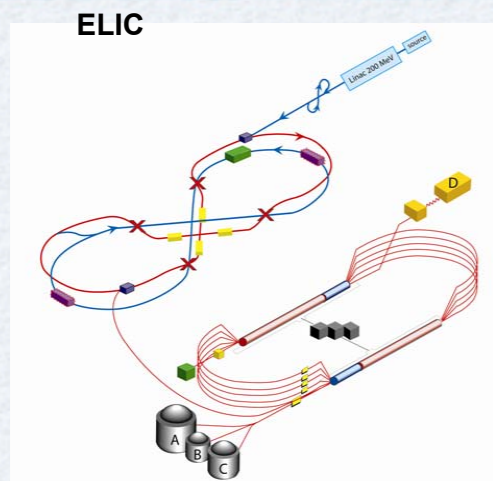


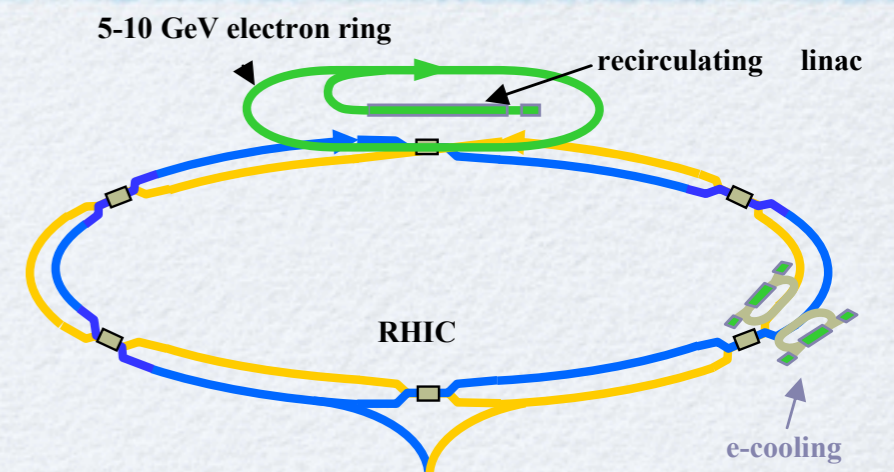
Nucleon Form Factors at the (M)EIC/MeRHIC

Guy Ron

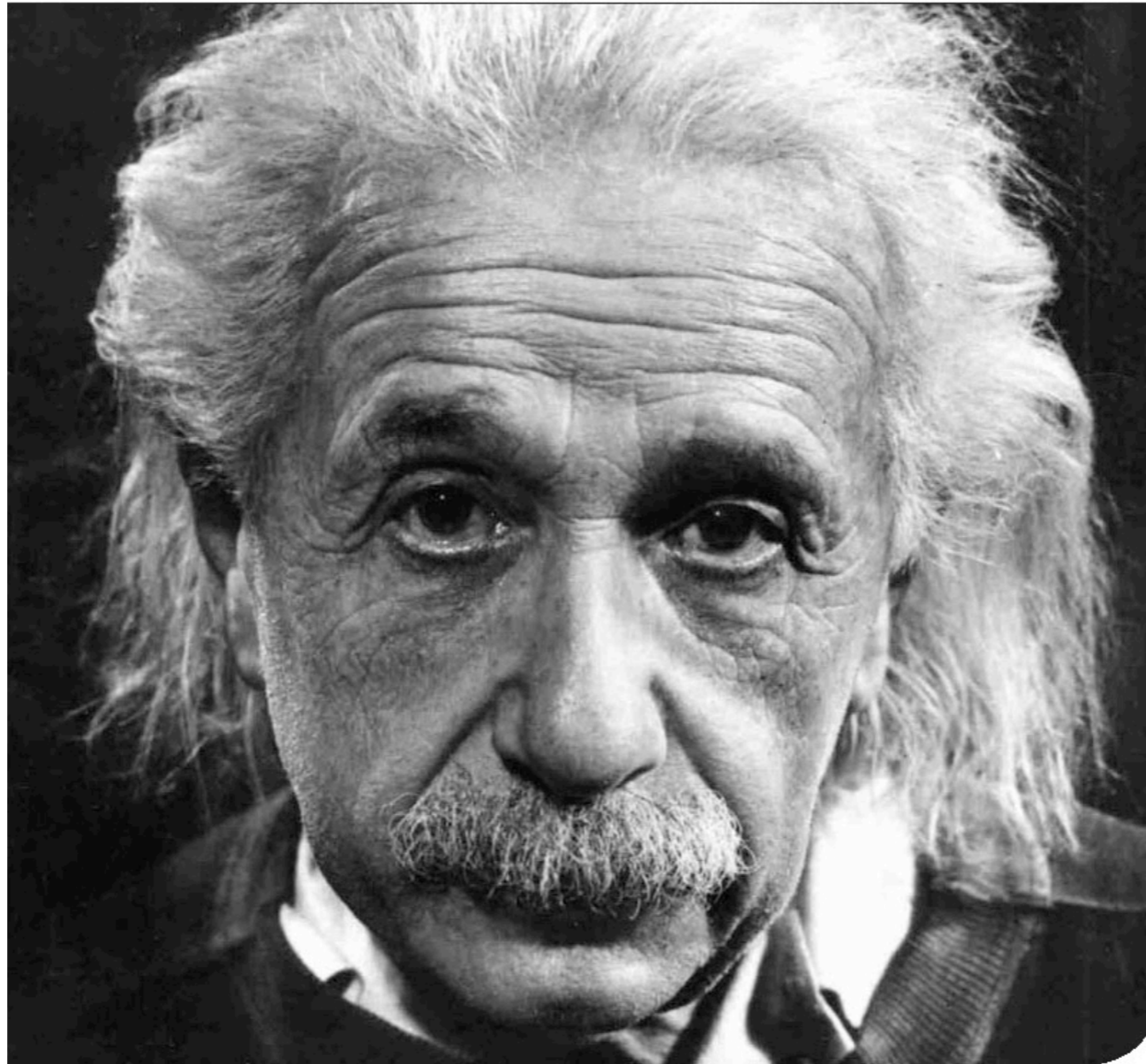
Nuclear Science Division
Lawrence Berkeley Lab



EIC Workshop
Rutgers University
Mar. 14, 2010



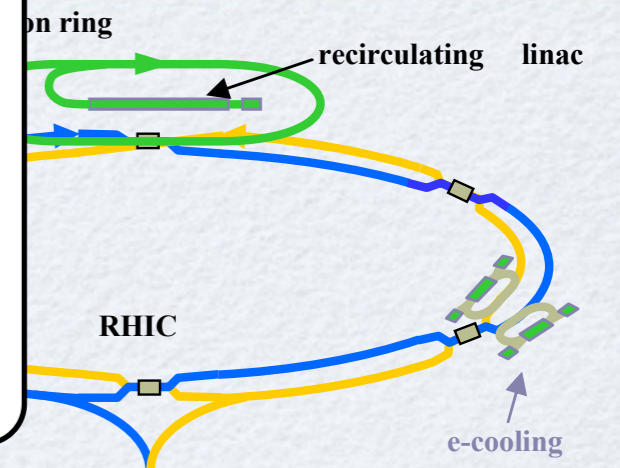
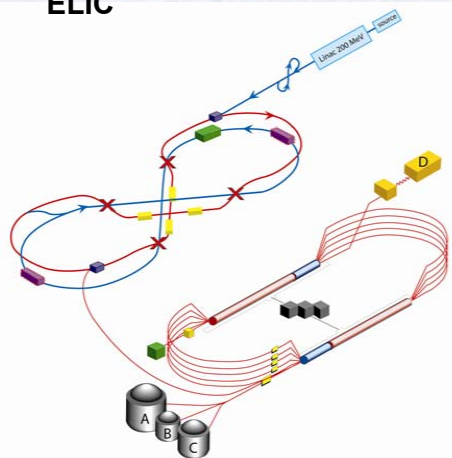
Nucle



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Happy π Day

ELIC



OUTLINE

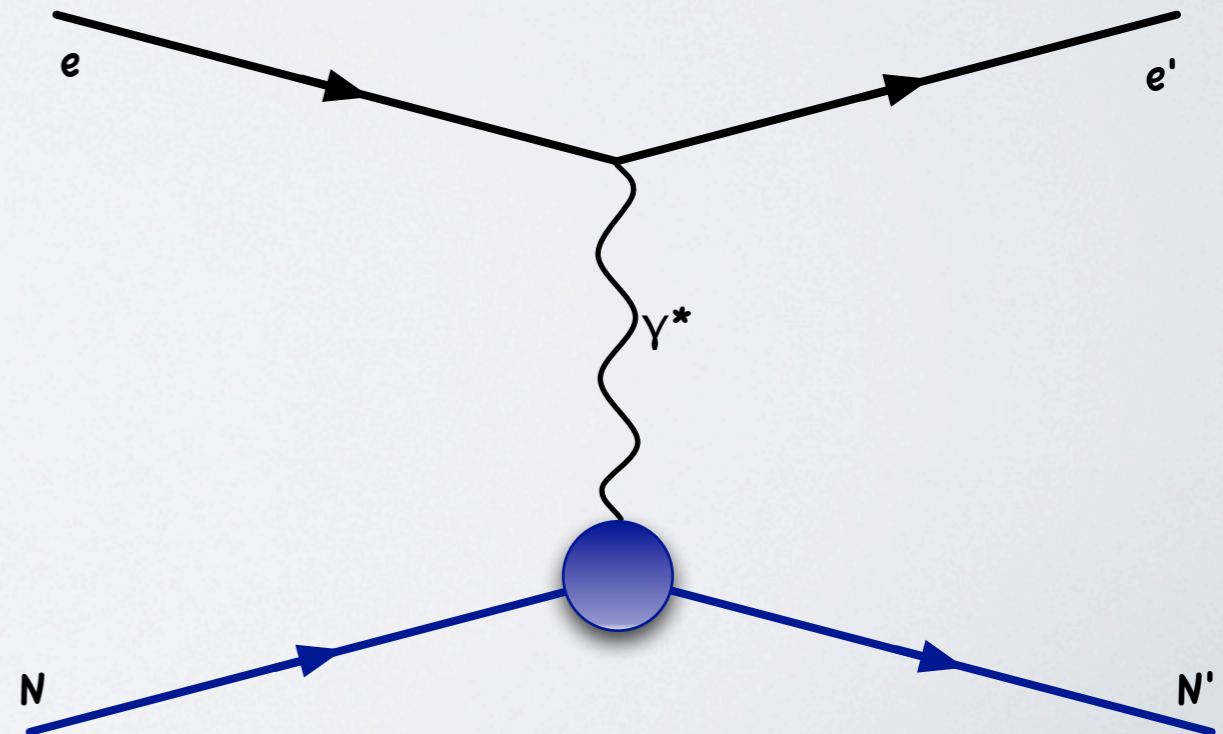
- **Form Factors 101.**
- **High Q^2**
 - **Motivation**
 - **Possibilities**
- **Low Q^2**
 - **Motivation**
 - **Possibilities**
- **Summary**

ELECTRON SCATTERING CROSS-SECTION (1- γ)

$$\frac{d\sigma_R}{d\Omega} = \frac{\alpha^2}{Q^2} \left(\frac{E'}{E} \right)^2 \frac{\cot^2 \frac{\theta_e}{2}}{1 + \tau}$$

Rutherford - Point-Like

$$\tau = \frac{Q^2}{4M^2}, \quad \varepsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right]^{-1}$$



ELECTRON SCATTERING CROSS-SECTION (1- γ)

