Multi-Photon Entanglement Quantum Non-Locality And One Way Computing

H.S. Eisenberg's QUANTUM OPTICS group seminar 2008

Part of the slide are adaptations taken from talks by: *Andreas Reinhard; Kevin Resch; Dan Browne; Sean Clark* (no slide was bluntly stolen it is states explicitly)

GHZ-A new state (*of mind*)

Going beyond Bell's Theorem 1989

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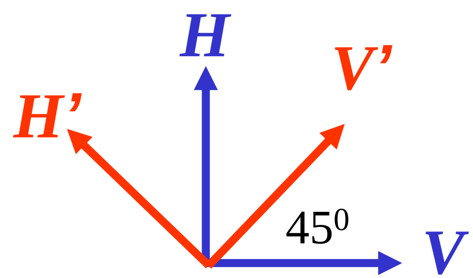
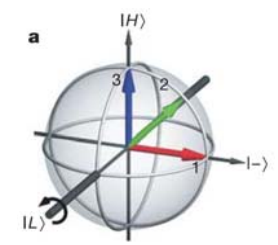
GHZ state: $$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle\right)$$





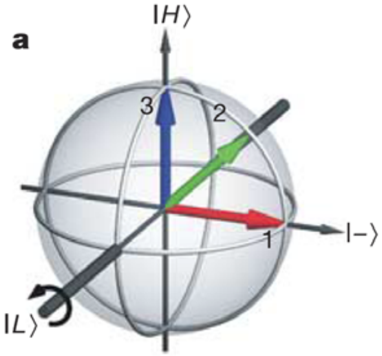
$$|H'\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \quad |V'\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$$

$$|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + \mathrm{i}\,|V\rangle), \quad |L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - \mathrm{i}\,|V\rangle)$$

$$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

x measurement: $|H'\rangle \qquad |V'\rangle \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

y measurement: $|R\rangle \qquad |L\rangle \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$



$$|\Psi\rangle = \frac{1}{2}(|R\rangle_1 |L\rangle_2 |H'\rangle_3 + |L\rangle_1 |R\rangle_2 |H'\rangle_3 + |R\rangle_1 |R\rangle_2 |V'\rangle_3 + |L\rangle_1 |L\rangle_2 |V'\rangle_3)$$

1. Any specific result obtained in any individual or in any two-photon joint measurement is maximally random
2. given any two results of measurements on any two photons, we can predict with certainty the result of the corresponding measurement performed on the third photon

Same can be done for *H'*/V'

In every one of the three *yyx*, *yxy* and *xyy* experiments, Third photon measurement (*circular and linear polarization*) is predicted with ***certainty***

local realism

Assume we did the measurement and found perfect correlations

➡ Each photon carries elements of reality for both *x* and *y*

Elements of reality

$X_i \in \{(-1,1)\}$ for H'/V' polarization

$Y_i \in \{(-1,1)\}$ for R'/L' polarization

$$|\Psi\rangle = \frac{1}{2}(|R\rangle_1|L\rangle_2|H'\rangle_3 + |L\rangle_1|R\rangle_2|H'\rangle_3 + |R\rangle_1|R\rangle_2|V'\rangle_3 + |L\rangle_1|L\rangle_2|V'\rangle_3)$$

➡

$$X_1Y_2Y_3 = -1$$
$$Y_1Y_2X_3 = -1$$
$$Y_1X_2Y_3 = -1$$

But what if we decide to measure *xxx*?

local realism

- x is independent the measurement performed on the other photon.

- And since always: $Y_i Y_i = +1$

$$X_1 X_2 X_3 = (X_1 Y_2 Y_3)\cdot(Y_1 X_2 Y_3)\cdot(Y_1 Y_2 X_3)$$

$$X_1 X_2 X_3 = -1$$

Odd number of *V's*

The possible results:

$$V'_1 V'_2 V'_3$$

$$H'_1 H'_2 V'_3$$

$$H'_1 V'_2 H'_3$$

$$V'_1 H'_2 H'_3$$

Quantum Mechanics?

$$|\Psi\rangle = \frac{1}{2}\left(|H'\rangle_1 |H'\rangle_2 |H'\rangle_3 + |H'\rangle_1 |V'\rangle_2 |V'\rangle_3 + |V'\rangle_1 |H'\rangle_2 |V'\rangle_3 + |V'\rangle_1 |V'\rangle_2 |H'\rangle_3\right)$$

local realism

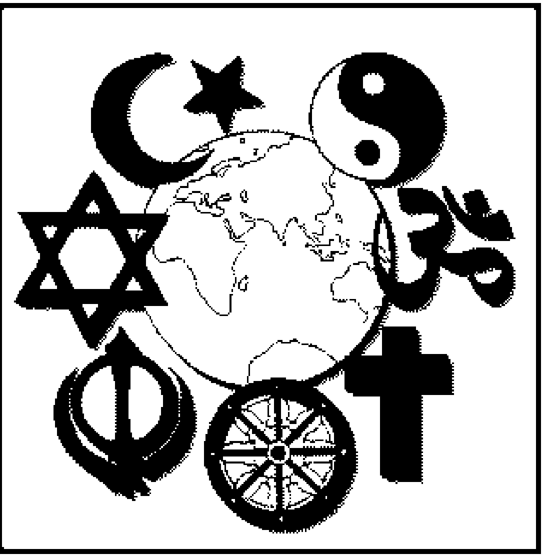
$V'_1 V'_2 V'_3$

$H'_1 H'_2 V'_3$

$H'_1 V'_2 H'_3$

$V'_1 H'_2 H'_3$

One measurement decides who is right



10 years later:

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Observation of Three-Photon Greenberger-Horne-Zeilinger Entanglement

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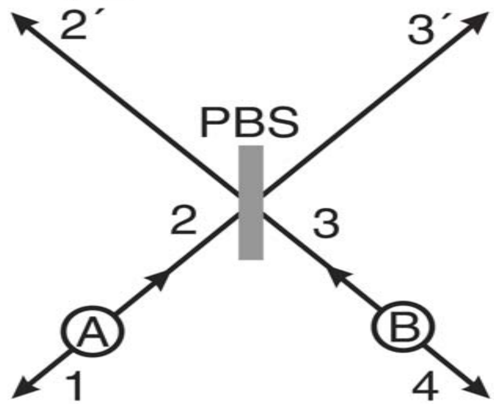
(Received 6 October 1998)

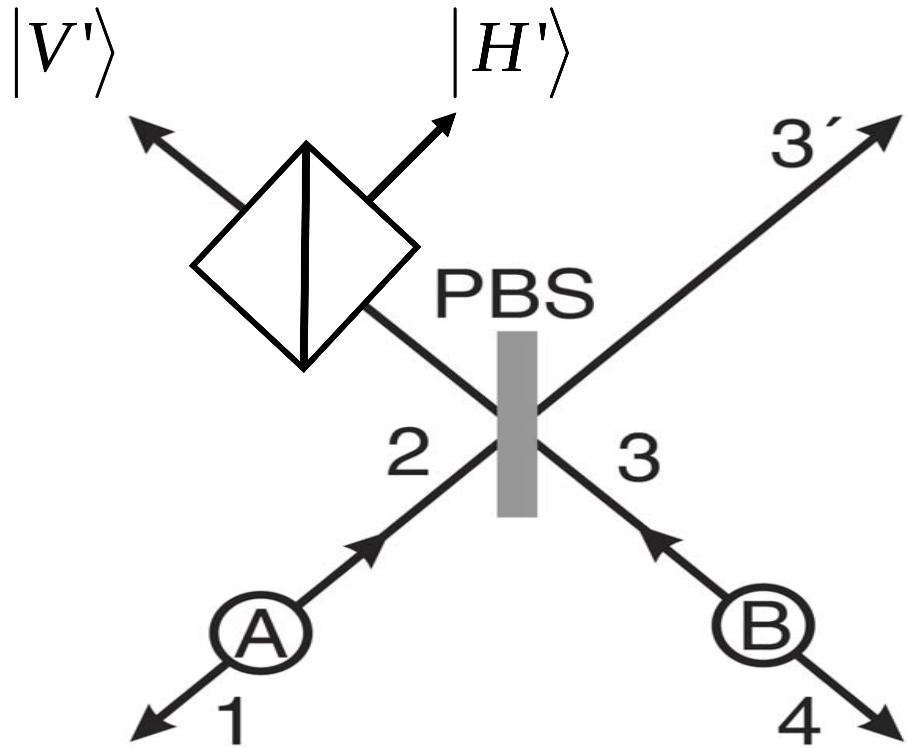
$$\left|\Psi^i\right\rangle_{1234} = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2\right) \otimes \frac{1}{\sqrt{2}}\left(|H\rangle_3|V\rangle_4 - |V\rangle_3|H\rangle_4\right)$$



