Violation of the action-reaction principle in the van der Waals interaction of excited atoms

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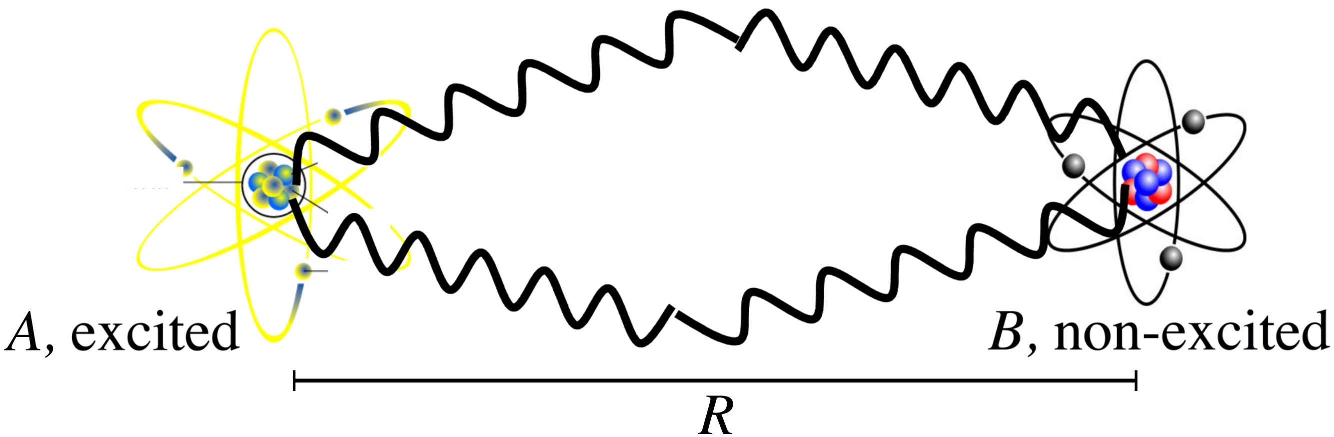
M. Donaire,
arXiv:1604.07071 (2016)

M. Donaire,
Phys. Rev. A**93**, 052706 (2016)

M. Donaire, R. Guerout, A. Lambrecht,
Phys.Rev.Lett. **115**, 033201 (2015)

Outline



0. Two-atom phase shift ≠ single atom phase shifts ≠ vdW potentials ≠ photon frequency shift

P.R. Berman,
Phys. Rev. A**91**, 042127 (2015)

P.Milonni & S.Rafsanjani,
Phys. Rev. A**92**, 062711 (2015)

M.Donaire, R.Guerout, A.Lambrecht,
Phys.Rev.Lett. **115**, 033201 (2015)

M. Donaire,
Phys. Rev. A**93**, 052706 (2016)

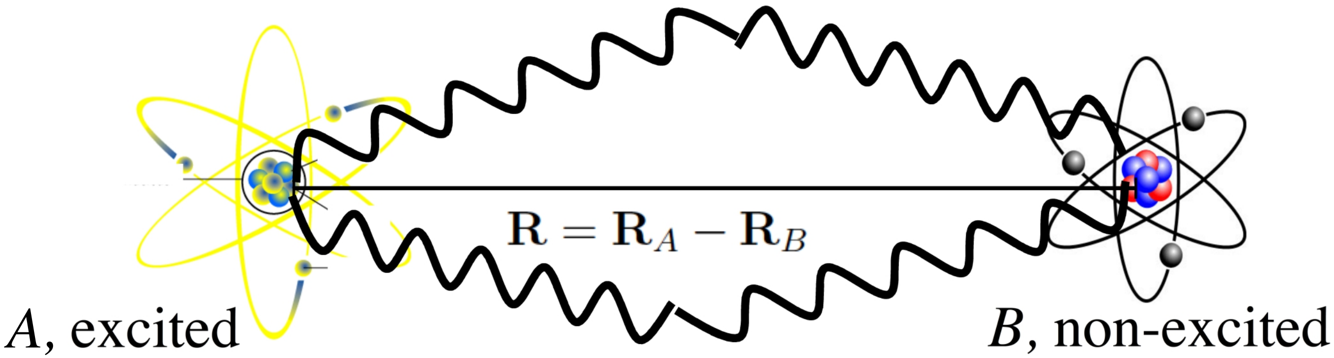
i. Computation of the dynamical vdW forces

M. Donaire,
Phys. Rev. A**93**, 052706 (2016)

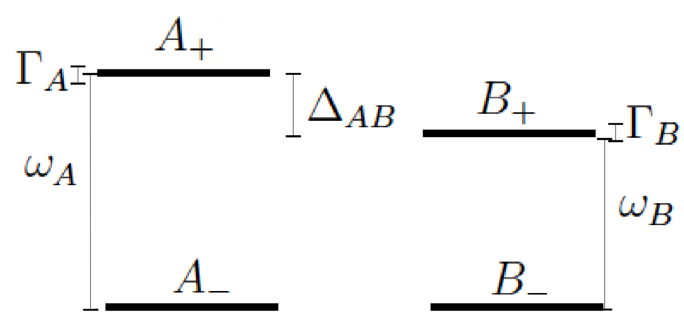
ii. Violation of the action-reaction principle

M. Donaire,
arXiv:1604.07071 (2016)

i- Computation of the dynamical vdW forces



$$H_0 = \hbar\omega_A |A_+\rangle\langle A_+| + \hbar\omega_B |B_+\rangle\langle B_+| + \sum_{\mathbf{k},\epsilon} \hbar\omega (a^\dagger_{\mathbf{k},\epsilon} a_{\mathbf{k},\epsilon} + 1/2)$$



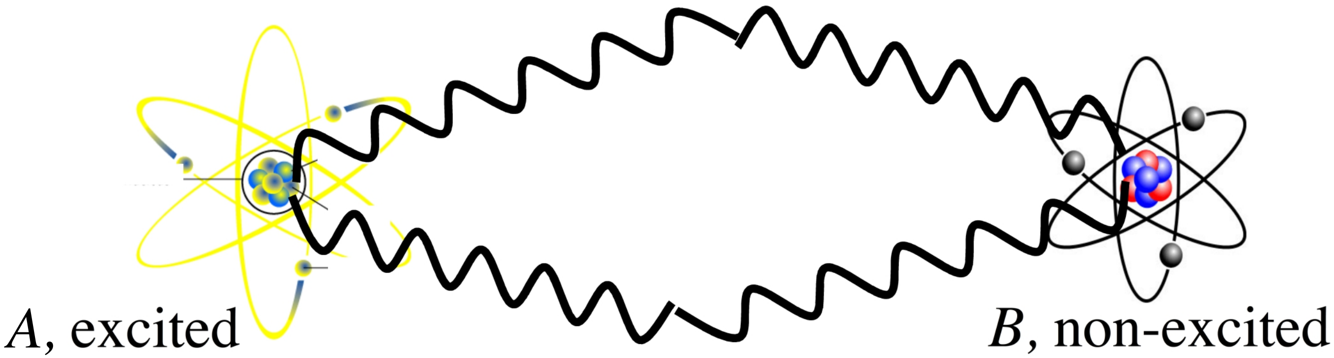
$$W = W_A + W_B = -\mathbf{d}_A \cdot \mathbf{E}(\mathbf{R}_A) - \mathbf{d}_B \cdot \mathbf{E}(\mathbf{R}_B)$$

$$\Delta_{AB} \equiv \omega_A - \omega_B$$

$$\Gamma_A \ll |\Delta_{AB}|$$

$$\mathbf{E}(\mathbf{R}_{A,B}) = i \sum_{\mathbf{k},\epsilon} \sqrt{\frac{\hbar c k}{2\mathcal{V}\epsilon_0}} [\epsilon a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}_{A,B}} - \epsilon^* a^\dagger_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}_{A,B}}]$$

i- Computation of the dynamical vdW forces



Why dynamical?

1. Finite life-times $\sim 1/\Gamma_A$

2. Periodic excitation transfer with probability $\sim \langle W \rangle / \hbar\Delta_{AB} \ll 1$

i- Computation of the dynamical vdW forces – Time-dependent pert. Th.

Wave-function formalism in the Schrödinger picture $|\Psi(T)\rangle = \mathbb{U}(T,0)|\Psi(0)\rangle$

$$\langle\mathcal{O}(T)\rangle = \langle\Psi(T)|\mathcal{O}|\Psi(T)\rangle = \langle\Psi(0)|\mathbb{U}^{\dagger}(T,0)\mathcal{O}\mathbb{U}(T,0)|\Psi(0)\rangle$$

$$\mathbb{U}(T,0) = \mathcal{T}\exp\left\{-i\hbar^{-1}\int_0^T \mathrm{d}t[H_0 + W]\right\}$$

i- Computation of the dynamical vdW forces – Time-dependent pert. Th.

Wave-function formalism in the Schrödinger picture $|\Psi(T)\rangle = \mathbb{U}(T,0)|\Psi(0)\rangle$

$$\langle \mathcal{O}(T)\rangle = \langle \Psi(T)|\mathcal{O}|\Psi(T)\rangle = \langle \Psi(0)|\mathbb{U}^{\dagger}(T,0)\mathcal{O}\mathbb{U}(T,0)|\Psi(0)\rangle$$

$$\mathbb{U}(T,0) = \mathcal{T}\exp\left\{-i\hbar^{-1}\int_0^T \mathrm{d}t[H_0 + W]\right\}$$

$$\mathbb{U}(T,0) = \mathbb{U}_0(T,0) + \delta\mathbb{U}(T,0)$$

$$\mathbb{U}_0(T,0) = \exp\left\{-i\hbar^{-1}H_0 T\right\}$$

$\delta\mathbb{U}^{(1)}(T)$

$$\delta\mathbb{U}(T) \simeq (-i/\hbar)\int_0^T \mathrm{d}t\,\mathbb{U}_0(T-t)W\mathbb{U}_0(t)$$

$\delta\mathbb{U}^{(2)}(T)$

$$+ (-i/\hbar)^2\int_0^T \mathrm{d}t\int_0^t \mathrm{d}t'\mathbb{U}_0(T-t)W\mathbb{U}_0(t-t')W\mathbb{U}_0(t')$$

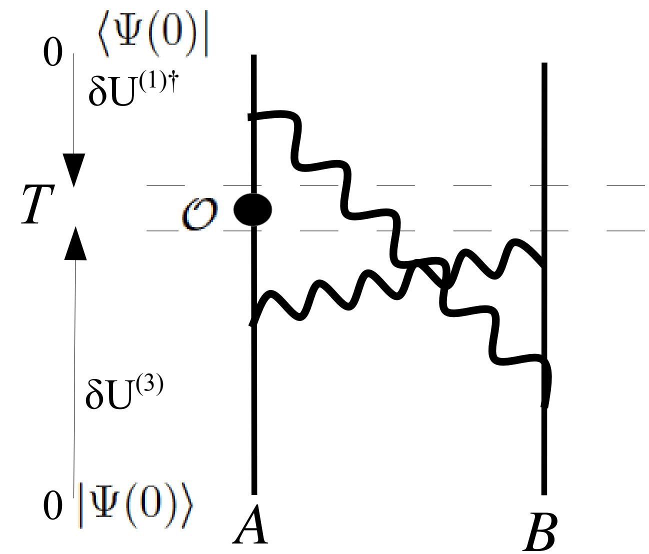
$$+ (-i/\hbar)^3\int_0^T \mathrm{d}t\int_0^t \mathrm{d}t'\int_0^{t'} \mathrm{d}t''\mathbb{U}_0(T-t)W\mathbb{U}_0(t-t')W\mathbb{U}_0(t'-t'')W\mathbb{U}_0(t'')$$

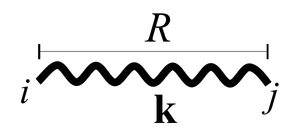
$\delta\mathbb{U}^{(3)}(T)$

i- Computation of the dynamical vdW forces – Time-dependent pert. Th.

