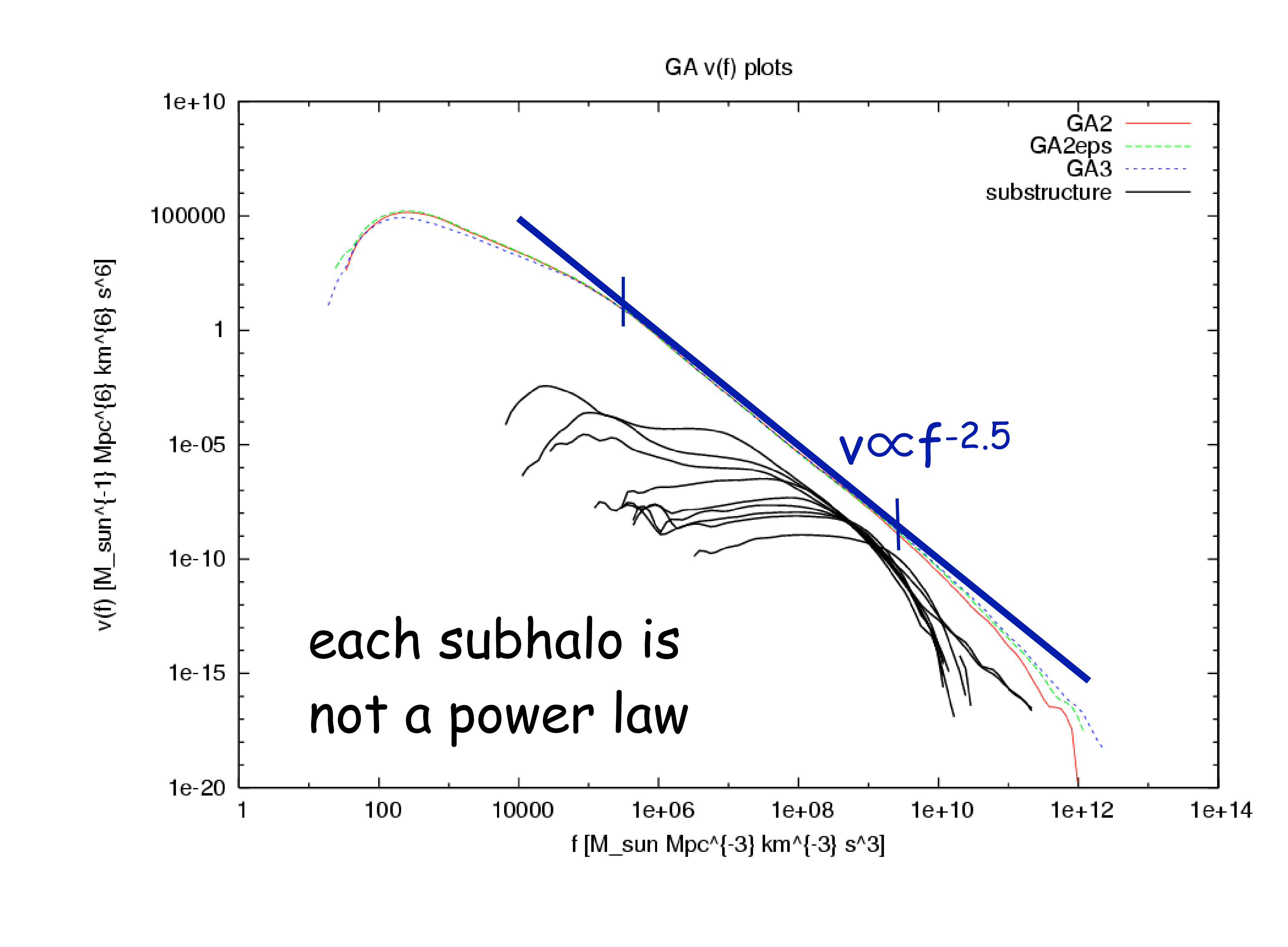


radius

