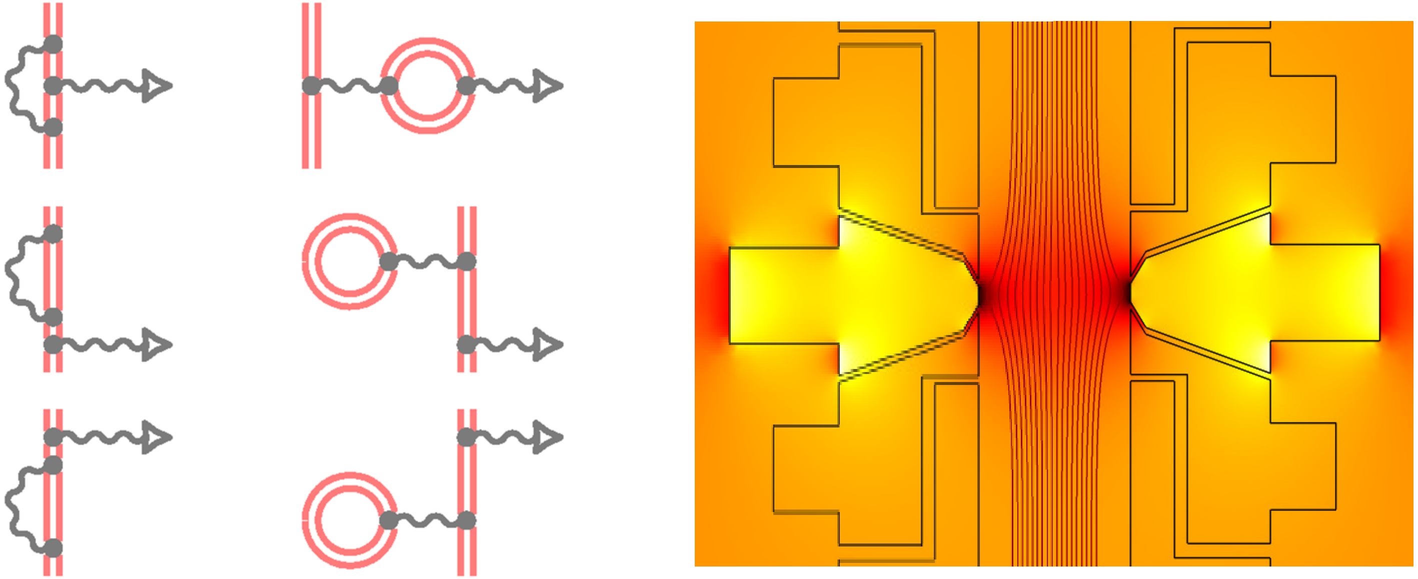
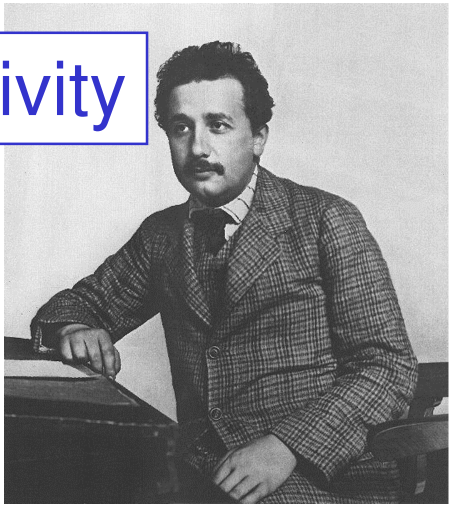
Bound-State Quantum Electrodynamics in Strong Fields Beyond the Furry Picture by g-Factor Measurements



Wolfgang Quint
GSI Darmstadt and Univ. Heidelberg

Quantum mechanics, Relativity, and P.A.M. Dirac



Quantum mechanics → Dirac theory

Special Relativity → Dirac theory

Dirac theory →

- electron magnetic moment
- energy levels of bound elec.
- negative energy states
- existence of anti-matter

g-factor

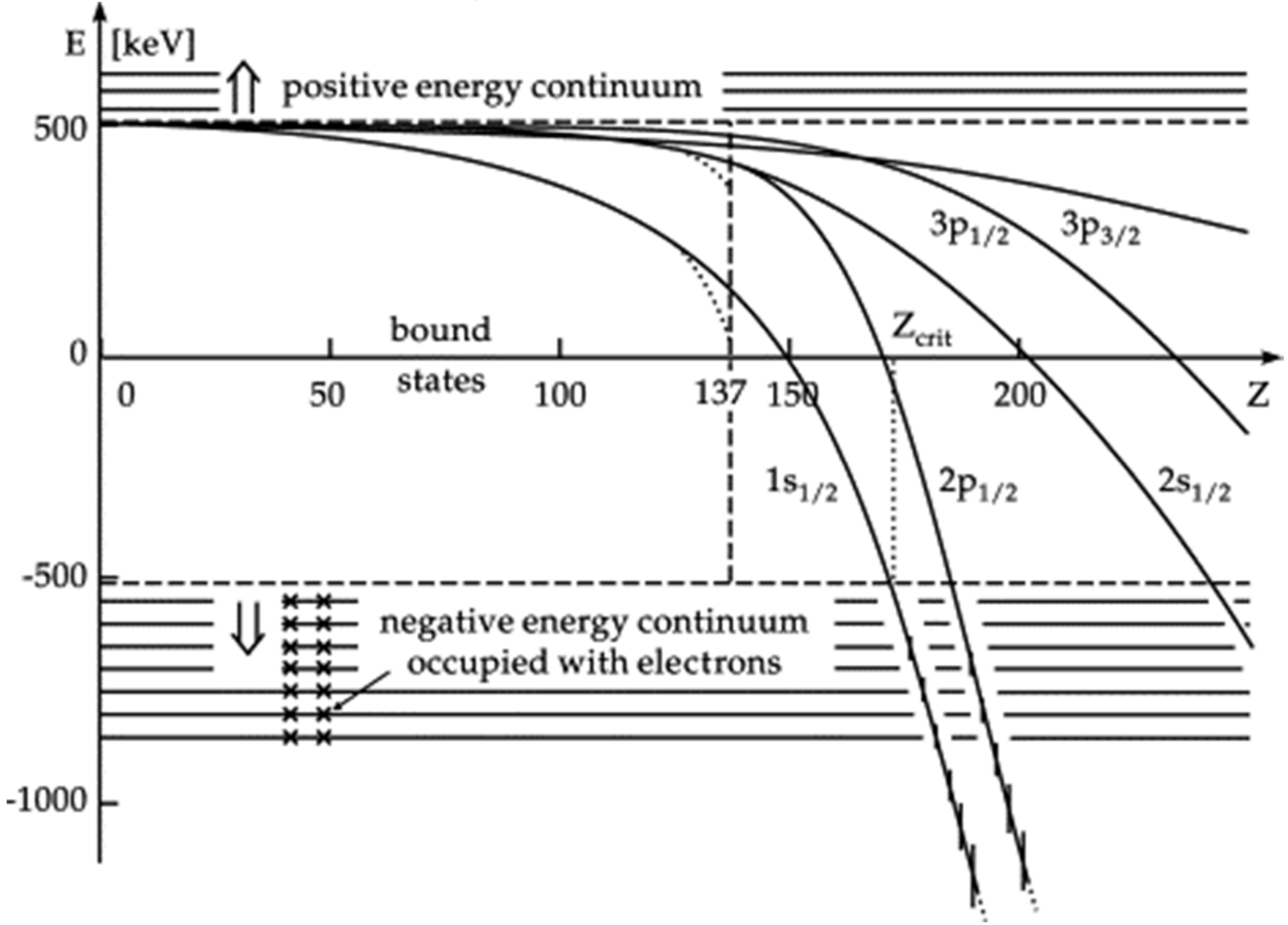
Lamb-shift

few-el. ions

CPT tests

Dirac sea of negative energy states

- Negative energy states: observable in energy levels and g-factors of few-electron ions

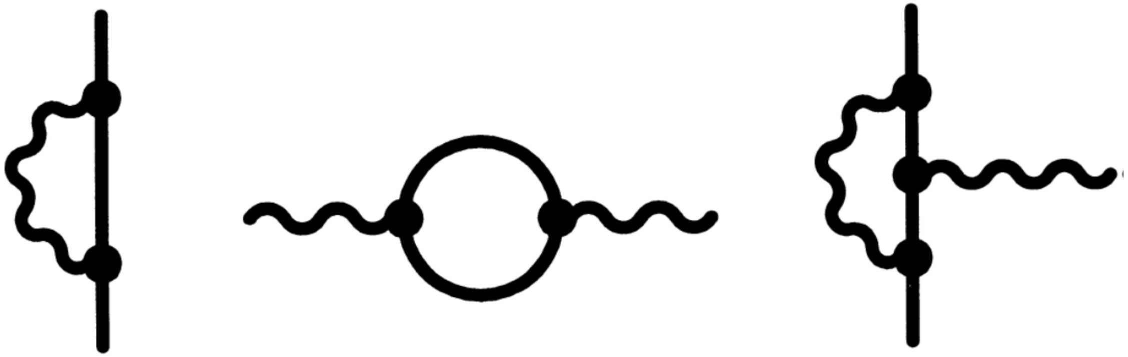


Ref.:
W. Greiner, Adv. in Quantum Chemistry, vol. 53, 99 (2008)

Quantum Electrodynamics (QED)

QED = Dirac theory + quantized radiation field

basic processes in QED:



self energy **vacuum polarization** **vertex correction**

QED coupling parameter: fine-structure constant $\alpha = e^2/2\varepsilon_0 hc \approx 1/137 \approx 0.007$

Ref.: T. Beier, Physics Reports 339, 79 (2000)

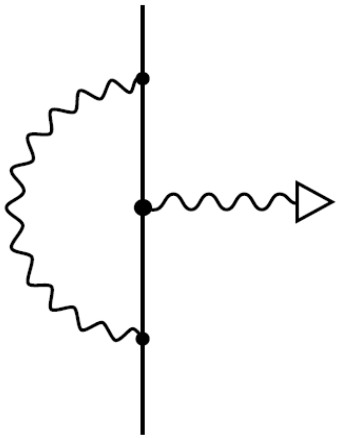
Quantum Electrodynamics

The *g*-factor:

$$\vec{\mu} = -\boxed{g}\,\mu_B \frac{\vec{s}}{\hbar}$$

Bohr magneton: $\mu_B \equiv \frac{e\hbar}{2m_e}$

Schwinger term:



Most stringent test:

***g*-factor of the free electron:**

Theory

$$\frac{g_e}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right)^1 + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

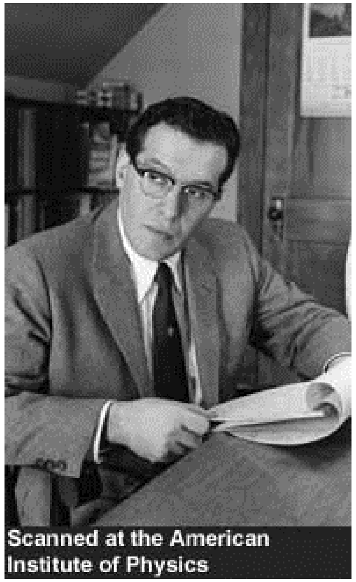
$$= 1.001\;159\;\;652\;\;181\;\;78\;\;(77)$$

[T. Aoyama et al., PRL **109**, 111807 (2012)]

Exp.