Wave-function formalism in the Schrödinger picture $|\Psi(T)\rangle = \mathbb{U}(T,0)|\Psi(0)\rangle$

$$\langle\mathcal{O}(T)\rangle = \langle\Psi(T)|\mathcal{O}|\Psi(T)\rangle = \langle\Psi(0)|\mathbb{U}^{\dagger}(T,0)\mathcal{O}\mathbb{U}(T,0)|\Psi(0)\rangle$$





$\mathrm{Im}G_{ij}(kR)$

$\delta\mathbb{U}^{(1)}(T)$

$$\delta\mathbb{U}(T) \simeq (-i/\hbar)\int_0^T \mathrm{d}t\, \mathbb{U}_0(T-t)W\mathbb{U}_0(t)$$

$\delta\mathbb{U}^{(2)}(T)$

$$+ (-i/\hbar)^2\int_0^T \mathrm{d}t\int_0^t \mathrm{d}t'\mathbb{U}_0(T-t)W\mathbb{U}_0(t-t')W\mathbb{U}_0(t')$$

$$+ (-i/\hbar)^3\int_0^T \mathrm{d}t\int_0^t \mathrm{d}t'\int_0^{t'} \mathrm{d}t''\mathbb{U}_0(T-t)W\mathbb{U}_0(t-t')W\mathbb{U}_0(t'-t'')W\mathbb{U}_0(t'')$$

$\delta\mathbb{U}^{(3)}(T)$

i- Computation of the dynamical vdW forces

Van der Waals force and potential on atom *A*

$$\begin{aligned}\langle \mathbf{F}_A(T)\rangle &= \partial_T\langle \mathbf{P}_A(T)\rangle = -i\hbar\partial_T\langle\Psi(0)|\mathbb{U}^\dagger(T)\nabla_{\mathbf{R}_A}\mathbb{U}(T)|\Psi(0)\rangle + \cancel{O(v/c)}\\ &= -\langle\nabla_{\mathbf{R}_A}W_A(T,\mathbf{R}_A)\rangle\end{aligned}$$

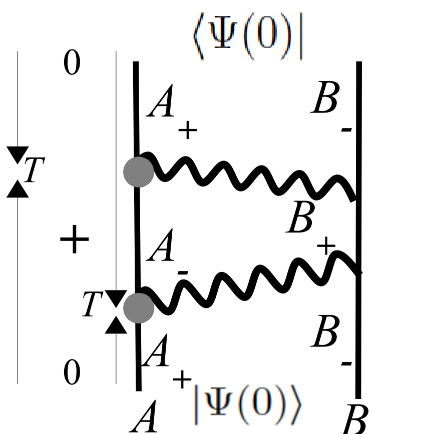
Idem for atom B, vdW potential on atom B

$$\langle \mathbf{F}_B(T)\rangle = -\langle\nabla_{\mathbf{R}_B}W_B(T,\mathbf{R}_B)\rangle$$

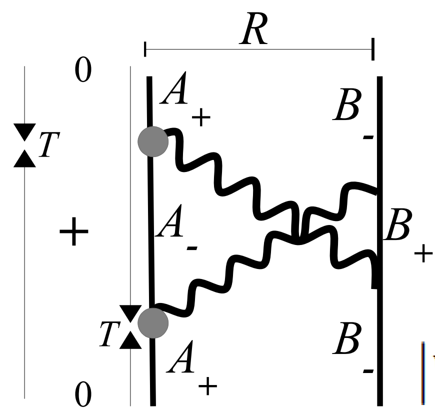
i- Computation of the dynamical vdW forces

● $= -\nabla_{\mathbf{R}_A} W_A$

$$\langle \mathbf{F}_A \rangle =$$

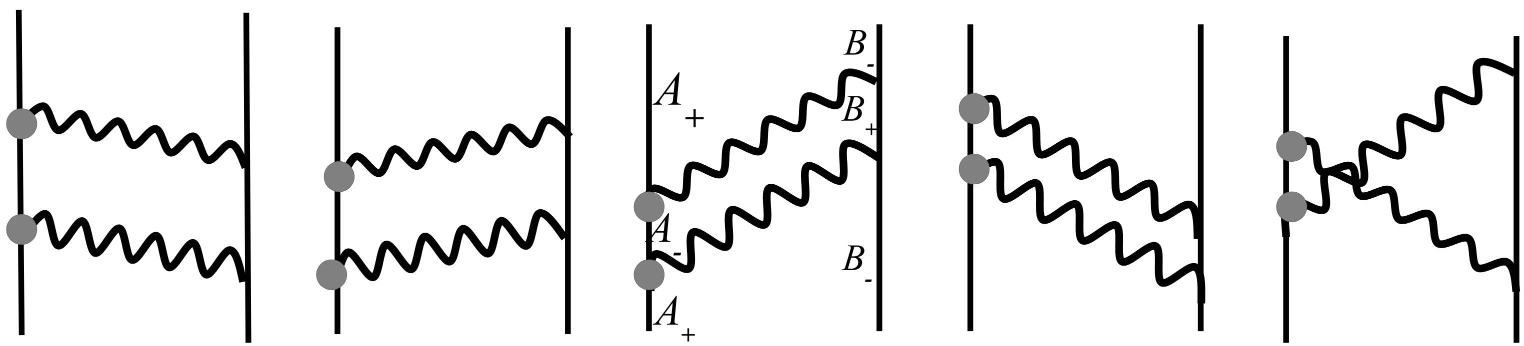


two doubly-resonant photons

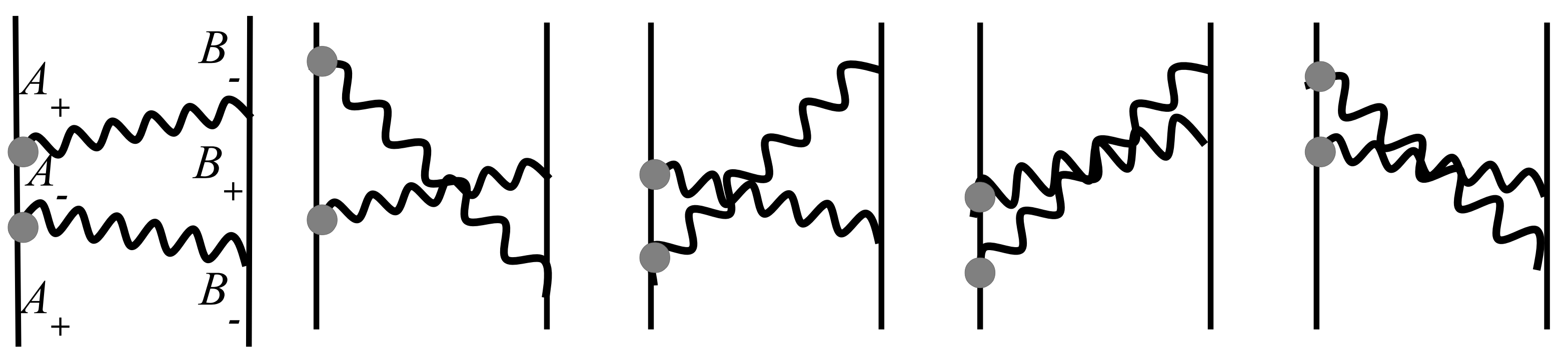


two simply-resonant photons

$$|\Psi(0)\rangle = |A_+\rangle \otimes |B_-\rangle \otimes |0_\gamma\rangle$$



one doubly-resonant photons

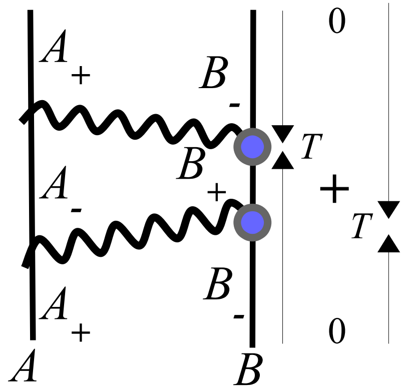


no resonant photons

i- Computation of the dynamical vdW forces

● $= -\nabla_{\mathbf{R}_B} W_B$

$\langle \mathbf{F}_B \rangle =$



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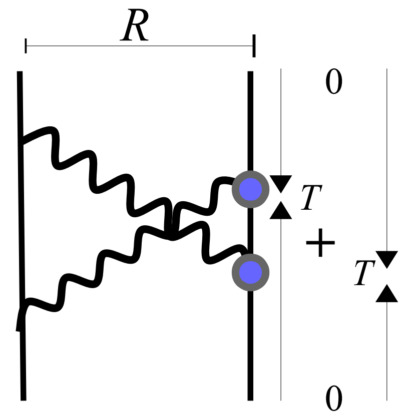

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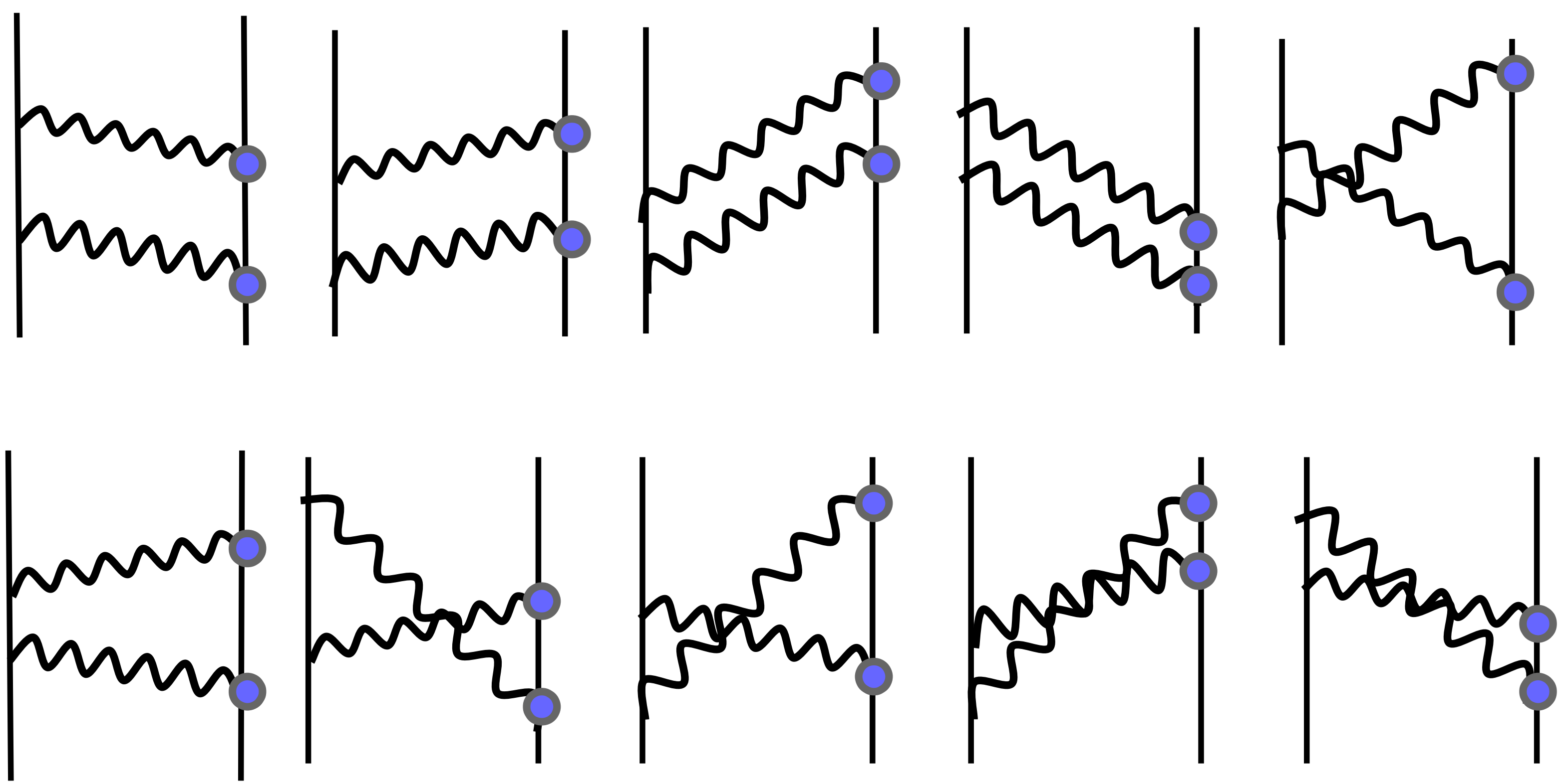


i- Computation of the dynamical vdW forces

Relevant diagrams for action-reaction violation

1. Non-resonant photons mediate the reciprocal exchange of momentum between the atoms, and generate the usual London and Casimir-Polder forces,

whereas

2. resonant photons mediate the transfer of the excitation between the atoms, and 'may' carry momentum off the two-atom system–see later–