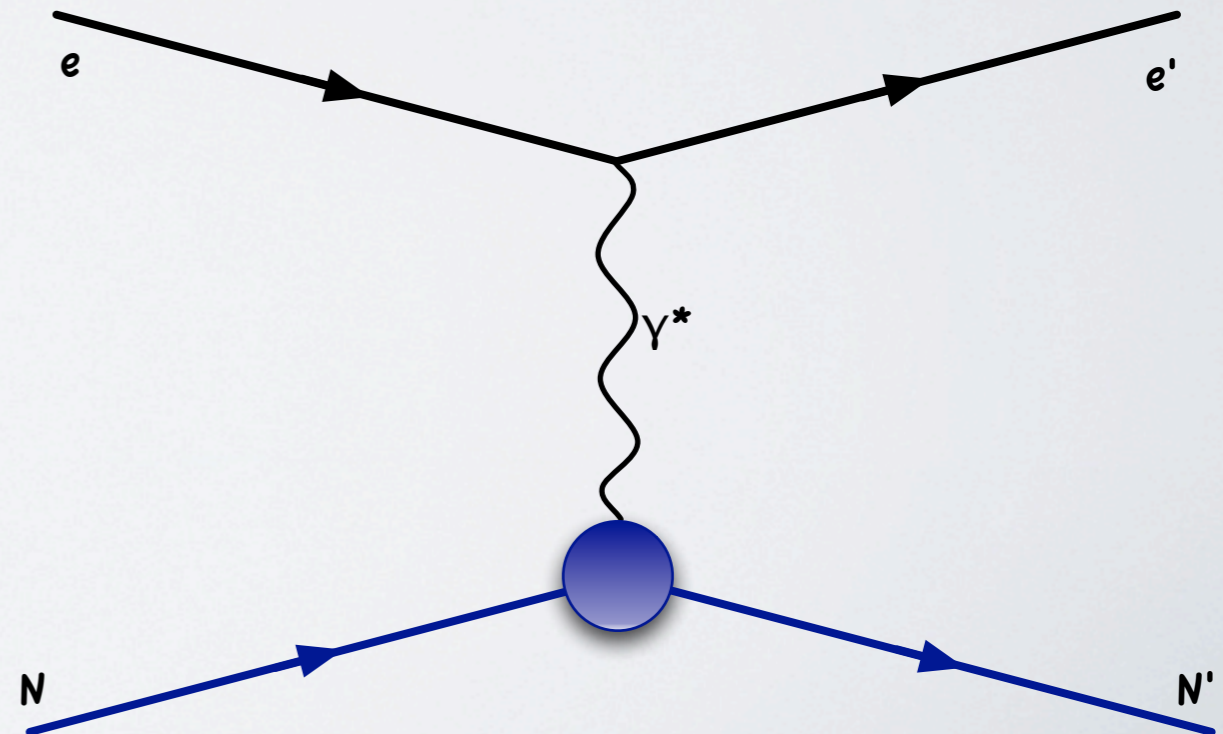
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Rutherford - Point-Like

$$\frac{d\sigma_M}{d\Omega} = \frac{d\sigma_R}{d\Omega} \times \left[1 + 2\tau \tan^2 \frac{\theta}{2} \right]$$

Mott - Spin-1/2

$$\tau = \frac{Q^2}{4M^2}, \quad \varepsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right]^{-1}$$



ELECTRON SCATTERING CROSS-SECTION (1- γ)

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Mott - Spin-1/2

$$\frac{d\sigma_{Str}}{d\Omega} = \frac{d\sigma_M}{d\Omega} \times \left[G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2) \right]$$

Rosenbluth -
Spin-1/2 with
Structure

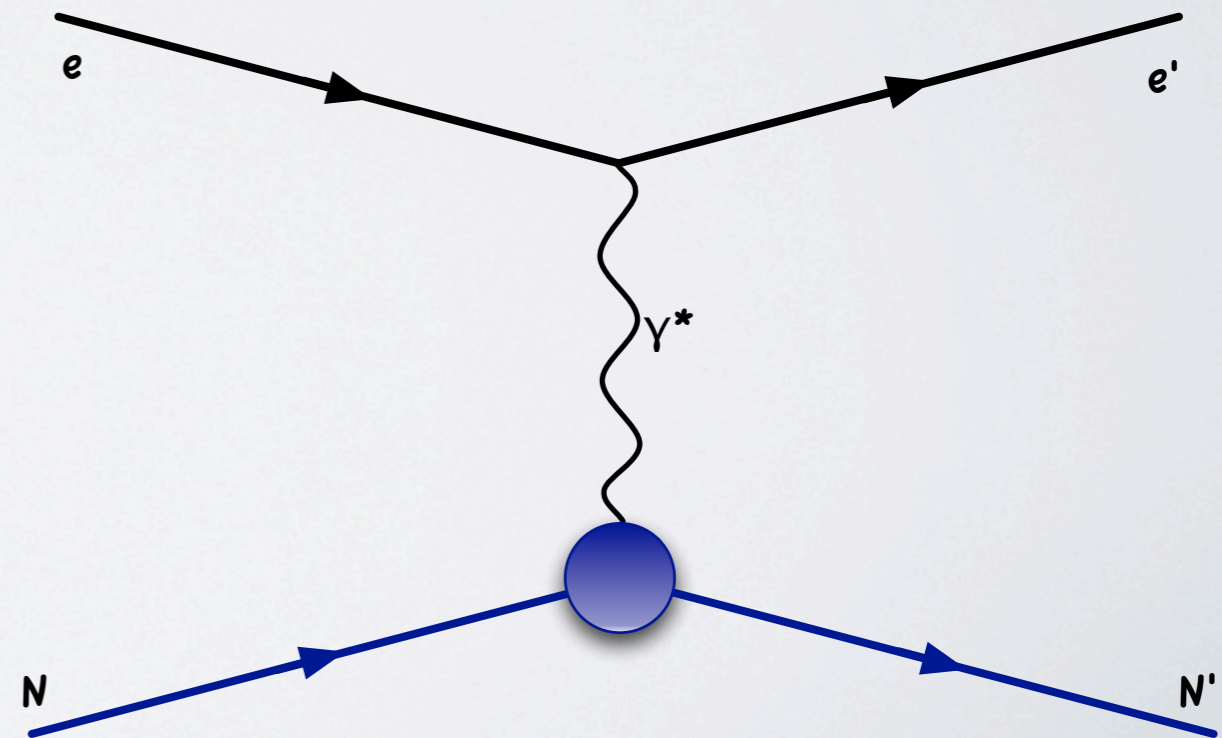
$$\tau = \frac{Q^2}{4M^2}, \quad \varepsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right]^{-1}$$

$$G_E^p(0) = 1 \quad G_E^n(0) = 0$$

$$G_M^p = 2.793 \quad G_M^n = -1.91$$

Sometimes
written using:

$$G_E = F_1 - \tau F_2$$

$$G_M = F_1 + F_2$$


ELECTRON SCATTERING CROSS-SECTION (1- γ)

$$\frac{d\sigma_R}{d\Omega} = \frac{\alpha^2}{Q^2} \left(\frac{E'}{E} \right)^2 \frac{\cot^2 \frac{\theta_e}{2}}{1 + \tau}$$

$$\frac{d\sigma_M}{d\Omega} = \frac{d\sigma_R}{d\Omega} \times \left[1 + 2\tau \tan^2 \frac{\theta}{2} \right]$$

$$\frac{d\sigma_{Str}}{d\Omega} = \frac{d\sigma_M}{d\Omega} \times \left[G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2) \right]$$

Everything we don't know goes here!

Rosenbluth -
Spin-1/2 with
Structure

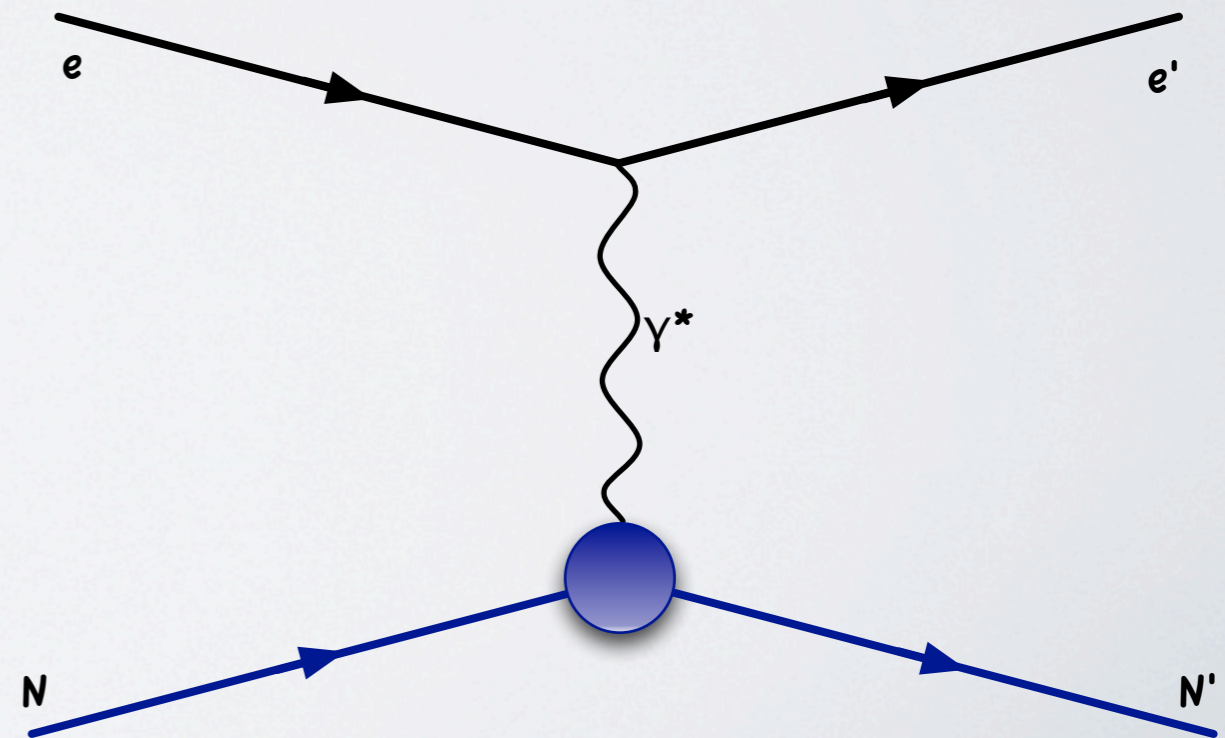
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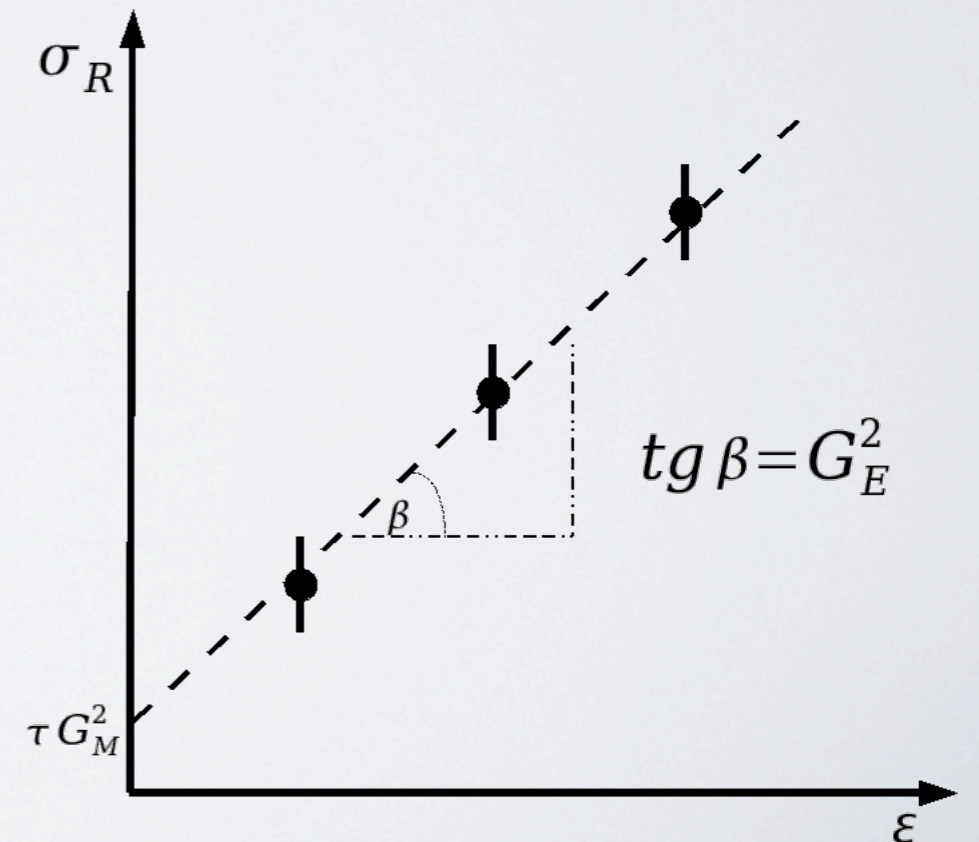
$$G_M = F_1 + F_2$$


Measurement Techniques

Rosenbluth Separation

$$\sigma_R = (d\sigma/d\Omega)/(d\sigma/d\Omega)_{\text{Mott}} = \tau G_M^2 + \varepsilon G_E^2$$

- Measure the reduced cross section at several values of ε (angle/beam energy combination) while keeping Q^2 fixed.
- Linear fit to get intercept and slope.
- **But...** G_M suppressed for low Q^2 (and G_E for high).
- Also normalization issues/acceptance issues/etc. make it hard to get high precision.