$$\left|\Psi^f\right\rangle_{12'3'4} = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_{2'}|V\rangle_{3'}|H\rangle_4 + |V\rangle_1|H\rangle_{2'}|H\rangle_{3'}|V\rangle_4\right)$$

A 4 photon GHZ state

$$\left|\Psi^f\right\rangle_{12'3'4} = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_{2'} |V\rangle_{3'} |H\rangle_4 + |V\rangle_1 |H\rangle_{2'} |H\rangle_{3'} |V\rangle_4\right)$$

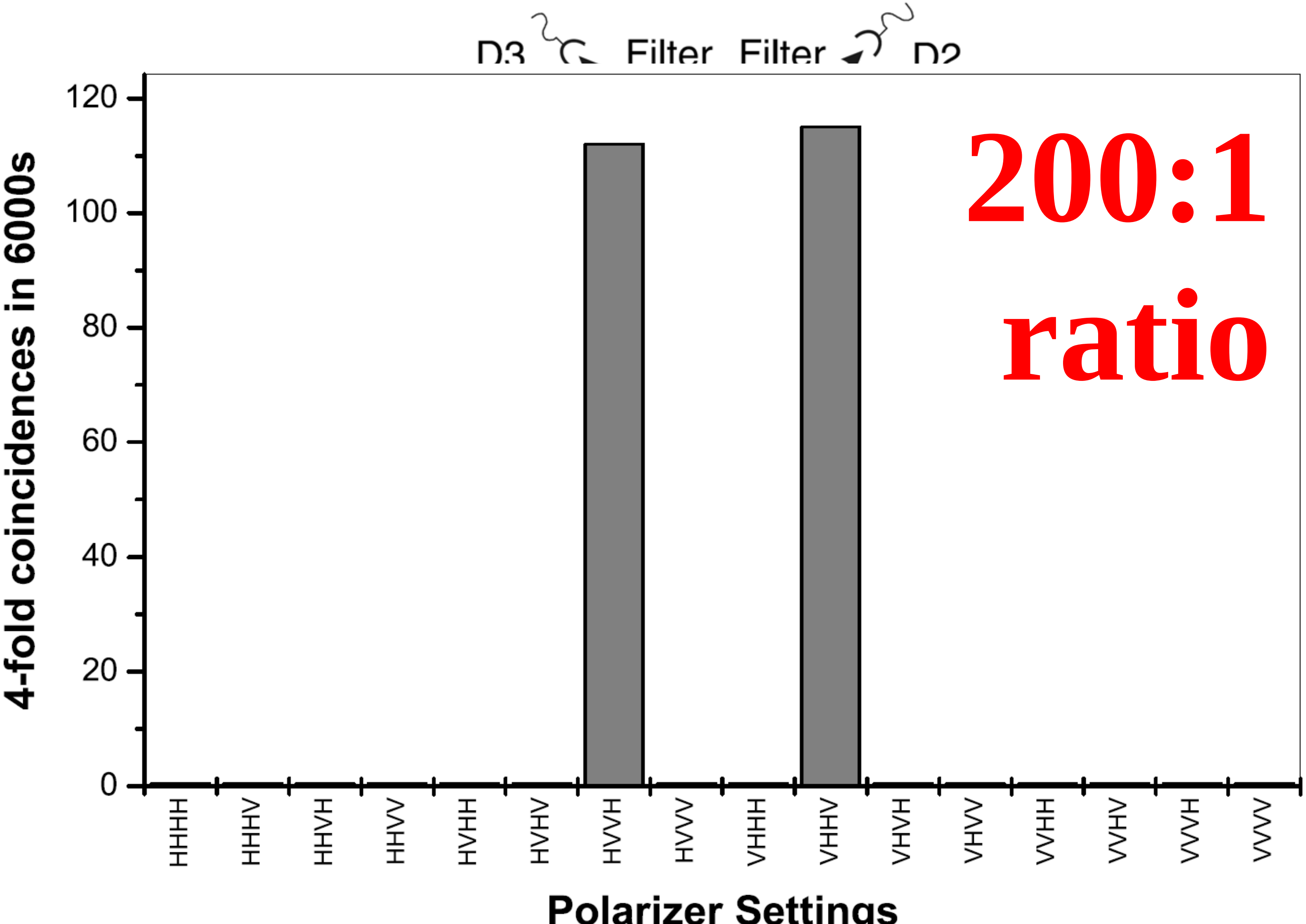


$|H'\rangle$ ➡ $$|\Psi\rangle_{13'4} = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_{3'} |H\rangle_4 + |V\rangle_1 |H\rangle_{3'} |V\rangle_4\right)$$

$|V'\rangle$ ➡ $$|\Psi\rangle_{13'4} = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_{3'} |H\rangle_4 - |V\rangle_1 |H\rangle_{3'} |V\rangle_4\right)$$

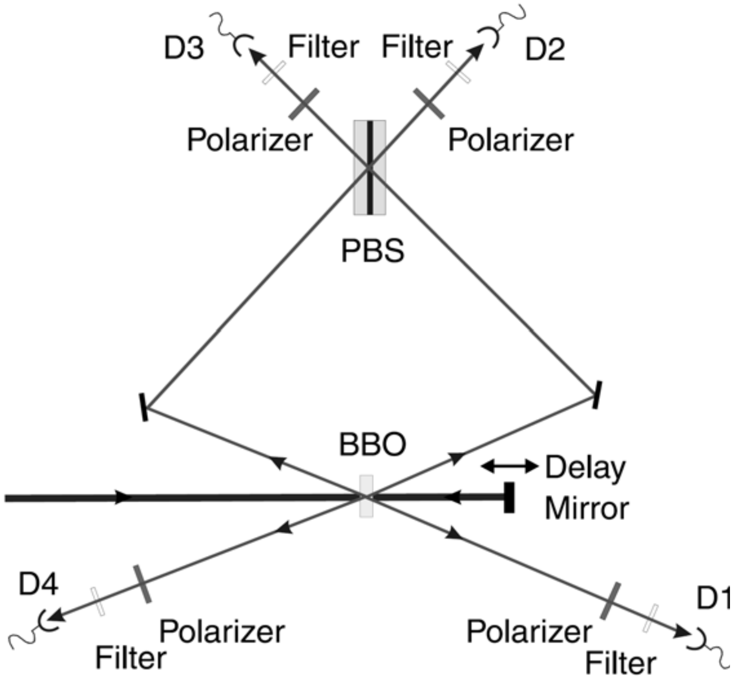
Verification of multi-photon entanglement