1.&2. => Only diagrams with resonant photons may lead to action-reaction violation

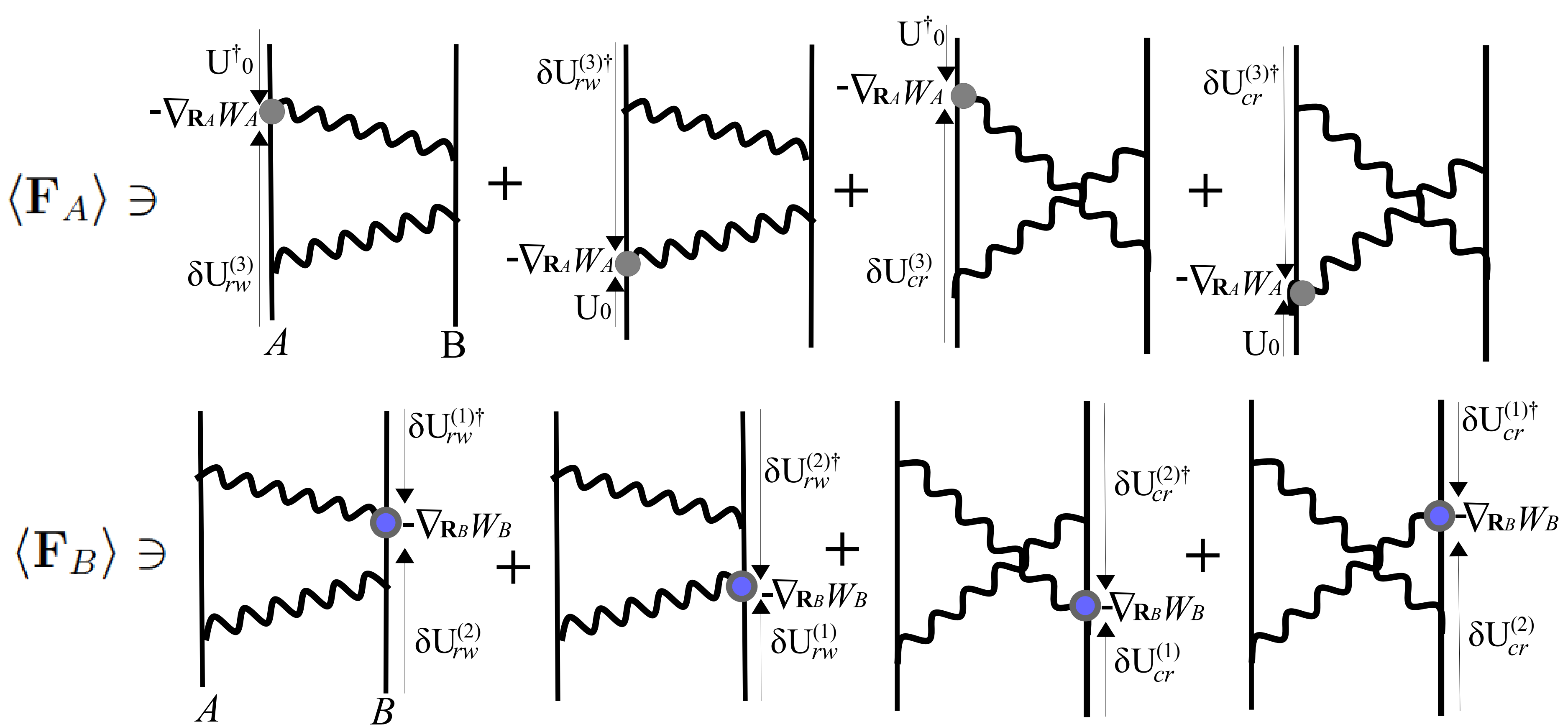
i- Computation of the dynamical vdW forces

Relevant diagrams for action-reaction violation

3. Under realistic conditions (adiabatic excitation), only the following diagrams survive

Rotating wave (*rw*) diagram $\sim \Delta_{AB}^{-1}$

Counter-rotating (*cr*) diagram $\sim (\omega_A + \omega_B)^{-1}$



i- Computation of the dynamical vdW forces

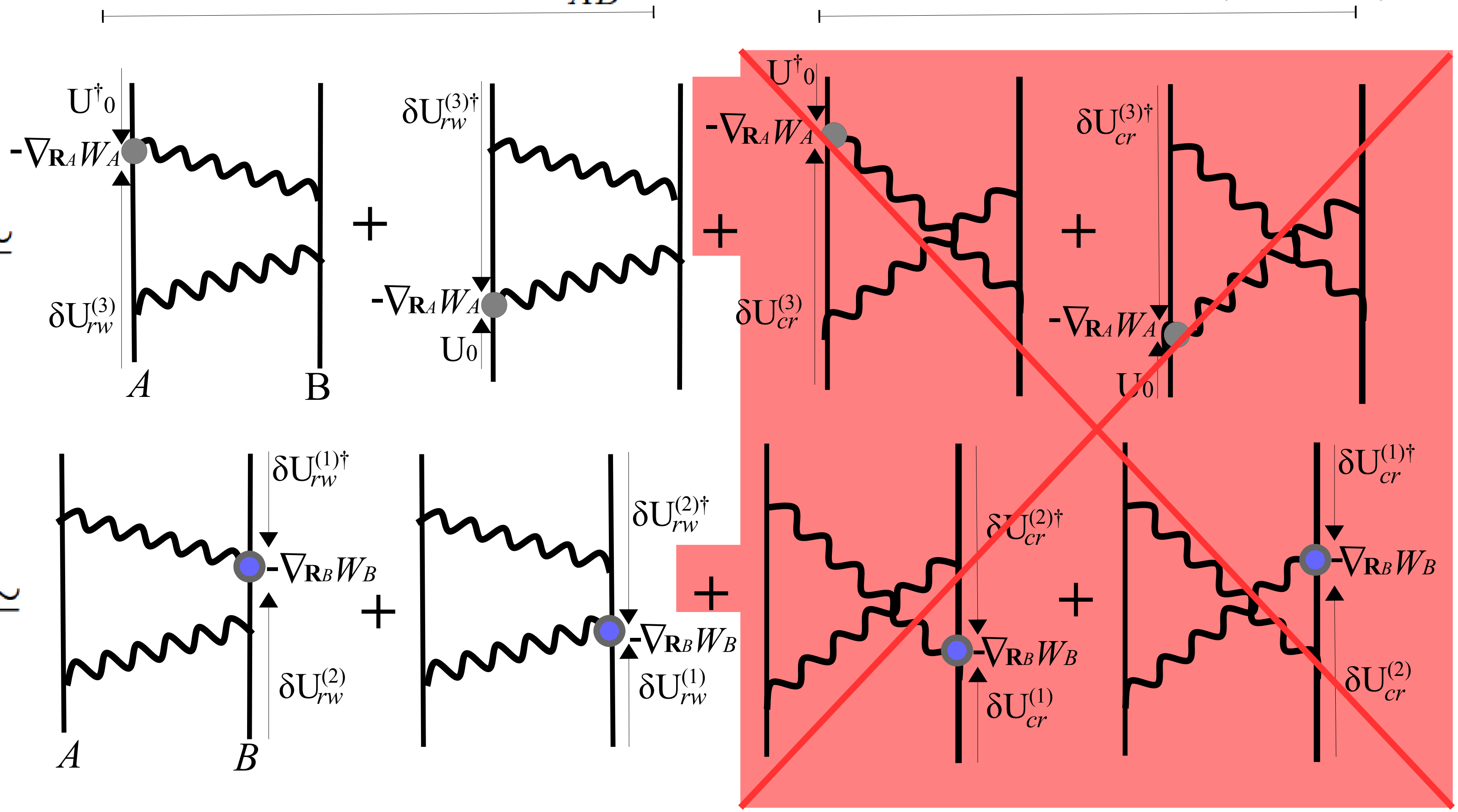
Quasiresonant approximation ~Rotating wave approximation

$$\Delta_{AB} \ll \omega_A, \omega_B$$

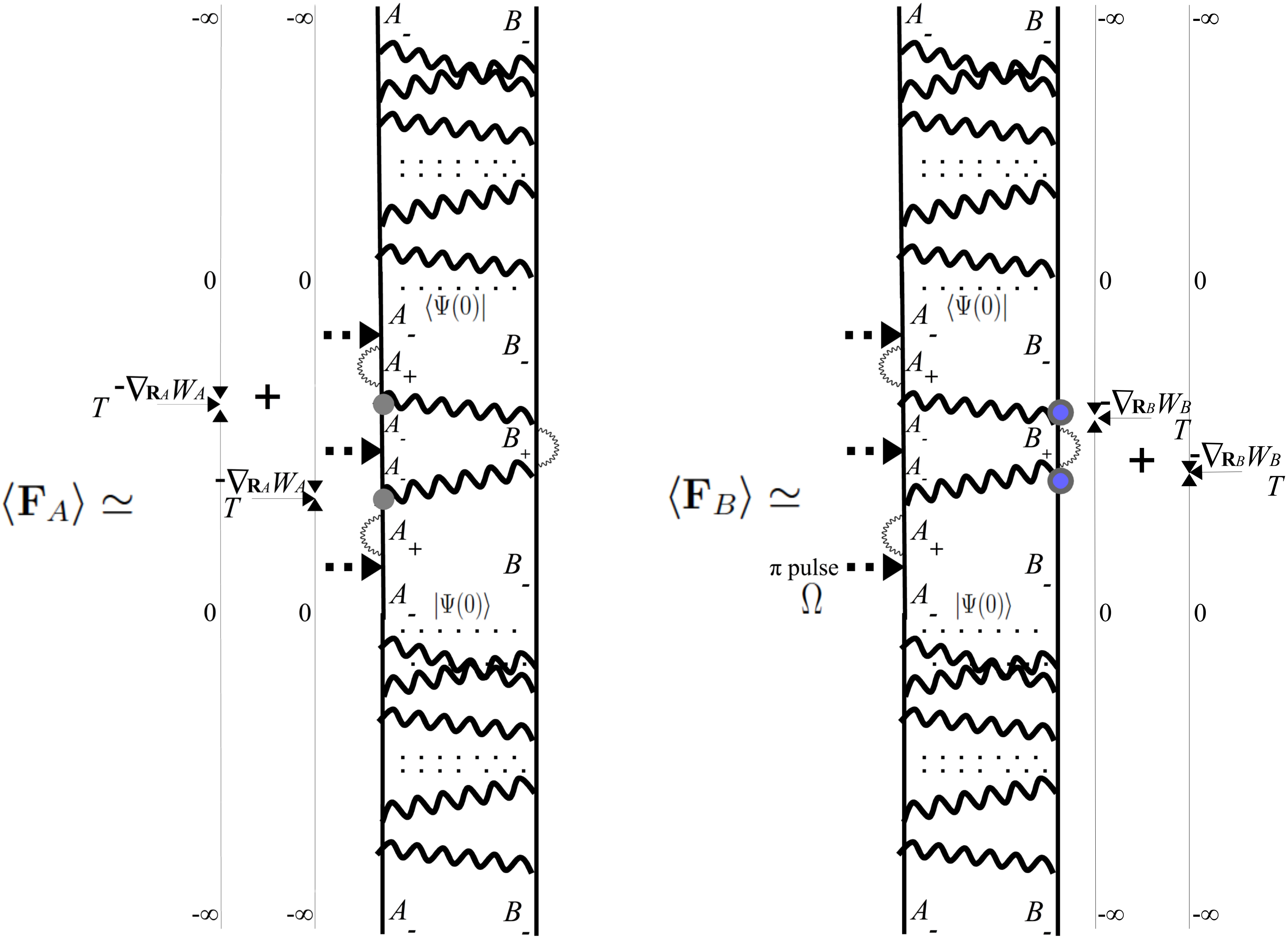
Rotating wave (*rw*) diagram $\sim \Delta_{AB}^{-1}$ Counter-rotating (*cr*) diagram $\sim (\omega_A + \omega_B)^{-1}$