Measurement Techniques

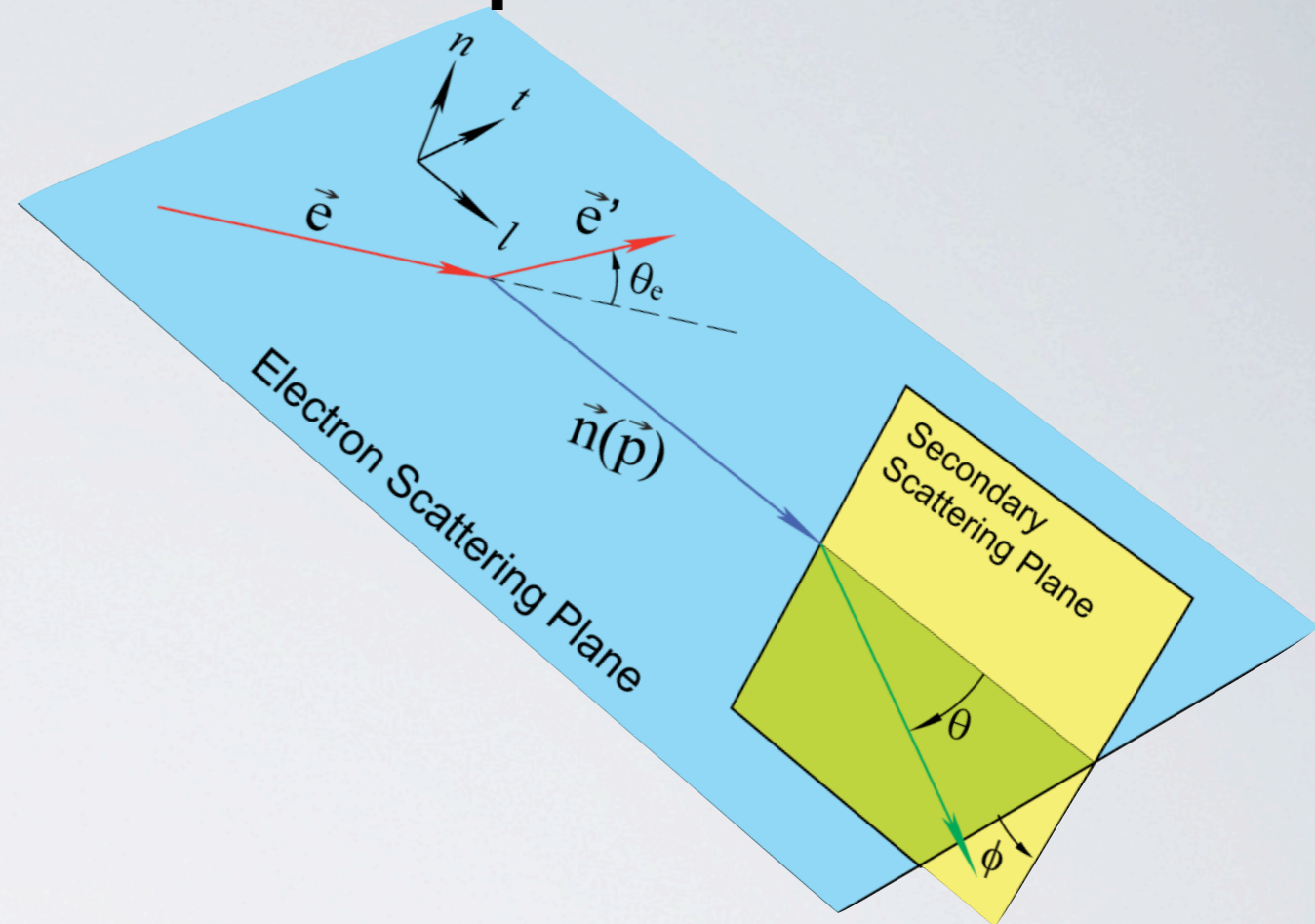
Recoil Polarization

(secondary scattering of nucleon)

$$I_0 P_t = -2\sqrt{\tau(1+\tau)} G_E G_M \tan \frac{\theta_e}{2}$$

$$I_0 P_l = \frac{E_e + E_{e'}}{M} \sqrt{\tau(1+\tau)} G_M^2 \tan^2 \frac{\theta_e}{2}$$

$$P_n = 0 \quad (1\gamma)$$



$$\mathcal{R} \equiv \mu_p \frac{G_E}{G_M} = -\mu_p \frac{P_t}{P_l} \frac{E_e + E_{e'}}{2M} \tan \frac{\theta_e}{2}$$

- A single measurement gives ratio of form factors.
- Interference of "small" and "large" terms allow measurement at practically all values of Q^2 .

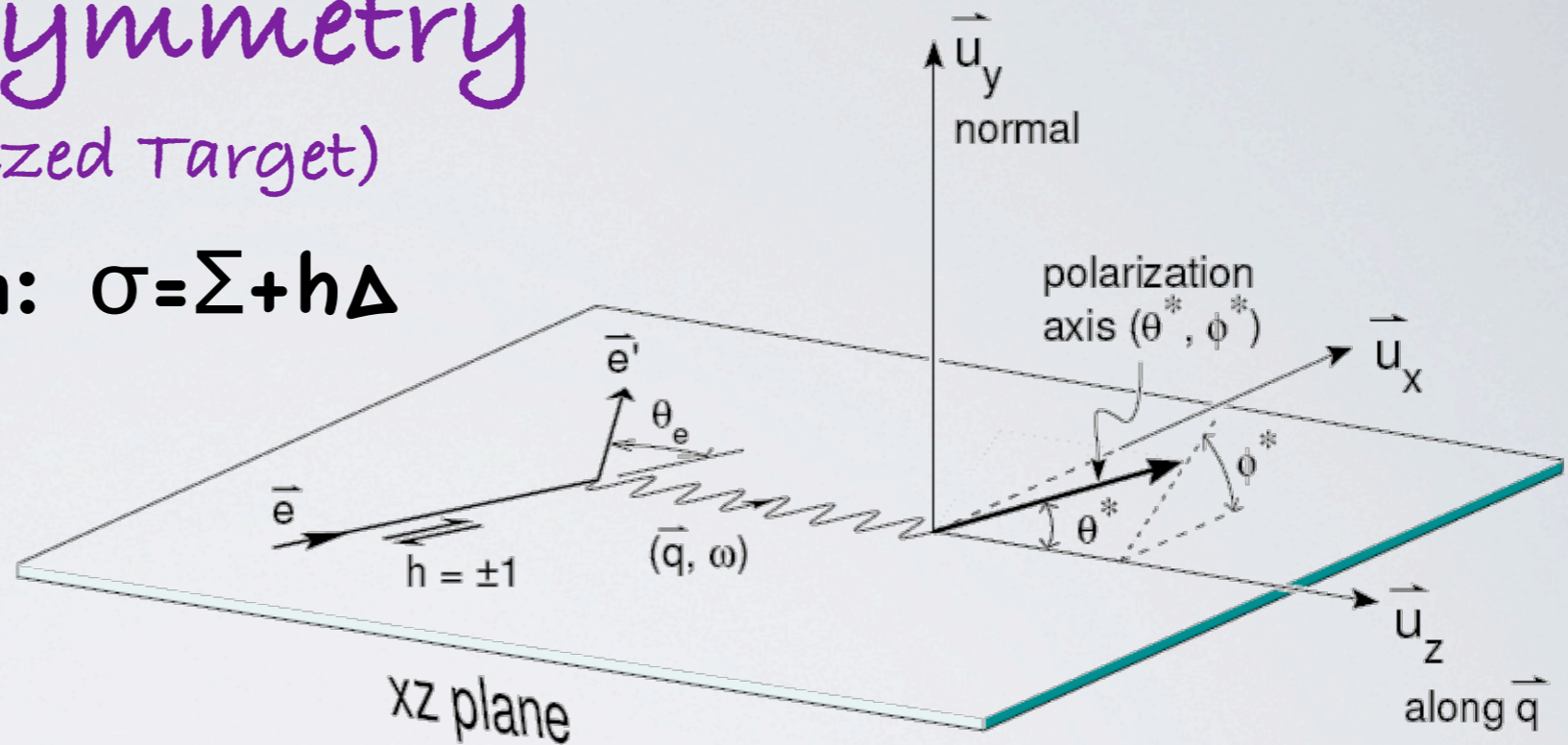
Measurement Techniques

Beam-Target Asymmetry

(Polarized Beam Polarized Target)

Polarized Cross Section: $\sigma = \Sigma + h\Delta$

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$



$$A = P_b P_t \frac{\overbrace{a \cos \theta^* G_M^2}^{A_T} + \overbrace{b \sin \theta^* \cos \phi^* G_E G_M}^{A_{LT}}}{c G_M^2 + d G_E^2}$$

Measure asymmetry at two different target settings, say $\theta^* = 0, 90$.

Ratio of asymmetries gives ratio of form factors.

Functionally identical to recoil polarimetry measurements.

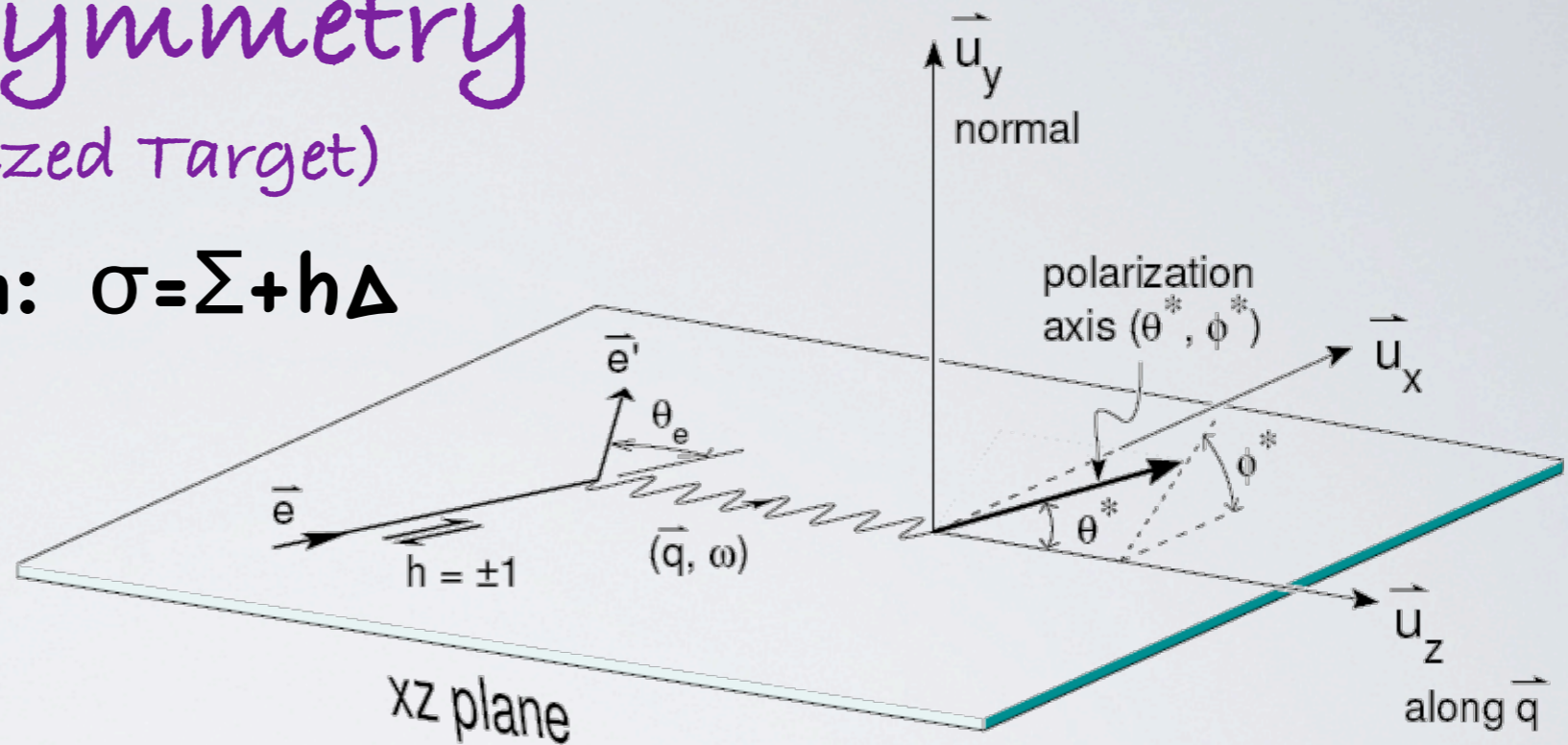
Measurement Techniques

Beam-Target Asymmetry

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Relevant for EIC

$$A = P_b P_t \frac{\overbrace{a \cos \theta^* G_M^2}^{A_T} + \overbrace{b \sin \theta^* \cos \phi^* G_E G_M}^{A_{LT}}}{c G_M^2 + d G_E^2}$$