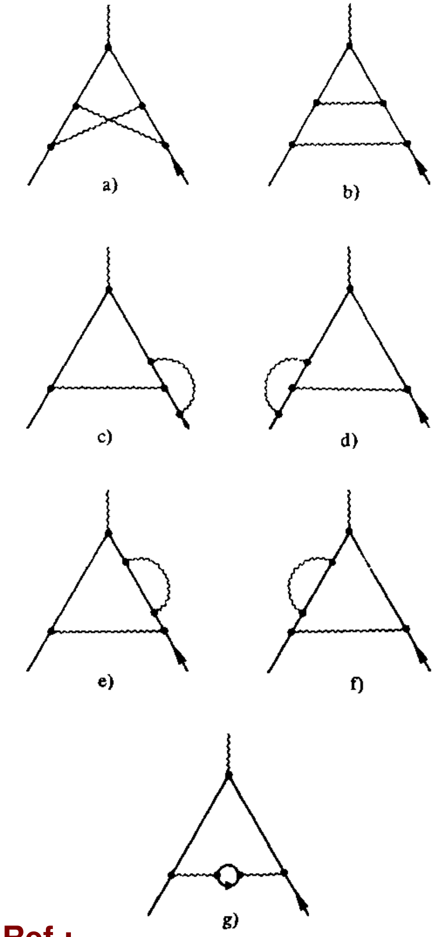
$$\frac{g_e}{2} = 1.001\;159\;\;652\;\;180\;\;73\;\;(28)$$

[D. Hanneke et al., Phys. Rev. A 83, 052122 (2011)]

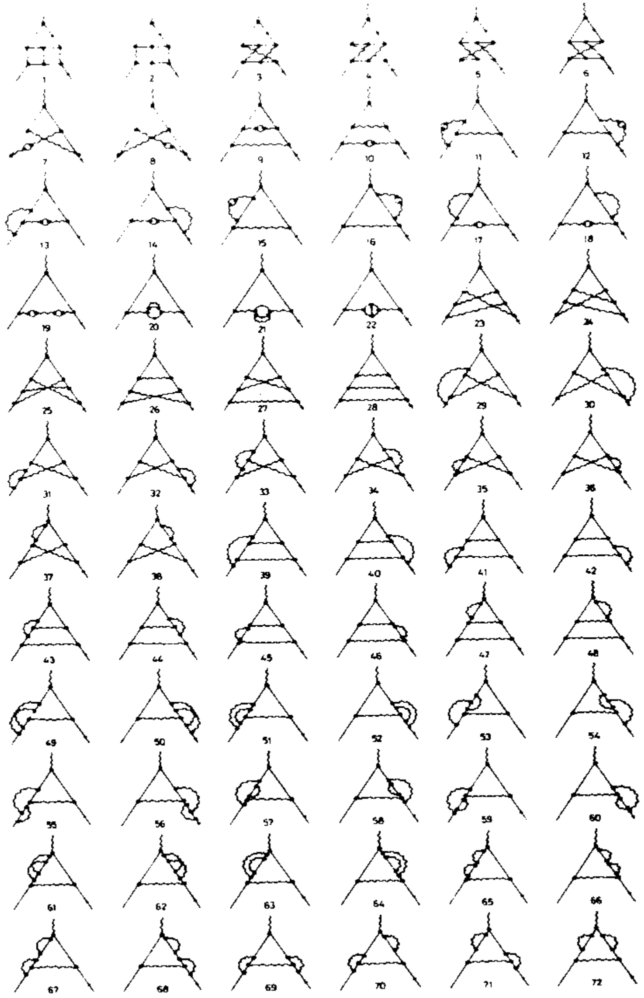


Free electron: QED contributions of 2^{nd} and 3^{rd} order

$$g_{free} = 2\,(1 + C_1\alpha/\pi + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + C_5(\alpha/\pi)^5 + \ldots.$$



2^{nd} order in α:
$C_2 = -\,0.328\;478\;966$
7 graphs



3^{rd} order in α:
$C_3 = 1.1765$
72 graphs

not shown:
4^{th} order in α:
$C_4 = -1.9108$
891 graphs

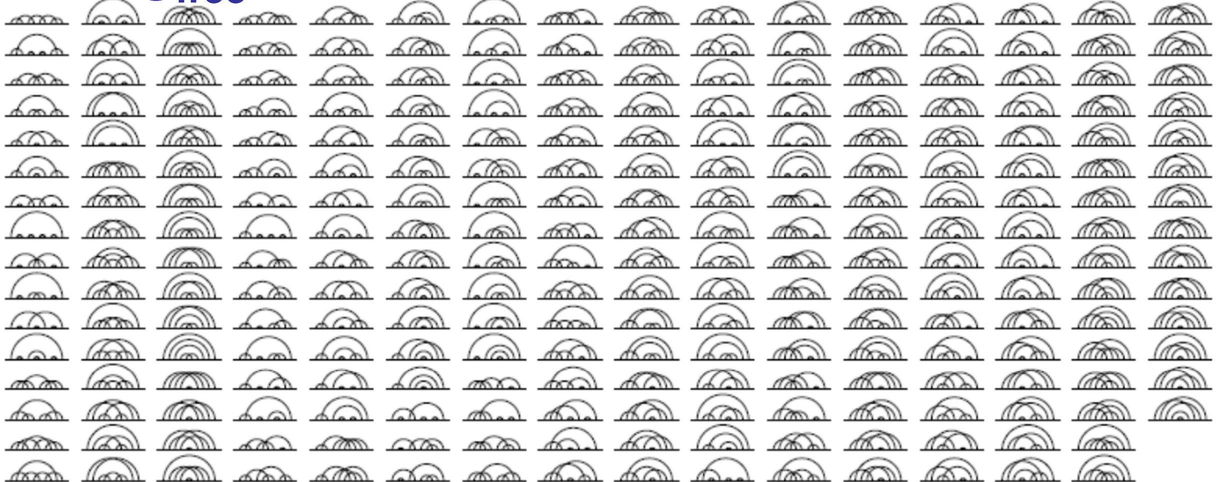
Ref.:
B. Lautrup et al., Phys. Rep. 3, 193 (1972)

Free electron: QED contributions of 5th order

$$g_{free} = 2\,(1 + C_1\alpha/\pi + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + C_5(\alpha/\pi)^5 + \ldots.$$

Harvard g-2 measurement 2011:

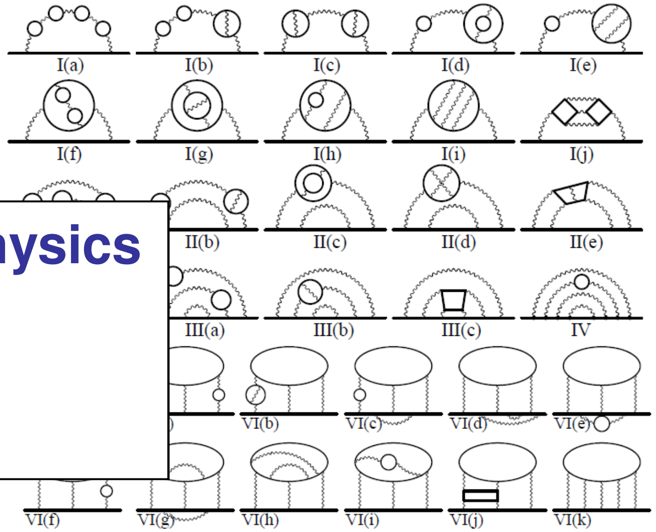
$$g_{free} = 2\,(1.001\ 159\ 652\ 180\ 73\ (28)) \rightarrow \text{determination of } \alpha$$



5th order in α:

$C_5 = 9.16$

12672 graphs



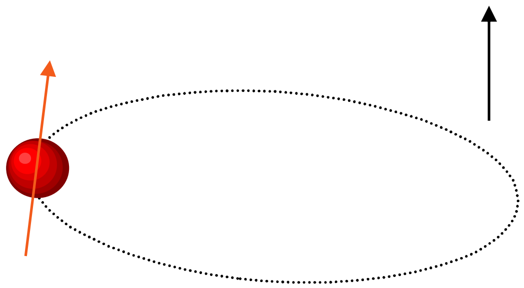
> **„I am digging at the roots of physics to see whether there is some treasure there.“**
>
> ***Toichiro Kinoshita***

Ref.:
Kinoshita et al., arXiv:1205.5368v1 [hep-ph] 24 May 2012

g-Factor of the free electron

Larmor precession frequency:

$$\omega_L^e = \frac{g}{2}\,\frac{e}{m_e}B$$



B: magnetic field in Penning trap

cyclotron frequency:

$$\omega_c^e = \frac{e}{m_e}B$$

$$\boxed{g_e = 2\cdot\frac{\omega_L^e}{\omega_c^e}}$$

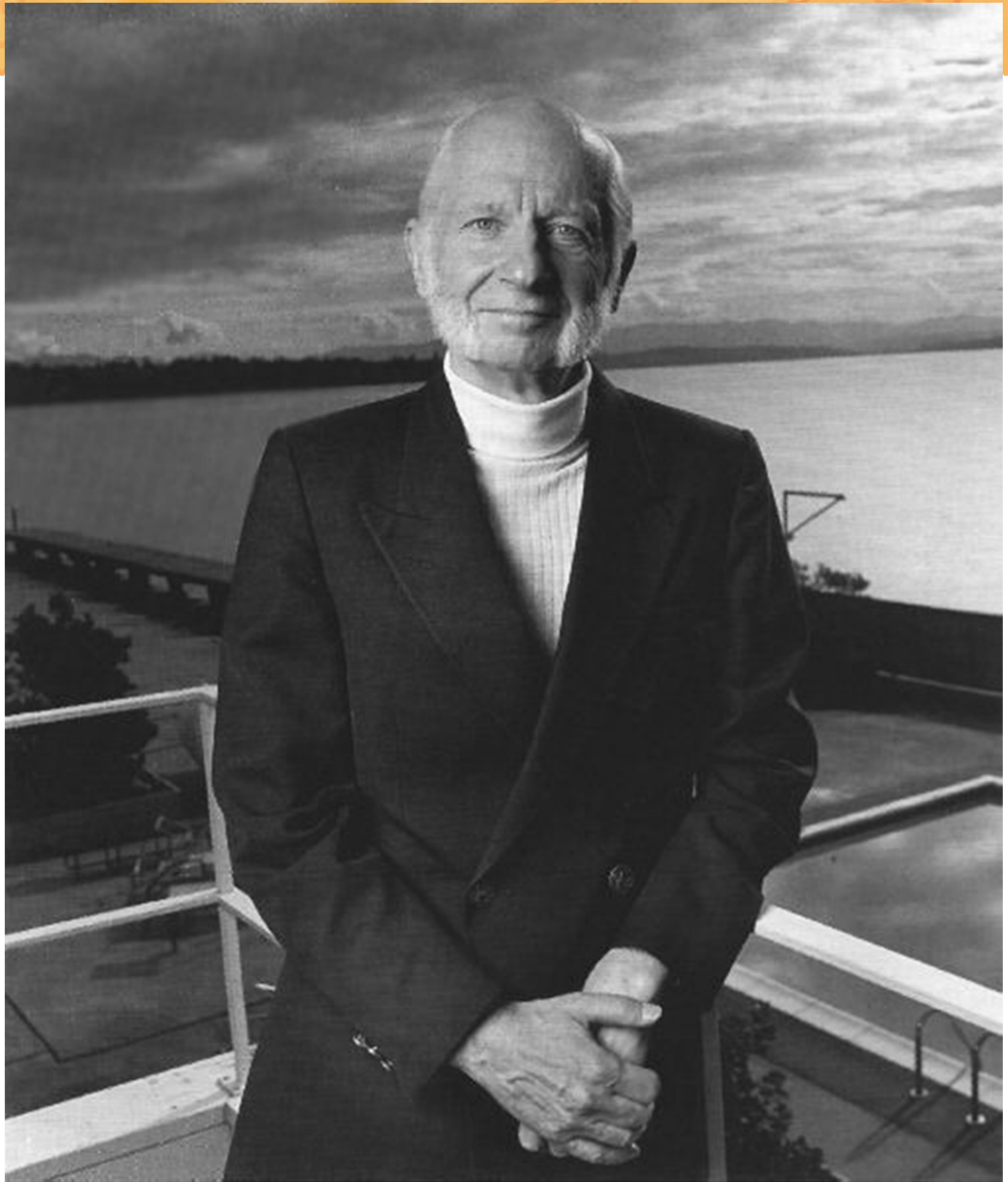
or rather

$$\boxed{g_e - 2 = 2\cdot\frac{\omega_a^e}{\omega_c^e}}$$

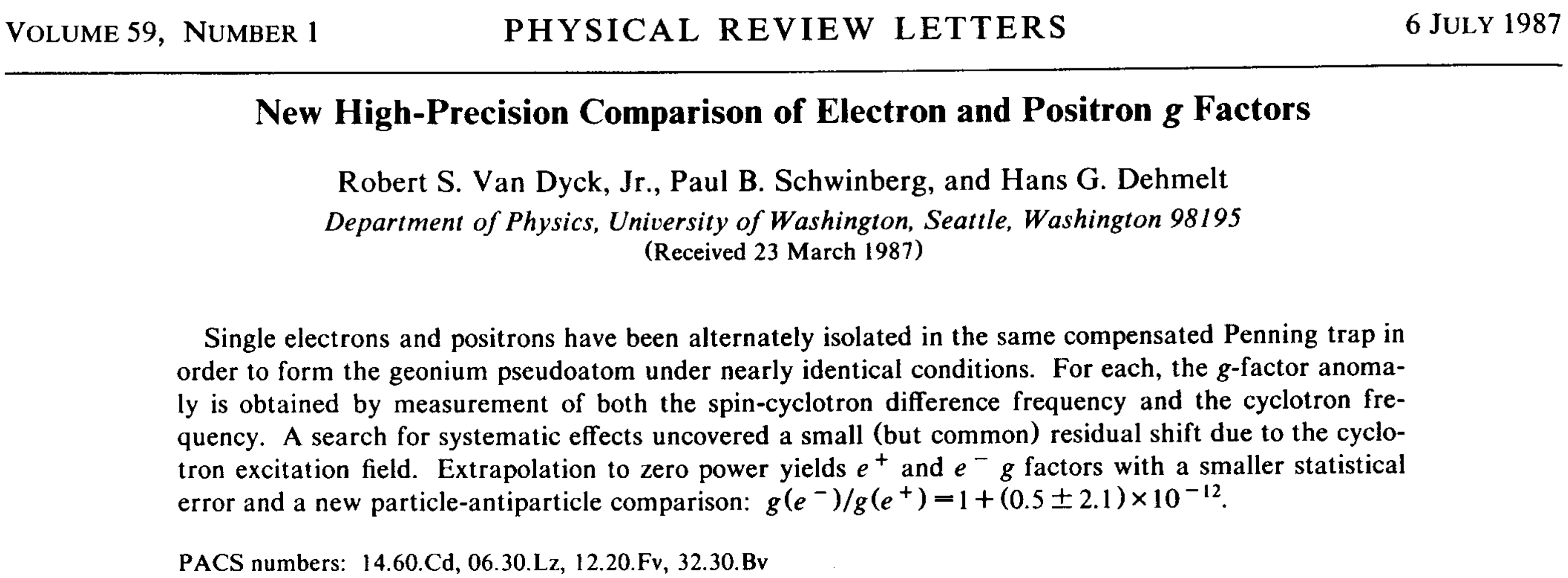
(get 3 orders of magnitude in accuracy by Nature)

Hans Dehmelt

Nobel Prize 1989 "for the development of the ion trap technique"



g-Factor of the electron and positron



VOLUME 59, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JULY 1987

New High-Precision Comparison of Electron and Positron g Factors

Robert S. Van Dyck, Jr., Paul B. Schwinberg, and Hans G. Dehmelt
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 23 March 1987)

Single electrons and positrons have been alternately isolated in the same compensated Penning trap in order to form the geonium pseudoatom under nearly identical conditions. For each, the g-factor anomaly is obtained by measurement of both the spin-cyclotron difference frequency and the cyclotron frequency. A search for systematic effects uncovered a small (but common) residual shift due to the cyclotron excitation field. Extrapolation to zero power yields e^+ and e^- g factors with a smaller statistical error and a new particle-antiparticle comparison: $g(e^-)/g(e^+) = 1 + (0.5 \pm 2.1) \times 10^{-12}$.

PACS numbers: 14.60.Cd, 06.30.Lz, 12.20.Fv, 32.30.Bv

Electron: $g = 2\times 1.001\ 159\ 652\ 188\ 4\ (43)$*

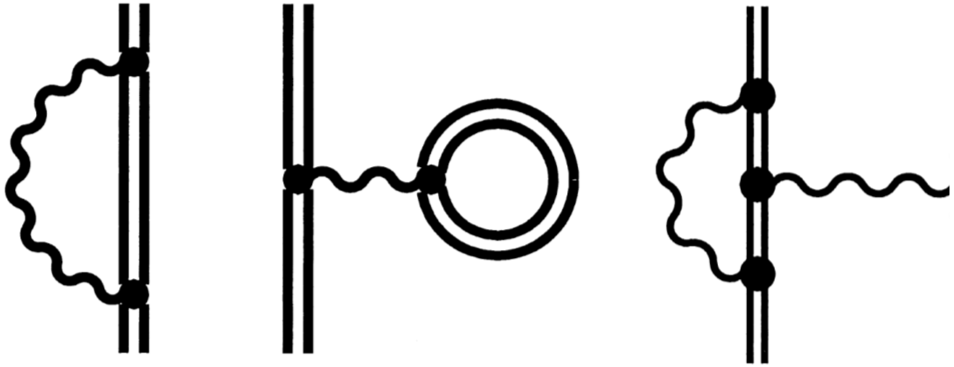
Positron: $g = 2\times 1.001\ 159\ 652\ 187\ 9\ (43)$*

*CODATA

QED and highly charged ions (HCI)

bound-state QED: quantum physics in strong fields

basic processes in bound-state QED:



self energy **vacuum polarization** **vertex correction**

bound-state QED coupling parameter for H-like uranium U^{91+}: $Z\alpha \approx 0.67$

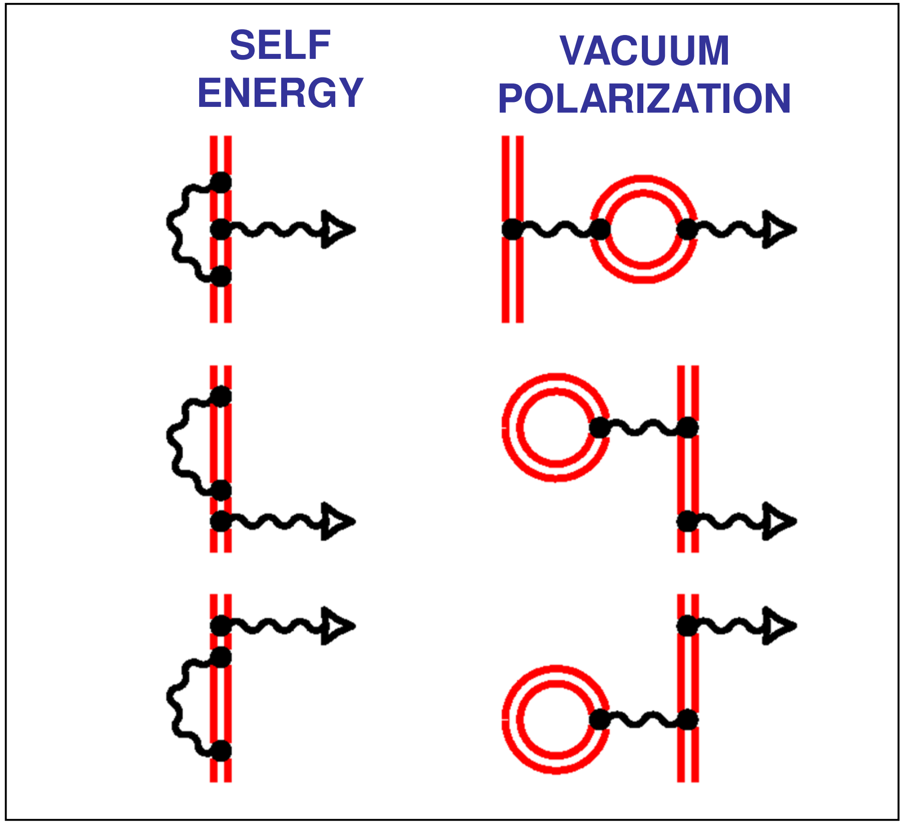
Ref.:
T. Beier, Physics Reports 339, 79 (2000)

Bound-electron g-factor: Feynman graphs 1st order in α/π

$$g_{bound}/g_{free} \approx 1 - (Z\alpha)^2/3 + \alpha(Z\alpha)^2/4\pi + \ldots.$$