$\langle \mathbf{F}_A \rangle \simeq$

$\langle \mathbf{F}_B \rangle \simeq$

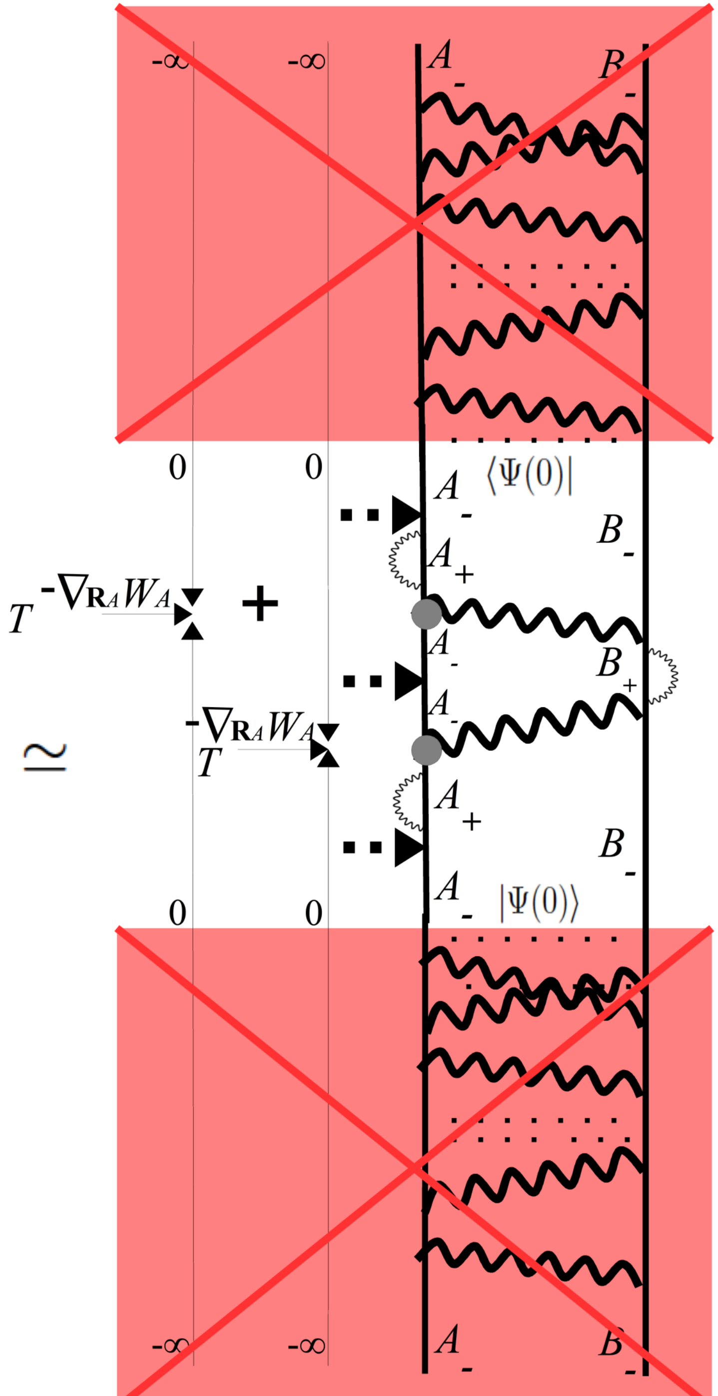


i- Computation of the dynamical vdW forces



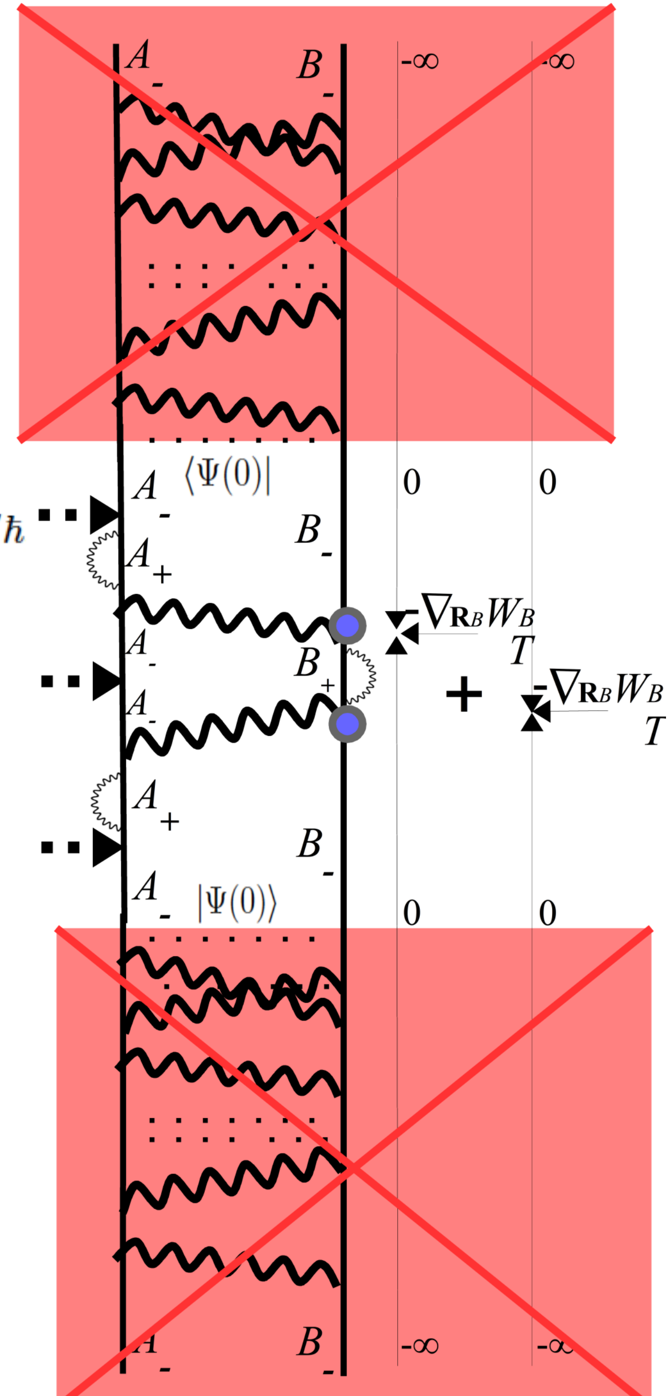
i- Computation of the dynamical vdW forces

$\langle \mathbf{F}_A \rangle \simeq$



weak coupling,
$|\Delta_{AB}| \gg \langle W(T)\rangle/\hbar$

$\langle \mathbf{F}_B \rangle \simeq$



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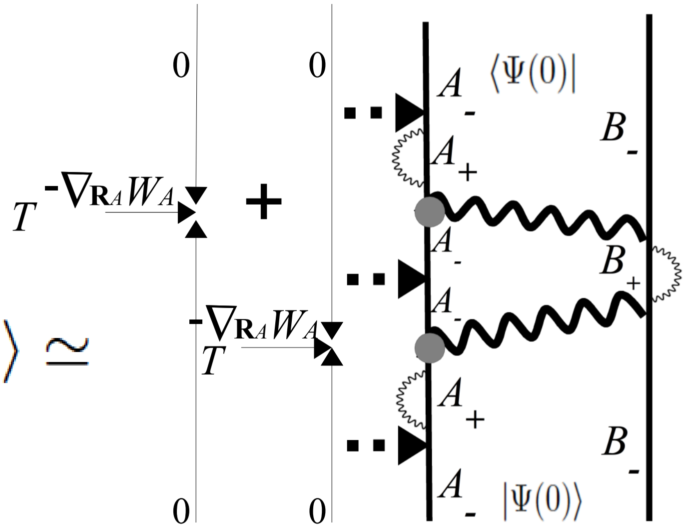
i- Computation of the dynamical vdW forces

$\langle \mathbf{F}_A \rangle \simeq$

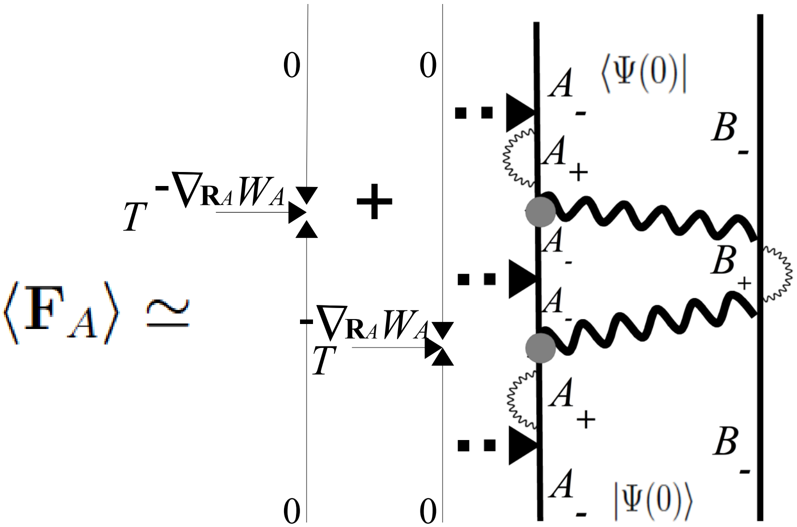


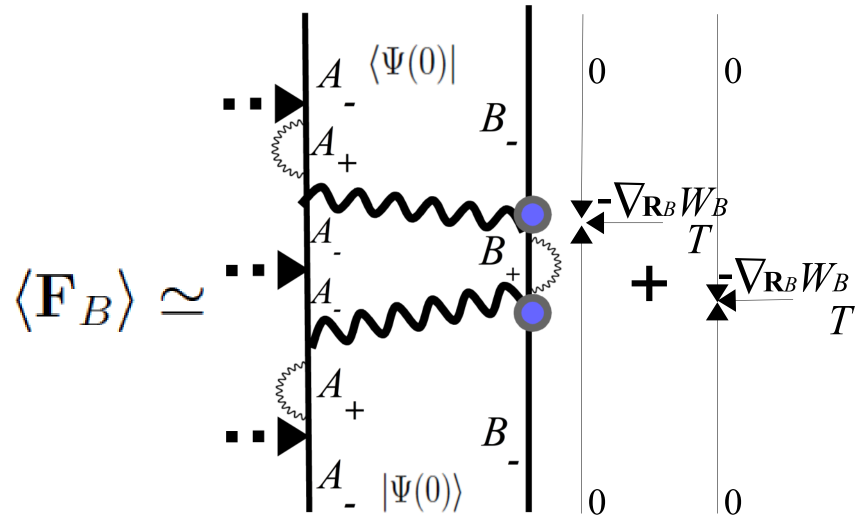
$$\langle \mathbf{F}_A(T)\rangle \simeq \mathcal{U}^{ijpq}\Big\{k_A^4 e^{-\Gamma_A T}\nabla_{\mathbf{R}}\Big[\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R},\omega_A) - \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_A)\Big]$$
$$+\frac{k_B^4\Omega^2 e^{-(\Gamma_A+\Gamma_B)T/2}}{2(\Delta_{AB}^2-\Omega^2)}\nabla_{\mathbf{R}}\Big[\Big(\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_B)\mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R},\omega_B) - \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R},\omega_B)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_B)\Big)$$
$$\times\left(\cos\left(\Delta_{AB}T\right)+\cos\left[\Delta_{AB}(T-\pi/\Omega)\right]\right) - 2\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_B)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_B)$$
$$\times\left(\sin\left(\Delta_{AB}T\right)+\sin\left[\Delta_{AB}(T-\pi/\Omega)\right]\right)\Big]\Big\}$$

