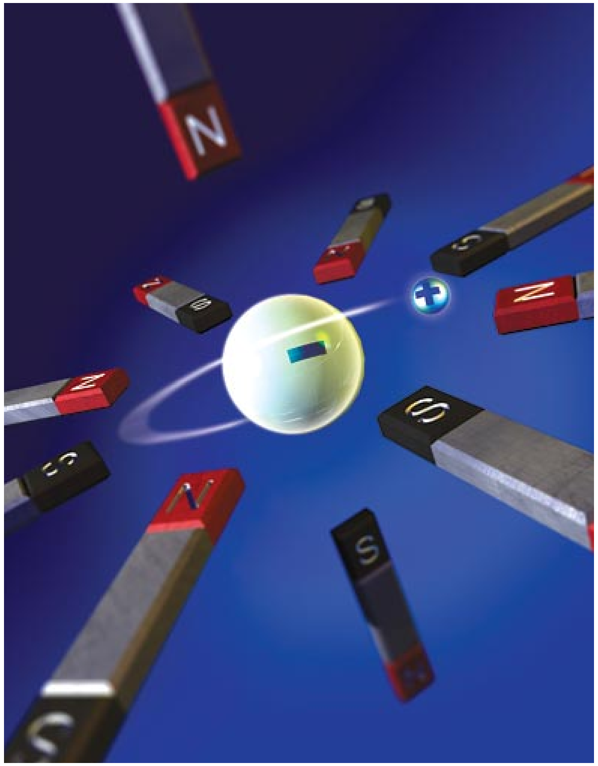
Properties of bound electrons and muons



Picture: http://news.berkeley.edu/2010/11/17

PSAS'2016
Jerusalem, May 23, 2016

Andrzej Czarnecki University of Alberta

with R. Szafron, M. Dowling, J. Piclum

Outline

Muon decay in orbit

- two energy regions
- different approaches to radiative corrections

g-factor of a bound electron

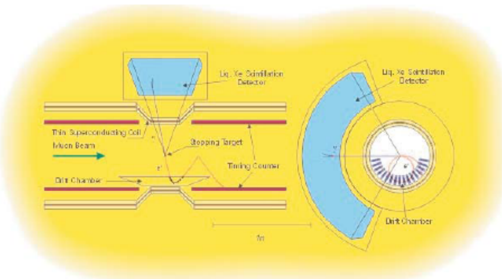
- towards $(Z\alpha)^5$ effects

Lepton flavor violation: $\mu \to e\gamma$

Previous experiment (MEGA @ Los Alamos): $\mathrm{BR}\,(\mu \to e\gamma) < 10^{-11}$

New bound (MEG @ Paul Scherrer Institute) $< 4.2 \cdot 10^{-13}$

arXiv:1605.05081



This corresponds to the transition dipole moment

$$d_{\mu \to e} \lesssim 3.5 \cdot 10^{-27}\ e \cdot \mathrm{cm}$$

Sensitive to the heaviest "new physics"

For comparison: electron EDM $d_e < 0.87 \cdot 10^{-28}\, e \cdot \mathrm{cm}$ 10.1126/science.1248213

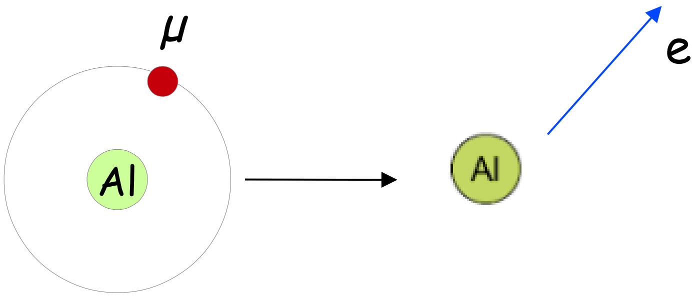
muon g-2 $d_\mu < 3 \cdot 10^{-22}\, e \cdot \mathrm{cm}$

Muon-electron conversion:
probes various types of interactions

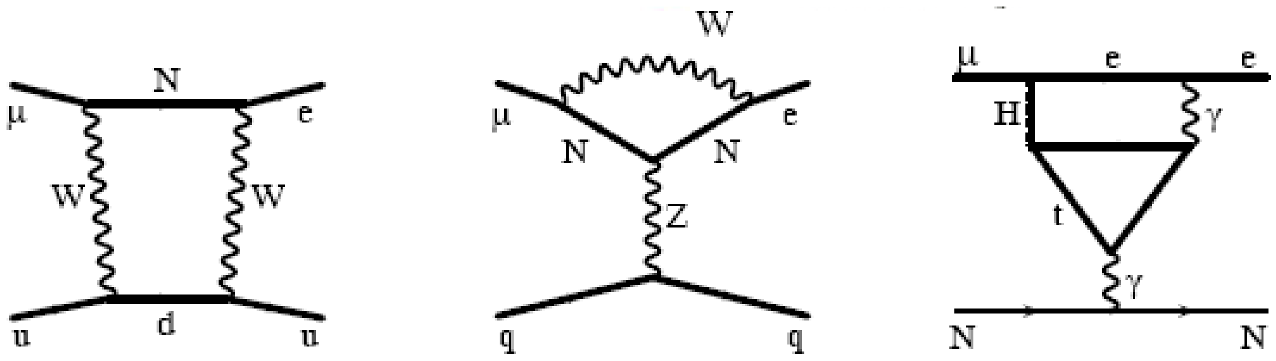
Non-dipole interactions are not (directly) probed by
processes with external photons,
by gauge invariance requirements.

New process: muon-electron conversion

(as well as mu --> eee)



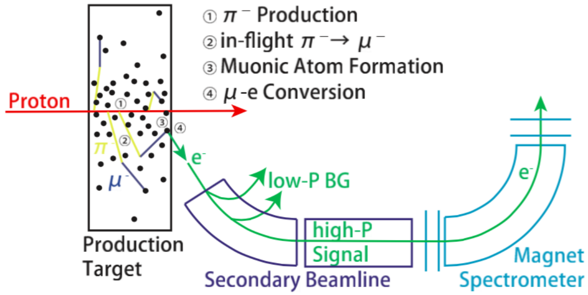
Variety of mechanisms:



Muon-electron conversion plans (The Next Big Thing in muon physics)

from Hiroaki Natori JPS Conf. Proc. 8, 025017 (2015)

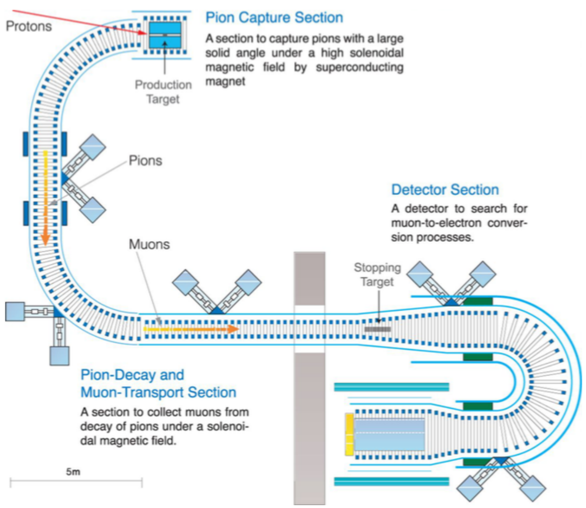
DeeMe
J-PARC



starts 2016;
aims for 1e-13 (graphite target),
followed by 1e-14 (SiC target)