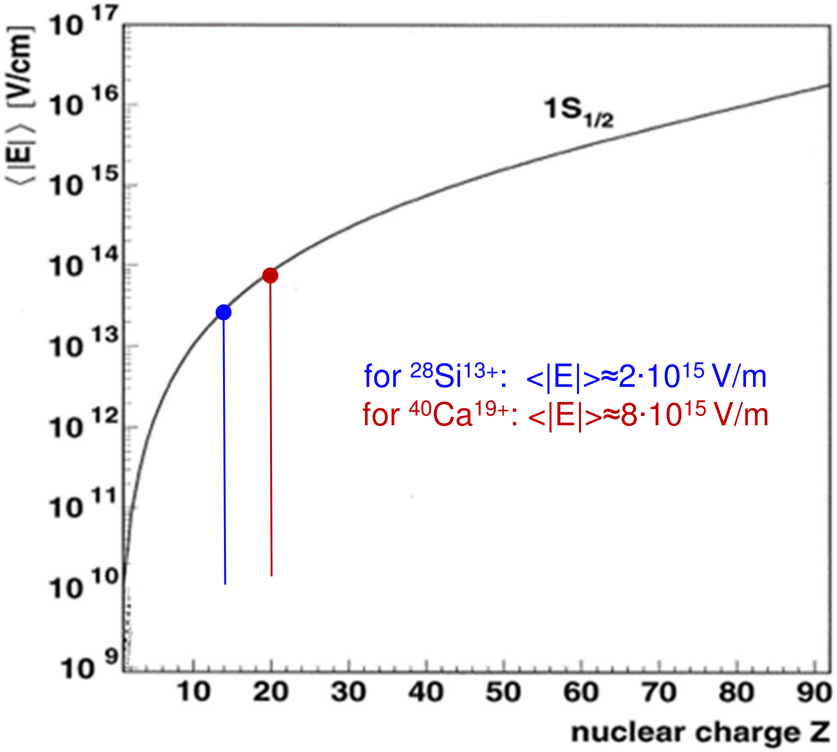
Dirac theory

bound-state QED



Ref.:
T. Beier, Physics Reports 339, 79 (2000)

QED in strong fields

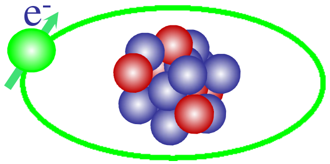


for $^{28}Si^{13+}$: $<|E|> \approx 2 \cdot 10^{15}$ V/m

for $^{40}Ca^{19+}$: $<|E|> \approx 8 \cdot 10^{15}$ V/m

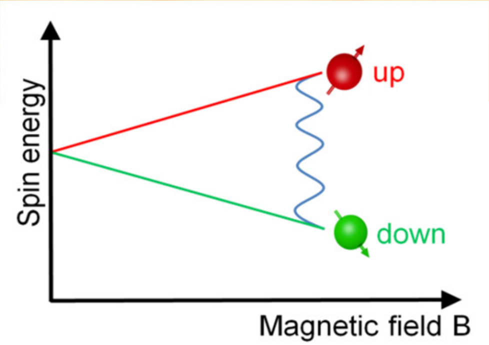
Common theoretical approach:

Furry picture **of bound-state QED**



Wendell Hinkle Furry
1907-1984
Prof. at Harvard University

Measurement principle

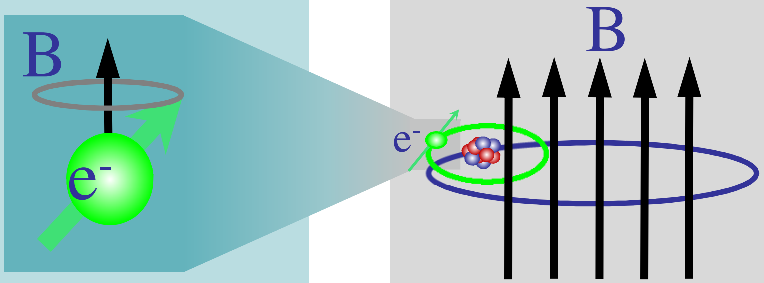


$$\Delta E_{spin} = h\nu_L$$

Larmor precession frequency

$$\nu_L = \frac{g}{4\pi}\frac{e}{m_e}B$$

Cyclotron frequency of the ion



$$\nu_c = \frac{q_{ion}}{2\pi m_{ion}}B$$

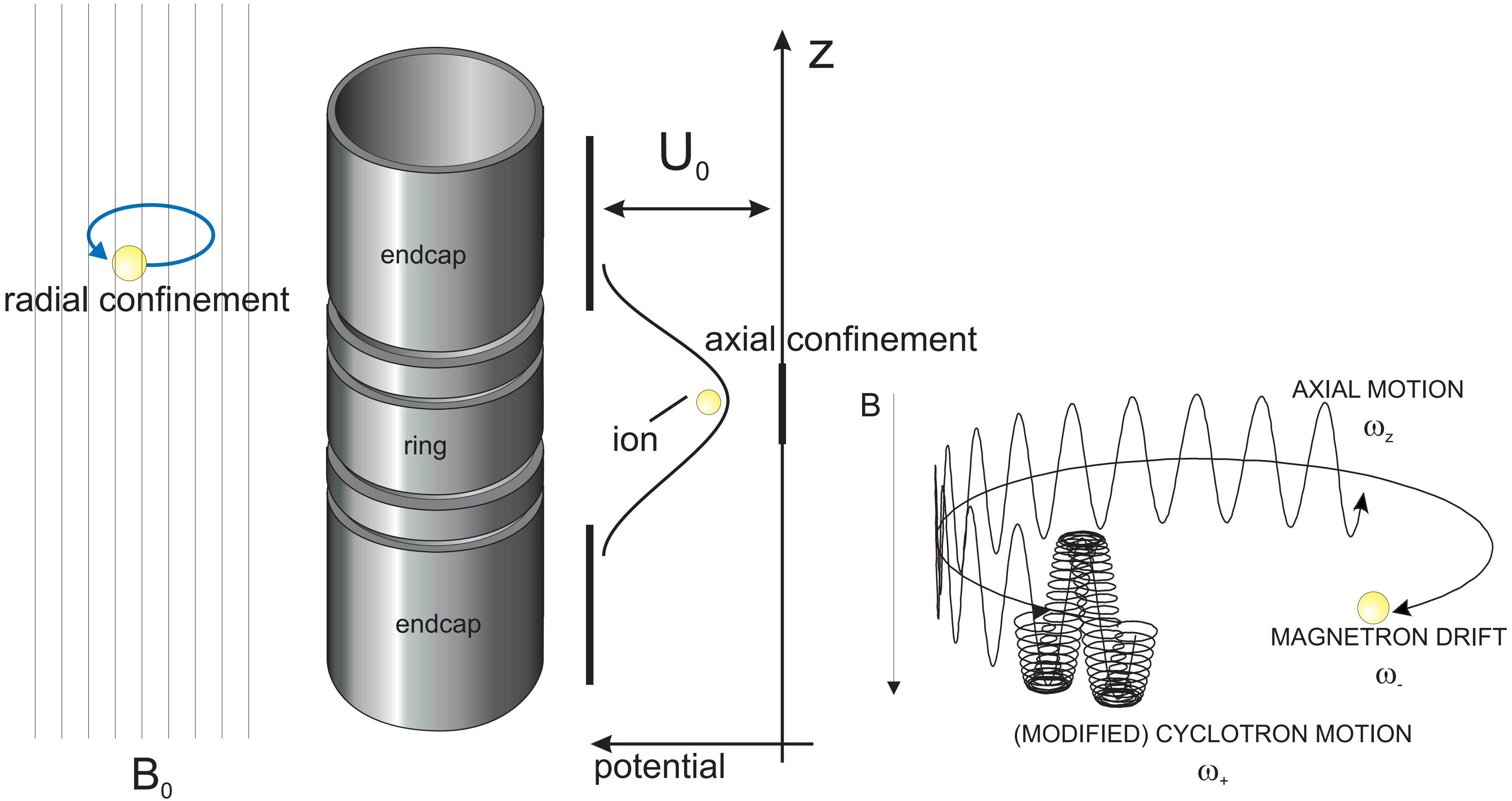
$$g = 2\frac{m_e}{m_{ion}}\frac{q_{ion}}{e}\boxed{\frac{\nu_L}{\nu_c}}$$

Our task:

Measurement of

$\Gamma = \nu_L/\nu_c$

A single highly charged ion stored in a Penning trap



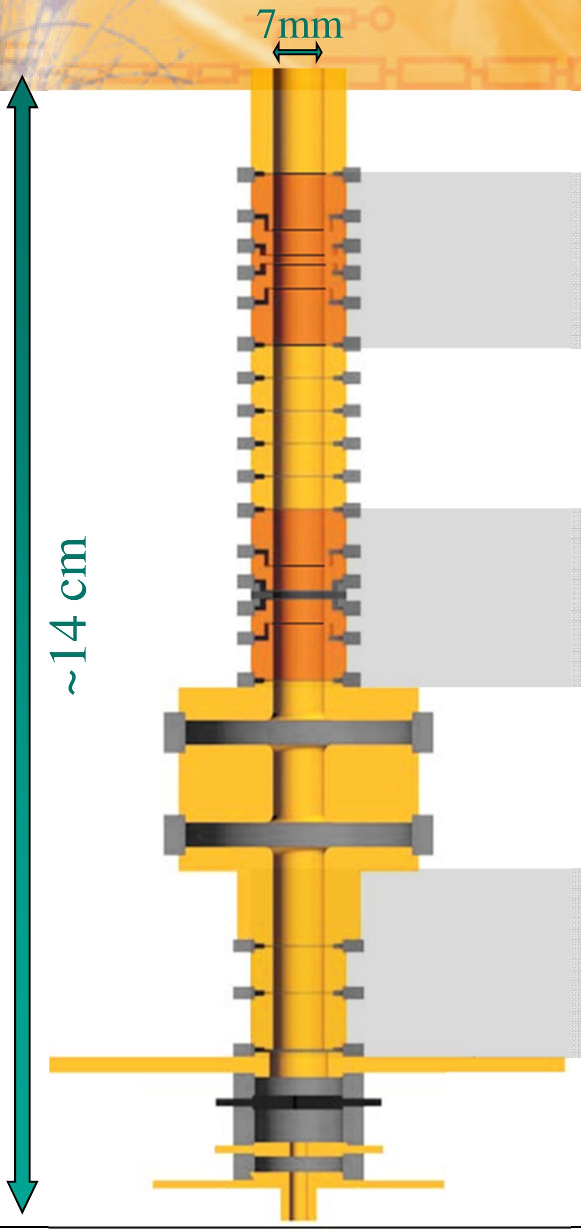
magnetic

physical

electric

combined ion motion

Triple Penning trap system



Precision trap (PT)