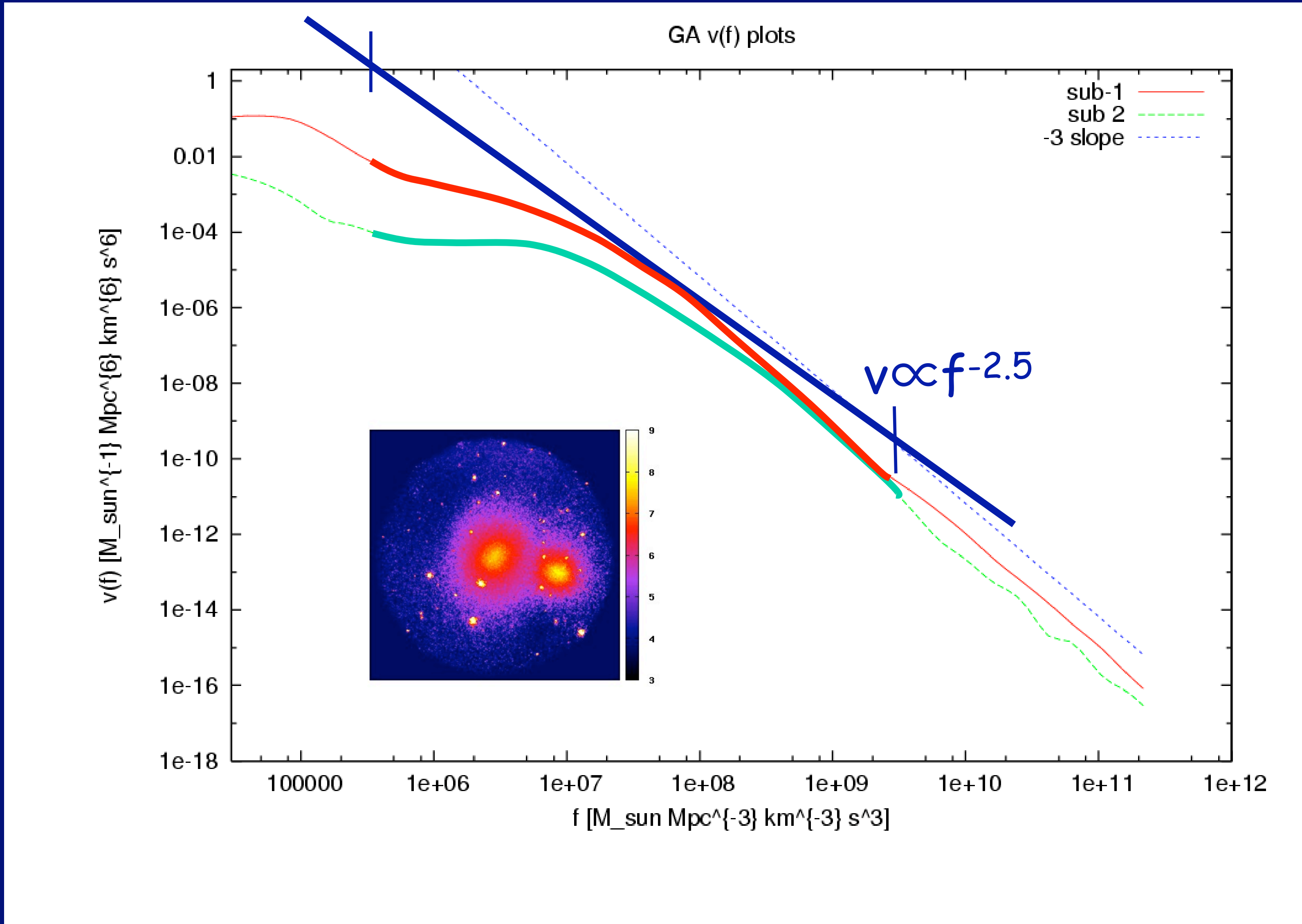
Same power law v(f)?