COMET
Phase 1
J-PARC

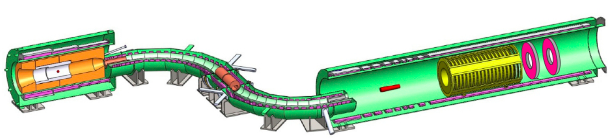
7e-15



COMET
Phase 2
J-PARC

2.6e-17

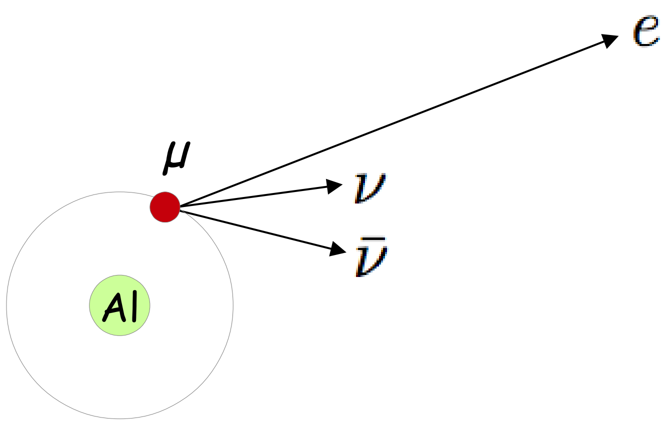
Mu2e
Fermilab



2e-17

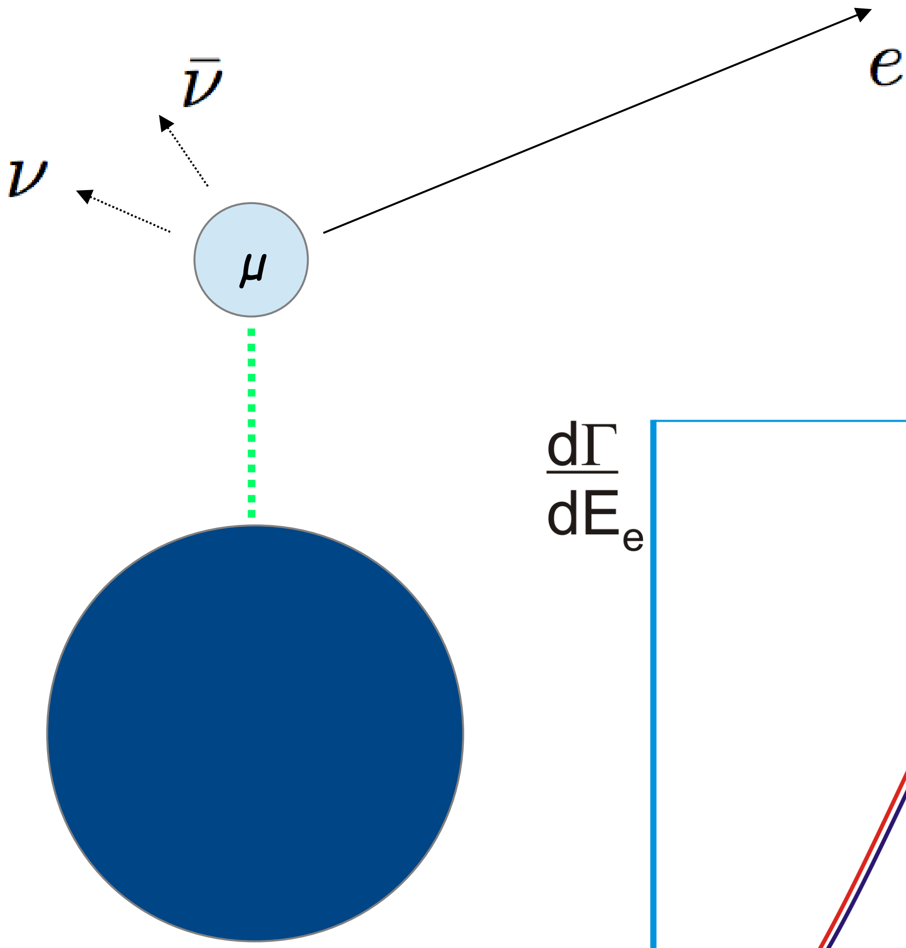
Background for the conversion search

Normal decay of the muon bound in the atom
can produce high-energy electron,

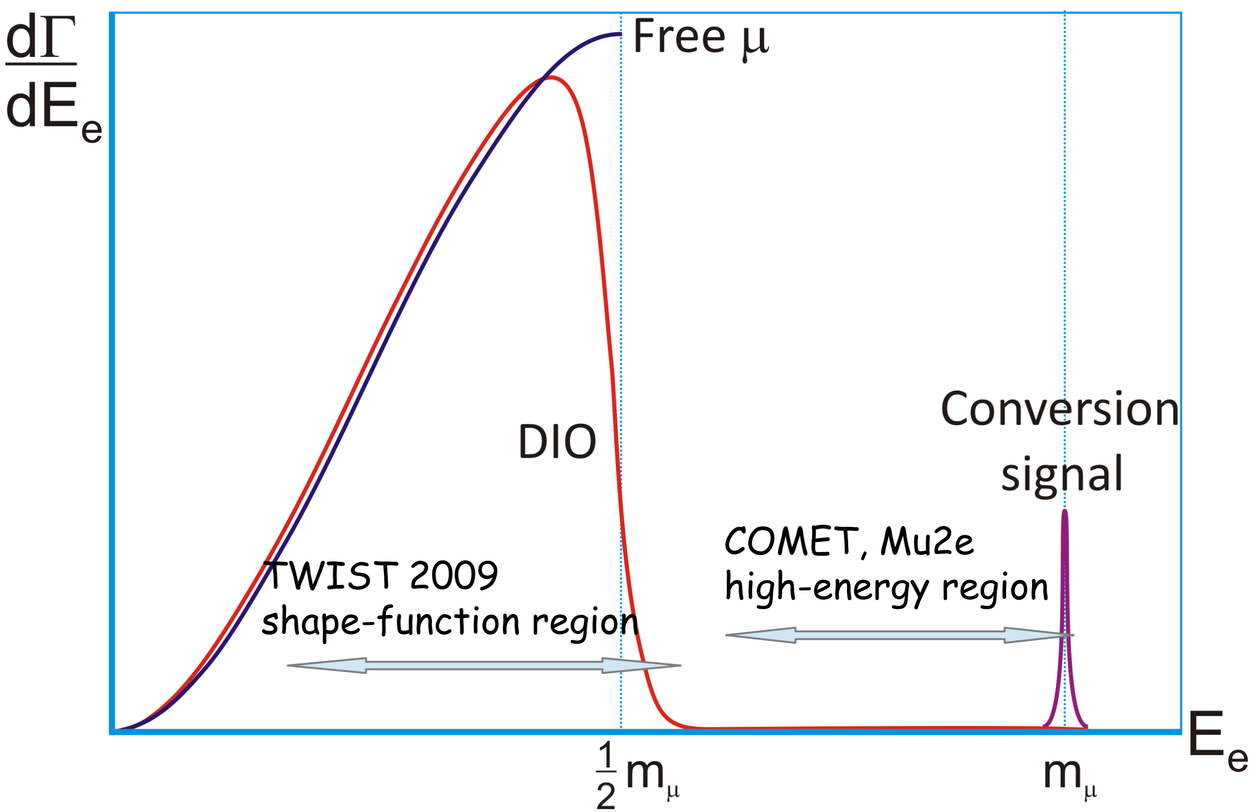


Spectrum has to be well understood.

Electron spectrum in a bound muon decay



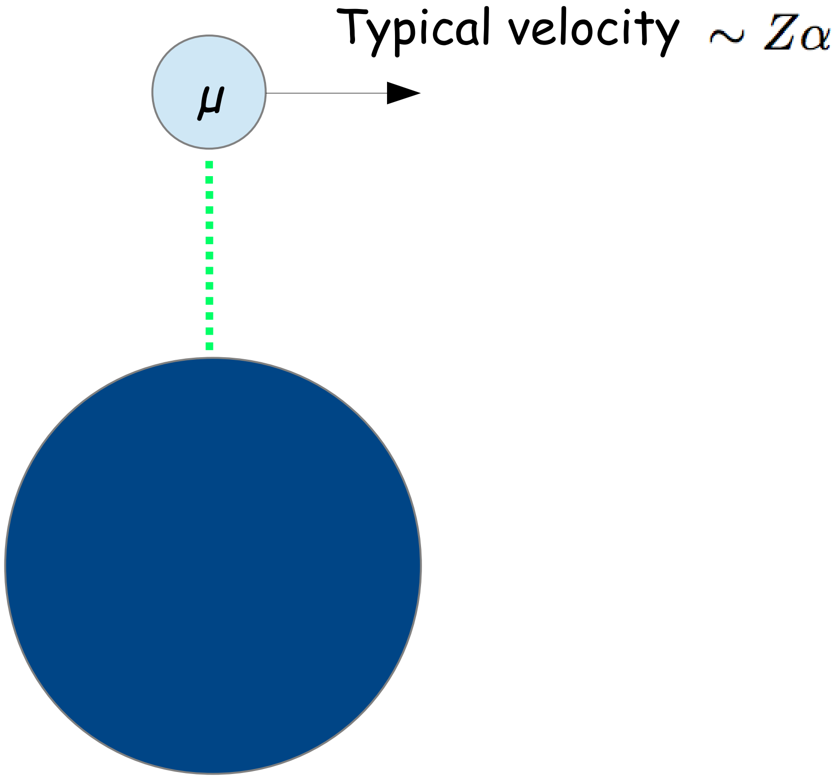
Electron energy can be as large as the whole muon mass