- Very homogeneous magnetic field

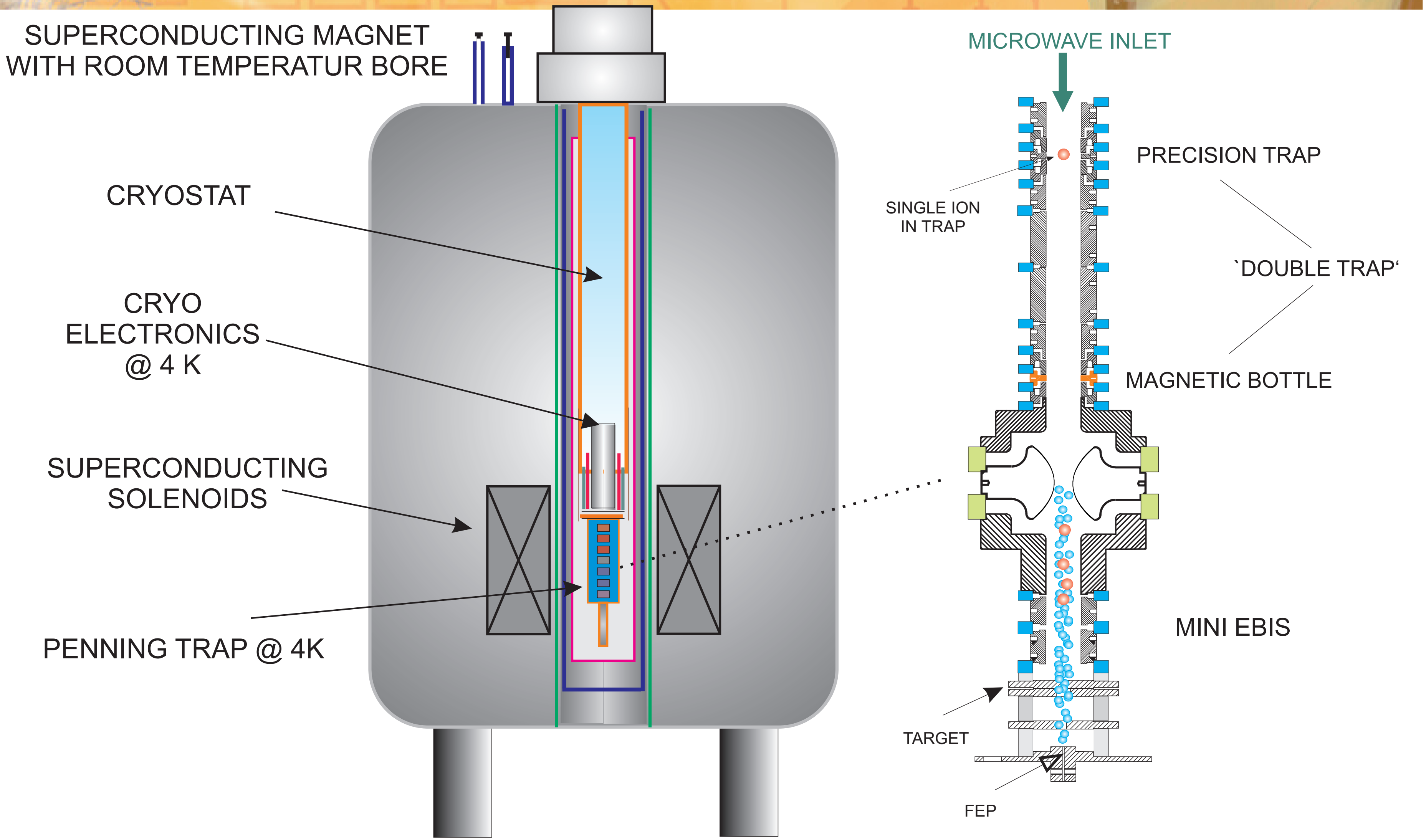
Analysis trap (AT)

- Magnetic bottle for spin detection

Creation trap (CT)

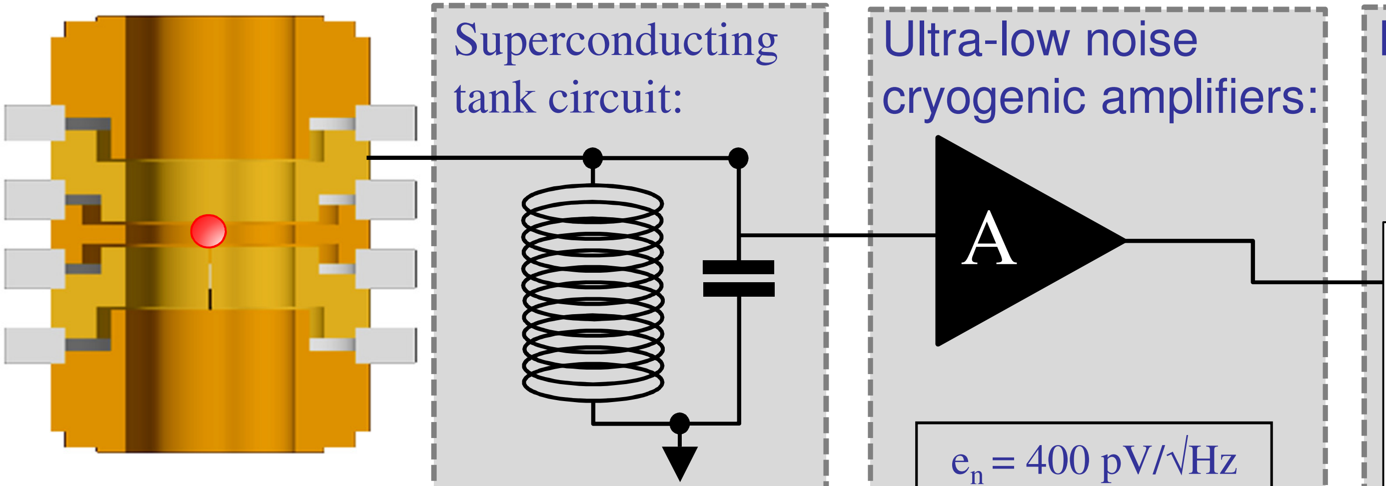
- In-trap ion creation of highly charged ions

Highly charged ion g-factor apparatus



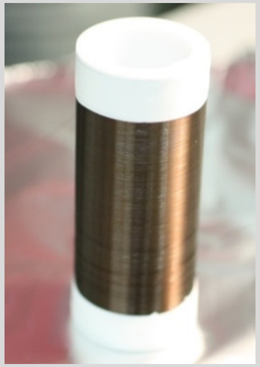
Ion oscillation frequencies

- Measurement of the tiny image currents (~ fA) on the trap electrodes requires:



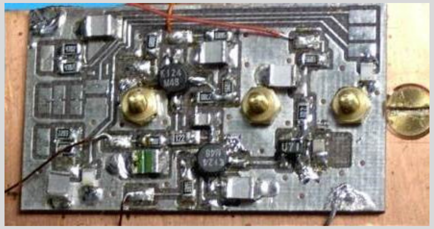
Superconducting tank circuit:

Helical resonator:
Q= 3200

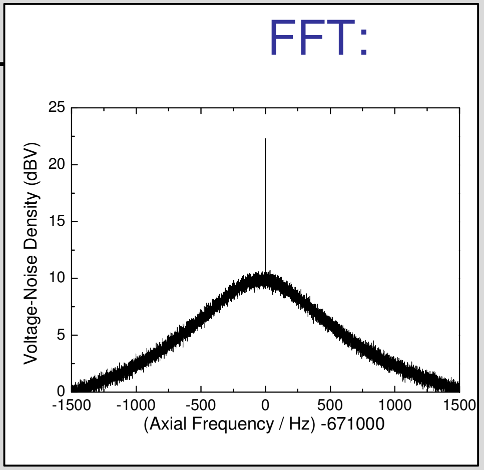


Ultra-low noise cryogenic amplifiers:

$e_n = 400$ pV/√Hz
$i_n \leq 10$ fA/ √Hz

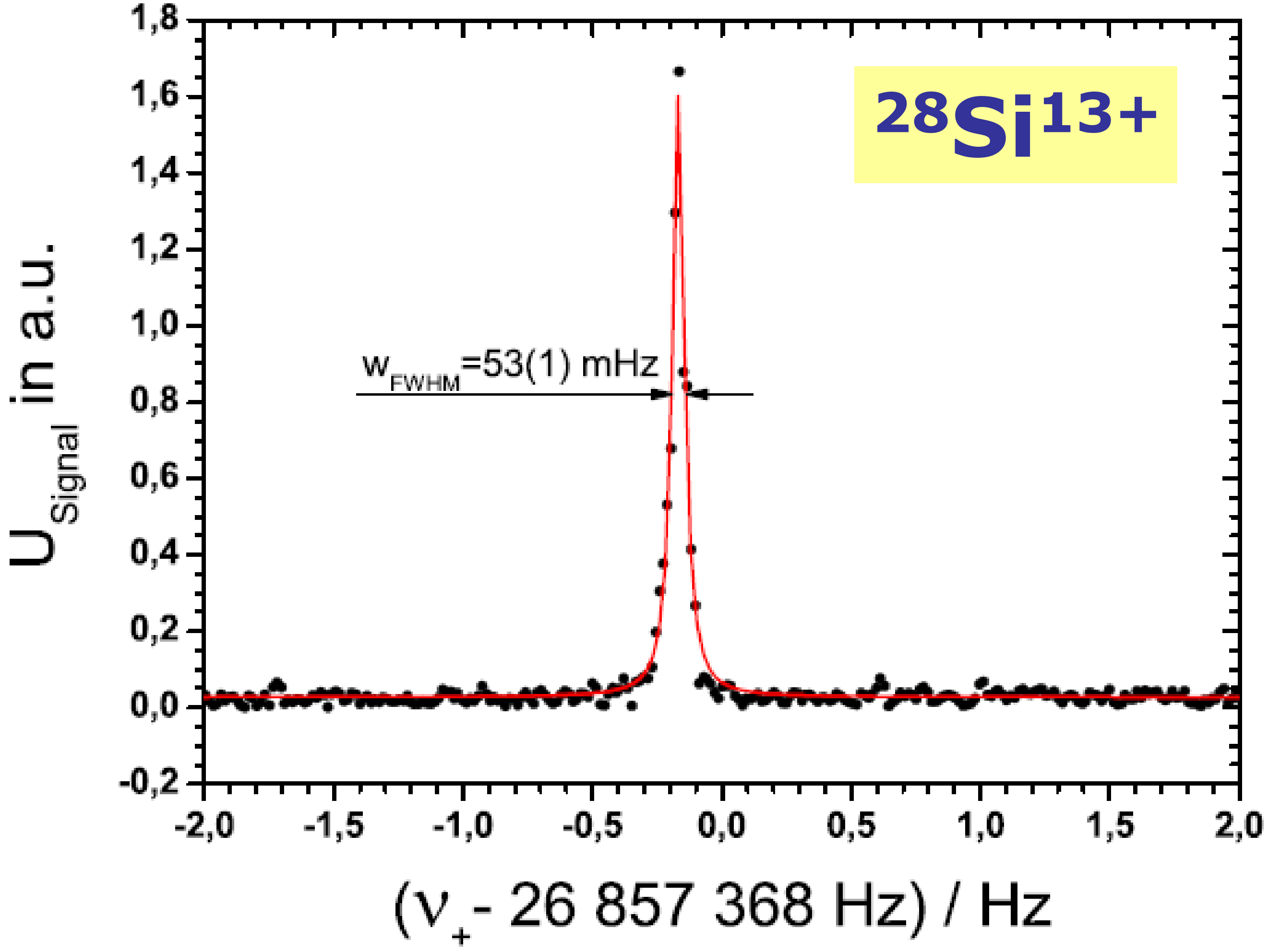


Fourier transformation:



A “hot“ ion is detected as a peak above the Johnson noise of the resonator.