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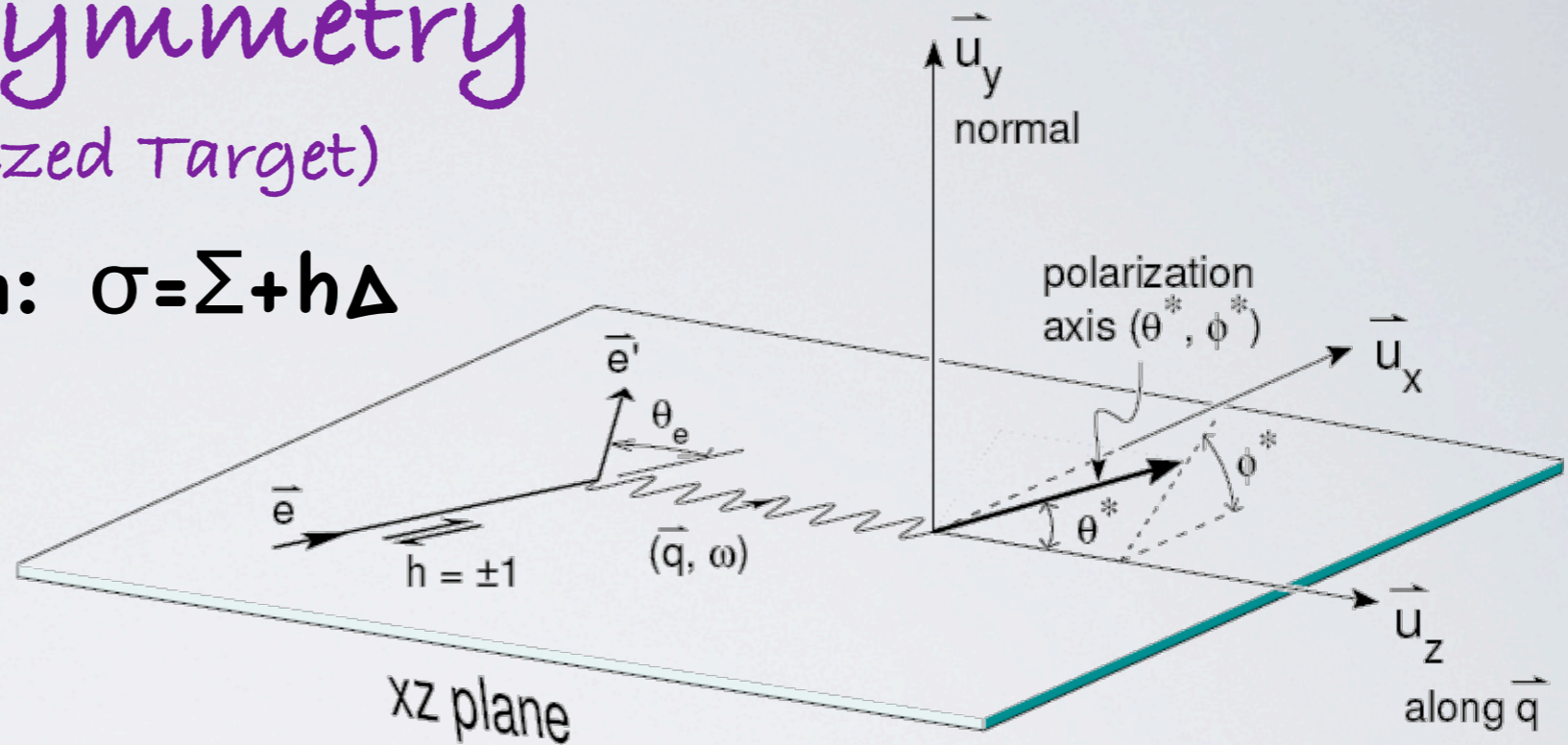
Canceling (some of) the uncertainties

Beam-Target Asymmetry

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Simultaneous measurement with two different values of θ^* .
 Ratio of asymmetries related to FF ratio and cancels systematics.

$$\frac{G_E}{G_M} = - \frac{a(\tau, \theta_e) \cos \theta_1^* - \Gamma a(\tau, \theta_e) \cos \theta_2^*}{\cos \phi_1^* \sin \theta_1^* - \Gamma \cos \phi_2^* \sin \theta_2^*}$$

$$a(\tau, \theta_e) = \sqrt{\tau(1 + (1 + \tau) \tan^2(\theta_e/2))}$$

$$\Gamma = \mathcal{A}_1 / \mathcal{A}_2$$

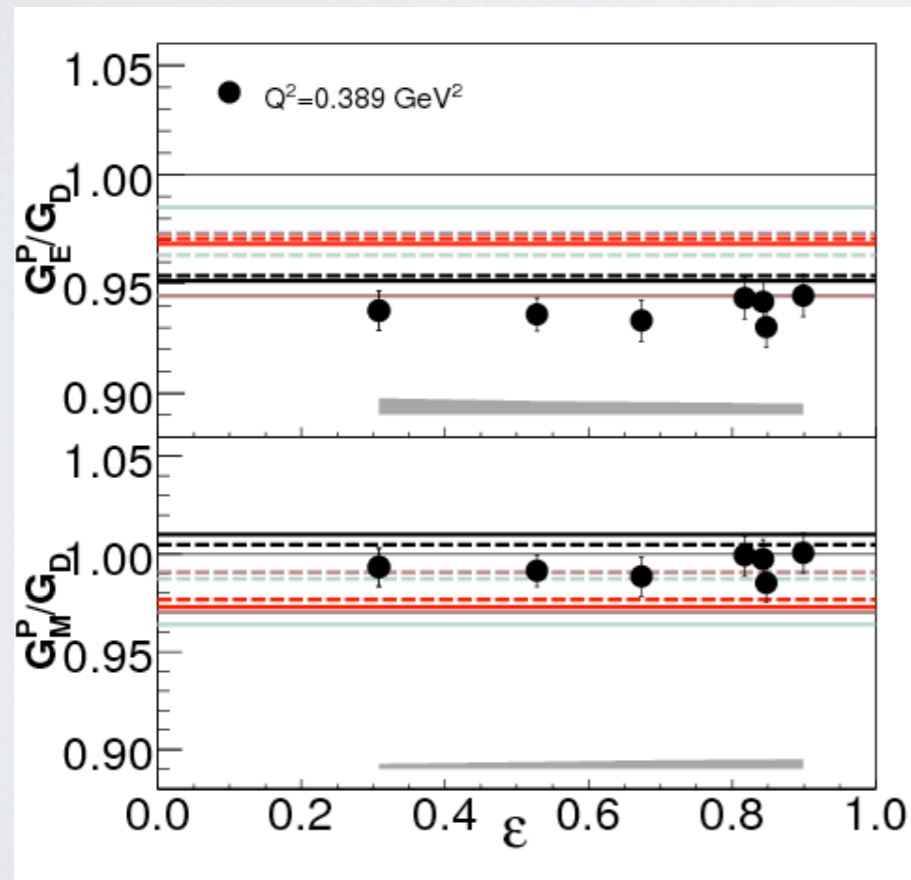
Set p beam polarization to intermediate angle such that $\theta_1^* \neq \theta_2^*$.

Getting G_E & G_M from Ratios + Cross Sections

Cross section at high/low Q^2 dominated by one term (Rosenbluth separation not feasible).

Ratio gives second equation

→ Can now solve 2 equations in two variables to get both ffs.
Multiple cross section measurements at the same Q^2 give cross check.



$$\sigma_R = \tau G_M^2 + \epsilon G_E^2$$

$$\mathcal{R} = \mu \frac{G_E}{G_M}$$

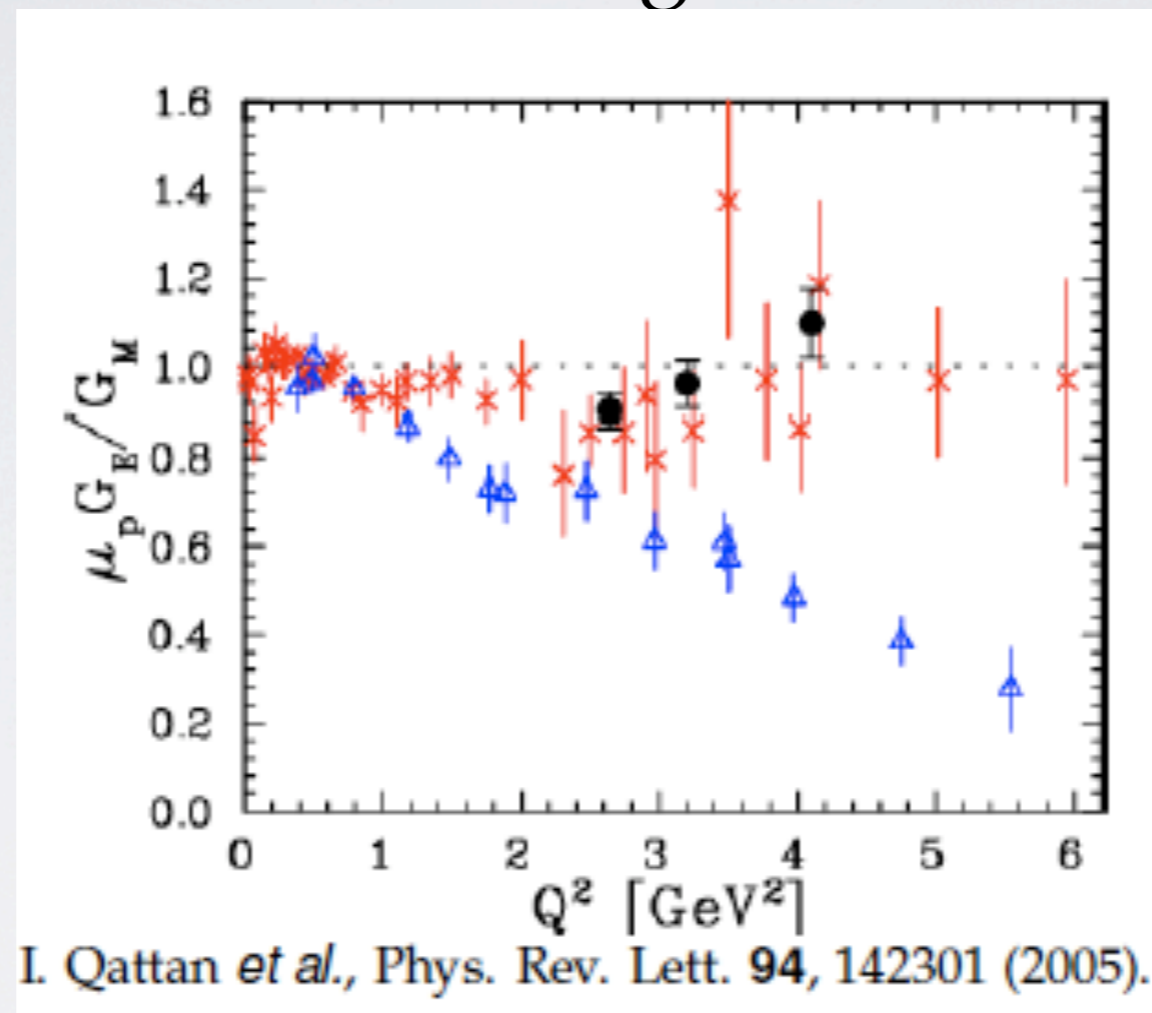
$$\sigma_R = \tau G_M^2 + \epsilon \frac{G_M^2 \mathcal{R}^2}{\mu^2}$$

$$G_M^2 = \sigma_R / \left(\tau + \epsilon \mathcal{R}^2 / \mu^2 \right)$$

High Q^2 Measurements

The high Q^2 discrepancy

- At high Q^2 Rosenbluth and polarization measurements for the proton are in violent disagreement.



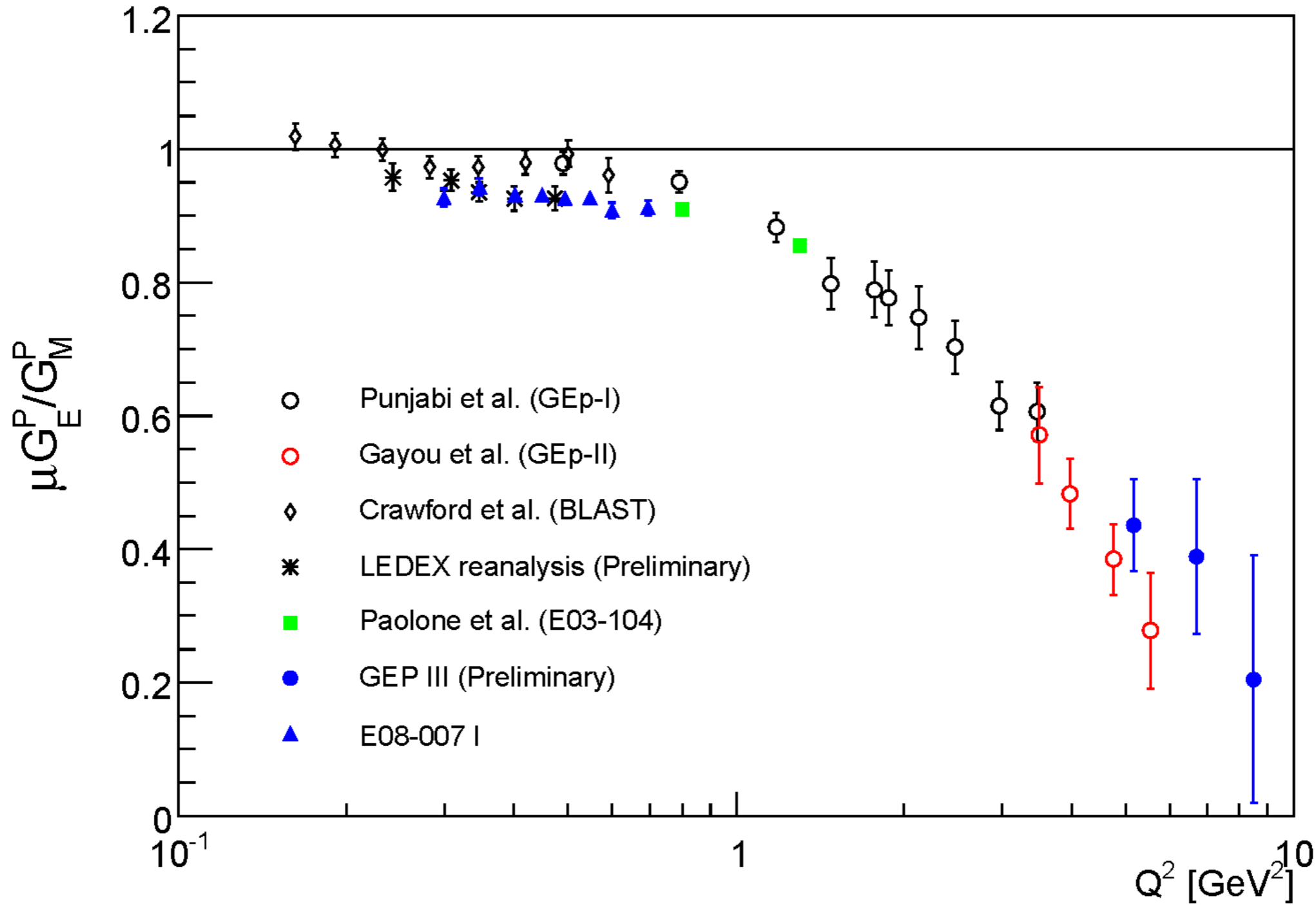
- Almost certainly explained by multi- γ effects.