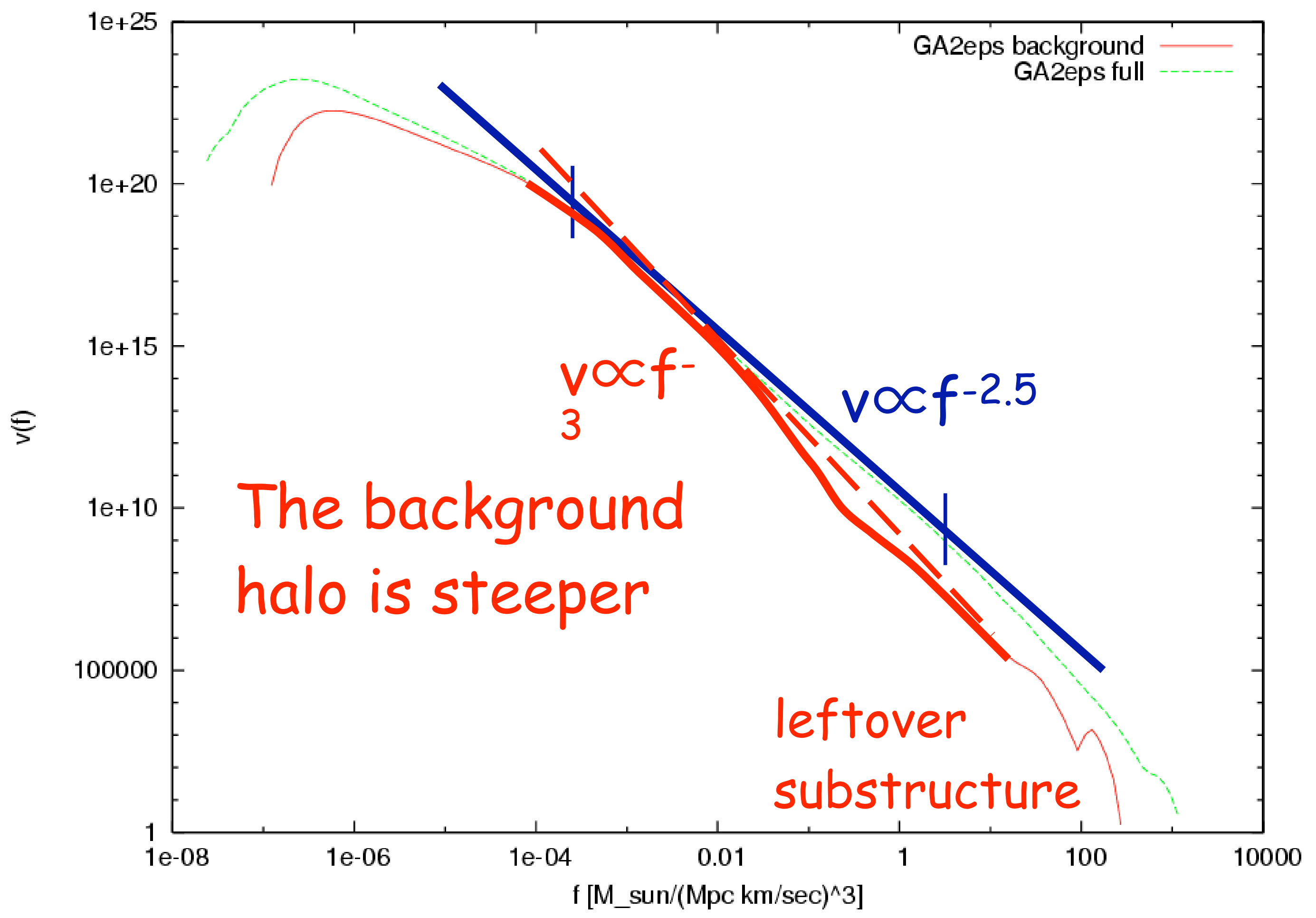
Additive Contribution of Subhalos



The Two Most Massive Subhalos

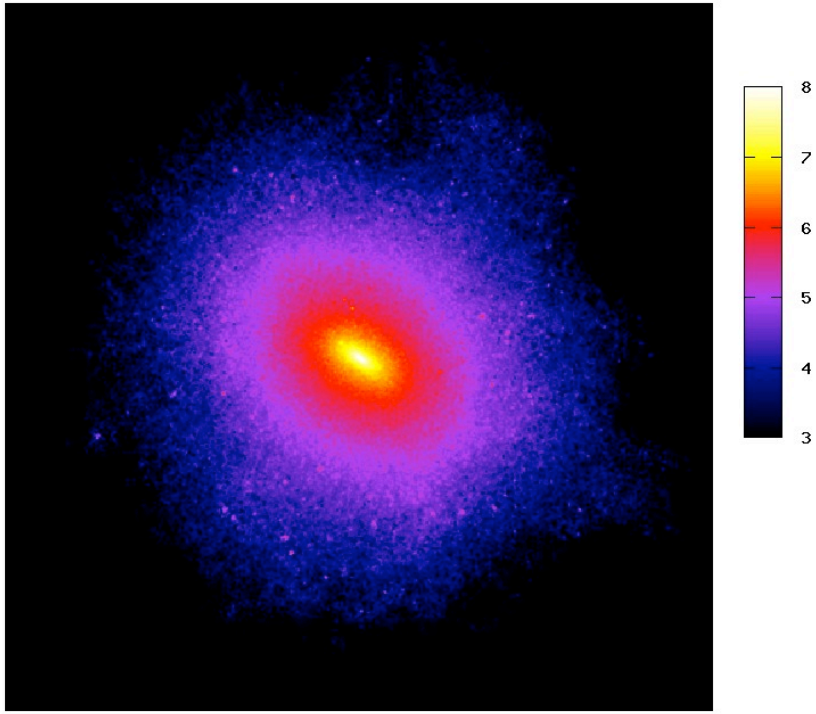


Background Halo - Subhalos Removed

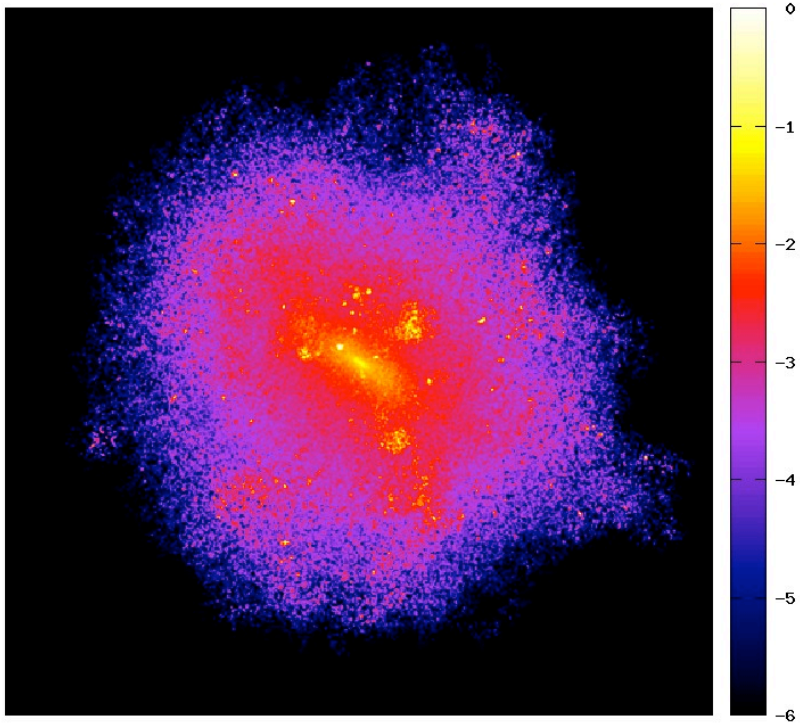


Background Halo

real space



phase space



Toy model: adding up small halos