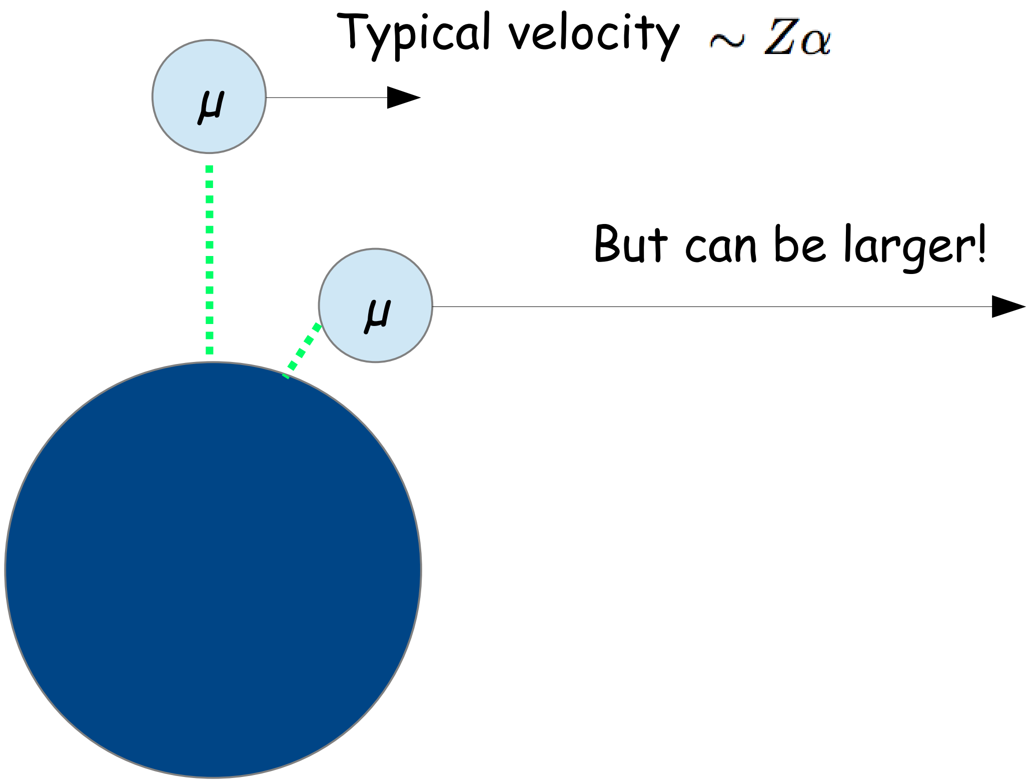
Muon decay-in-orbit spectrum:
the shape-function region

Experiment: TWIST

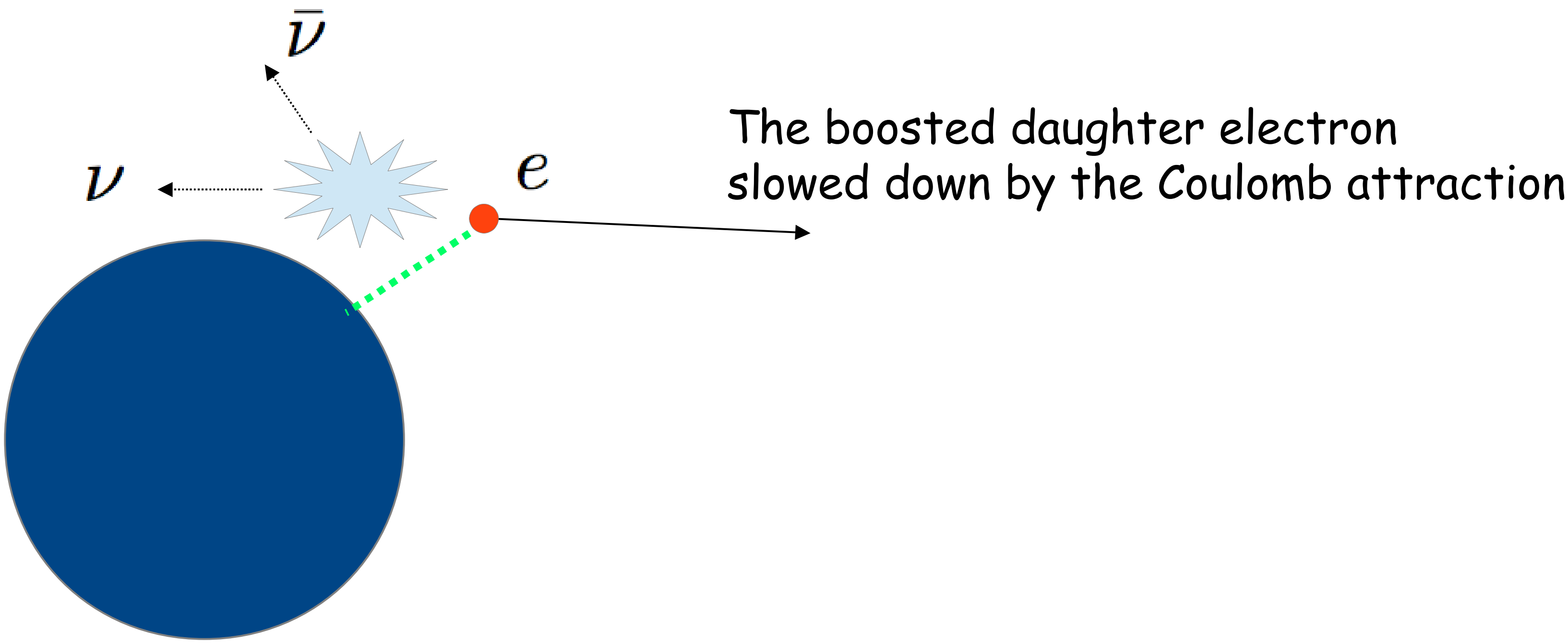
Two effects: muon motion & Coulomb attraction