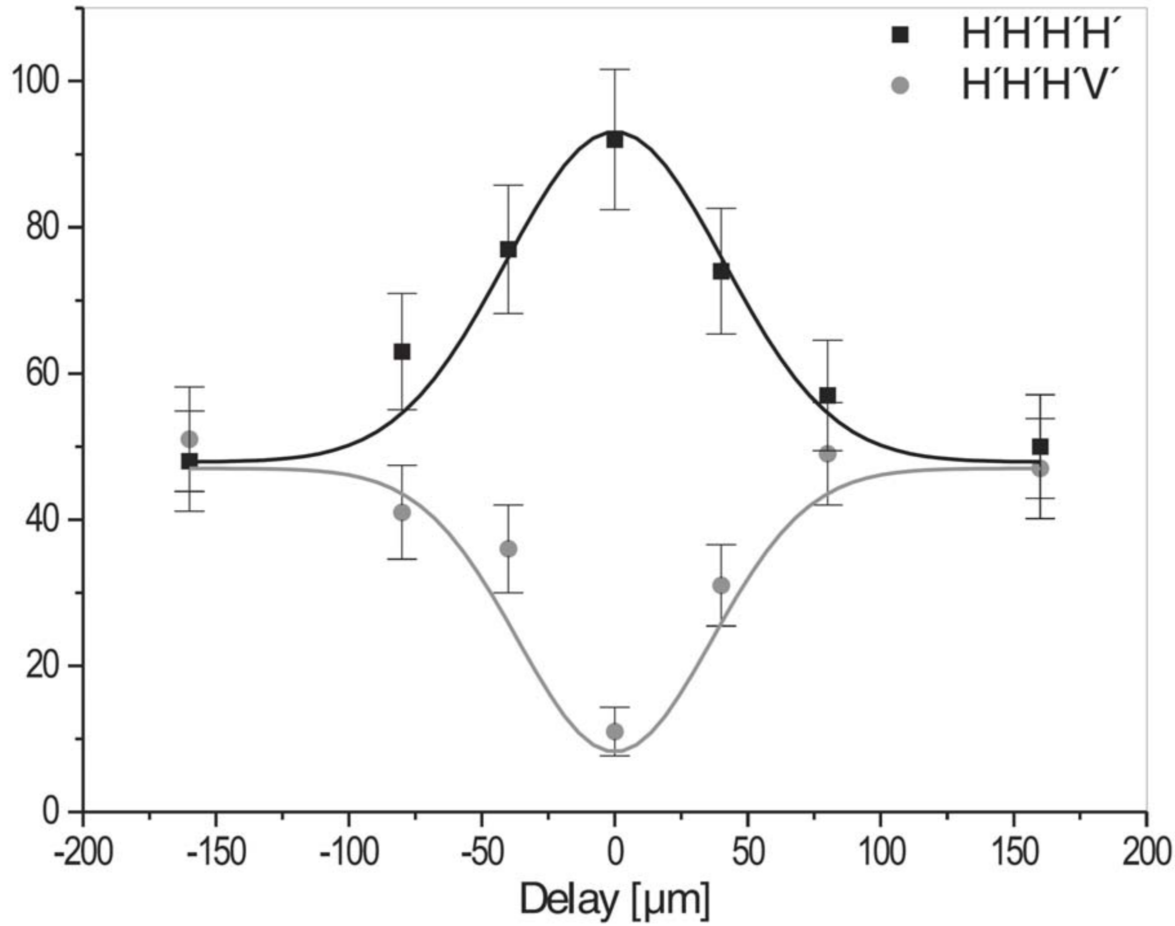
Observation of the *HVVH* & *VHHV* components



Verification of actual entanglement by performing polarization test at a *V'/H'* basis

8 out of 16 combinations are possible all with even number of *H'*





HHHV is suppressed with a visibility of 0.79±0.06

Experimental Test of Quantum Non-Locality

First: perform *yyx*, *yxy*, and *xyy* experiments

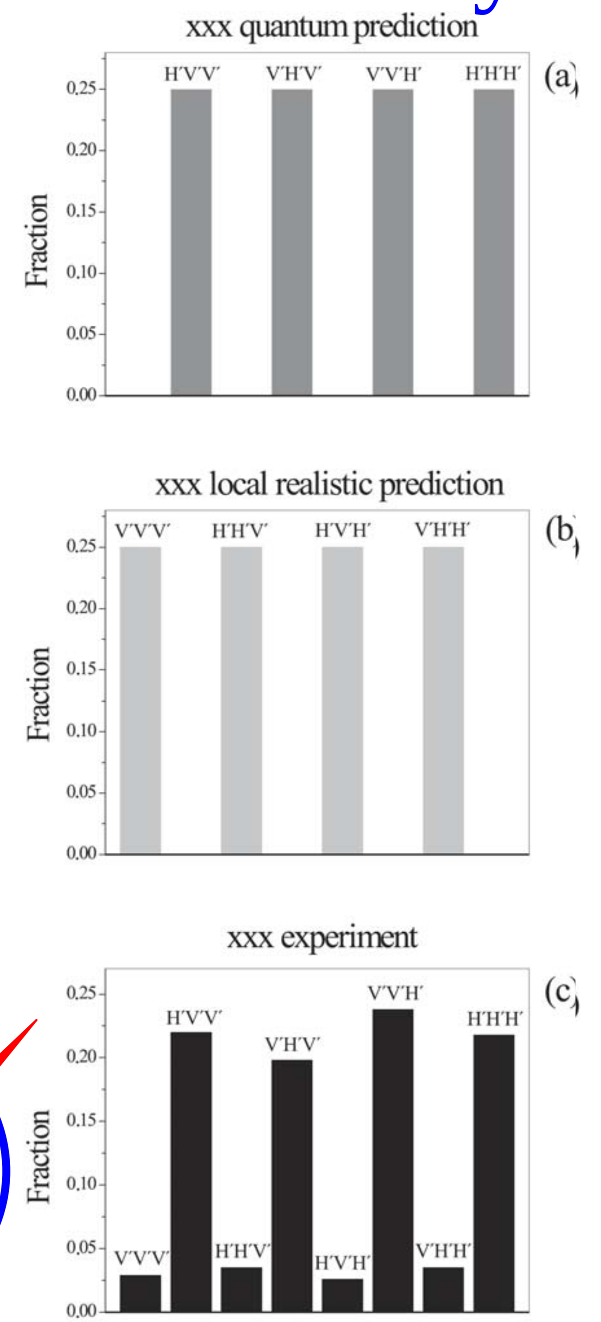
Second : perform *xxx* experiments:

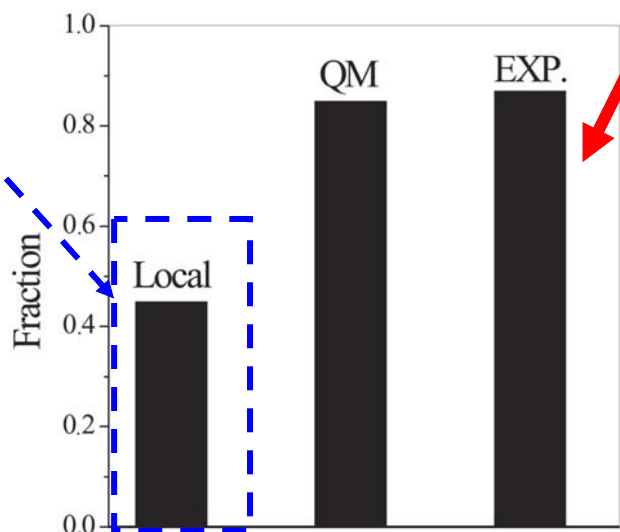
Q-M is 'right' 85% of the time

But… Are we sure that this means Q-M is right???