Halo mass function: $\phi(m) \propto m^{-\gamma} \quad \gamma \approx 2$

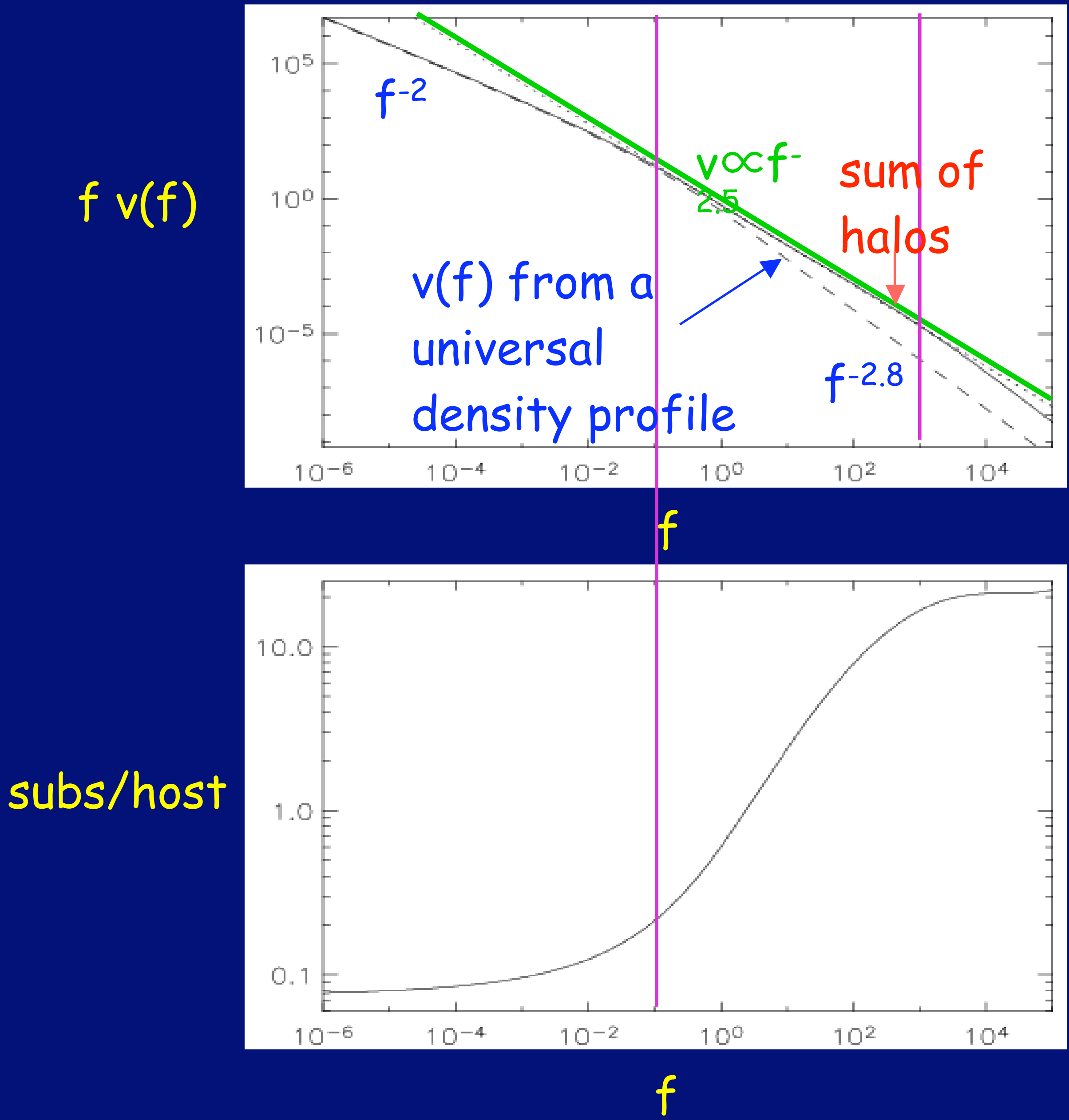
Scaling of halos:

$$\rho \propto m / r^3 = const. \quad r \propto m^{1/3} \quad \sigma \propto m^{1/3}$$

→ $V(f) \propto f^{-(4-\gamma)} \approx f^{-2}$ vs. $V(f) \propto f^{-2.5}$

→ subhalos evolve !

Adding up Sub-halos



_(r)°v(f)

halo mass function:

$\phi(m) \propto m^{-\gamma} \quad \gamma \approx 1.8$

Scaling of halos:

$\rho \propto m / r^3 = const.$

$r \propto m^{1/3} \qquad \sigma \propto m^{1/3}$

Boylan-Kolchin, Ma, Arad, Dekel

Tentative Conclusions

In hierarchical clustering, robust PDF: $v(f) \propto f^{-2.5}$ doesn't depend on power-spectrum slope, or on method of simulation