$$\mathbb{G}^{(0)}(kR) = \frac{k\, e^{ikR}}{-4\pi}[\gamma/kR + i\beta/(kR)^2 - \beta/(kR)^3] \qquad \gamma = \mathbb{I} - \mathbf{RR}/R^2,\ \beta = \mathbb{I} - 3\mathbf{RR}/R^2$$

$$\mathcal{U}^{ijpq} = d_A^i d_B^j d_B^p d_A^q/\epsilon_0^2\hbar\Delta_{AB}$$

i- Computation of the dynamical vdW forces





$$
\begin{aligned}
\langle \mathbf{F}_A(T)\rangle \simeq \mathcal{U}^{ijpq}\Big\{ & k_A^4 e^{-\Gamma_A T}\nabla_{\mathbf{R}}\Big[\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R},\omega_A) - \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_A)\Big] \\
& + \frac{k_B^4\Omega^2 e^{-(\Gamma_A+\Gamma_B)T/2}}{2(\Delta_{AB}^2-\Omega^2)}\nabla_{\mathbf{R}}\Big[\Big(\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_B)\mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R},\omega_B) - \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R},\omega_B)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_B)\Big) \\
& \times \big(\cos(\Delta_{AB}T) + \cos[\Delta_{AB}(T-\pi/\Omega)]\big) - 2\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_B)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_B) \\
& \times \big(\sin(\Delta_{AB}T) + \sin[\Delta_{AB}(T-\pi/\Omega)]\big)\Big]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathbf{F}_B(T)\rangle \simeq -\mathcal{U}^{ijpq}\Big\{ & k_A^4 e^{-\Gamma_A T}\nabla_{\mathbf{R}}\Big[\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R},\omega_A) + \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_A)\Big] \\
& + \frac{k_A^2 k_B^2\Omega^2 e^{-(\Gamma_A+\Gamma_B)T/2}}{2(\Delta_{AB}^2-\Omega^2)}\nabla_{\mathbf{R}}\Big[\Big(\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R},\omega_B) + \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_B)\Big) \\
& \times \big(\cos(\Delta_{AB}T) + \cos[\Delta_{AB}(T-\pi/\Omega)]\big) - \Big(\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R},\omega_B) \\
& - \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R},\omega_A)\mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R},\omega_B)\Big)\big(\sin(\Delta_{AB}T) + \sin[\Delta_{AB}(T-\pi/\Omega)]\big)\Big]\Big\}
\end{aligned}
$$

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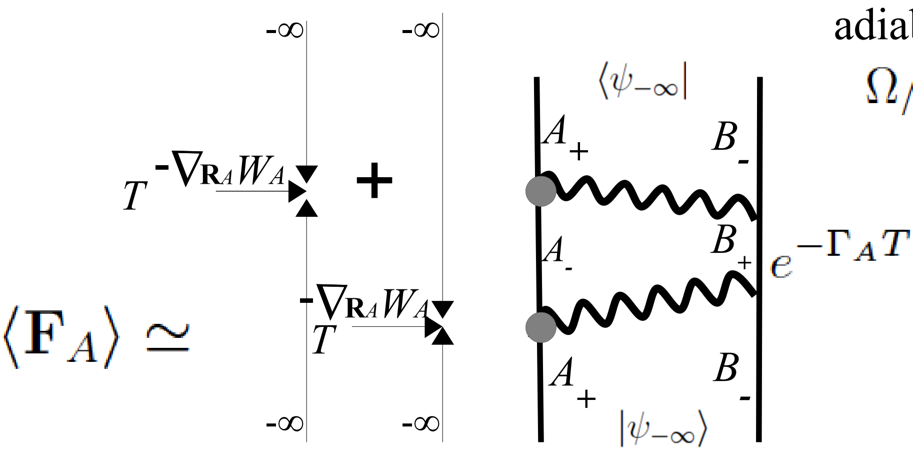
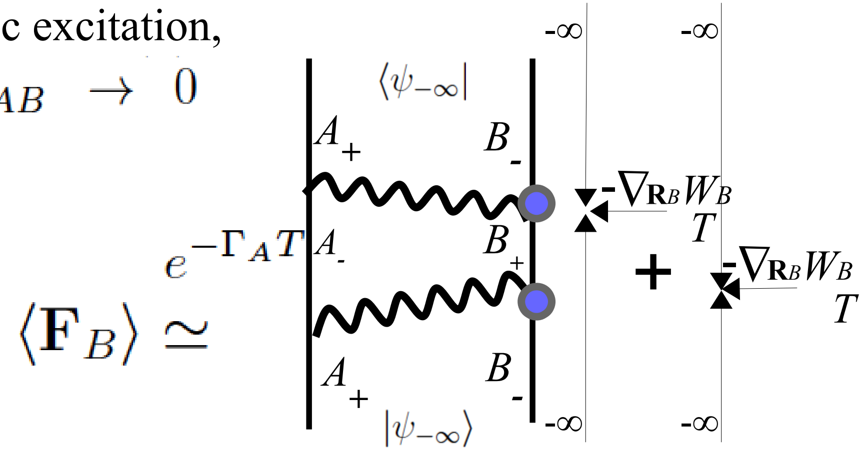
i- Computation of the dynamical vdW forces

adiabatic excitation,

$\Omega/\Delta_{AB} \rightarrow 0$





$$W \rightarrow W \exp \eta t, \eta \rightarrow 0^+$$

$$\langle \mathbf{F}_A(T) \rangle \simeq \mathcal{U}^{ijpq} \; k_A^4 e^{-\Gamma_A T} \nabla_{\mathbf{R}} \left[\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R}, \omega_A) \mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R}, \omega_A) - \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R}, \omega_A) \mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R}, \omega_A) \right]$$

$$\langle \mathbf{F}_B(T) \rangle \simeq -\mathcal{U}^{ijpq} \; k_A^4 e^{-\Gamma_A T} \nabla_{\mathbf{R}} \left[\mathrm{Re}\, G_{ij}^{(0)}(\mathbf{R}, \omega_A) \mathrm{Re}\, G_{pq}^{(0)}(\mathbf{R}, \omega_A) + \mathrm{Im}\, G_{ij}^{(0)}(\mathbf{R}, \omega_A) \mathrm{Im}\, G_{pq}^{(0)}(\mathbf{R}, \omega_A) \right]$$

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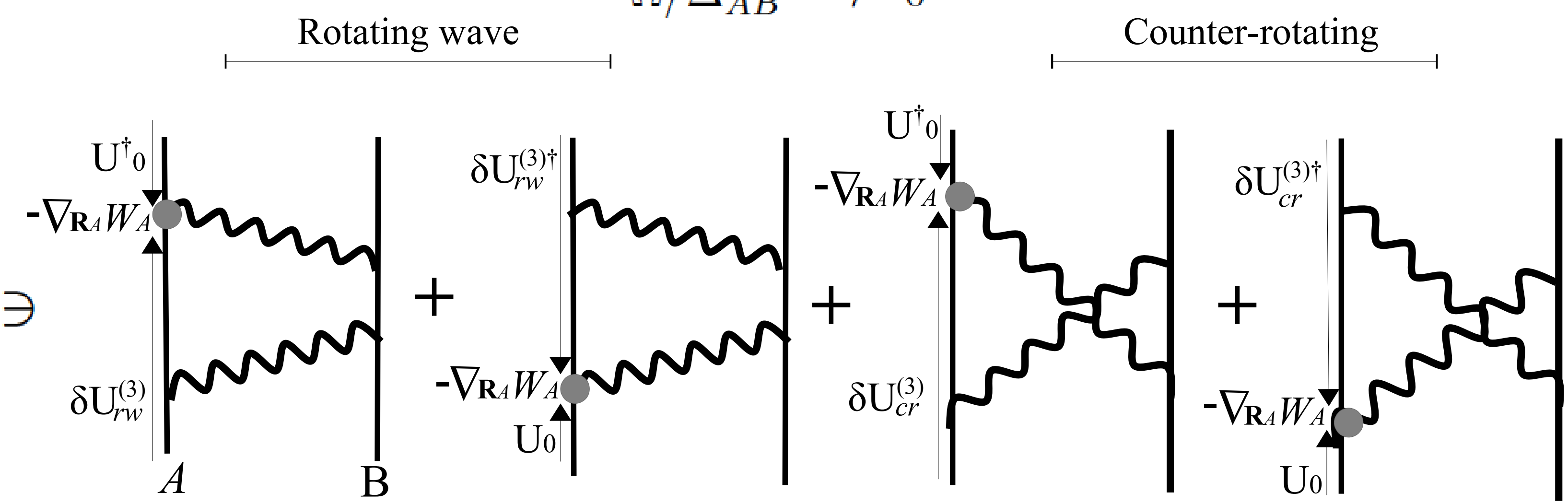
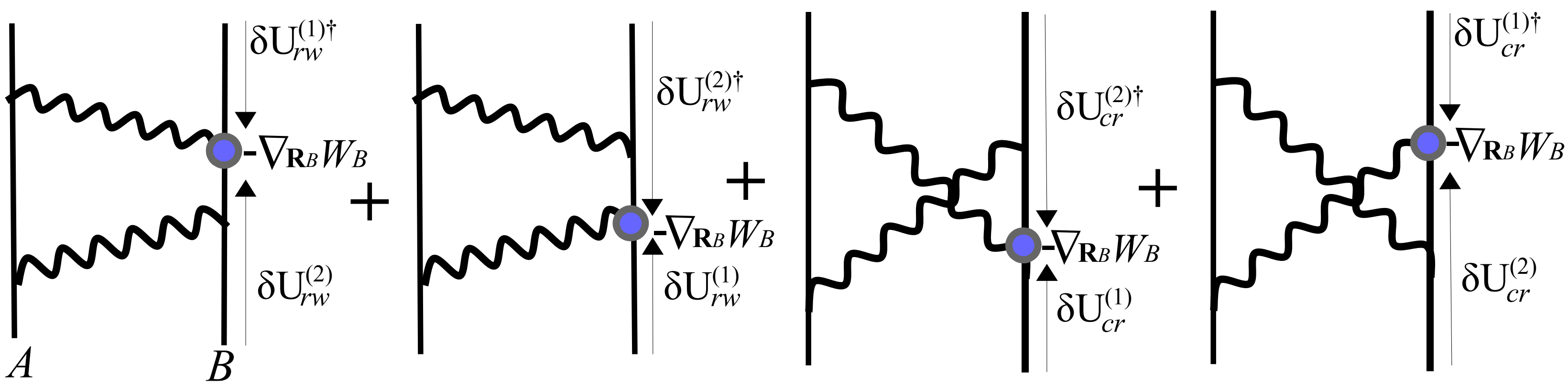
i- Computation of the dynamical vdW forces

adiabatic excitation,
$\Omega/\Delta_{AB} \;\to\; 0$

$\langle \mathbf{F}_A \rangle \ni$



$\langle \mathbf{F}_B \rangle \ni$



$$\langle \mathbf{F}_A + \mathbf{F}_B \rangle = \frac{2\omega_B \Delta_{AB}}{\omega_A^2 - \omega_B^2} \mathcal{U}^{ijpq} k_A^4 e^{-\Gamma_A T} \nabla_{\mathbf{R}} [\mathrm{Im} G^{(0)}_{ij}(\mathbf{R}, \omega_A) \mathrm{Im} G^{(0)}_{pq}(\mathbf{R}, \omega_A)]$$

=> **Violation of action-reaction**: Q1. Why the rhs is non-zero?
Q2. Where momentum goes?