If our visibility is 74% P(xxx = +1) = 0.87 ± 0.04

Were does this prediction come from





To address this argument, a number of inequalities for N-particle *GHZ* states have been derived. For instance, Mermin's inequality for a threeparticle *GHZ* state reads as follows: $|\sigma_x\sigma_y\sigma_y + \sigma_y\sigma_x\sigma_y + \sigma_y\sigma_y\sigma_x - \sigma_x\sigma_x\sigma_x| \leq 2$, where symbol · denotes the expectation value of a specific physical quantity. The necessary visibility to violate this inequality is 50%. The visibility observed in our *GHZ* experiment is 71±4% and obviously surpasses the 50% limitation. Substituting our results measured in the *yyx*, *yxy* and *xyy* experiments into the left-hand side of, we obtain the following constraint: $\sigma_x\sigma_x\sigma_x \leq -0.1$, by which a local realist can thus predict that in an *xxx* experiment the probability fraction for the outcomes yielding a +1 product, denoted by *P(xxx = +1)*, should be no larger than 0.45±0.03 (also refer to the first bar in …..

Bla Bla Bla...

6 photon *GHZ*

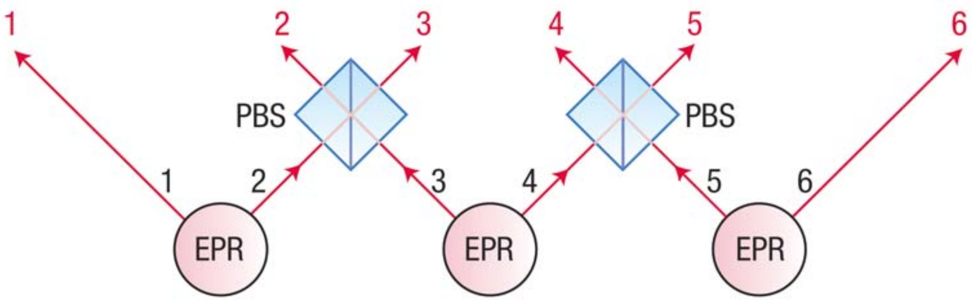
LETTERS

Experimental entanglement of six photons in graph states

CHAO-YANG LU[1]*, XIAO-QI ZHOU[1], OTFRIED GÜHNE[2], WEI-BO GAO[1], JIN ZHANG[1], ZHEN-SHENG YUAN[1], ALEXANDER GOEBEL[3], TAO YANG[1] AND JIAN-WEI PAN[1,3]*

Start by preparing 3 EPRs'

$$|\Phi^+\rangle_{ij} = \frac{1}{\sqrt{2}}(|H\rangle_i|H\rangle_j + |V\rangle_i|V\rangle_j),$$



only if both incoming photons have the same polarization they can go to different outputs. Thus, a coincidence detection of all six outputs corresponds to the state

$$|G_6\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4|H\rangle_5|H\rangle_6 + |V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4|V\rangle_5|V\rangle_6),$$

Characterization

Entanglement witness = An observable that has a positive expectation value on all biseparable states

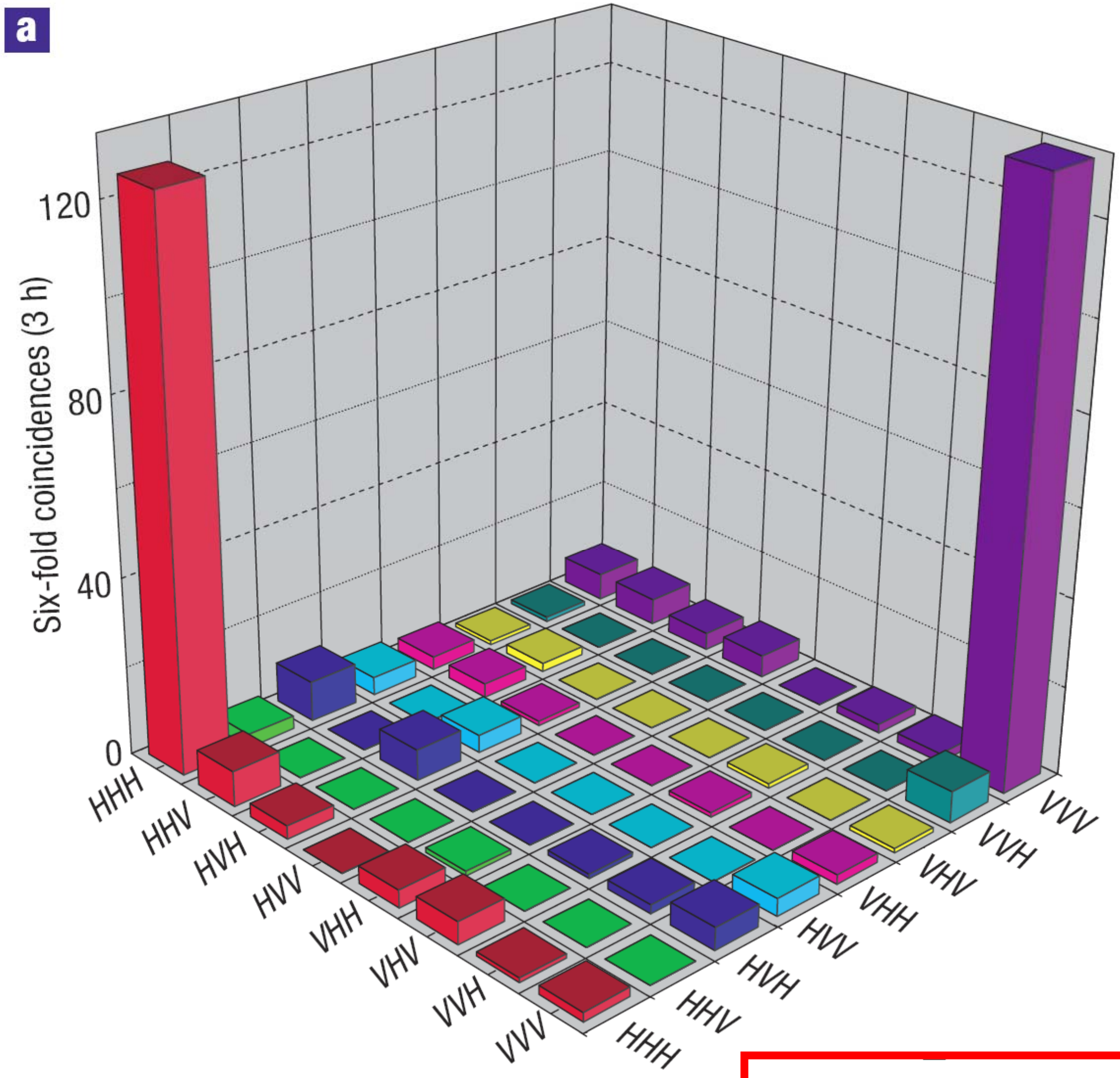
For the six-photon GHZ state: $$W_G = \frac{I}{2} - |G_6\rangle\langle G_6|,$$

Re-writing the state:

$$|G_6\rangle\langle G_6| = \frac{1}{2}[(|H\rangle\langle H|)^{\otimes 6} + (|V\rangle\langle V|)^{\otimes 6}] + \frac{1}{12}\sum_{n=-2}^{3}(-1)^n M_{(n)}^{\otimes 6},$$

$M_{(n)} = \cos(n\pi/6)\sigma_x + \sin(n\pi/6)\sigma_y$ Are measurament on the *x*-*y* plane

seven measurement settings are required



$$\mathrm{Tr}(W_G \rho_{exp}) = -0.093 \pm 0.025$$

W-*STATES*

GHZ state: $$|\psi\rangle = \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle)$$

W state: $$|W\rangle = \frac{1}{\sqrt{3}}(|HHV\rangle_{abc} + |HVH\rangle_{abc} + |VHH\rangle_{abc})$$

Which one is better?