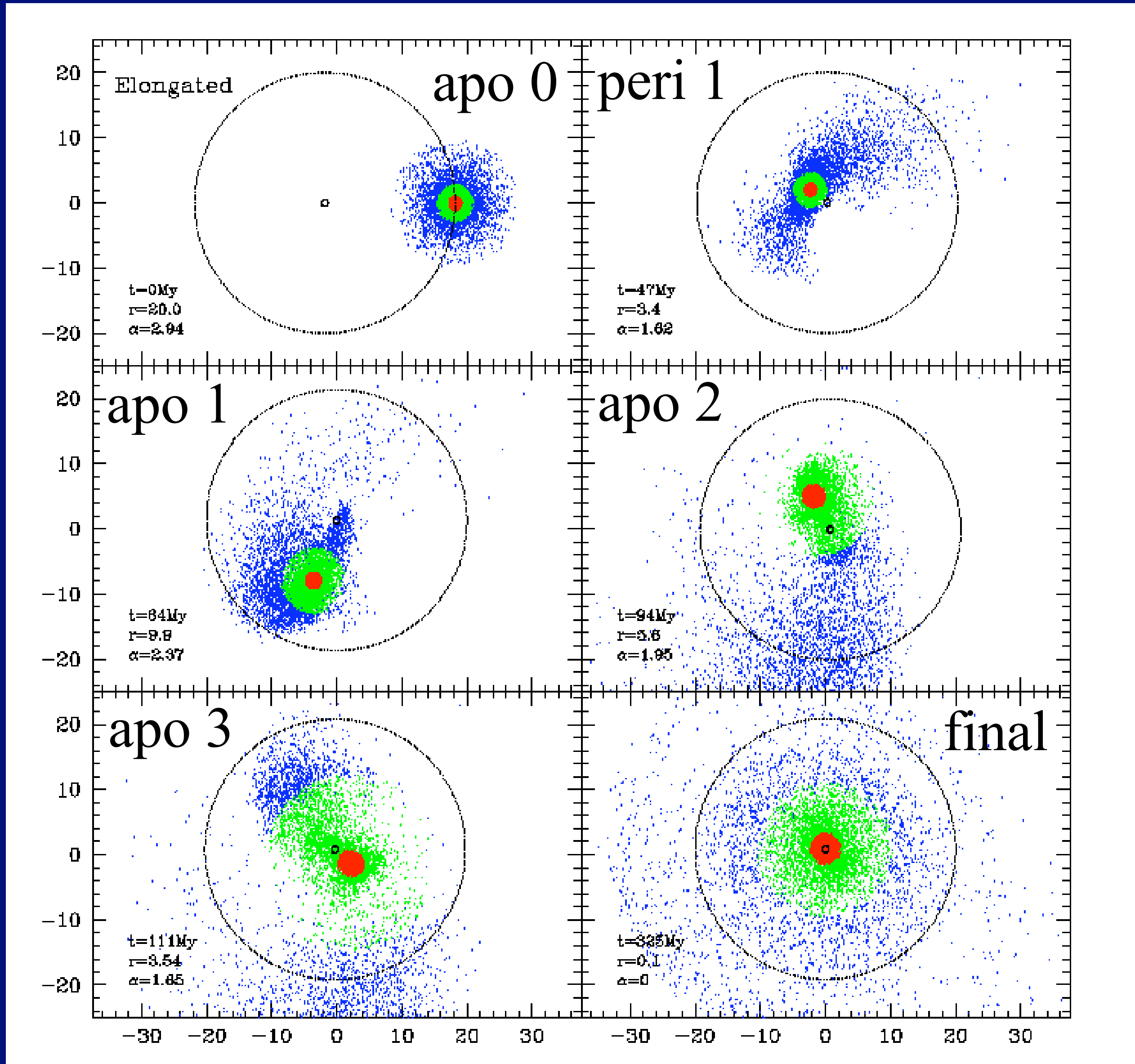
The power-law $v(f)$ is driven by substructure.

How exactly? Yet to be understood !