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ii- Violation of action-reaction. Q2. Where does the momentum go?

A2. ….. to the EM vacuum (total momentum conservation)

Time-variation of vacuum momentum = - Total force on the system

$$\mathbf{P} = \sum_{\mathbf{k},\epsilon} \hbar\mathbf{k}(a^{\dagger}_{\mathbf{k},\epsilon}a_{\mathbf{k},\epsilon} + 1/2) \qquad \langle\dot{\mathrm{P}}\rangle = -\langle\mathbf{F}_A + \mathbf{F}_B\rangle \qquad \langle\dot{\mathrm{P}}\rangle = \langle\dot{\mathrm{P}}^{1\gamma}_{rw}\rangle + \langle\dot{\mathrm{P}}^{1\gamma}_{cr}\rangle + \langle\dot{\mathrm{P}}^{2\gamma}_{cr}\rangle$$

ii- Violation of action-reaction. Q2. Where does momentum go?

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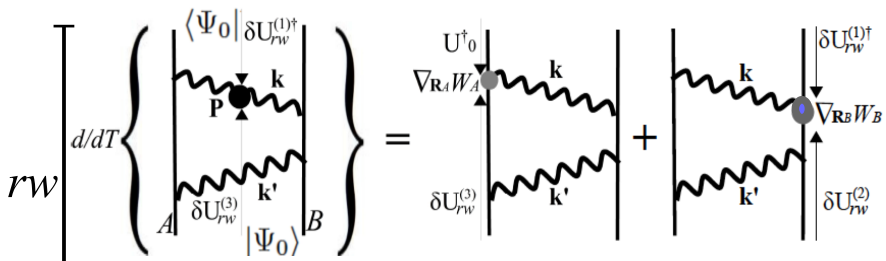
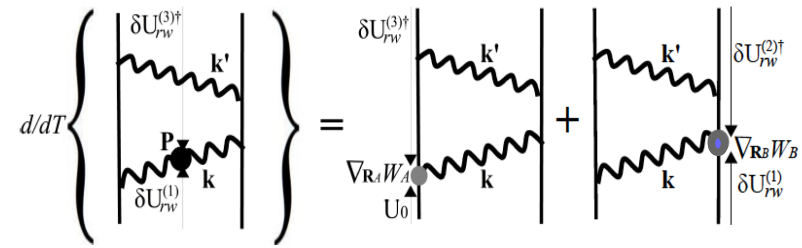
- $\mathbf{P} = \sum_{\mathbf{k},\epsilon} \hbar\mathbf{k}(a^{\dagger}_{\mathbf{k},\epsilon} a_{\mathbf{k},\epsilon} + 1/2)$ $\quad \langle\dot{\mathbf{P}}\rangle = -\langle\mathbf{F}_A + \mathbf{F}_B\rangle \quad \langle\dot{\mathbf{P}}\rangle = \langle\dot{\mathbf{P}}^{1\gamma}_{rw}\rangle + \langle\dot{\mathbf{P}}^{1\gamma}_{cr}\rangle + \langle\dot{\mathbf{P}}^{2\gamma}_{cr}\rangle$





$$\langle\dot{\mathrm{P}}\rangle^{1\gamma}_{rw} = \sum_{\mathrm{k}} \partial_T \langle\Psi_0|\delta\mathbb{U}^{\dagger(1)}_{rw}(T,\mathrm{k})\hbar\mathrm{k}\delta\mathbb{U}^{(3)}_{rw}(T,\mathrm{k})|\Psi_0\rangle + \mathrm{c.c.} = \sum_{\mathrm{k}}\Big[\langle\Psi_0|\mathbb{U}^{\dagger}_0(T)\nabla_{\mathrm{R}_A}W_{A,\mathrm{k}}\delta\mathbb{U}^{(3)}_{rw}(T,\mathrm{k})|\Psi_0\rangle + \langle\Psi_0|\delta\mathbb{U}^{\dagger(1)}_{rw}(T,\mathrm{k})\nabla_{\mathrm{R}_B}W_{B,\mathrm{k}}\delta\mathbb{U}^{(2)}_{rw}(T)|\Psi_0\rangle\Big] + \mathrm{c.c.}$$

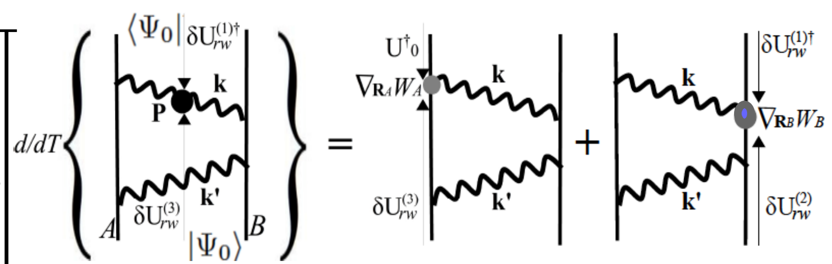
ii- Violation of action-reaction. Q2. Where does momentum go?

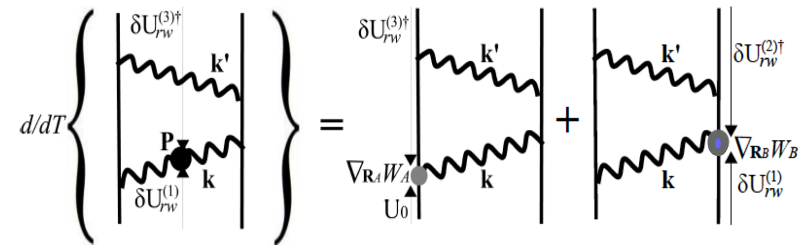
A2. ….. to the EM vacuum (total momentum conservation)

Time-variation of vacuum momentum = - Total force on the system

$\bullet\ \mathbf{P} = \sum_{\mathbf{k},\epsilon} \hbar\mathbf{k}(a^\dagger_{\mathbf{k},\epsilon} a_{\mathbf{k},\epsilon} + 1/2)$ $\qquad \langle \dot{\mathbf{P}}\rangle = -\langle \mathbf{F}_A + \mathbf{F}_B\rangle$ $\qquad \langle \dot{\mathbf{P}}\rangle = \langle \dot{\mathbf{P}}^{1\gamma}_{rw}\rangle + \langle \dot{\mathbf{P}}^{1\gamma}_{cr}\rangle + \langle \dot{\mathbf{P}}^{2\gamma}_{cr}\rangle$

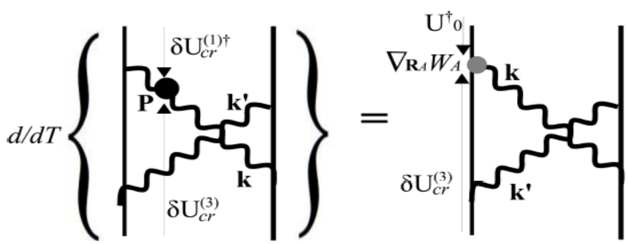
rw

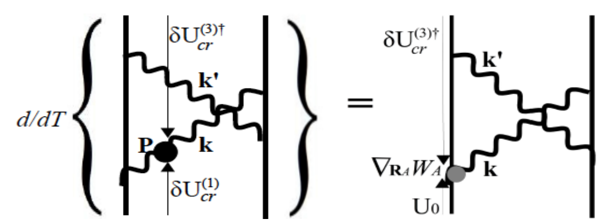




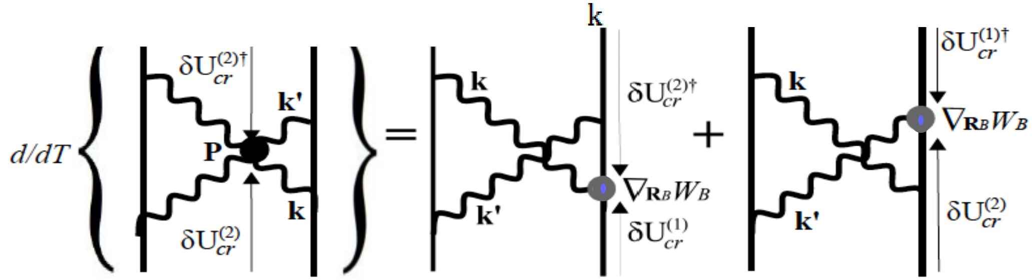
$$\langle\dot{\mathrm{P}}\rangle^{1\gamma}_{rw} = \sum_{\mathrm{k}} \partial_T \langle\Psi_0|\delta\mathbb{U}^{\dagger(1)}_{rw}(T,\mathrm{k})\hbar\mathrm{k}\,\delta\mathbb{U}^{(3)}_{rw}(T,\mathrm{k})|\Psi_0\rangle + \mathrm{c.c.} = \sum_{\mathrm{k}}\Big[\langle\Psi_0|\mathbb{U}^\dagger_0(T)\nabla_{\mathrm{R}_A}W_{A,\mathrm{k}}\delta\mathbb{U}^{(3)}_{rw}(T,\mathrm{k})|\Psi_0\rangle + \langle\Psi_0|\delta\mathbb{U}^{\dagger(1)}_{rw}(T,\mathrm{k})\nabla_{\mathrm{R}_B}W_{B,\mathrm{k}}\delta\mathbb{U}^{(2)}_{rw}(T)|\Psi_0\rangle\Big] + \mathrm{c.c.}$$

cr





$$\langle\dot{\mathrm{P}}\rangle^{1\gamma}_{cr} = \sum_{\mathrm{k}} \partial_T \langle\Psi_0|\delta\mathbb{U}^{\dagger(1)}_{cr}(T,\mathrm{k})\hbar\mathrm{k}\,\delta\mathbb{U}^{(3)}_{cr}(T,\mathrm{k})|\Psi_0\rangle + \mathrm{c.c.} = \sum_{\mathrm{k}}\langle\Psi_0|\mathbb{U}^\dagger_0(T)\nabla_{\mathrm{R}_A}W_{A,\mathrm{k}}\delta\mathbb{U}^{(3)}_{cr}(T,\mathbf{k})|\Psi_0\rangle + \mathrm{c.c.}$$