Two effects: muon motion & Coulomb attraction



Two effects: muon motion & Coulomb attraction

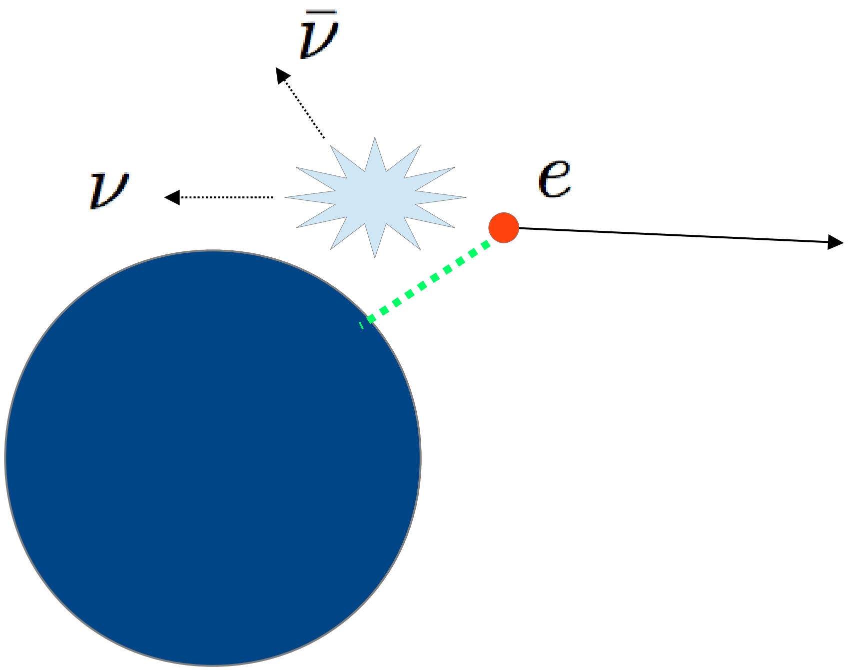


Two effects: muon motion & Coulomb attraction

Net effect:
- In the decay rate: almost none;
  only time dilation

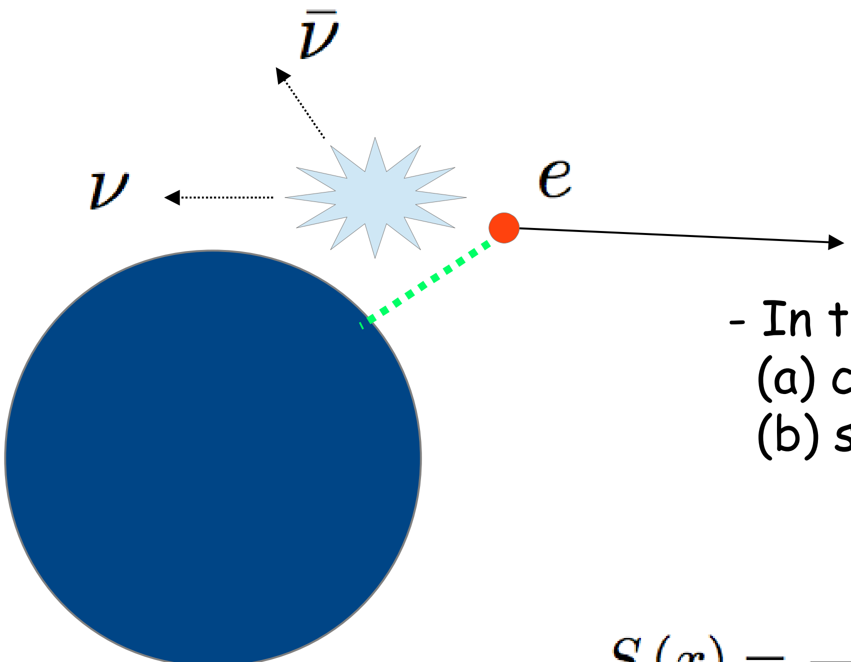
$$\Gamma \to \left(1 - \frac{(Z\alpha)^2}{2}\right)\Gamma$$

Überall, Phys. Rev. 119, 365 (1960)



Two effects: muon motion & Coulomb attraction

Szafron, AC, PRD 92 (2015) 053004



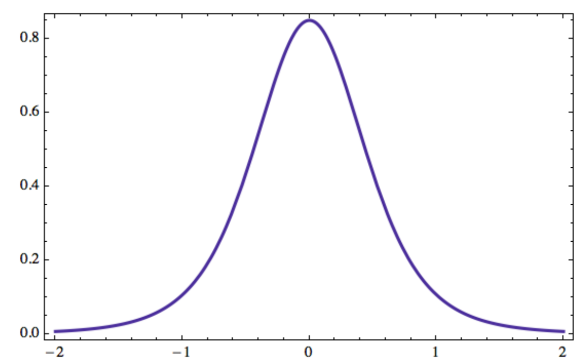
Net effect:

- In the decay rate: almost none; only time dilation

$$\Gamma \to \left(1 - \frac{(Z\alpha)^2}{2}\right)\Gamma$$

- In the electron energy spectrum:
  (a) computable shift
  (b) smearing -> "shape function"

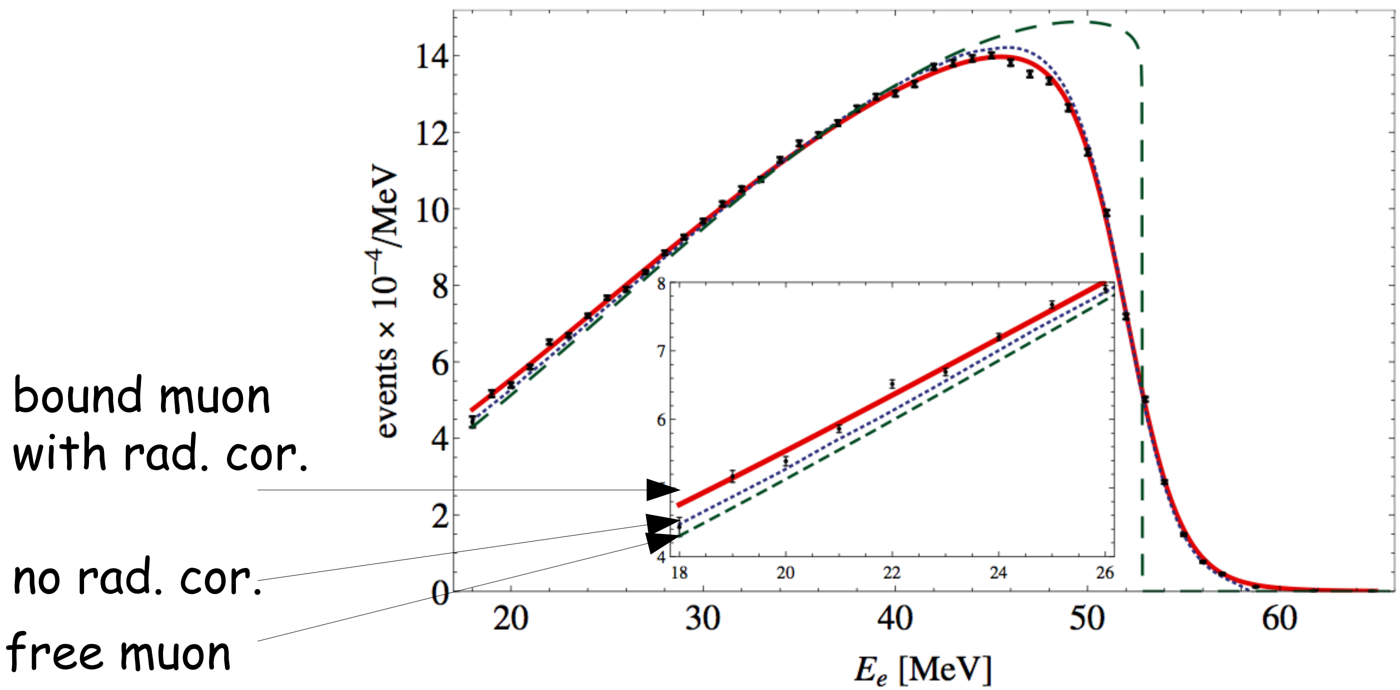
$$S(x) = \frac{8}{3\pi\left[1+x^2\right]^3}$$



Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.