- *GHZ violates Mermin (Bell?) inequalities more (what does that mean?)*
- *W-States are less fragile then GHZ states*

Experimental Realization of a Three-Qubit Entangled *W* State

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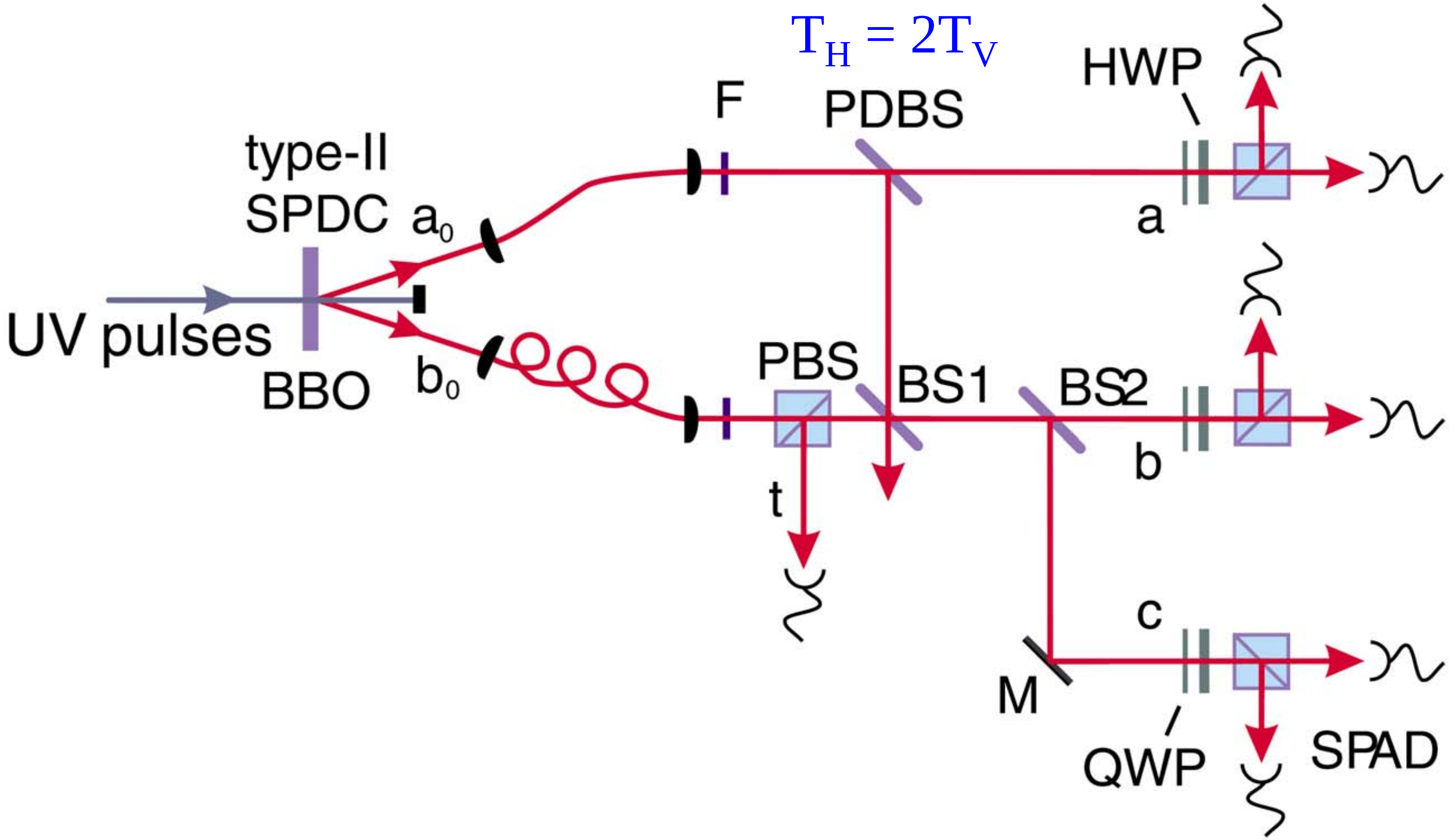
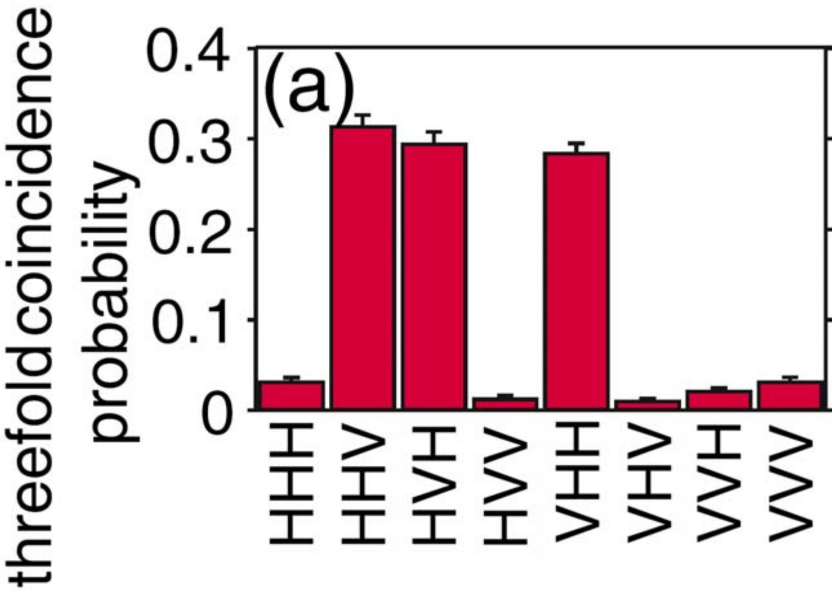