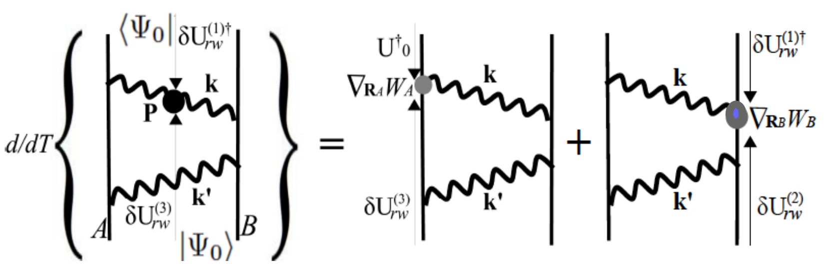


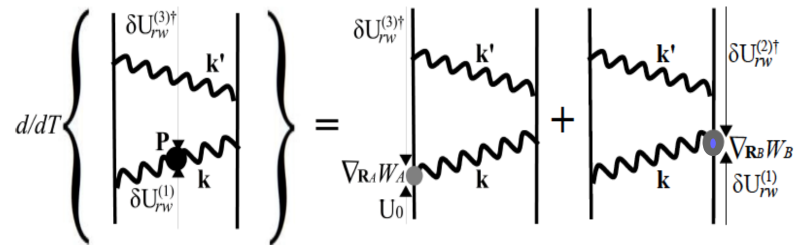
$$\langle\dot{\mathbf{P}}\rangle^{2\gamma}_{cr} = \sum_{\mathbf{k},\mathbf{k}'}\partial_T\langle\Psi_0|\delta\mathbb{U}^{\dagger(2)}_{cr}(T,\mathbf{k}',\mathbf{k})\hbar(\mathbf{k}+\mathbf{k}')\delta\mathbb{U}^{(2)}_{cr}(T,\mathbf{k},\mathbf{k}')|\Psi_0\rangle = \sum_{\mathrm{k},\mathrm{k}'}\langle\Psi_0|\delta\mathbb{U}^{\dagger(2)}_{cr}(T,\mathrm{k}',\mathrm{k})\nabla_{\mathrm{R}_B}W_{B,\mathrm{k}}\delta\mathbb{U}^{(1)}_{cr}(T,\mathrm{k})|\Psi_0\rangle + \mathrm{c.c.}$$

ii- Violation of action-reaction. Q1. Why does vacuum momentum not vanish?

A1. ….Optical theorem on 'will-be' emitted photons (conservation of total emission probability)

● $\mathbf{P} = \sum_{\mathbf{k},\epsilon} \hbar\mathbf{k}(a^{\dagger}_{\mathbf{k},\epsilon} a_{\mathbf{k},\epsilon} + 1/2)$ $\quad \langle \dot{\mathbf{P}} \rangle = -\langle \mathbf{F}_A + \mathbf{F}_B \rangle$ $\quad \langle \dot{\mathbf{P}} \rangle = \langle \dot{\mathbf{P}}^{1\gamma}_{rw} \rangle + \langle \dot{\mathbf{P}}^{1\gamma}_{cr} \rangle + \langle \dot{\mathbf{P}}^{2\gamma}_{cr} \rangle$





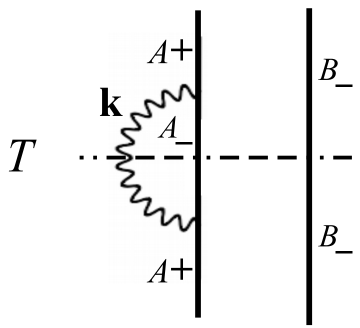
$$\langle \dot{\mathbf{P}} \rangle^{1\gamma}_{rw} = \sum_{\mathbf{k}} \hbar\mathbf{k}\partial_T \langle \Psi_0 | \delta\mathbb{U}^{\dagger(1)}_{rw}(T,\mathbf{k}) \delta\mathbb{U}^{(3)}_{rw}(T,\mathbf{k}) | \Psi_0 \rangle + \mathrm{c.c.} = \underbrace{\sum_{\mathbf{k}} \hbar\mathbf{k}\dot{\mathcal{P}}^{1\gamma}_{rw}(\mathbf{k})}$$

$\neq 0 \Rightarrow \dot{\mathcal{P}}^{1\gamma}(\mathbf{k})$ non-invariant under **k** → **-k**

ii- Violation of action-reaction. Q1. Why does vacuum momentum not vanish?

One-photon emission probability $\mathcal{P}_{rw}^{1\gamma} = 1$

P.R. Berman,
Phys. Rev. A**76**, 043816 (2007)



$$+ \underbrace{\cdots\cdots}_{0}$$

$T \rightarrow \infty$

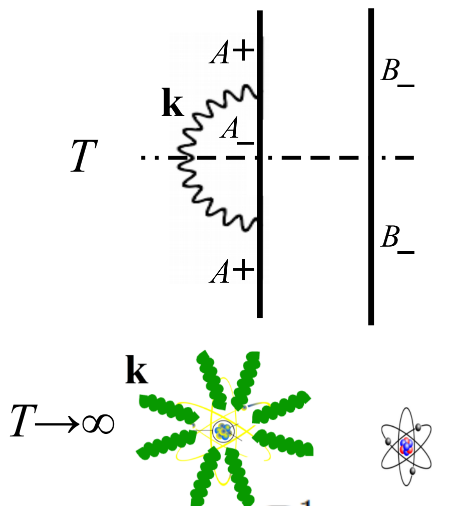
$\mathcal{P}^{1\gamma} = 1$

free-space emission

ii- Violation of action-reaction. Q1. Why does vacuum momentum not vanish?

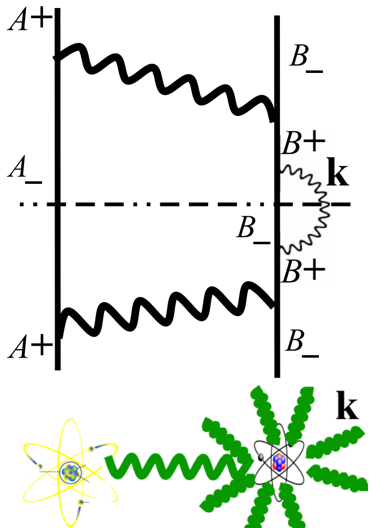
One-photon emission probability $\mathcal{P}^{1\gamma}_{rw} = 1$

P.R. Berman,
Phys. Rev. A**76**, 043816 (2007)



$\mathcal{P}^{1\gamma}_{rw} = 1$

free-space emission



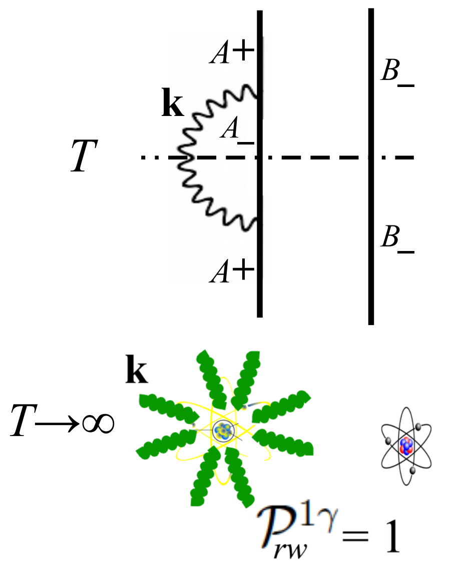
$\mathcal{O}(\Gamma^2_A/\Delta^2_{AB})$

scattering off B

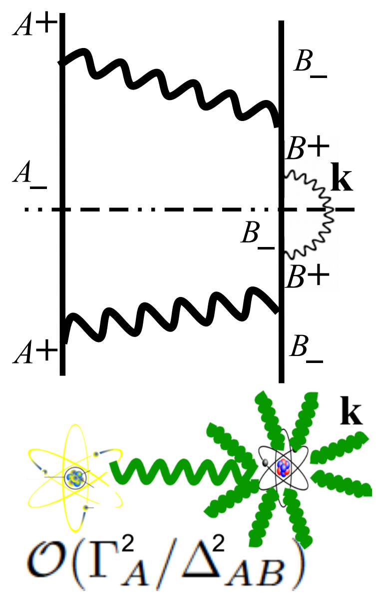
ii- Violation of action-reaction. Q1. Why does vacuum momentum not vanish?

One-photon emission probability $\mathcal{P}^{1\gamma}_{rw} = 1$

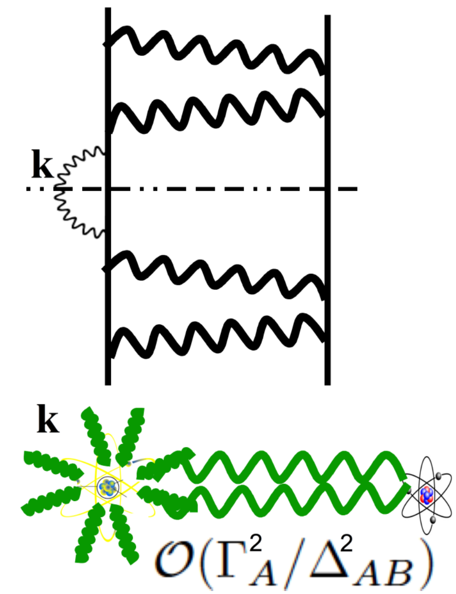
P.R. Berman,
Phys. Rev. A**76**, 043816 (2007)