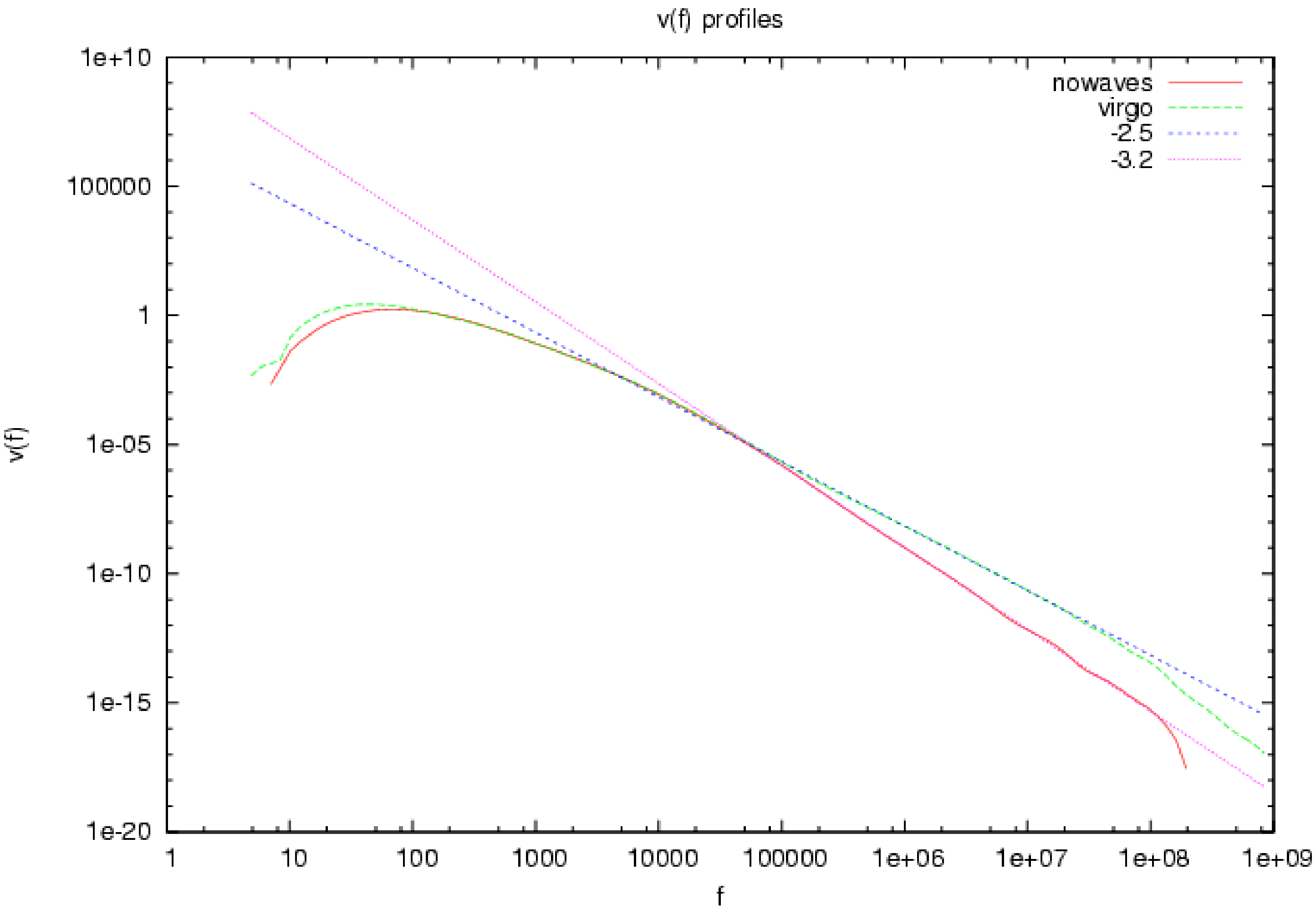
Phase-space density is a unique tool for studying substructure and its evolution

Adding up small CDM halos leads to $v(f) \propto f^{-2.5}$? How robust? How dependent on subhalo density profile and mass function?

Satellite merging into a halo



Dekel, Devor & Hetzroni 2003


v(f) profiles
nowaves
virgo
-2.5
-3.2
1e+10
100000
1
1e-05
1e-10
1e-15
1e-20
v(f)
1
10
100
1000
10000
100000
1e+06
1e+07
1e+08
1e+09
f