Mannel, Neubert,
Bigi, Shifman, Uraltsev, Vainshtein

Comparison with measurement: TWIST

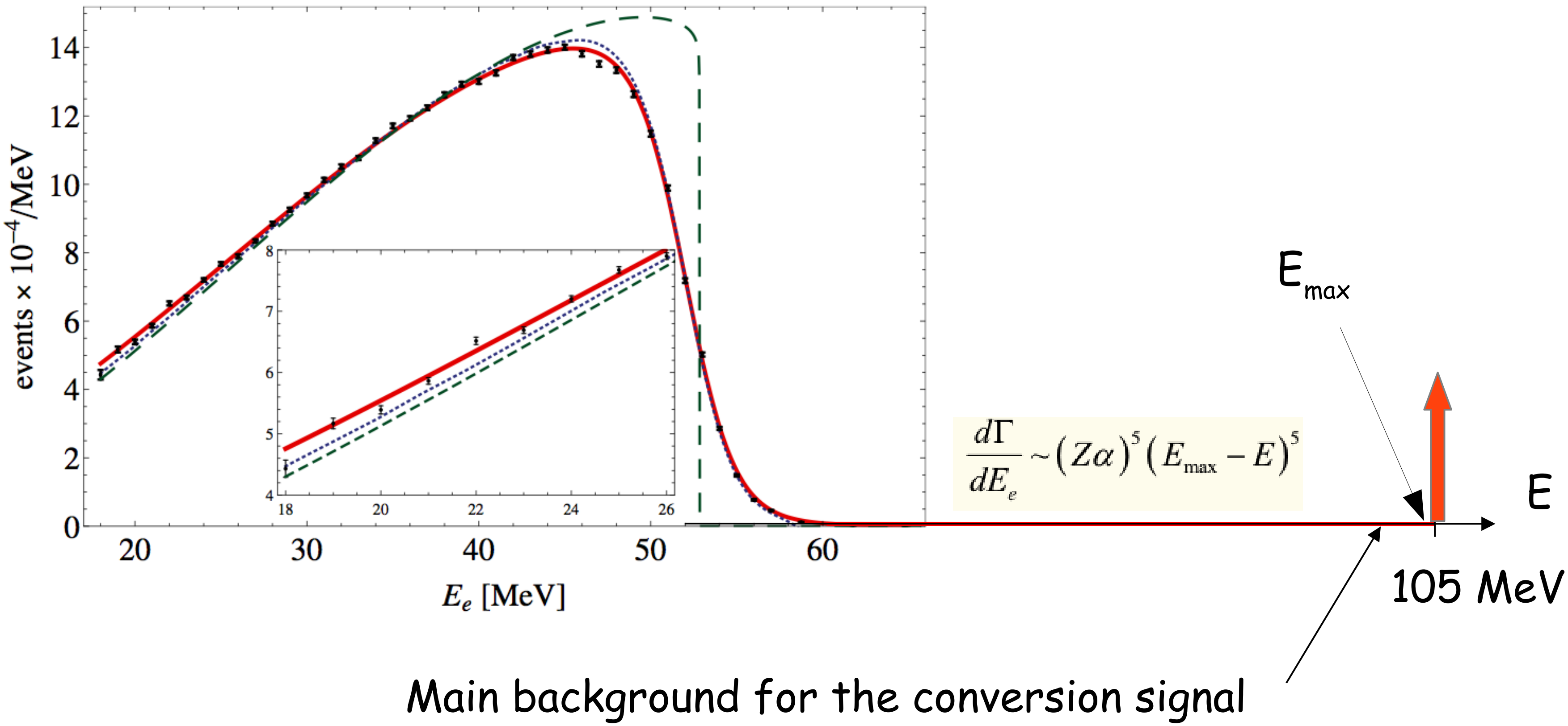


The spectrum is modified very significantly: effects ~ 1/Zα

Muon decay-in-orbit spectrum: the high-energy region

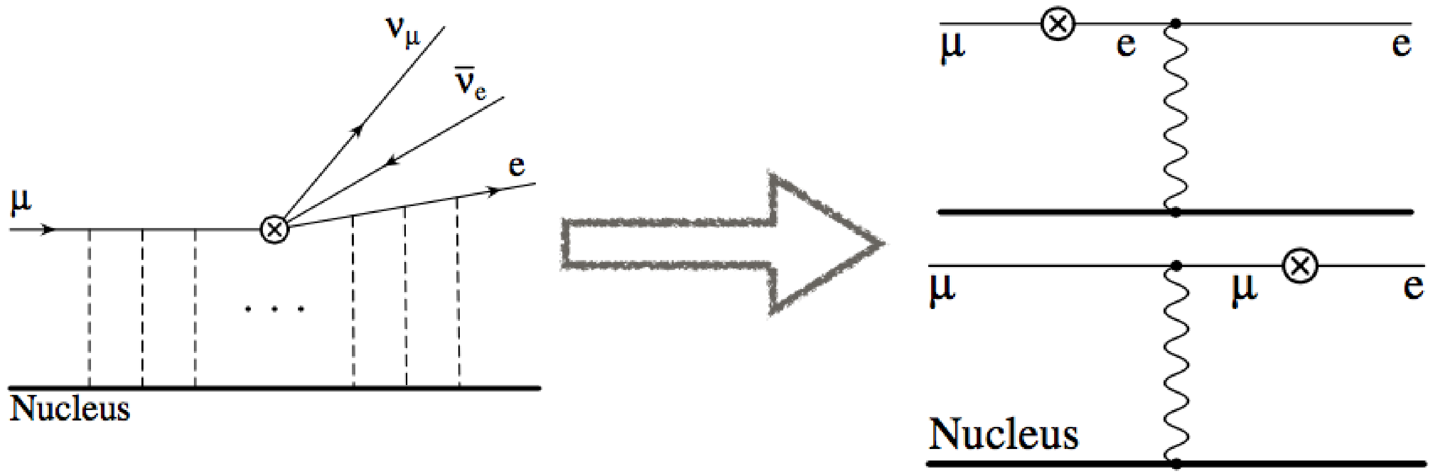
Experiments: Mu2e and COMET

Spectrum of the bound muon decay



AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron
R. Szafron, AC

Origin of the $(E_{max}-E)^5$ suppression

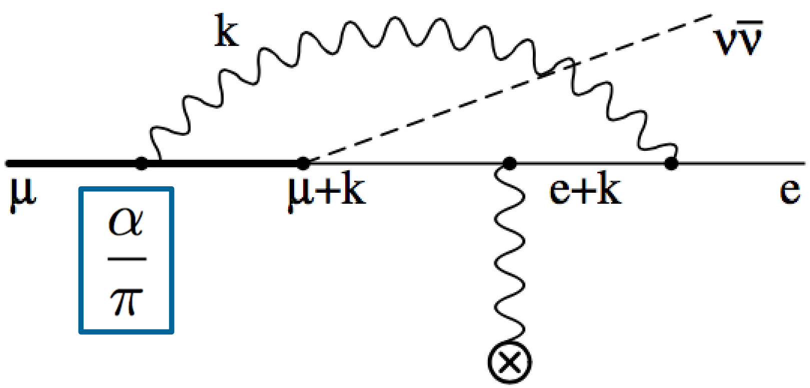


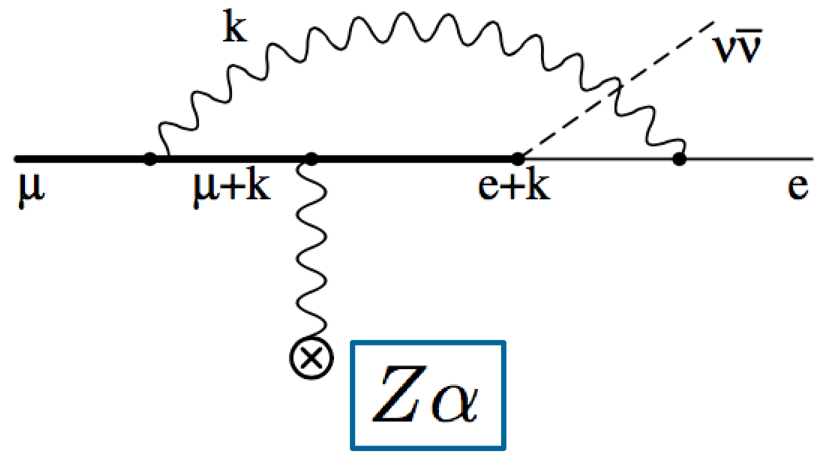
Neutrinos get no energy;
The nucleus balances electron's momentum, takes no energy.
Near the end point:

Radiative corrections to the electron spectrum

Expansion near the end-point $$\frac{m_\mu}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}E} = \sum_{ijk} B_{ijk}\Delta^i(\pi Z\alpha)^j\left(\frac{\alpha}{\pi}\right)^k$$

Three "small" parameters: $$\Delta = \frac{E_{\max}-E}{m_\mu}$$

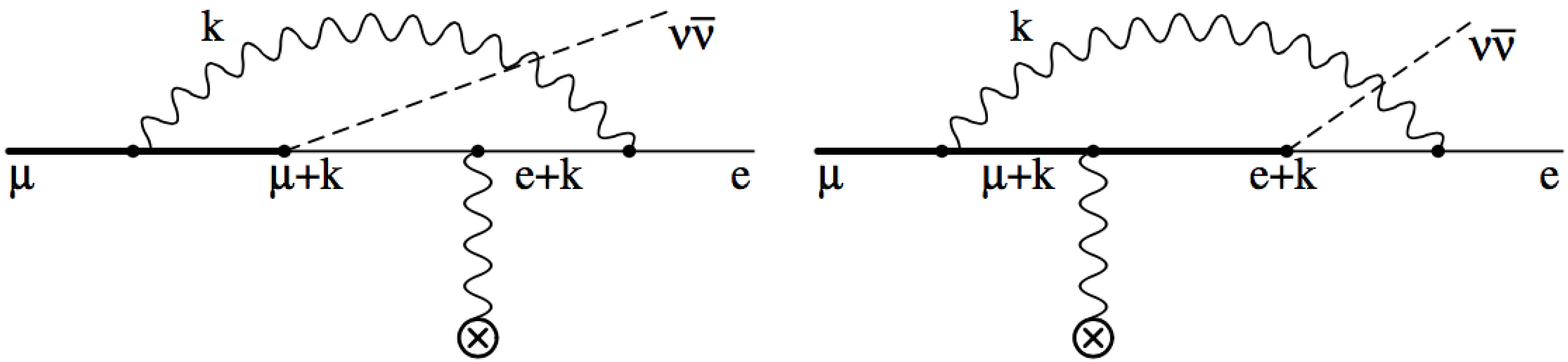




The expansion starts with B_{550}

The first radiative correction is B_{551}

Radiative corrections to the electron spectrum



Expansion near the end-point $$\frac{m_\mu}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}E} = \sum_{ijk} B_{ijk}\Delta^i(\pi Z\alpha)^j\left(\frac{\alpha}{\pi}\right)^k$$