$$P_{HHH} = C_{HHH} / \sum_{i,j,k=\{H,V\}} C_{ijk}$$

C_{HHH} is the number of recorded *HHH* events



incoherent mixture

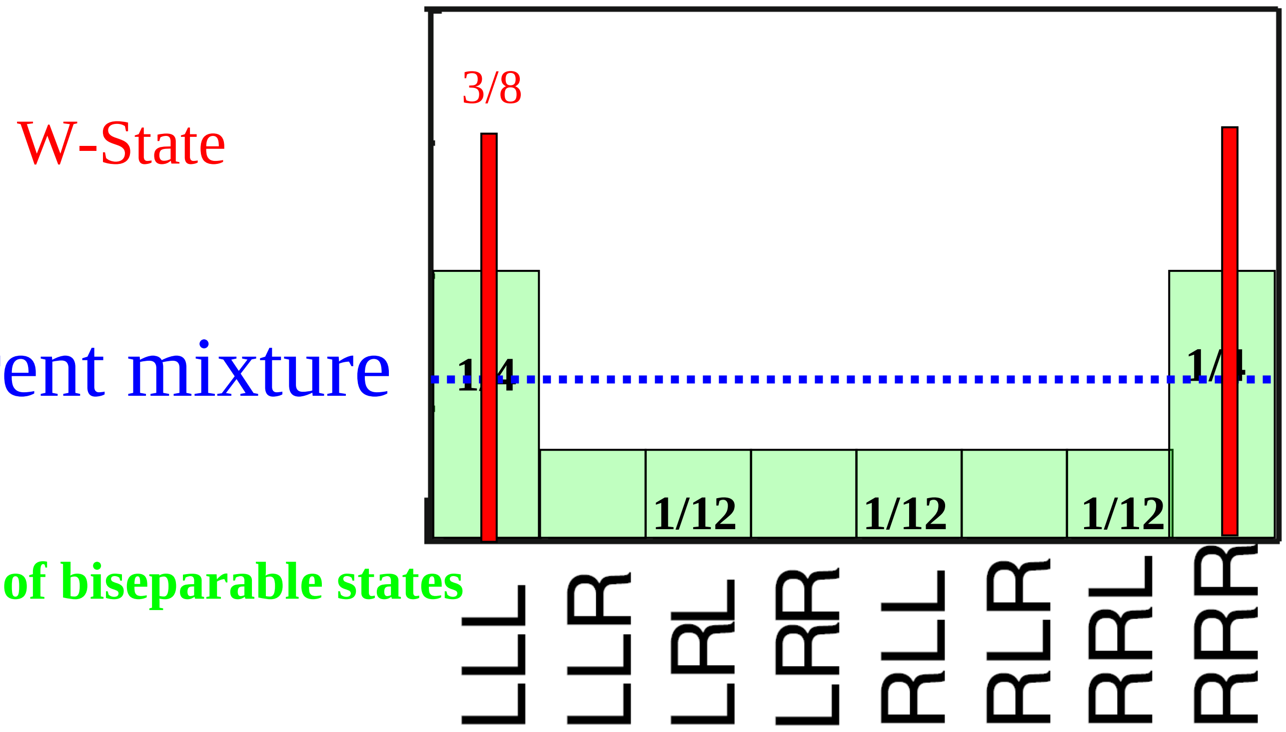
$$\rho_M = 1/3(|HHV\rangle\langle HHV| + |HVH\rangle\langle HVH| + |VHH\rangle\langle VHH|)$$

equally weighted mixture of biseparable states

$$\rho_B = 1/3\rho_a \otimes \rho_{bc} + 1/3\rho_b \otimes \rho_{ac} + 1/3\rho_c \otimes \rho_{ab}$$

$\rho_a = |H\rangle\langle H|$ ρ_{bc} = bell state between modes *b* and *c*

L/R Basis



incoherent mixture

$$\rho_M = 1/3(|HHV\rangle\langle HHV| + |HVH\rangle\langle HVH| + |VHH\rangle\langle VHH|)$$

equally weighted mixture of biseparable states

$$\rho_B = 1/3\rho_a \otimes \rho_{bc} + 1/3\rho_b \otimes \rho_{ac} + 1/3\rho_c \otimes \rho_{ab}$$

Characterizing the Entanglement

Measurement Basis $|k_j, \phi_j\rangle = 1/\sqrt{2}(|R\rangle + k_j e^{i\phi_j}|L\rangle)$

$$\hat{\sigma}_j = \sum_{k_j} k_j |k_j, \phi_j\rangle\langle k_j, \phi_j|$$

$k_j=\pm 1$ $\quad$ $j=a,b,c$

correlation function

$$E(\phi_a, \phi_b, \phi_c) = \langle \hat{\sigma}_a(\phi_a)\hat{\sigma}_b(\phi_b)\hat{\sigma}_c(\phi_c)\rangle$$
$$= \sum_{k_a,k_b,k_c=\pm 1} k_a k_b k_c p_{k_a k_b k_c}(\phi_a, \phi_b, \phi_c)$$

$P k_a k_b k_c(\phi_a;\ \phi_b;\ \phi_{c)}$ is the probability for a threefold coincidence with the results k_a, k_b, and k_c for the specific setting of phases ϕ_j.

correlation function

$$E(\phi_a, \phi_b, \phi_c) = \langle \hat{\sigma}_a(\phi_a)\hat{\sigma}_b(\phi_b)\hat{\sigma}_c(\phi_c)\rangle$$

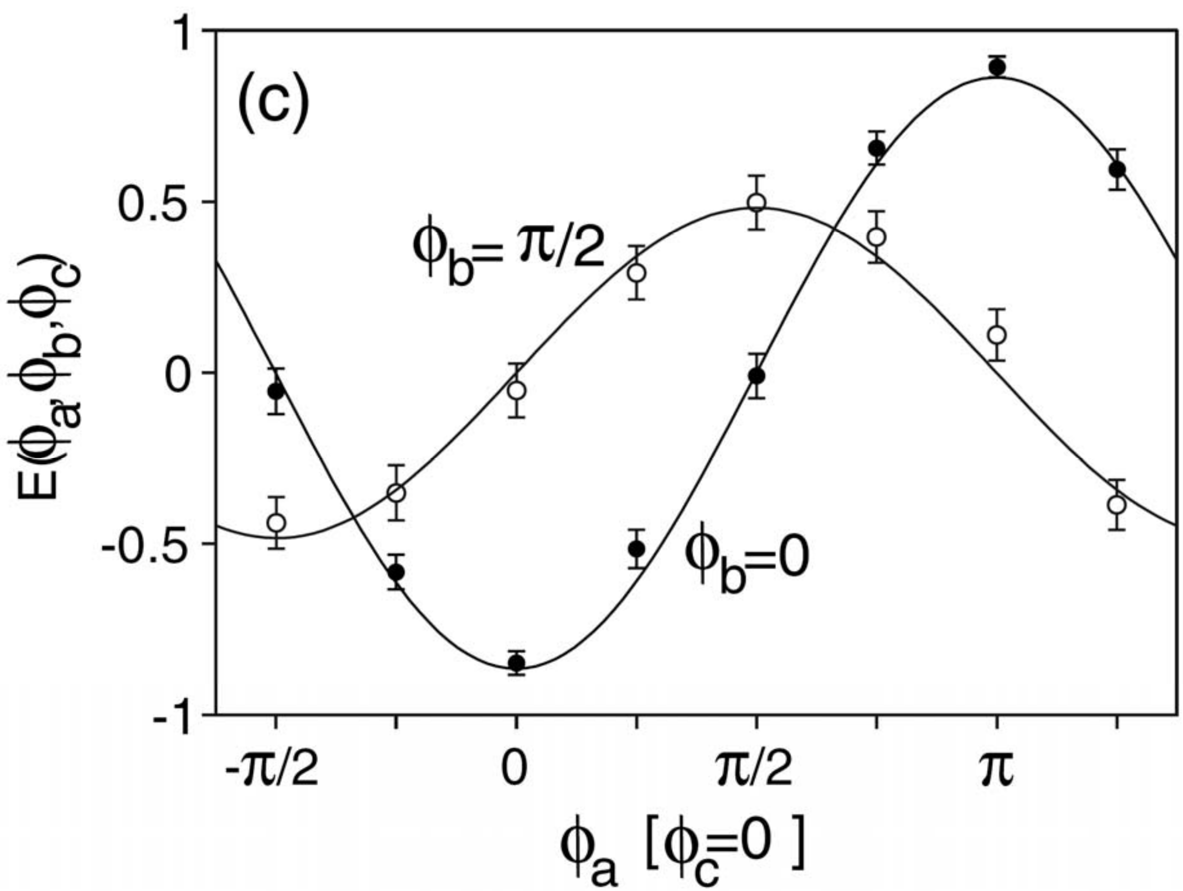
$$= \sum_{k_a, k_b, k_c = \pm 1} k_a k_b k_c \, p_{k_a k_b k_c}(\phi_a, \phi_b, \phi_c)$$

For a W-state

$$E(\phi_a, \phi_b, \phi_c) = -\frac{2}{3}\cos(\phi_a + \phi_b + \phi_c) - \frac{1}{3}\cos(\phi_a)\cos(\phi_b)\cos(\phi_c)$$