“As $G_E = F_1 - \tau F_2$, it is a priori quite likely that G_E becomes negative for large values of k^2 ” - *N. Dombey, Rev. Mod. Phys.* 41, 1 (1969). - **Not supported by new results.**

The high Q^2 discrepancy

- At the

values for



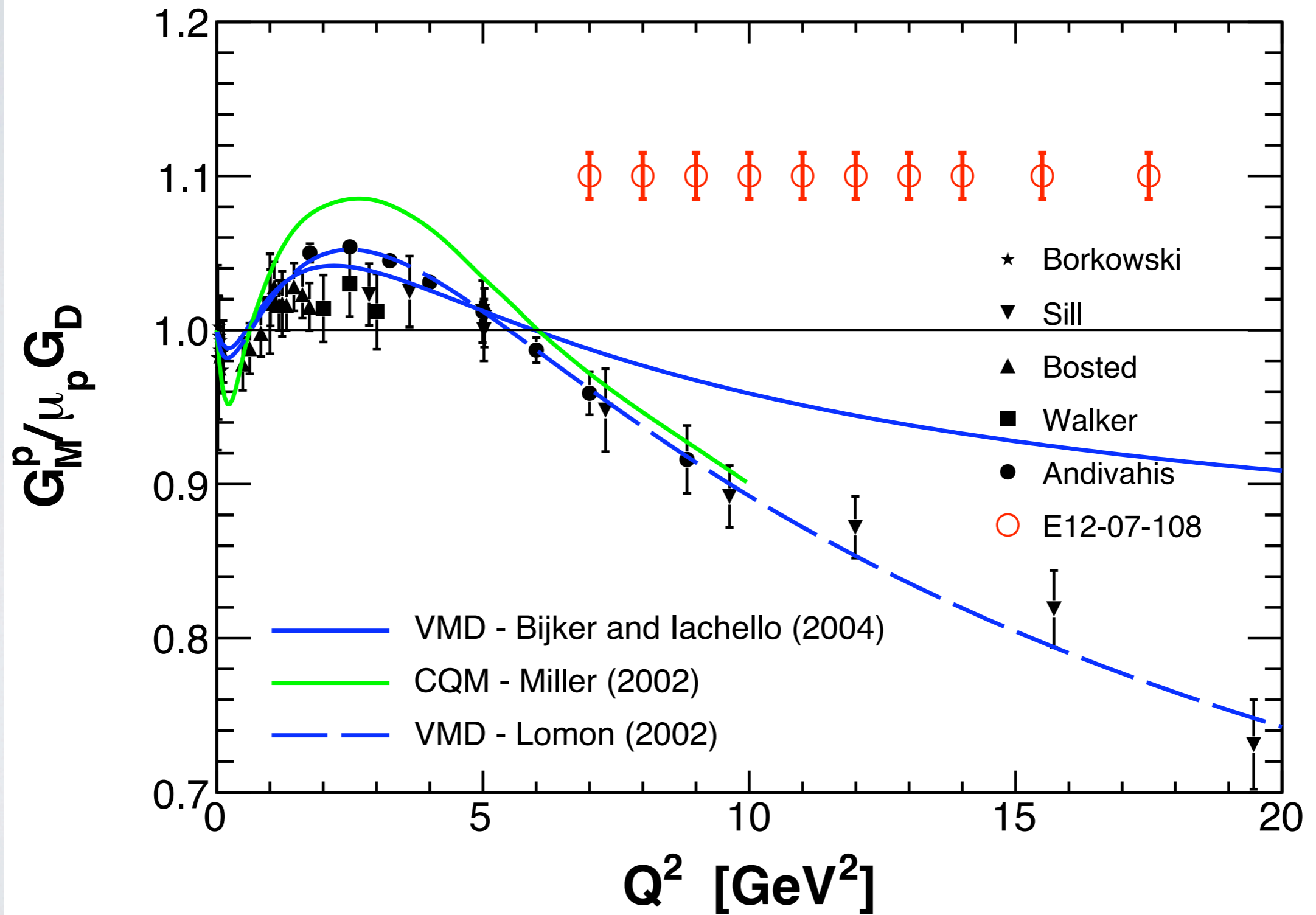
values

- All

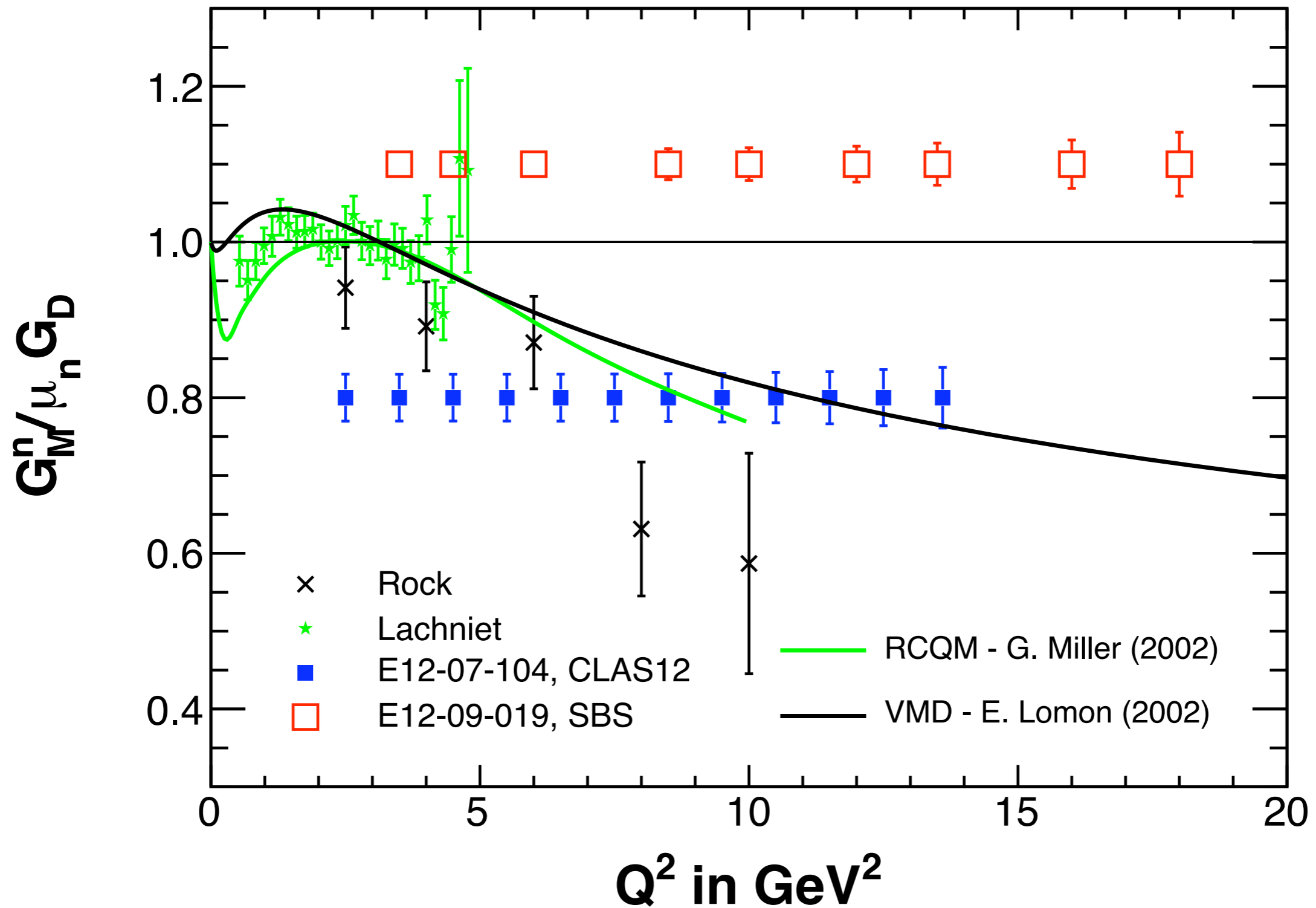
“As G

of k^2 ” - *N. Dombey, Rev. Mod. Phys.* 41, 1 (1969). - **Not supported by new results.**

12 GeV G_{MP} @ JLab



12 GeV G_{Mn} @ JLab

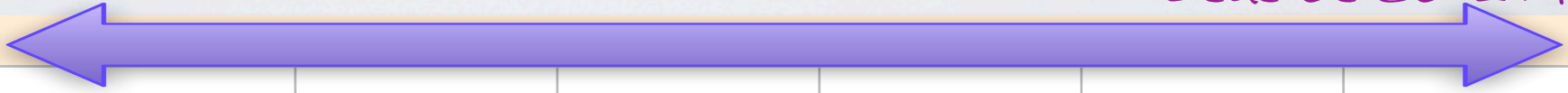


Prospects for High Q^2 ep with EIC

- 3 GeV Electron + 30 GeV Proton.
- $\mathcal{L} = 10^{34} \text{ cm}^{-1} \text{ sec}^{-1}$.
- Full angular (φ) detector coverage.
- $\Delta Q^2 / Q^2 = 0.1$.