Competing effects:
- vacuum polarization in the hard photon; and
- self-energy and real radiation

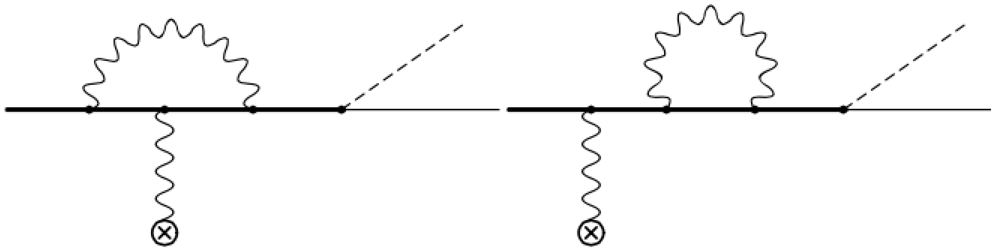
$$\frac{B_{551}}{B_{550}} = \delta_{\mathrm{H}} + \delta_{\mathrm{S}}\ln\Delta$$

$$\delta_{\mathrm{H}} = 6.31 - \tfrac{26}{15}\ln\tfrac{m_\mu}{m_e}.$$

$$\delta_{\mathrm{S}} = 2\ln 2 - 2 + 2\ln\frac{m_\mu}{m_e}$$

Szafron, AC, PLB753, 61 (2016)

Results from the first radiative corrections



The most important correction is due to soft photons.
Can be summed up to all orders (exponentiated).

$$B_{550} + \frac{\alpha}{\pi} B_{551} \to B_{550} \left[\Delta^{\frac{\alpha}{\pi}\delta_S} + \frac{\alpha}{\pi}\delta_H \right]$$

$$\delta_S = 10.1$$

number of electrons in the
end-point bin of 1 (0.1) MeV is reduced by 11% (16%)

Magnetic moment of a bound electron

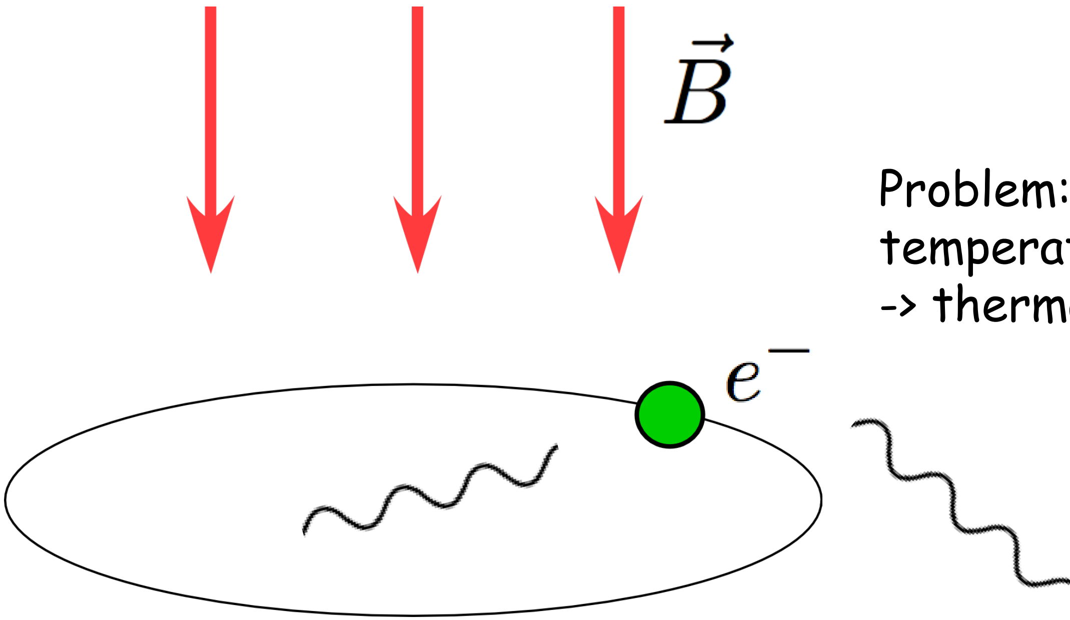
Why useful?

- determination of the electron mass
- future determination of alpha
- indirectly related to muon *g*-2 (muonium)

Why interesting?

- quantum effects in external field
- simple system, model for more complex ones
- numerical estimates exist for large Z
- should be analytically feasible for small Z

Determination of the electron mass (20th century)



Problem: finite temperature (low, 4K) -> thermal photons

$$\omega_{\text{cycl}} = \frac{eB}{m_e}$$

$$\frac{m_e}{\sqrt{1-v^2}}$$

$$\frac{\delta m_e}{m_e} \sim v^2 \sim \frac{kT}{m_e} \sim \frac{10^{-4}\,\frac{\text{eV}}{\text{K}} \cdot 4\ \text{K}}{5 \cdot 10^5\ \text{eV}} \simeq 10^{-9}$$

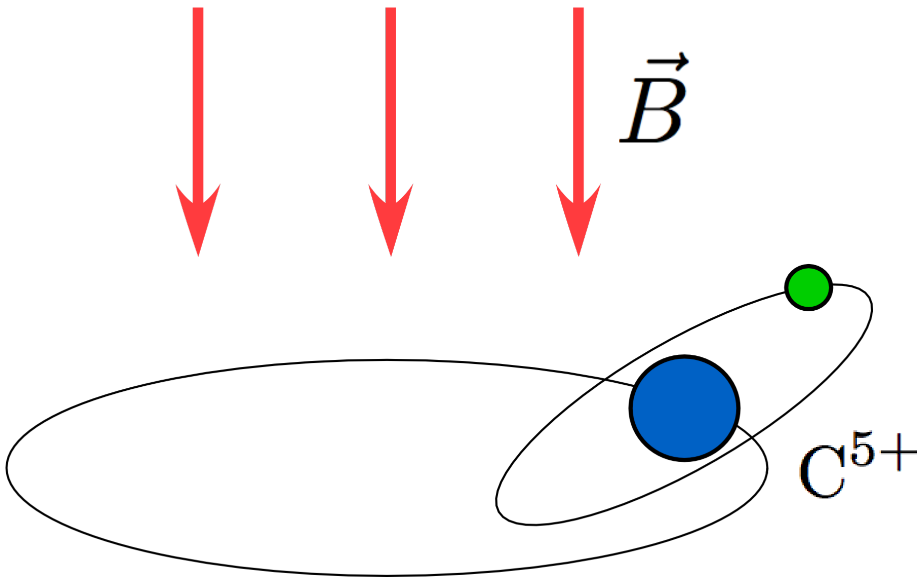
VOLUME 75, NUMBER 20 PHYSICAL REVIEW LETTERS 13 NOVEMBER 1995