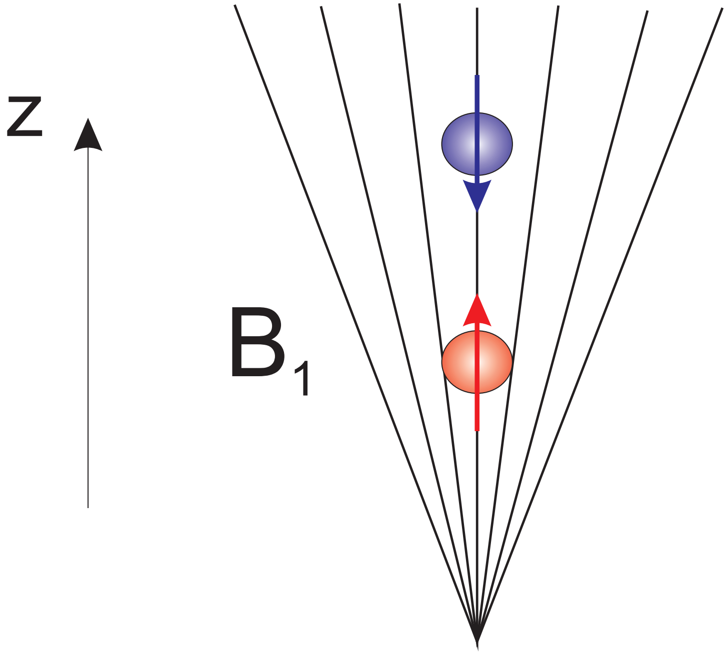
High-resolution cyclotron frequency measurement of a single highly charged silicon ion



Continuous Stern-Gerlach effect: Determination of spin direction

CLASSICAL STERN-GERLACH

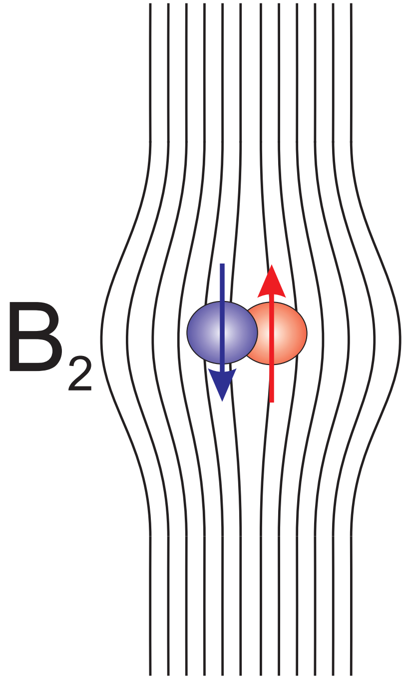
SEPARATION IN POSITION SPACE



$$\Delta z = \frac{\mu L^2}{2KE} B_1$$

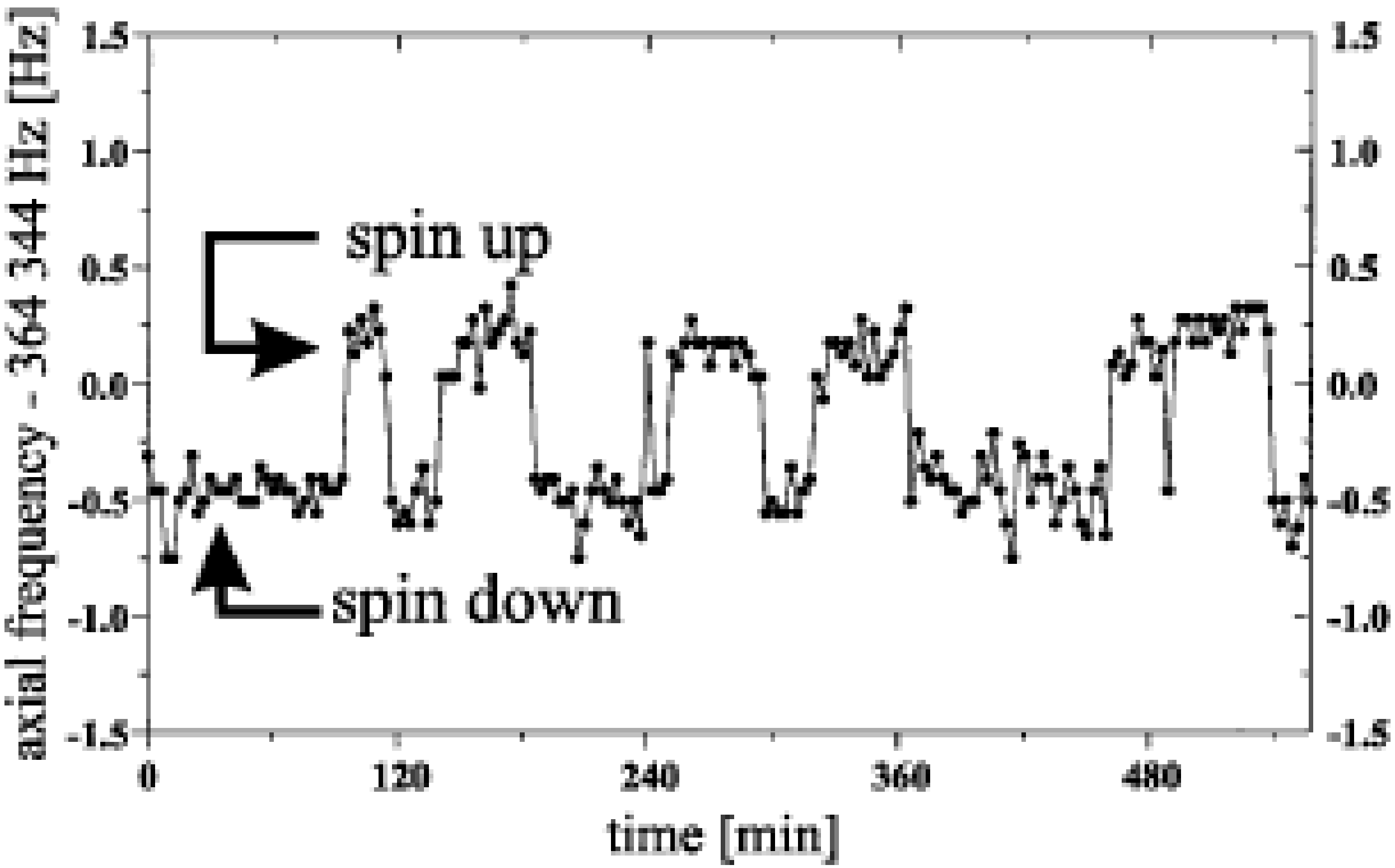
CONTINUOUS STERN-GERLACH

SEPARATION IN FREQUENCY SPACE

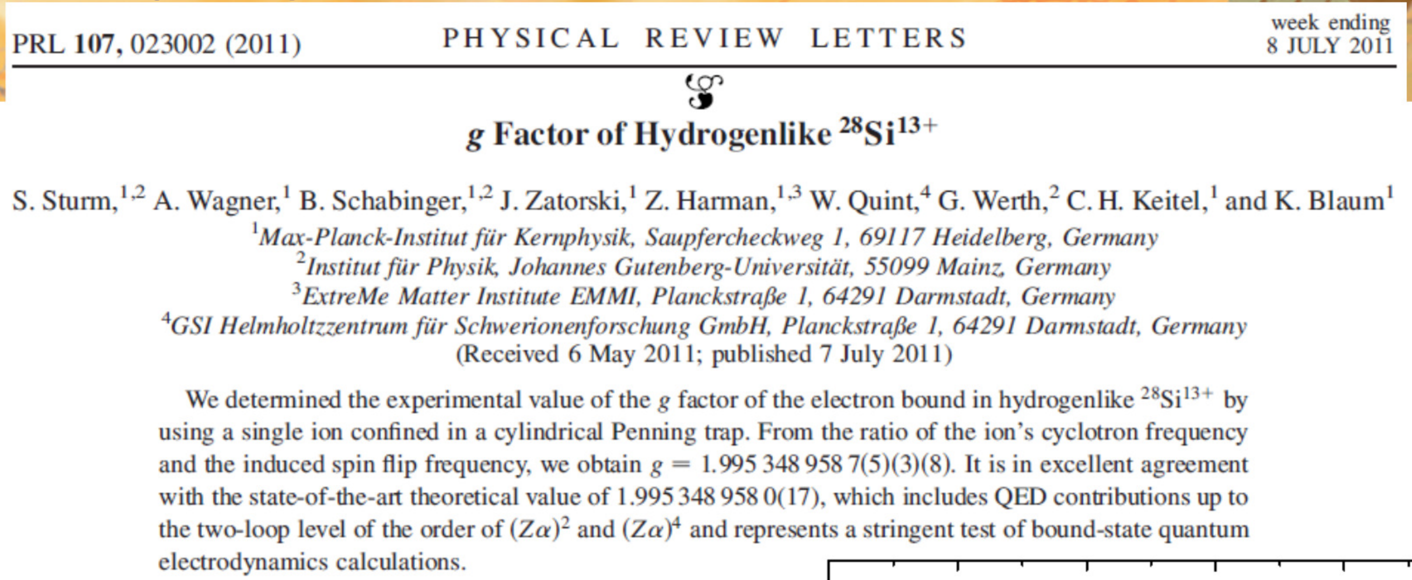


$$\Delta\omega_z = \frac{\mu}{m\omega_z} B_2$$

Quantum jump spectroscopy:
Spin-flip transitions in the analysis trap



Bound electron magnetic moment measurement on hydrogen-like silicon $^{28}Si^{13+}$



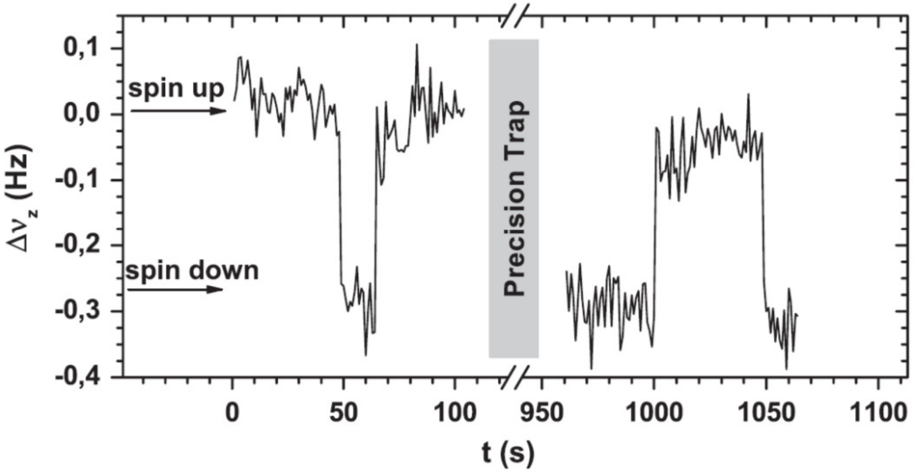
PRL **107,** 023002 (2011) PHYSICAL REVIEW LETTERS week ending 8 JULY 2011