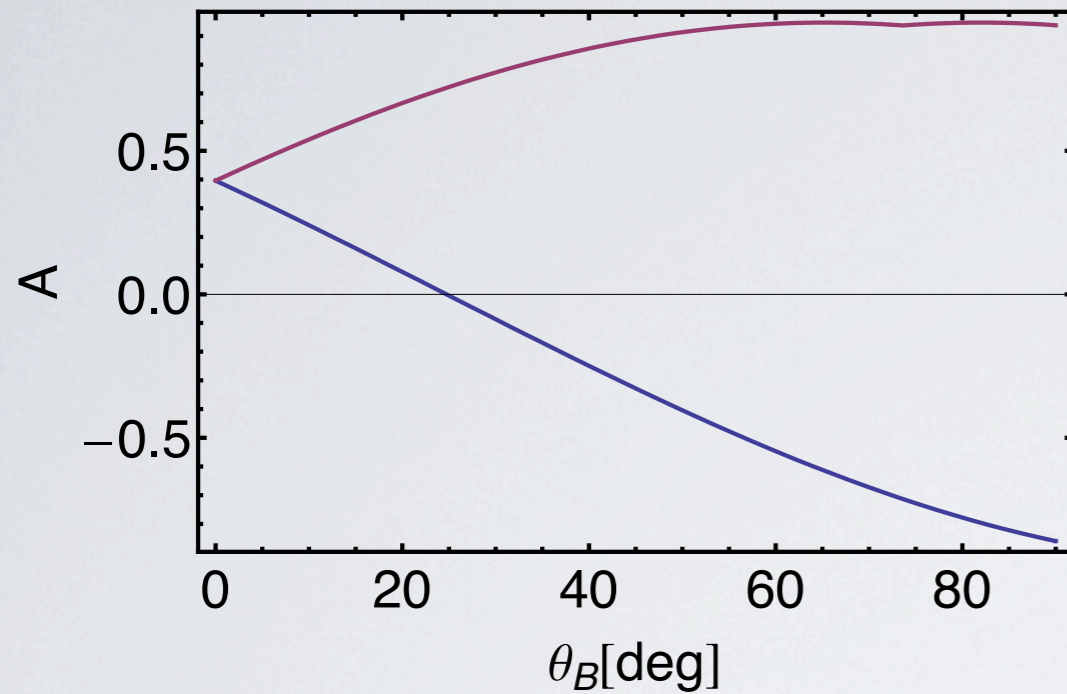
Systematics Limited

Statistics Limited

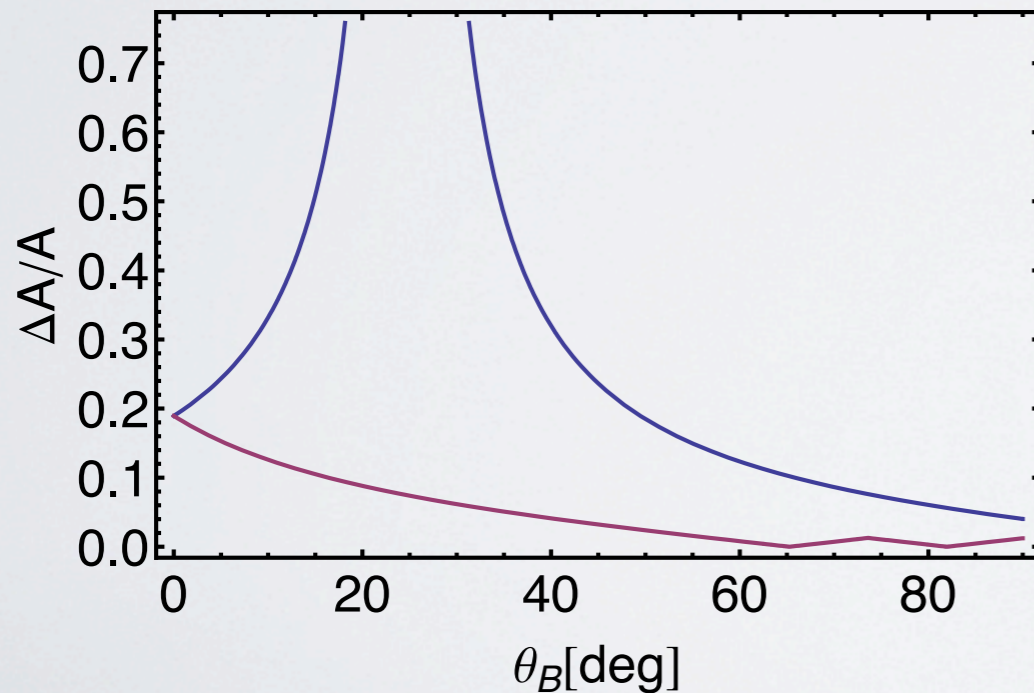


Q^2 (GeV^2)	10	20	30	40	50	60
θ_e	56.25	74.97	87.28	96.4	103.6	109.5
θ_p	6.12	8.77	10.9	12.76	14.5	16.1
E'_e	3.74	4.5	5.24	6	6.74	7.5
E'_p	28.3	27.56	26.81	26.06	25.31	24.56
Events / year	186000	7300	1000	250	80	30
$(\Delta \sigma / \sigma)_{stat}$	0.2%	1.2%	3.1%	6.3%	11.2%	18.2%
$(\Delta A / A)_{stat}$	0.3%	1.6%	4.4%	9%	15.8%	25.8%

Prospects for High Q^2 ep with EIC



Asymmetry as a function of proton polarization angle for $Q^2 = 10 \text{ GeV}^2$



Systematic uncertainty in asymmetry as a function of proton polarization angle for $Q^2 = 10 \text{ GeV}^2$.

$$\Delta\theta_{\text{pol}} = 5^\circ$$

Low Q^2 Measurements

Why LOW Q^2 ?

- Deviations from dipole form evident.
- Probe static properties ($Q^2 \rightarrow 0$) and peripheral structure.
- Small Q^2 does not allow for pQCD, many competing EFTs.
- Hitting the π mass region (2π -cut in Pauli / Dirac FFs).
- Potentially impacts many high precision measurements (nucleon GPDs, parity violation, Zemach radius,...).

Some Models

VMD

$$F(Q^2) = \sum \frac{C_{\gamma V_i}}{Q^2 + M_{V_i}^2} F_{V_i N}(Q^2)$$

Breaks down at high Q^2

Lattice QCD (*not really a model....*)

RCQM

*Point Form
Light Front*

di-Quark

CBM/LFCBM

pQCD

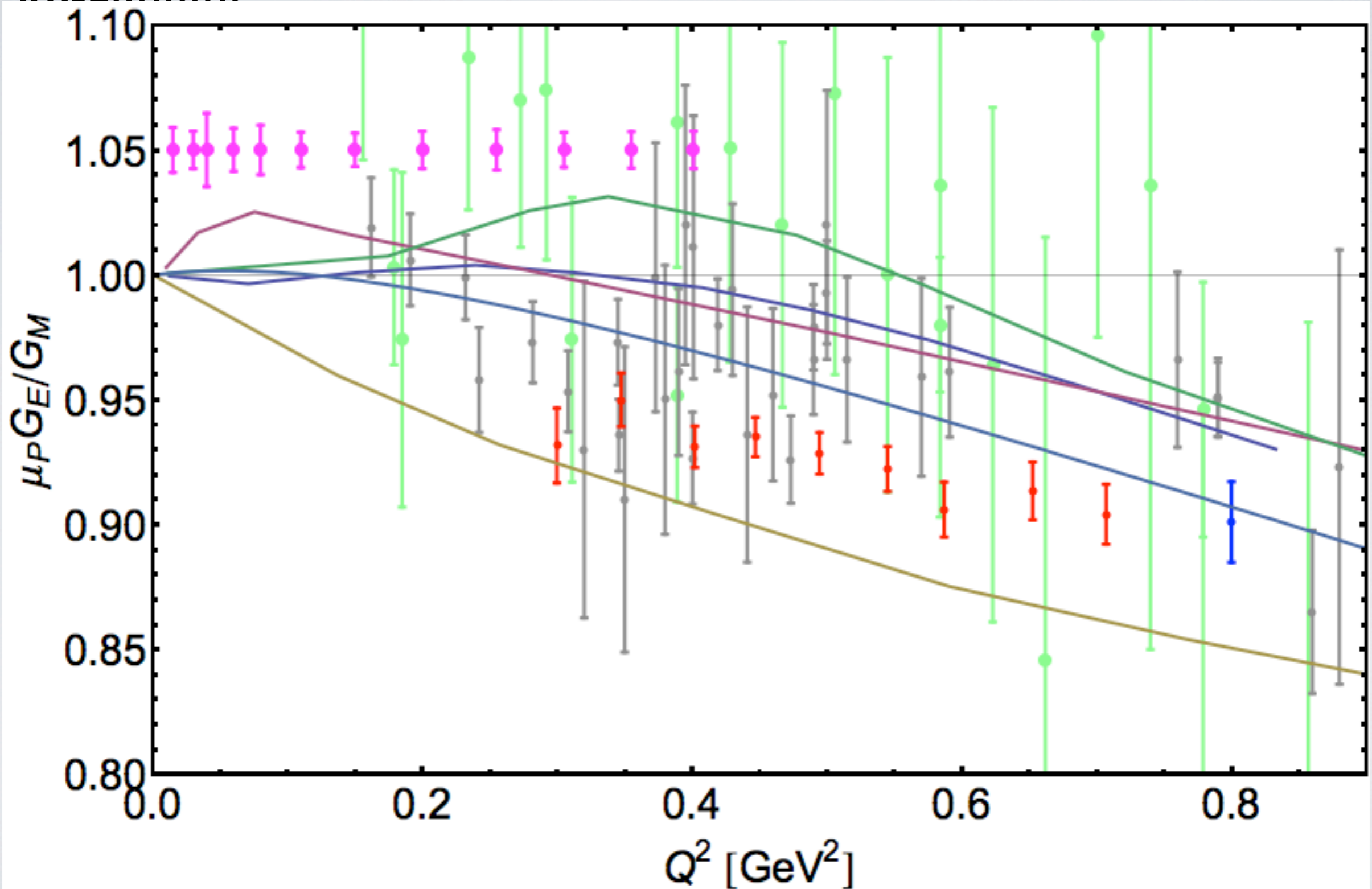
*Helicity Conservation
Counting rules $\frac{Q^2 F_2}{F_1} \rightarrow \text{Constant}$*

State of the Art

- **Rosenbluth:**
 - Mainz has concluded a high precision cross section survey.
 - Measured cross sections down to $Q^2 \approx 0.01 \text{ GeV}^2$.
- **Polarization Data (FF Ratio):**
 - **Bates BLAST** (Beam-Target Asymmetry) - $Q^2 = 0.16 - 0.6 \text{ GeV}^2$, C. B. Crawford et al., Phys. Rev. Lett. 98, 052301 (2007).
 - **JLab LEDEX** (Recoil Polarization) - $Q^2 = 0.22 - 0.5 \text{ GeV}^2$, G. Ron et al., Phys. Rev. Lett. 99, 202002 (2007).
 - **JLab E08007 Part I** (Recoil Polarization) - $Q^2 = 0.25 - 0.7 \text{ GeV}^2$ (Very high precision), X. Zhan PhD Thesis.
 - **JLab E08007 Part II** (Beam-Target Asymmetry) - $Q^2 = 0.01 - 0.4 \text{ GeV}^2$ (Very high precision) **Tentative 2012**.
- **Strong deviation from unity at low Q^2 .**
- **Attributed to $G_{Ep} < G_D$.**