$\phi_b = \phi_c = 0$ $\Rightarrow$ $E(\phi_a, 0, 0) = -\cos(\phi_a)$

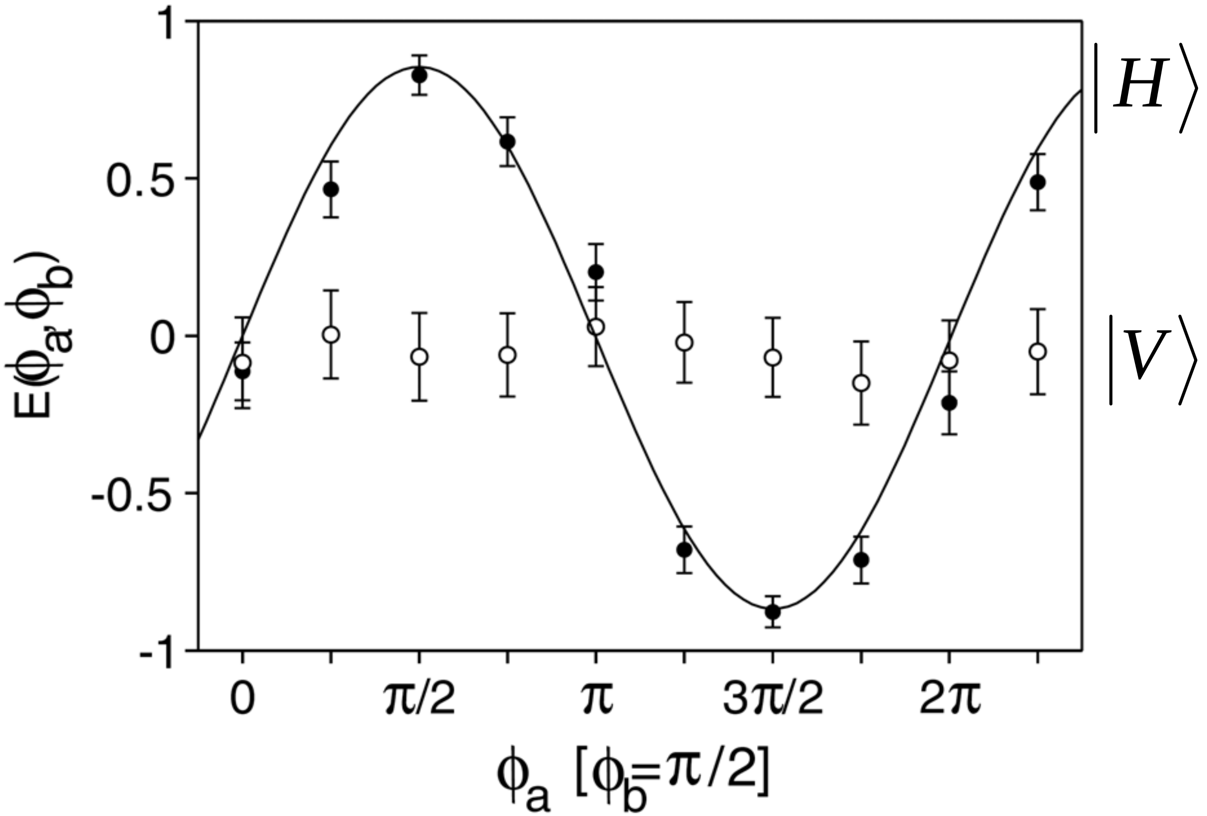
$\phi_b = \phi_c = 0$ → $E(\phi_a, 0, 0) = -\cos(\phi_a)$



Note that $E_{GHZ}(\phi_a, \phi_b, 0) = 0$ While $|E_w(\phi_a, \pi/2, \pi/2)| < 2/3$

Robustness of the entanglement

Correlation between a and *b*, depending on the measurement result of the photon in mode *c*



Quantum State Tomography

A test of the Peres-Horodecki criterion

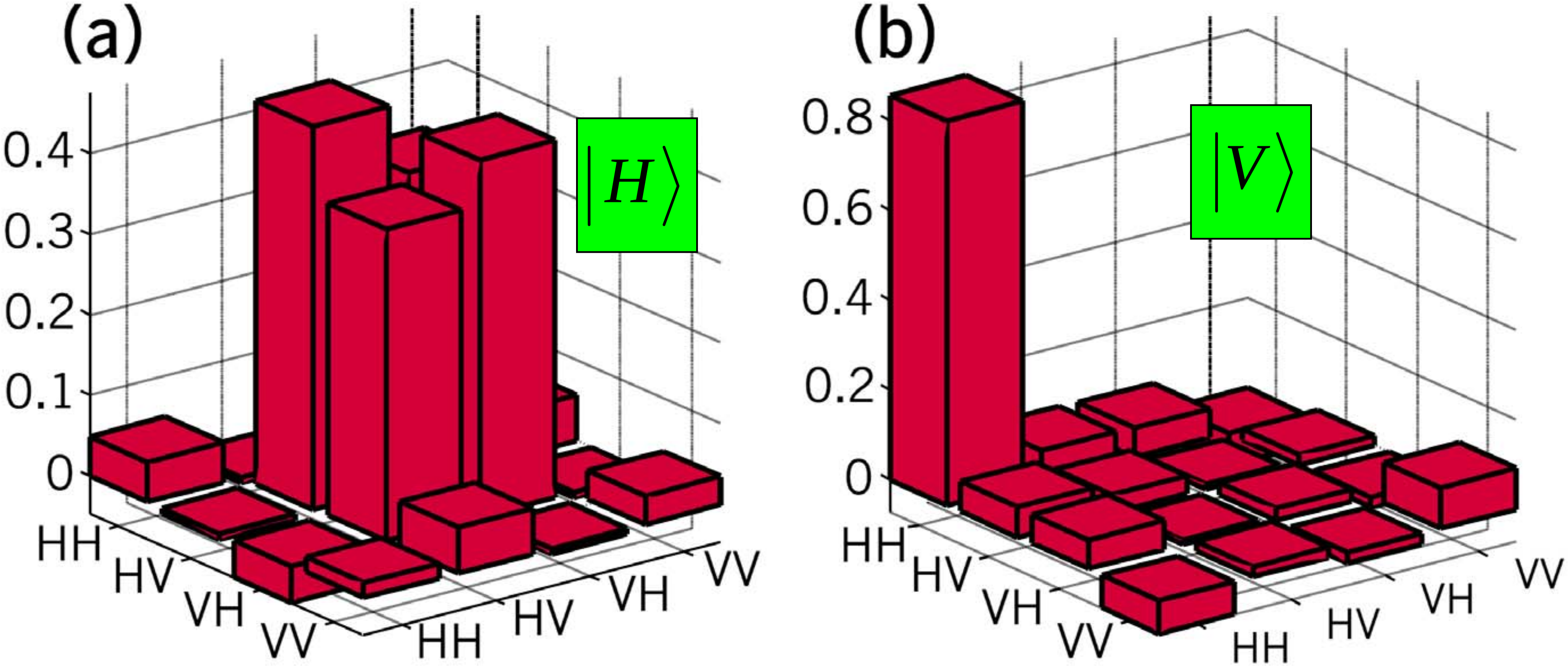
A separable state $\rho = \sum_A w_A \rho'_A \otimes \rho''_A$

$\lambda^H = -0.5$

$\lambda^H_{exp} = -0.348 \pm 0.019$

$\lambda^V = -0.5$

$\lambda^V_{exp} = -0.113 \pm 0.062$



W-States in multiqubit systems

The totally symmetric state including N-1 zeros and 1 ones

$$|W_N\rangle \equiv (1/\sqrt{N})|N-1,1\rangle$$

Example: $N=4$:

$$|W_4\rangle = (1/\sqrt{4})(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)$$

reduced density operators ρ_{km}:

$$\rho_{\kappa\mu} = \frac{1}{N}(2|\Psi^+\rangle\langle\Psi^+| + (N-2)|00\rangle\langle 00|)$$

