COSMOLOGY – PROBLEM 11

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11. Press-Schechter Formalism

a. Lets crudely simplify the linear power spectrum of CDM density fluctuations by

$$P_k \propto \begin{cases} A\,(k/k_{peak}) & k < k_{peak} \\ A\,(k/k_{peak})^{-2} & k \geq k_{peak} \end{cases}$$

with $k_{peak} = 0.05 Mpc^{-1}$. What is the physical meaning of the length scale associated with the wave number k_{peak}, and how does it give rise to a peak in the power spectrum?

b. Recall that the variance of density fluctuations in spheres of comoving radius R, containing on average mass M, is approximated by

$$\sigma^2(M) = \int_0^{K=2\pi/(2R)} P_k(k)\, d^3k \ .$$

This is using a top-hat window in k space. Compute $\sigma(M)$ for the power spectrum specified above. Determine the normalization constant A such that $\sigma_8 = 1$, the rms fluctuation in spheres of radius $R = 8\,Mpc$. What is the rms fluctuation σ_{30} in spheres of radius $R = 30\,Mpc$?

c. Assume an Einstein-deSitter (EdS) cosmological model. Write the equation and the solution for the linear fluctuation growth rate, $D(a)$.

d. Explain the concept of "the linearly extrapolated density fluctuation corresponding to collapse in the spherical model, δ_c". What is its numerical value?

e. Compute the Press-Schechter characteristic halo mass M_* at redshift z $[a = 1/(1+z)]$, defined such that

$$\sigma(M_*) = \delta_c / D(a) \ .$$

What is its value today, at $z = 0$ (given that the mean mass density today is $\rho_0 = 3 \times 10^{-30}\, g\, cm^{-3}$)? What was the typical halo mass at $z = 1$? At $z = 3$? At $z = 10$?

f. Given a halo virial velocity V at redshift z. Use the virial theorem and the top-hat spherical collapse model to obtain its virial radius (R_{200}) and virial mass M (EdS cosmology, ρ_0 is given above). The virial velocity of a Milky-Way type galaxy at $z = 0$ is $V = 120\,\mathrm{km\,s^{-1}}$. What are its mass and radius? Same for a Coma-like cluster of $V = 1200\,\mathrm{km\,s^{-1}}$.

g. Use the Press-Schechter formula to compute the mean comoving number density of dark halos like the Milky-Way and larger at $z = 0$. Same for a Coma-like clusters. Same at $z = 2$.