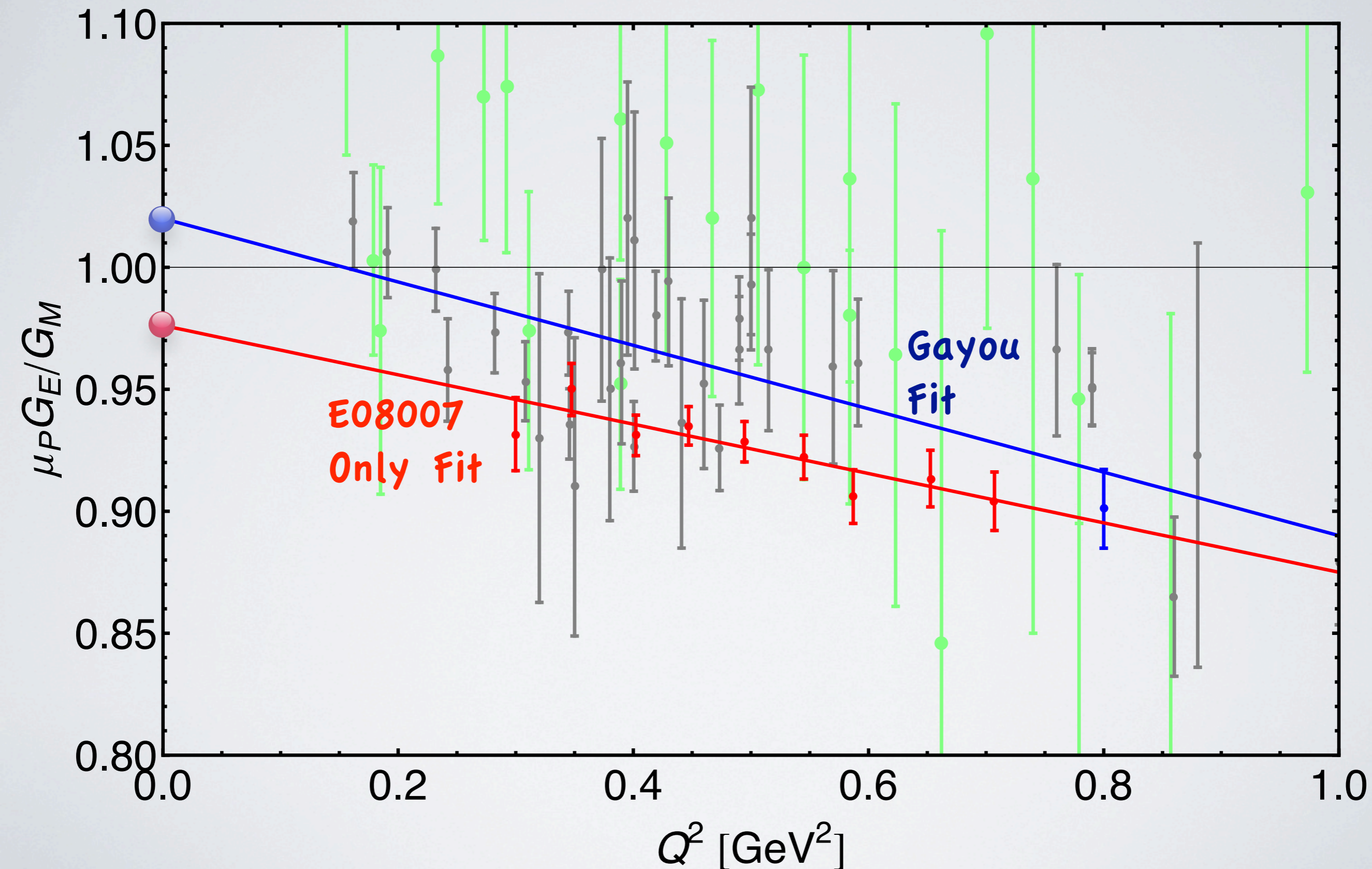
State of the Art

• Rosenbluth

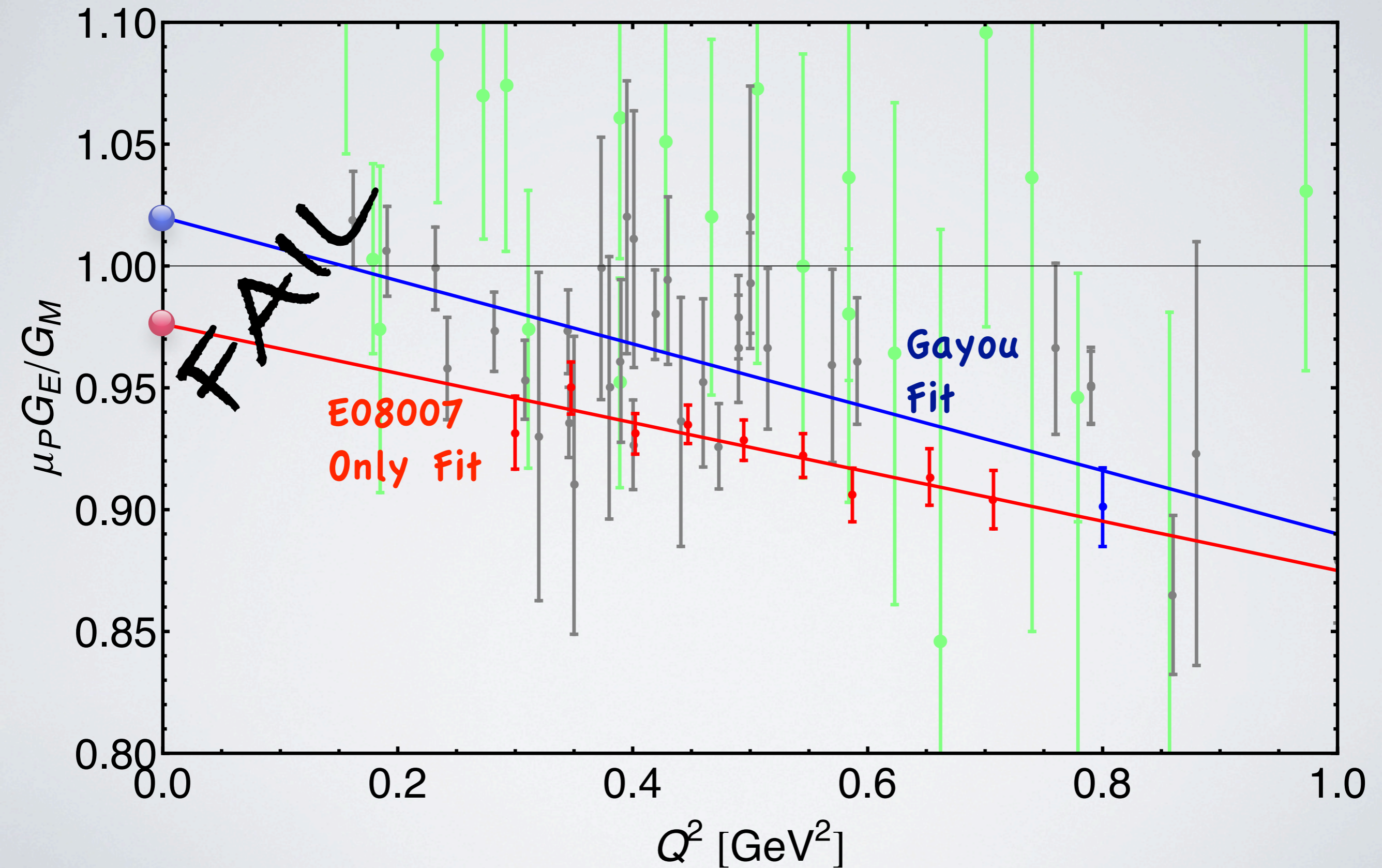


• Attributed to $G_{Ep} \setminus G_D$.

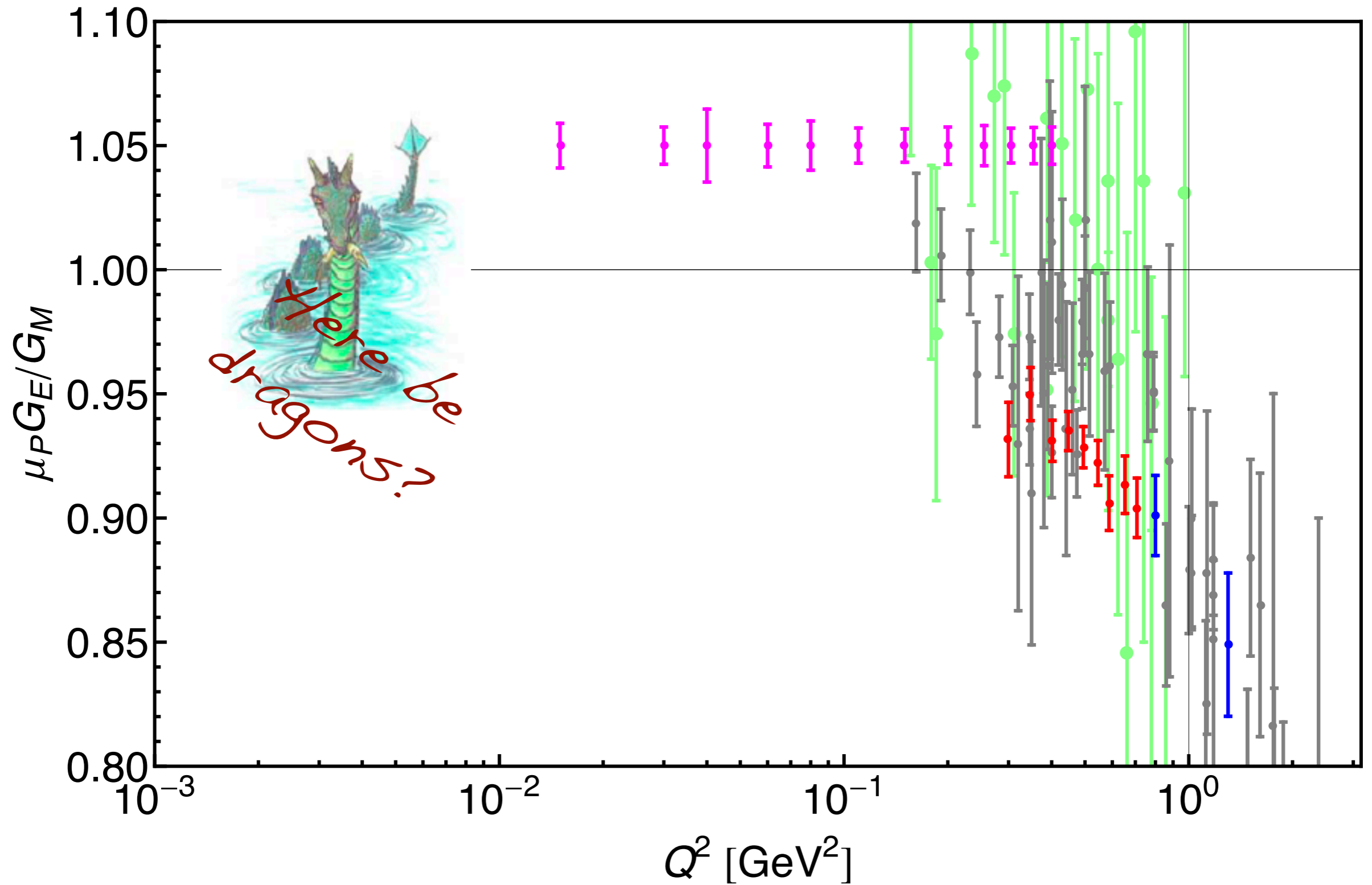
Low/High Q^2 Data Matching



Low/High Q^2 Data Matching



State of the Art



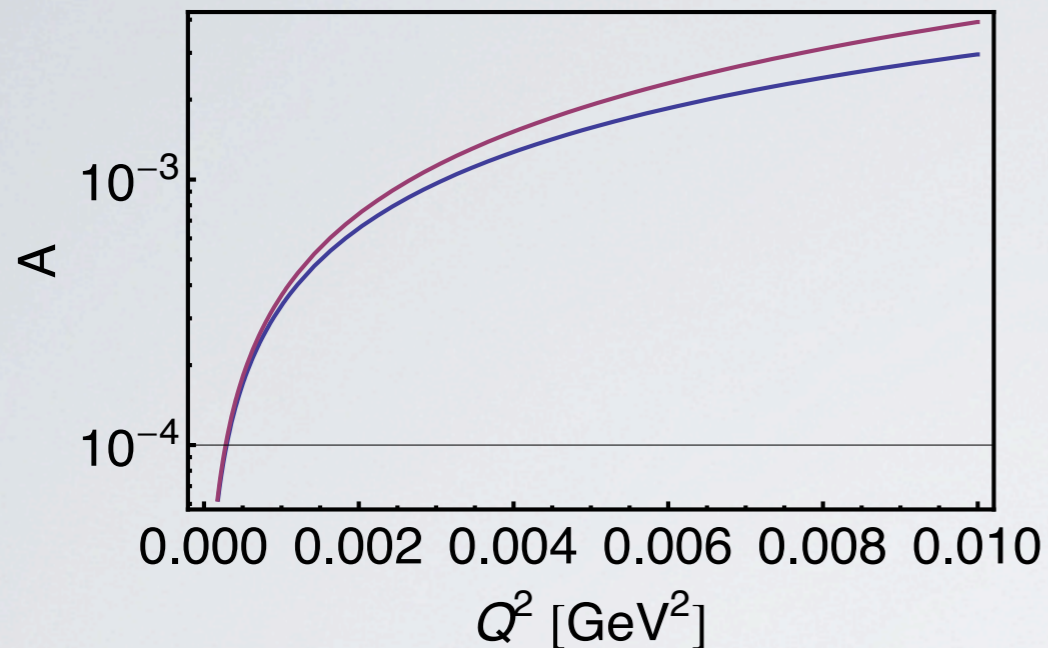
Prospects for **LOW** Q^2 **ep** with EIC

- Proton polarimetry not feasible for high proton beam energies ($T_p \sim T_{p'}$).
- Very forward scattered electron.
- Luminosity drop significantly when lowering beam energies.
- Cross section measurement gives essentially G_E (charge radius).
- But.... Statistics not an issue.
- **Limiting factor is systematic uncertainties (in particular proton beam polarization direction).**

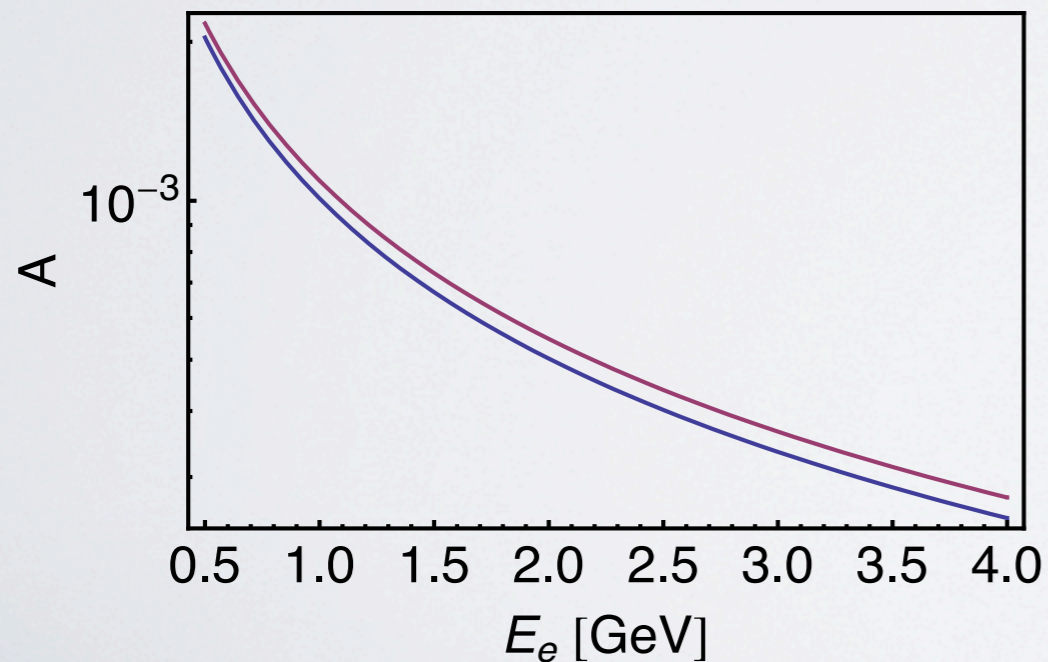
Q^2 (GeV^2)	10^{-4}	$5 \cdot 10^{-4}$	10^{-3}	$5 \cdot 10^{-3}$	0.01
θ_e	0.19	0.427	0.6	1.35	1.9
χS (cm^{-2})	2.60E-23	1.00E-24	2.50E-25	1.00E-26	2.50E-27
Rate (Hz)	9.1	1.75	0.875	0.175	0.0875
$T_{0.5\%}$ (hr)	1.22	6.35	12.7	63.5	127