free-space emission



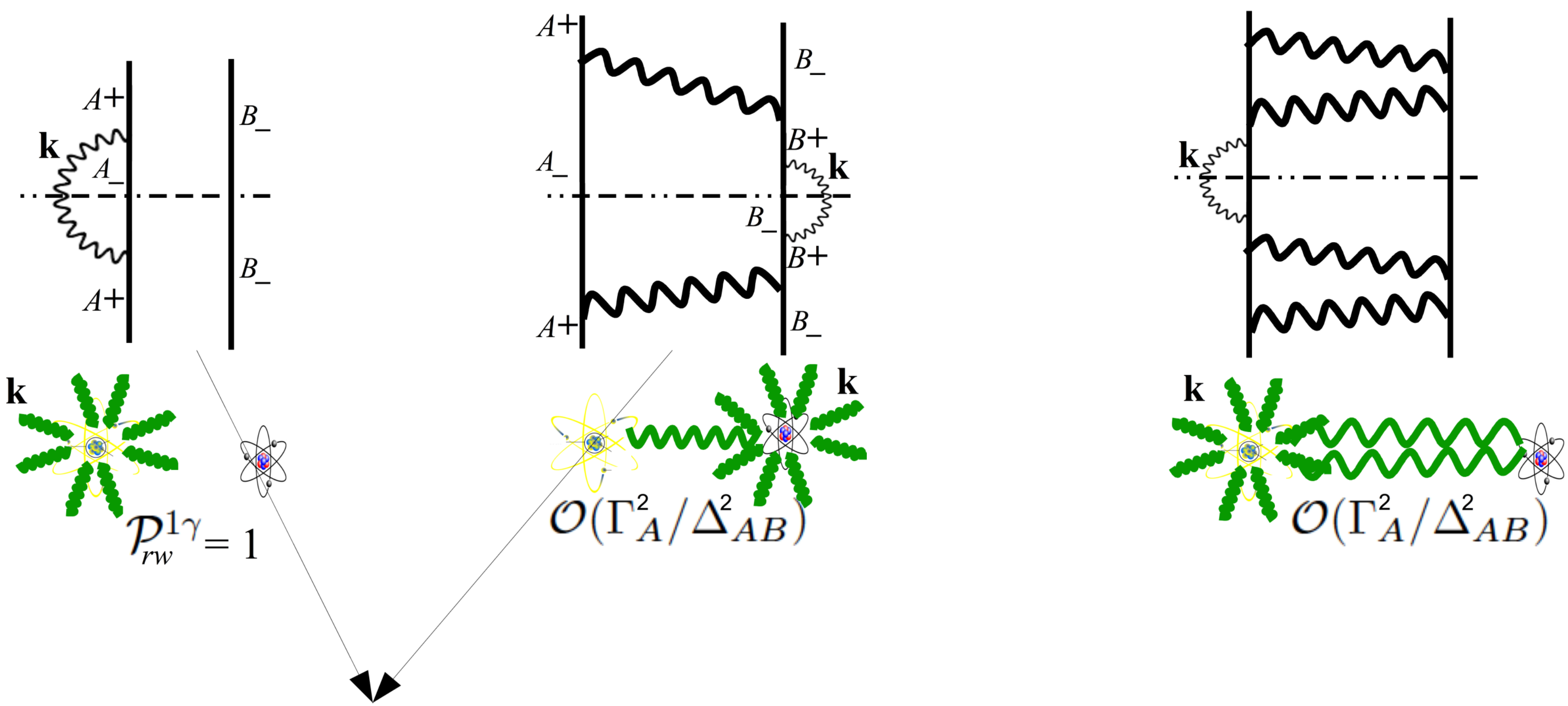
scattering off B



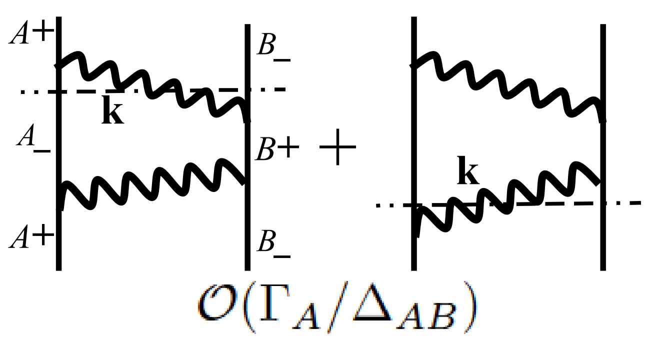
rescattering off A

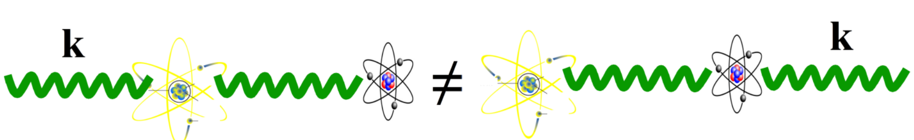
ii- Violation of action-reaction. Q1. Why does vacuum momentum not vanish?

One-photon emission probability $\mathcal{P}^{1\gamma}_{rw}$



Interference terms





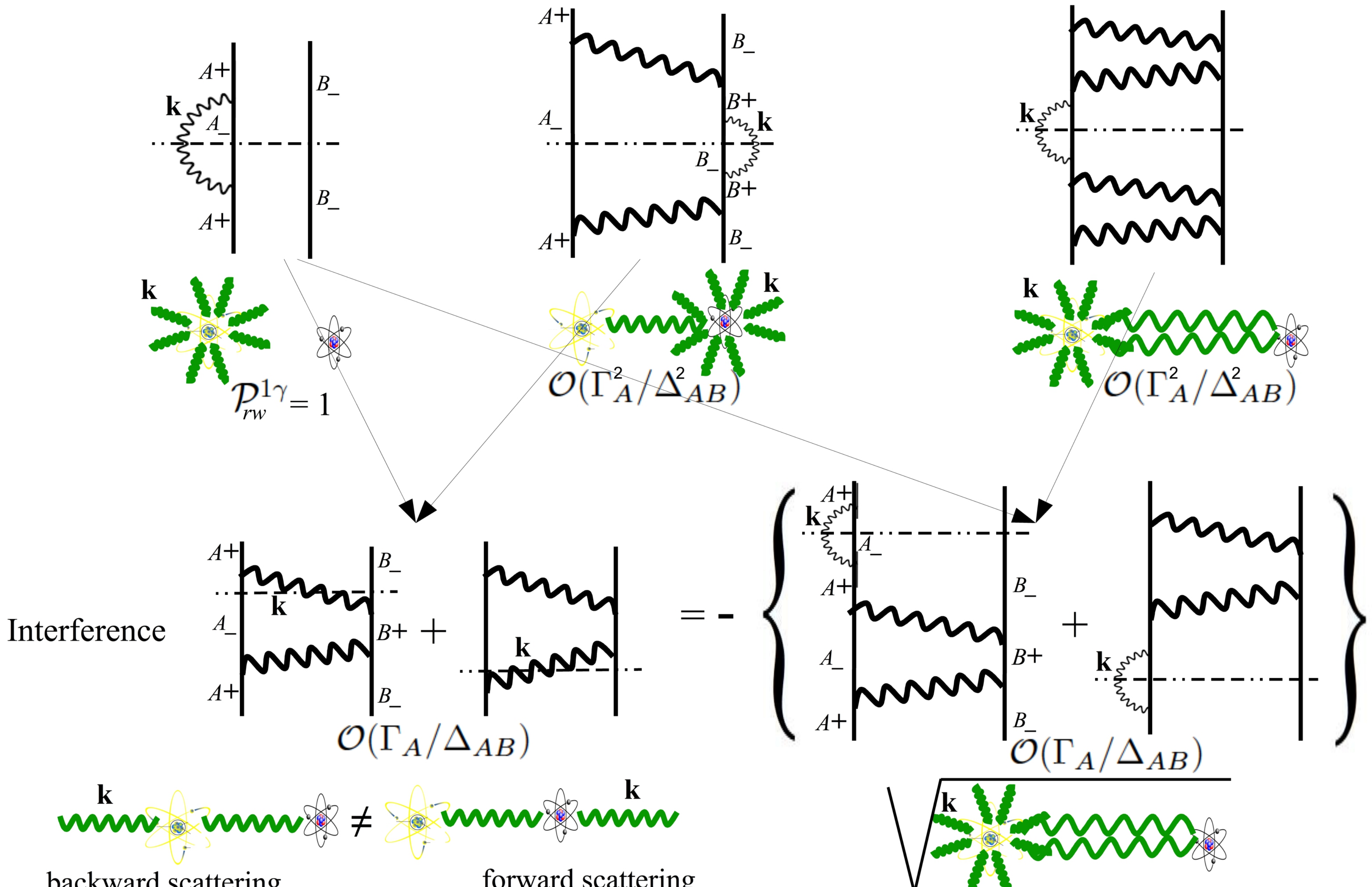
backward scattering

forward scattering

Asymmetric emission

ii- Violation of action-reaction. Q1. Why does vacuum momentum not vanish?

One-photon emission probability $\mathcal{P}^{1\gamma}_{rw}$ **Optical theorem on 'will-be' emitted photons**

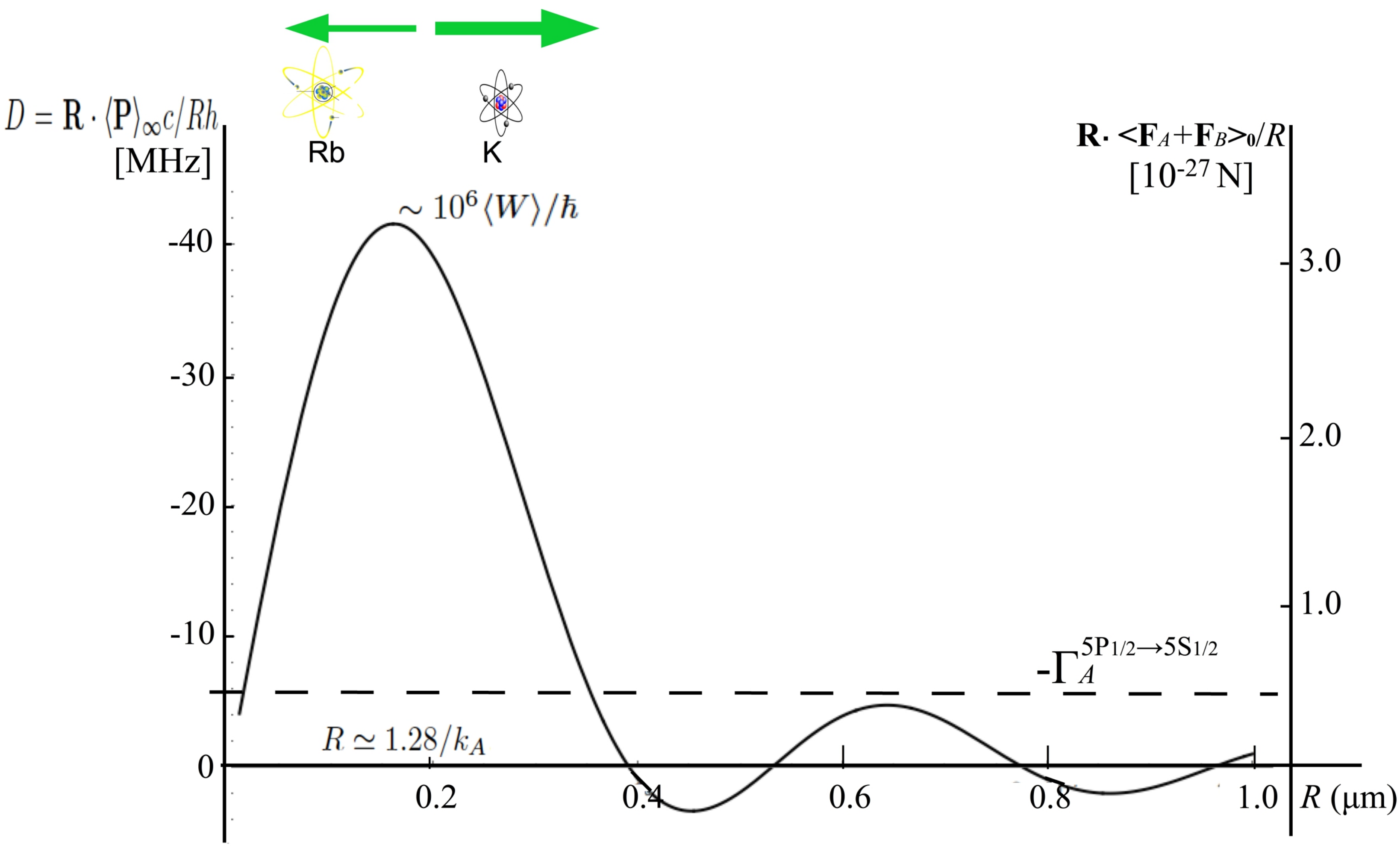


Asymmetry => **directional emission**

ii- Violation of action-reaction. Directional spontaneous emission

Experimental detection

Action-reaction violation <=> Directional spontaneous emission



Rubidium atom excited at $5P_{1/2}$ at a distance R from a potassium atom in its ground state

M. Donaire, arXiv:1604.07071 (2016)

Conclusions

. Violation of the action-reaction principles occurs in a two-atom system, with one atom excited.

$$(F_A + F_B)_{max} \Rightarrow D_{max} \sim \frac{d_B^2 \omega_B \omega_A^4}{(\omega_A^2 - \omega_B^2) c^3 \epsilon_0 \hbar}$$

M. Donaire,
arXiv:1604.07071 (2016)

M. Donaire,
Phys. Rev. A**93**, 052706 (2016)

M. Donaire, R. Guerout, A. Lambrecht,
Phys.Rev.Lett. **115**, 033201 (2015)

. It implies **i)** the transfer of momentum to the EM vacuum and **ii)** directional spontaneous emission, which are the result of, respectively,

1) Total momentum conservation

2) The optical theorem in asymmetrically excited system <= unitarity + locality

} common to any QFT => *Conjecture*: action-reaction violation holds in any asymmetrically excited system