Determination of the Electron's Atomic Mass and the Proton/Electron Mass Ratio via Penning Trap Mass Spectroscopy

$$\frac{m_e}{u} = 0.000\ 548\ 579\ 911\,(1)$$

2ppb relative error

21st century: anchor the electron in an ion

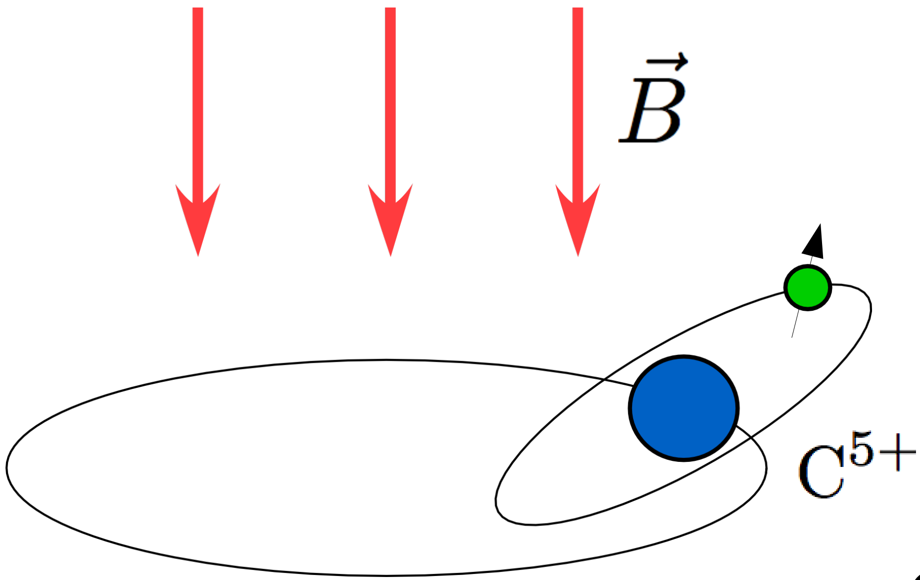


The speed of the electron becomes even higher,

$$v_e^2 \sim (Z\alpha)^2 \sim 10^{-3}$$

but is calculable in bound-state QED.

21^{st} century: anchor the electron in an ion



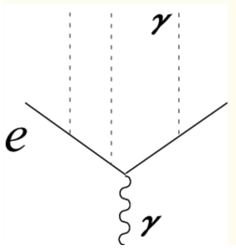
Larmor frequency $\omega_L = \frac{geB}{2m_e}$

Cyclotron frequency $\omega_{cycl} = \frac{ZeB}{M}$

$$m_e = \frac{g}{2Z} \frac{\omega_{cycl}}{\omega_L} M$$

Bound-electron g-2: the leading effect

Breit 1928: energy correction due to magnetic field in the hydrogen ground state.



$$\delta E = e \int d^3x f^2 v^* \left[1 - i\gamma \boldsymbol{\Sigma}\cdot\hat{\boldsymbol{r}}\gamma^5\right]\gamma^5 \boldsymbol{A}\cdot\boldsymbol{\Sigma}\left[1 + i\gamma \boldsymbol{\Sigma}\cdot\hat{\boldsymbol{r}}\gamma^5\right] v$$

$$g = 2\cdot\frac{1}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) \simeq 2\left(1 - \frac{(Z\alpha)^2}{3}\right)$$

Bound-electron g-2: the leading effect

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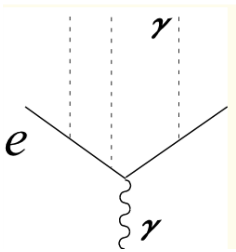


$$\delta E = e \int d^3x f^2 v^* \left[1 - i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] \gamma^5 \boldsymbol{A} \cdot \boldsymbol{\Sigma} \left[1 + i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] v$$

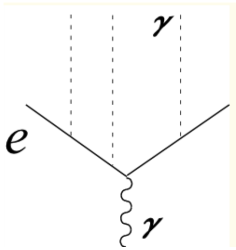
$$g = 2 \cdot \frac{1}{3} \left(1 + 2\sqrt{1 - (Z\alpha)^2}\right) \simeq 2 \left(1 - \frac{(Z\alpha)^2}{3}\right)$$

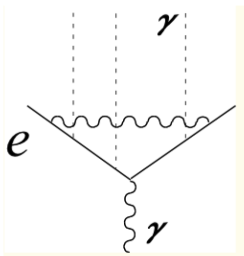
Important: dependence on alpha; may be exploited to determine its value. (Use ions with various Z)

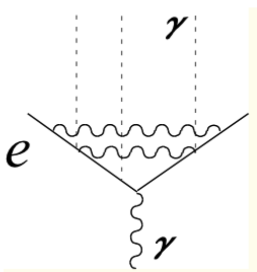
Shabaev, Glazov, Oreshkina, Volotka, Plunien, Kluge, Quint

Yerokhin, Berseneva, Harman, Tupitsyn, Keitel: PRL (2016)

Bound-electron g-2: binding and loops







$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots$$
$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \dots\right]$$
$$\underbrace{+\left(\frac{\alpha}{\pi}\right)^2 \left[-0.65..\left(1+\frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \dots\right]}_{\text{two-loop corrections}}$$

two-loop corrections

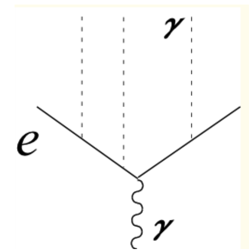
$$b_{41} = \frac{28}{9}$$
$$b_{40} = -16.4$$

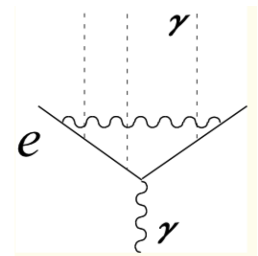
Pachucki, AC Jentschura, Yerokhin 2005

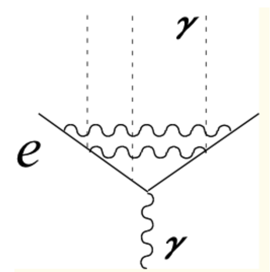
Together with experiments (Wolfgang Quint et al), this improved the accuracy of m_e by about a factor 3,

$$\frac{m_e}{u} = 0.000\,548\,579\,909\,32\,(29)\,(1)$$

Bound-electron g-2: binding and loops







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$$+ \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\left(b_{41}\ln Z\alpha + b_{40}\right) + \dots\right]}_{\text{two-loop corrections}}$$

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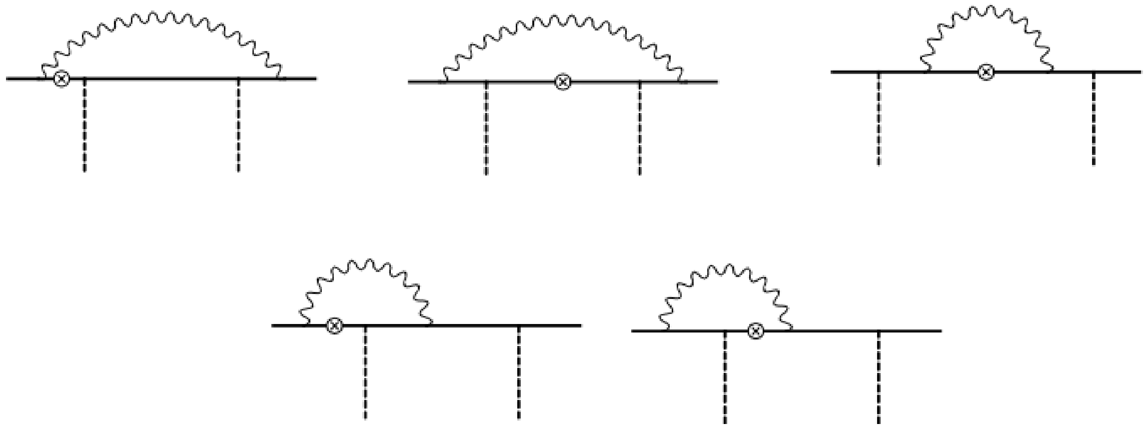
$$\frac{m_e}{u} = 0.000\ 548\ 579\ 909\ 32\,(29)\,(1)$$

$$\frac{m_e}{u} = 0.000\ 548\ 579\ 909\ 067\,(17)$$

Nature 2014
Sturm et al

Next theory challenge:
$(Z\alpha)^5$ effects.
Numerical result for $\alpha(Z\alpha)^5$ but not $\alpha^2(Z\alpha)^5$
Yerokhin, Indelicato, Shabaev

Towards an analytic result for $\alpha(Z\alpha)^5$



The result is *gauge-invariant*; but not yet complete.

What if the magnetic field couples to an external line?