- $\Delta Q^2/Q^2 = 0.01$.
- Assuming "CDF Style" roman pots detectors 1m from intersection point.
- Smallest possible angle $\sim 0.2\text{deg}$.
- Lowest possible $Q^2 \sim 10^{-4} \text{ GeV}^2$.
- Uncertainties always dominated by systematics - **In particular proton beam polarization direction.**

Prospects for **LOW** Q^2 **ep** with EIC



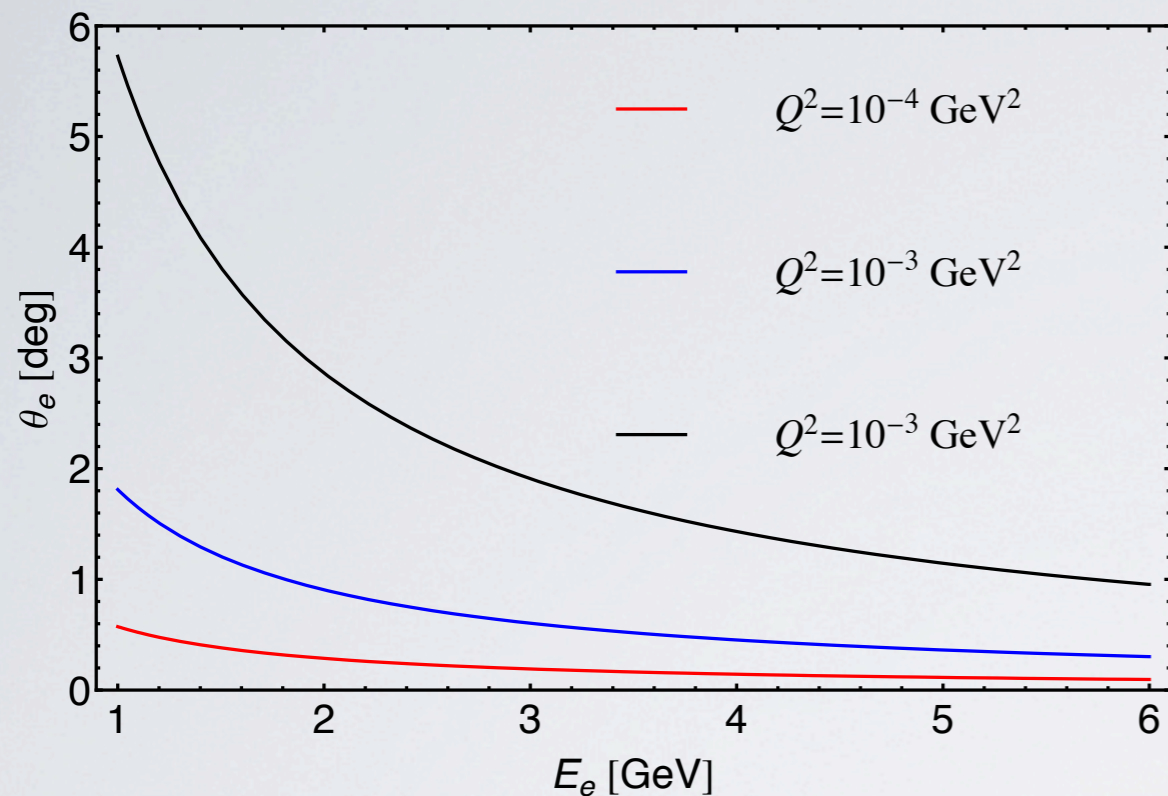
Asymmetry as a function of Q^2 ($\theta_{\text{pol}} = 45^\circ$).



Asymmetry as a function of electron beam energy ($Q^2 = 0.001 \text{ GeV}^2$, $\theta_{\text{pol}} = 45^\circ$).

Lower beam energy is better.

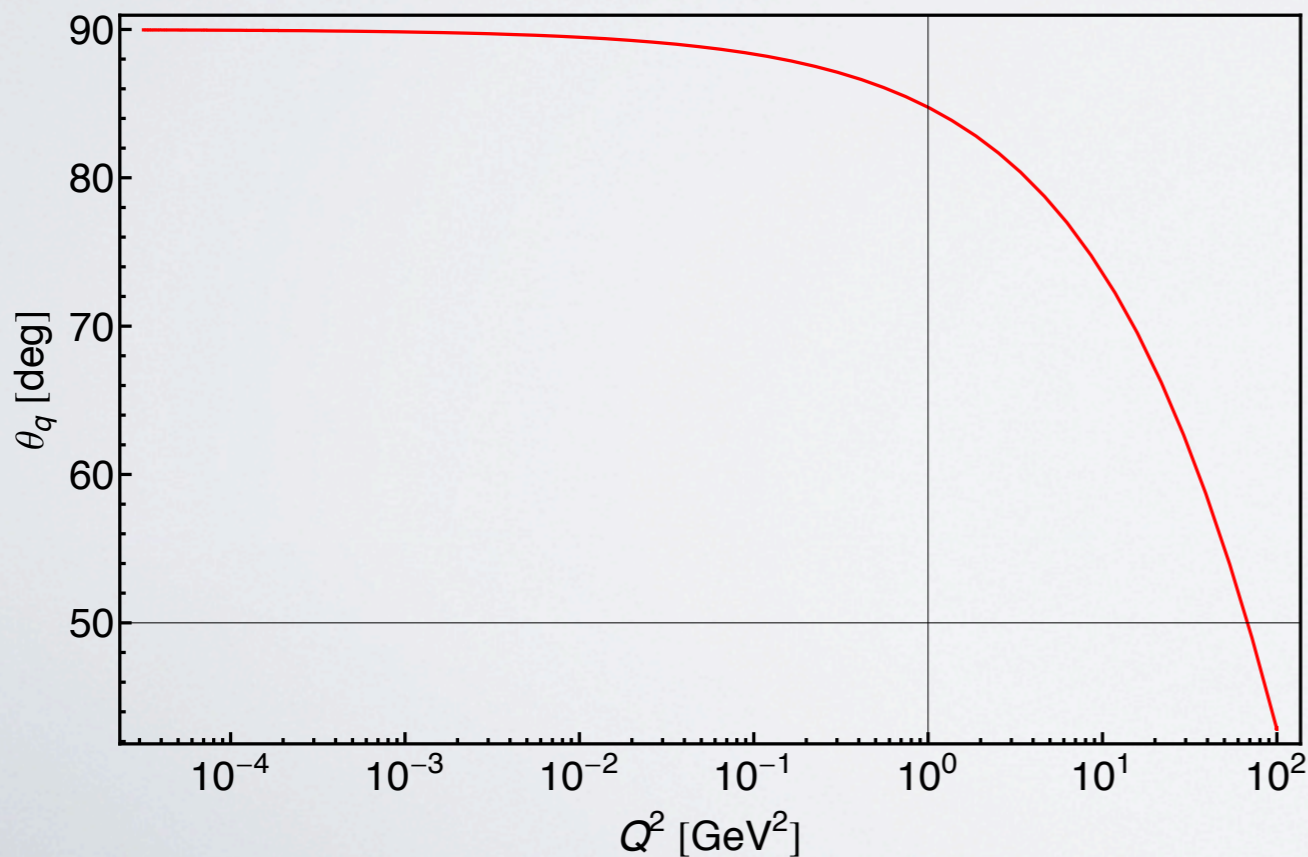
Prospects for **LOW** Q^2 ep with EIC



Electron angle as a function of electron beam energy.

Lower beam energy is better.

Negligible effect from proton beam energy.



q-vector angle as a function of Q^2 .

Since for low Q^2 $\theta_q \sim 90$ deg, need intermediate θ polarization.

Prospects for **LOW** Q^2 ep with EIC

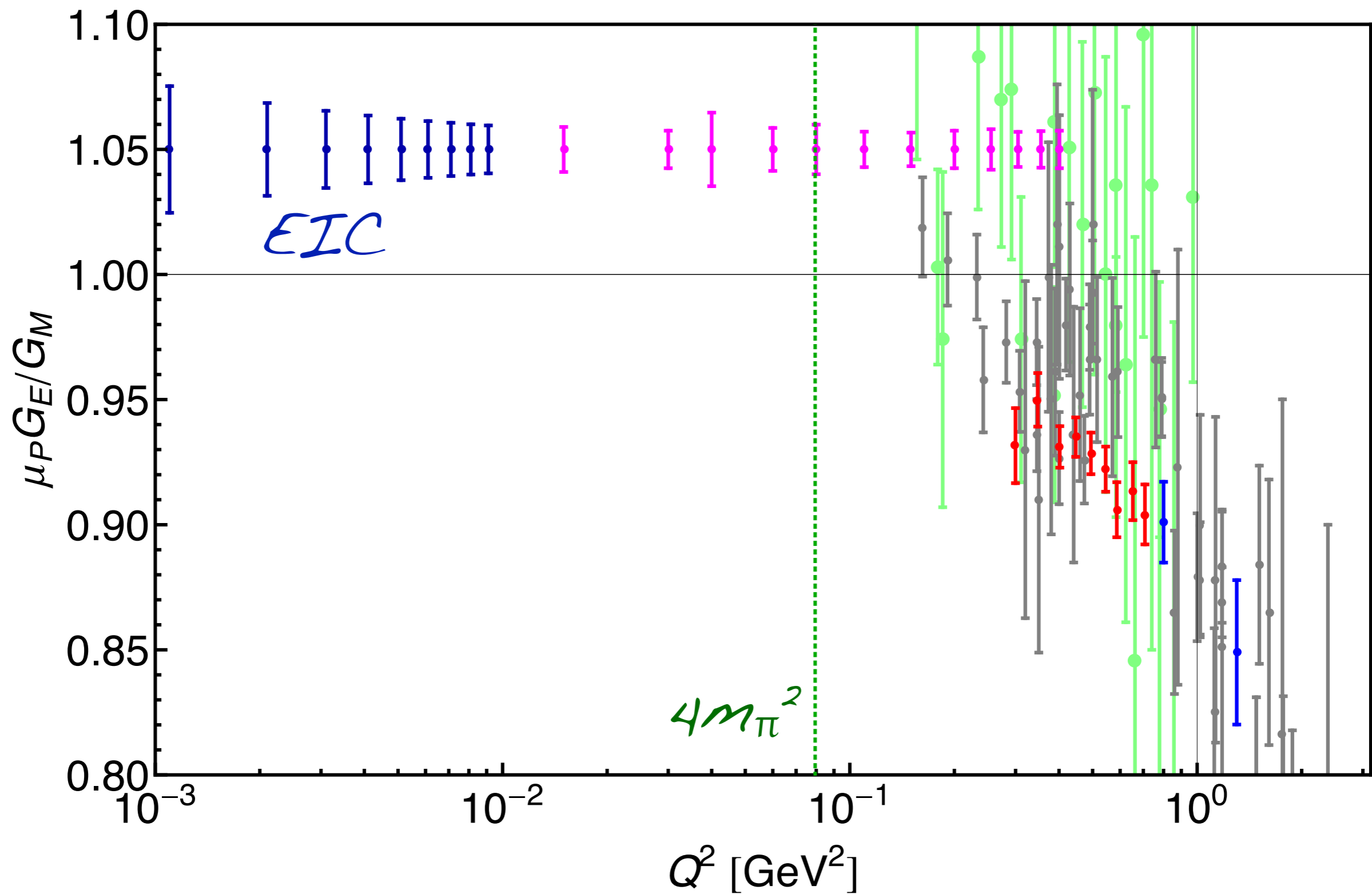
- Possible fix for beam polarization direction uncertainty \rightarrow Calibrate polarization direction online using measured intermediate Q^2 values.

Q^2 (GeV^2)	10^{-4}	$5 \cdot 10^{-4}$	10^{-3}	$5 \cdot 10^{-3}$	0.01	0.3	0.5
θ_e	0.19	0.427	0.6	1.35	1.9	10.43	13.45
χS (cm^{-2})	2.60E-23	1.00E-24	2.50E-25	1.00E-26	2.50E-27	1.00E-30	2.30E-31
Rate (Hz)	9.1	1.75	0.875	0.175	0.0875	3.25	1.13
$T_{0.5\%}$ (hr)	1.22	6.35	12.7	63.5	127		

Measurable online using standard “barrel” detector.
High precision FF ratio data available.

1% uncertainty on FFR \rightarrow 0.1 uncertainty on θ_B (at 10 degrees)

What it could look like....



SUMMARY

- EIC feasible for both high and low Q^2 measurements.
- Both ratio of FFs and cross section can (in principle) be simultaneously measured, giving individual form factors.
- Luminosity not an issue for low Q^2 - measurements better with lower electron beam energy.
- For High Q^2 we need $L \sim 10^{34} \text{ sec}^{-1} \text{ cm}^{-2}$.
- **Primary concerns:**
 - Polarization direction uncertainty for proton beam.
 - Design of "roman pot" style detector for small angles.
- Other things I'd like to see:
 - Polarized positrons for multi- γ studies.
 - Polarized D , ${}^3\text{He}$, ${}^7\text{Li}$ (compare quasi-free/elastic ep):
 - Is D really $p+n$?
 - Is ${}^3\text{He}(\text{pol})$ really $n(\text{pol})$?