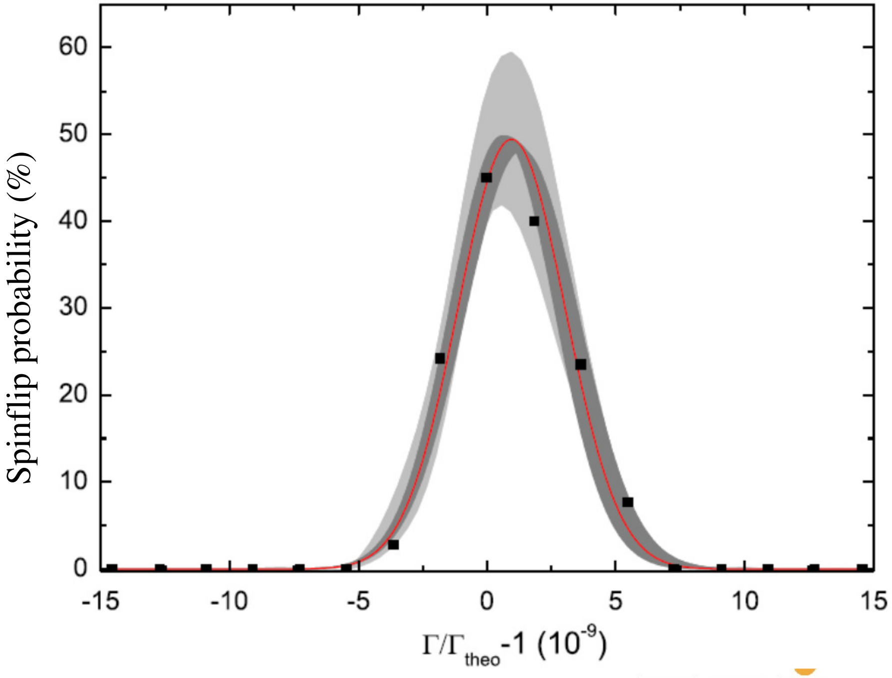
g Factor of Hydrogenlike $^{28}Si^{13+}$

S. Sturm,[1,2] A. Wagner,[1] B. Schabinger,[1,2] J. Zatorski,[1] Z. Harman,[1,3] W. Quint,[4] G. Werth,[2] C. H. Keitel,[1] and K. Blaum[1]
[1]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*
[2]*Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany*
[3]*ExtreMe Matter Institute EMMI, Planckstraße 1, 64291 Darmstadt, Germany*
[4]*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany*

(Received 6 May 2011; published 7 July 2011)

We determined the experimental value of the g factor of the electron bound in hydrogenlike $^{28}Si^{13+}$ by using a single ion confined in a cylindrical Penning trap. From the ratio of the ion's cyclotron frequency and the induced spin flip frequency, we obtain $g = 1.995\,348\,958\,7(5)(3)(8)$. It is in excellent agreement with the state-of-the-art theoretical value of $1.995\,348\,958\,0(17)$, which includes QED contributions up to the two-loop level of the order of $(Z\alpha)^2$ and $(Z\alpha)^4$ and represents a stringent test of bound-state quantum electrodynamics calculations.





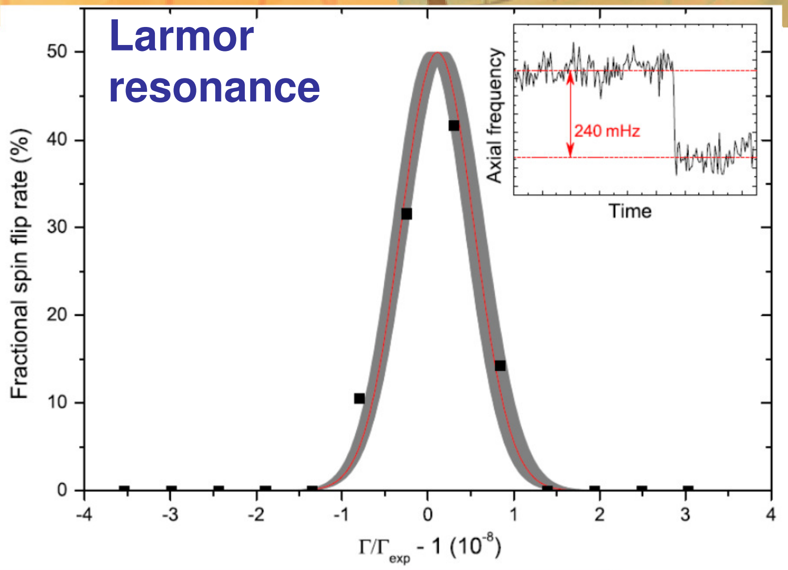
Bound electron magnetic moment measurement on lithium-like silicon $^{28}\mathrm{Si}^{11+}$

$g_{exp}(^{28}\mathrm{Si}^{11+})$ = 2.000 889 889 9(21)

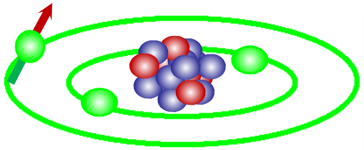
$g_{theo}(^{28}\mathrm{Si}^{11+})$ = 2.000 889 909 (51)

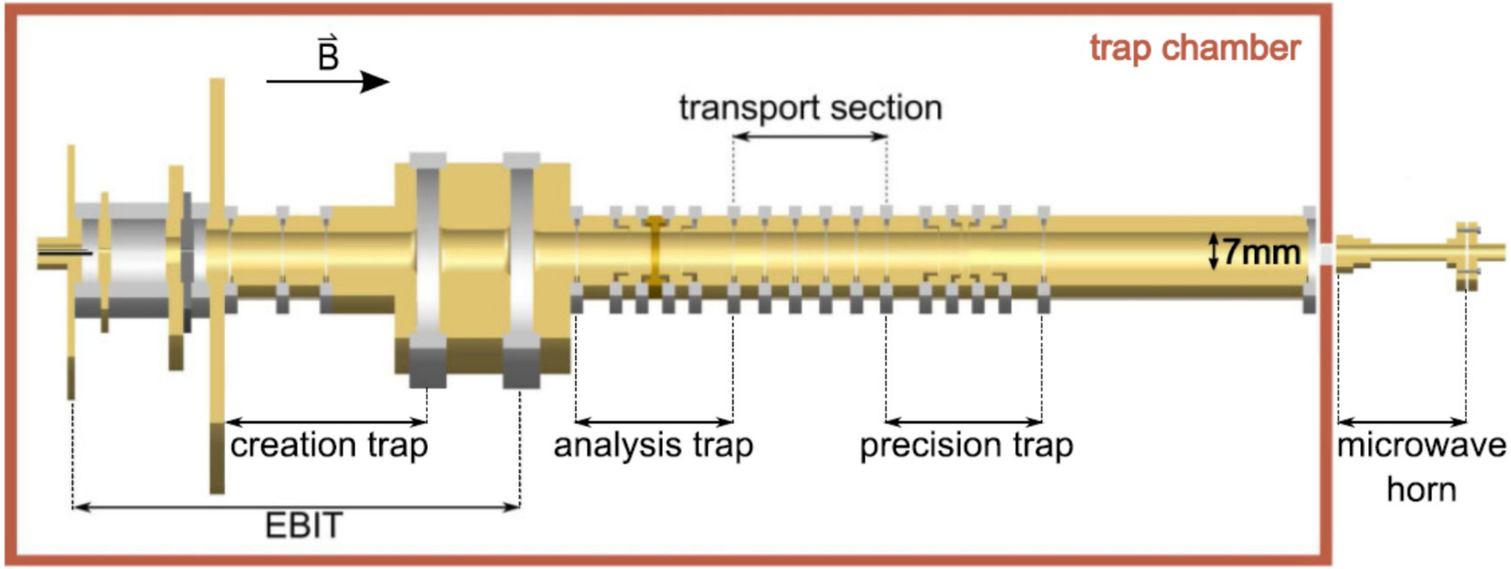
theoretical calculations by D.A. Glazov,

A.V. Volotka, V.M. Shabaev



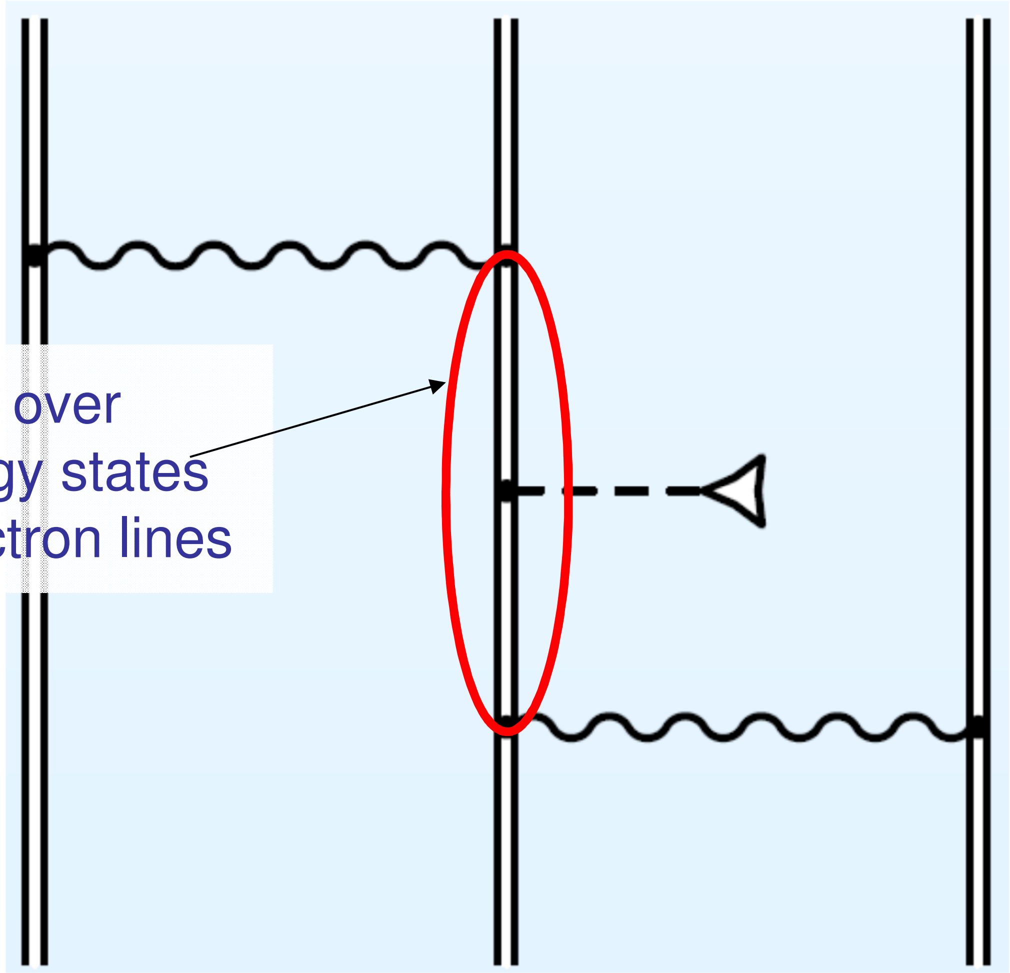
Precision test of
- electron-electron interaction
- screened QED contributions





Ref.:
A. Wagner et al.
PRL 110, 033003 (2013)

Dirac sea: contribution of negative energy states to bound electron magnetic moment in Li-like HCI



integration over
negative energy states
for internal electron lines

Ref.:
D. Glazov

Quantum Electrodanymics in strong fields: non-perturbative treatment in Zα beyond the Furry picture

- External field approximation:

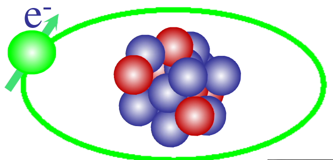
  Nucleus approximated as an external Coulomb field, V(r),

$$\left[-i\vec{\alpha}\nabla+\beta+V(\vec{r})\right]\psi(\vec{r})=E\,\psi(\vec{r})$$

reducing a 2-body system to a 1-body system

Common theoretical approach:

Furry picture **of bound-state QED**



Wendell Hinkle Furry
1907-1984
Prof. at Harvard University

Quantum Electrodanymics in strong fields: non-perturbative treatment in Zα beyond the Furry picture

Isotope shift in bound-electron *g*-factors

→ **All contributions cancel**

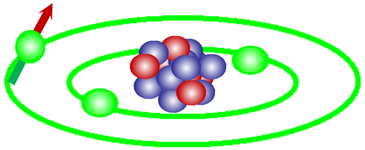
except for:

- Nuclear recoil
- Nuclear size

Not included in Furry picture

Our test system:

$g(^{40}\mathrm{Ca}^{17+}) \leftrightarrow g(^{48}\mathrm{Ca}^{17+})$



- nuclear recoil dominates the isotope shift (99.96%), since nuclear size:

$$r_{\mathrm{nucl.}}(^{40}\mathrm{Ca}) = 3.4776(19)\cdot 10^{-15}\ \mathrm{fm}$$
$$r_{\mathrm{nucl.}}(^{48}\mathrm{Ca}) = 3.4771(20)\cdot 10^{-15}\ \mathrm{fm}$$

[I. Angeli et al., Atomic Data and Nuclear Data Tables 99 (2013)]

Ref.:
F. Köhler, S. Sturm

g-Factors of lithium-like calcium $^{40}\mathrm{Ca}^{17+}$ and $^{48}\mathrm{Ca}^{17+}$ → sensitive test of QED beyond the Furry picture

$$g = 2\Gamma \frac{m_e}{m_{ion}} \frac{q_{ion}}{e}$$

Measured *g*-factors:

(stat) (syst) (m$_{ion}$)

$g(^{40}\mathrm{Ca}^{17+}) = 1.999\,202\,040\,55\,(10)\,(12)\,(110)\cdot 10^{-10} \rightarrow \delta g/g = 5.6\cdot 10^{-10}$

$g(^{48}\mathrm{Ca}^{17+}) = 1.999\,202\,028\,85\,(12)\,(13)\,(80)\cdot 10^{-10} \rightarrow \delta g/g = 4.1\cdot 10^{-10}$

$$\Delta g_{exp} = g(^{40}\mathrm{Ca}^{17+}) - g(^{48}\mathrm{Ca}^{17+}) = 11.70\,(16)\,(3)\,(138)\cdot 10^{-9}$$

✓ **Most precise *g*-factors of lithium-like ions!**

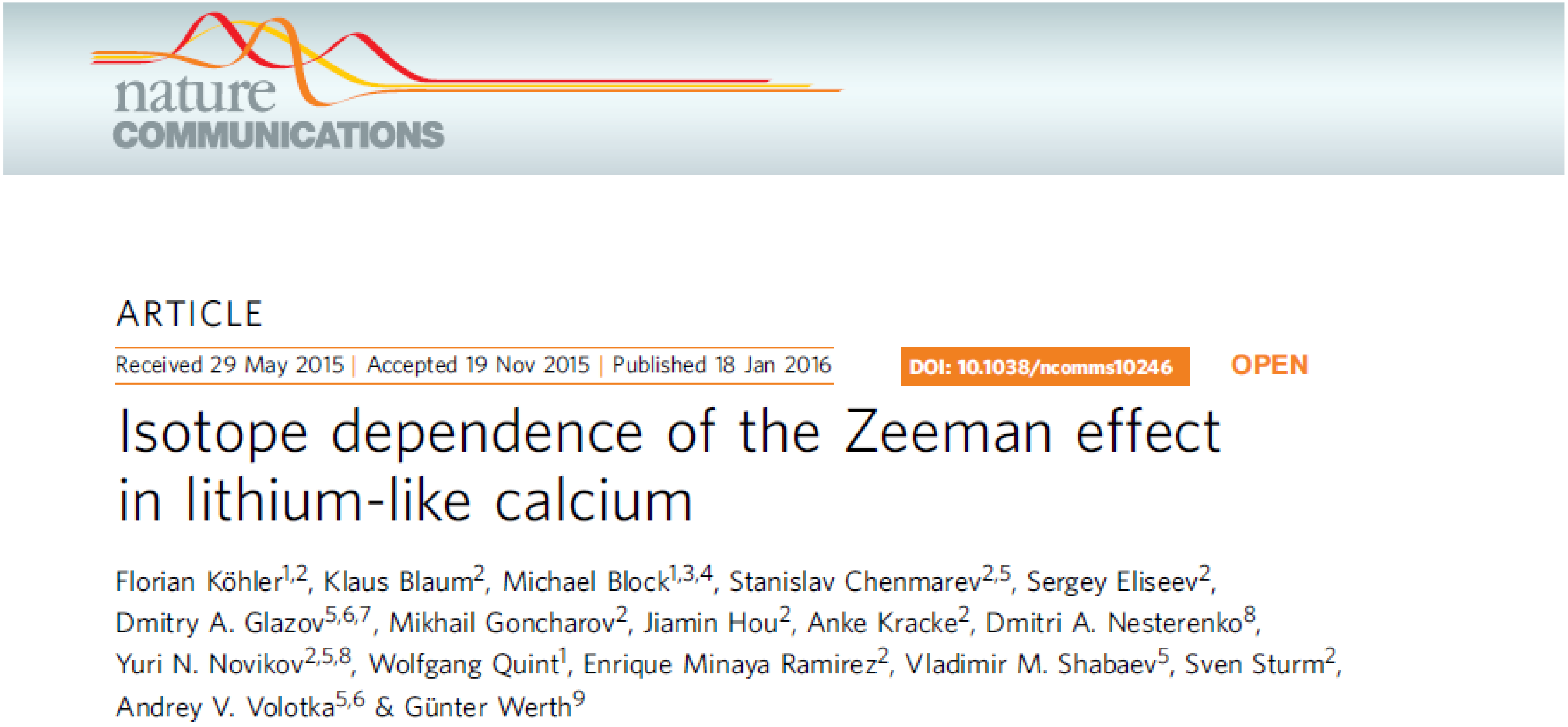
$$\Delta g_{theo} = g(^{40}\mathrm{Ca}^{17+}) - g(^{48}\mathrm{Ca}^{17+}) = 10.305\,(27)\cdot 10^{-9}$$

[Sz. Nagy et al. Eur. Phys. J. D 39, (2006)]

[A. V. Volotka et. al., PRL 112, 253004 (2014)]

comparison to theory:
V. Shabaev et al.

g-Factors of lithium-like calcium $^{40}\mathrm{Ca}^{17+}$ and $^{48}\mathrm{Ca}^{17+}$ → sensitive test of QED beyond the Furry picture



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OPEN

Isotope dependence of the Zeeman effect in lithium-like calcium

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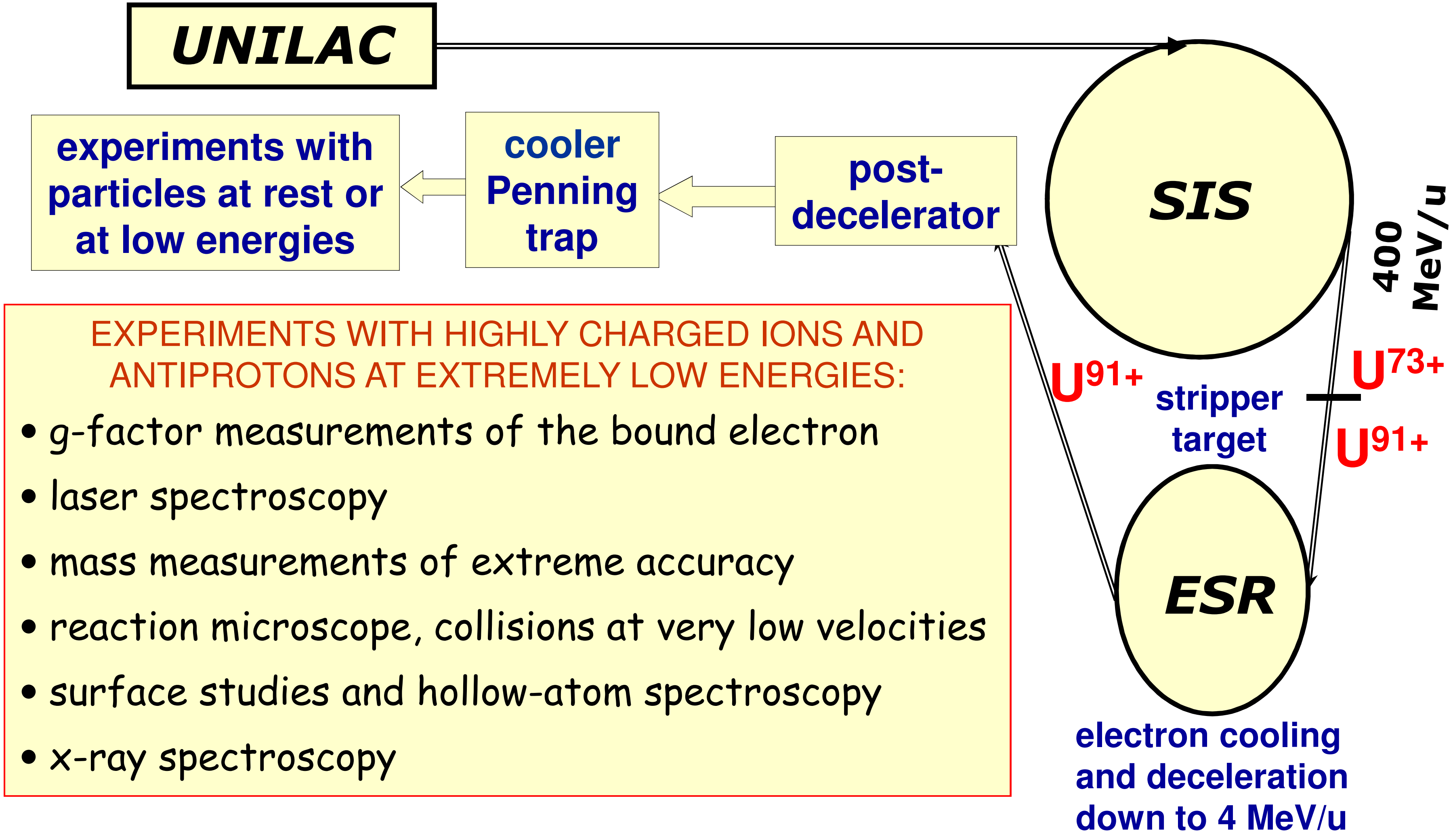
HITRAP at the Experimental Storage Ring ESR at GSI Darmstadt



EXPERIMENTS WITH HIGHLY CHARGED IONS AND ANTIPROTONS AT EXTREMELY LOW ENERGIES:

- g-factor measurements of the bound electron
- laser spectroscopy
- mass measurements of extreme accuracy
- reaction microscope, collisions at very low velocities
- surface studies and hollow-atom spectroscopy
- x-ray spectroscopy

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Thank you for your attention !