Magnetic correction to the wave function

$$\Delta g = \frac{2\left|e\right|}{\mu_0 m B} \sum_{n}^{n\neq a} \frac{\langle a\left|\delta U\right|n\rangle \left\langle n\left|\vec{\alpha}\cdot\vec{A}\right|a\right\rangle}{E_a - E_n}$$

This sum can be done exactly with help of virial identities

S. Karshenboim, V. Ivanov, and V. Shabaev

$$\Delta g = \frac{2\kappa m}{j\left(j+1\right)} \langle n\kappa|\, \delta U \frac{\kappa}{m^2} \left(I - |n\kappa\rangle\langle n\kappa|\right) \left[\left(E_{n\kappa} - \frac{m}{2\kappa}\right) r i \sigma_y + m r \sigma_x + \alpha Z i \sigma_y - \kappa \sigma_z\right] |n\kappa\rangle$$

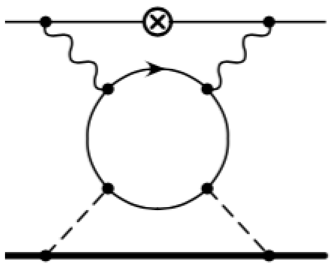
The result is especially simple if the short-distance perturbation is energy-independent (like the Uehling potential)

$$\Delta g = \tfrac{16}{3}\left(1 - \tfrac{1-\sqrt{1-(Z\alpha)^2}}{2(Z\alpha)^2}\right)\left\langle \delta^3\left(r\right)\right\rangle_\psi \simeq\ 4\left\langle \delta^3\left(r\right)\right\rangle_\psi$$

This is just Lamb

Next goal: $\alpha^2(Z\alpha)^5$ corrections to g

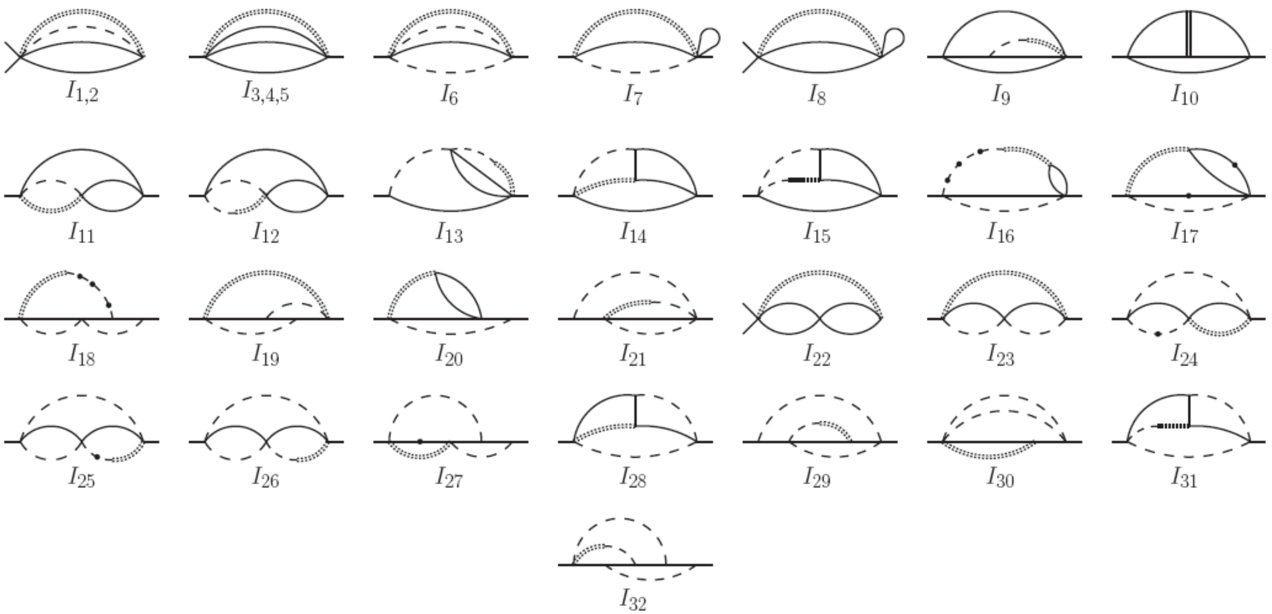
Example:



Related diagrams:
Karshenboim and Milstein, 2000-2002

More than 300 contributions.

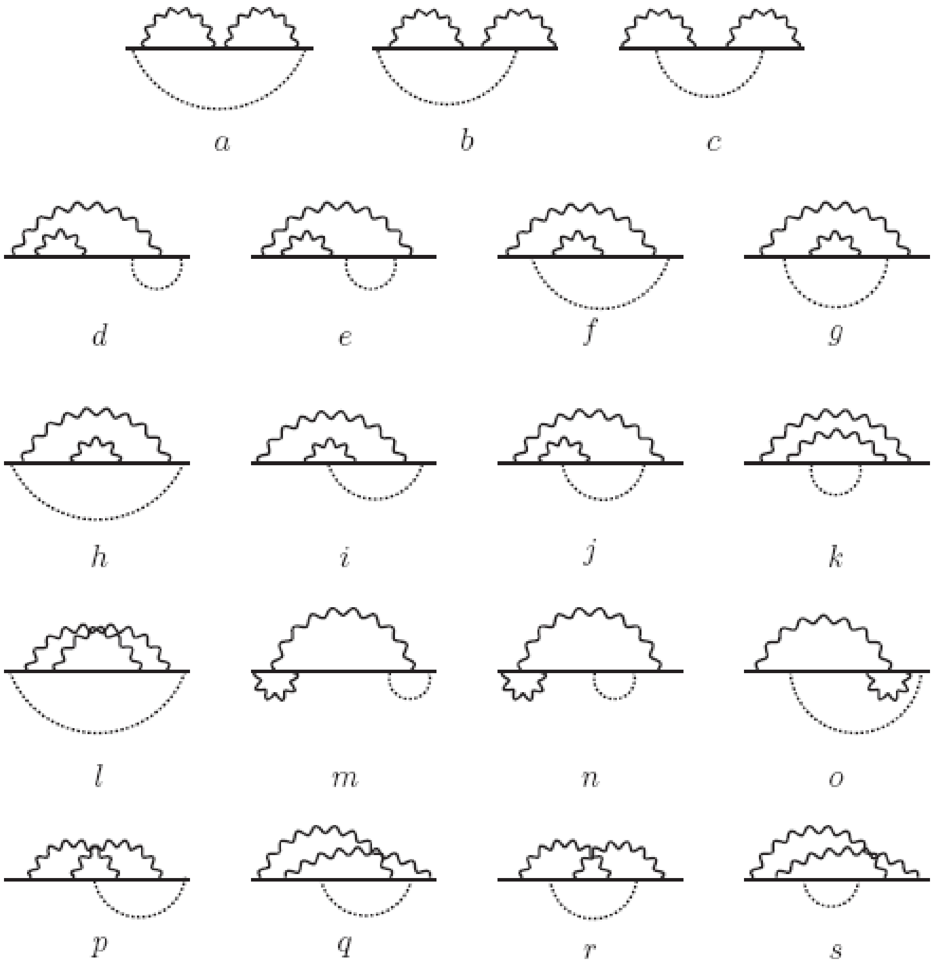
A set of 32 master integrals



Typical expression

$$I_{24} = G(0,1,2,1,0,1,0) = \frac{2\pi^2}{\epsilon} - 162.745878930257(1) + 640.681562239(2)\epsilon$$
$$-9490.745115169417(3)\epsilon^2 + \mathcal{O}(\epsilon^3)\,,$$

Reevaluation of the $\alpha^2(Z\alpha)^5$ Lamb shift



$$\delta E_{a-s} = \frac{\alpha^2 (Z\alpha)^5}{\pi n^3} \left(\frac{\mu}{m}\right)^3 m \left[-7.72381(4)\right]$$

Dowling, Mondejar, Piclum, AC, PRA 81, 022509

Previous results

-7.61(16) Pachucki 1994
-7.724(1) Eides and Shelyuto, 1995

Summary

* Binding modifies the muon decay and the electron *g*-factor

* Theory of both effects is more fun than for free particles

* Synergy with beautiful experiments: lepton-flavor violation, mass of the electron and, in future, the fine structure constant.

* For *g*: $\alpha(Z\alpha)^5$ effects almost finished; $\alpha^2(Z\alpha)^5$ hopefully soon.

* Opportunities for more theoretical improvement...