No experimental W-state > 3 yet

Measures of entanglement using the density matrix

Fidelity -a measure of state overlap:

$$F(\rho_1, \rho_2) = \left(\mathrm{Tr} \left\{ \sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}} \right\} \right)^2$$

ρ_1 and ρ_2 pure - simplifies to $\mathrm{Tr}\{\rho_1\rho_2\} = |\langle\psi_1|\psi_2\rangle|^2$

Tangle - The *concurrence* and *tangle* are measures of the non-classical properties of a quantum state

Concurrence: For a non-Hermitian matrix $$\hat{R} = \hat{\rho}\hat{\Sigma}\hat{\rho}^{\mathrm{T}}\hat{\Sigma}$$

$$\hat{\Sigma} \equiv \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

For $r_1 < r_2 < r_3 < r_4$ eigenvalues of *R*

Concurrence:

$$C = \mathrm{Max}\{0, \sqrt{r_1} - \sqrt{r_2} - \sqrt{r_3} - \sqrt{r_4}\}$$

Tangle:

$$T \equiv C^2$$

For a product state: T=0

For a Bell state: T=1

Entropy and the Linear Entropy - The Von Neuman entropy quantifies the degree of mixture in a quantum state

$$S \equiv -\mathrm{Tr}\left\{\hat{\rho}\ln\left[\hat{\rho}\right]\right\} = -\sum_i p_i \ln\left\{p_i\right\}$$

eigenvalues of ρ

The linear entropy for a two-qubit system:

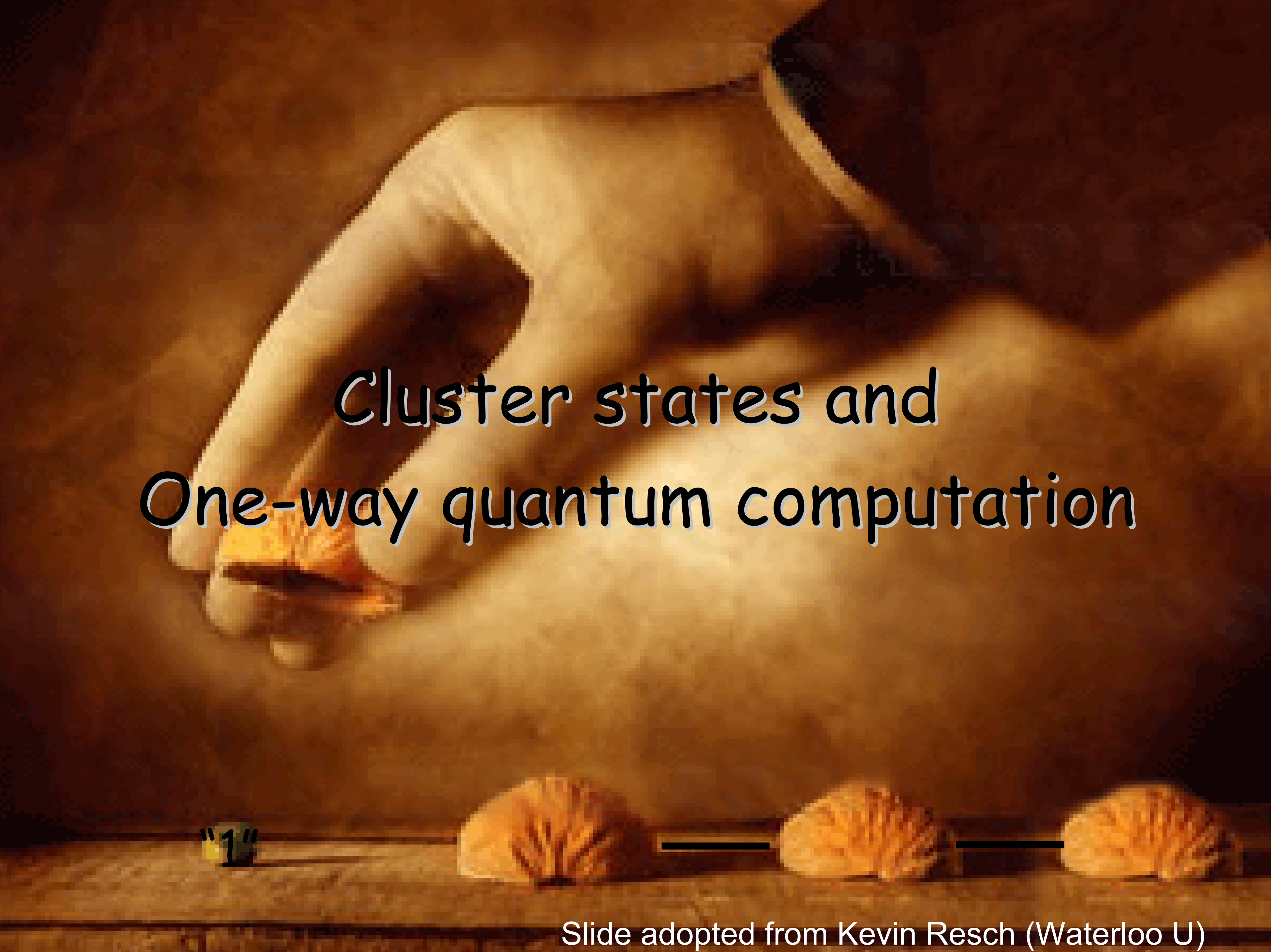
$$S_L = \frac{4}{3}\left(1 - \mathrm{Tr}\left\{\hat{\rho}^2\right\}\right) = \frac{4}{3}\left(1 - \sum_{a=1}^{4} p_a^2\right)$$

eigenvalues of ρ

For a pure state: $\rho^2 = \rho$ ➡ $\mathrm{Tr}[\rho] = 1$

➡ $S_L = 0$ for a pure state

$S_L = 1$ for a completely mixed state


Cluster states and
One-way quantum computation
"1"
Slide adopted from Kevin Resch (Waterloo U)

Cluster States

Examples

- In *two* qubits: Bell State
- In *three* qubits: GHZ state

In general, "*Cluster States*" have no simple state vector representation (no. of terms increases exponentially in no. of qubits).

Stabiliser formalism provides an easy and compact description.

Stabiliser Formalism

Operator ***O*** is stabiliser of state |ψ> if:

$$O|\psi\rangle = |\psi\rangle$$

Specifying multiple stabilisers can define a sub-space, or even a specific state.

Cluster States

Cluster states are pure quantum states of two level systems ~qubits! located on a cluster *C*.

This cluster is a connected subset of a simple cubic lattice Z_d in d>1

The cluster states $|\phi_{\{k\}}>_c$ obey the set of eigenvalue equations:

$$K^{(a)}|\phi_{\{\kappa\}}\rangle_C = (-1)^{\kappa_a}|\phi_{\{\kappa\}}\rangle_C$$

with the correlation operators:

$$K^{(a)} = \sigma_x^{(a)} \bigotimes_{b \in \mathrm{nghb}(a)} \sigma_z^{(b)}$$

$$\{\kappa\} := \{\kappa_a \in \{0,1\} | \, a \in C\}$$

Stabilizers for the Cluster State

A cluster state on a given qubit array A is defined by the following stabilisers.

$$-1^{\kappa_a} X^a \bigotimes_{i \in \mathrm{ngbr(a)}} Z^i$$

$\forall a \in A$ where *ngbr(a)* represents all nearest neighbours of qubit *a*.

$$k_a \in \{0,1\}$$

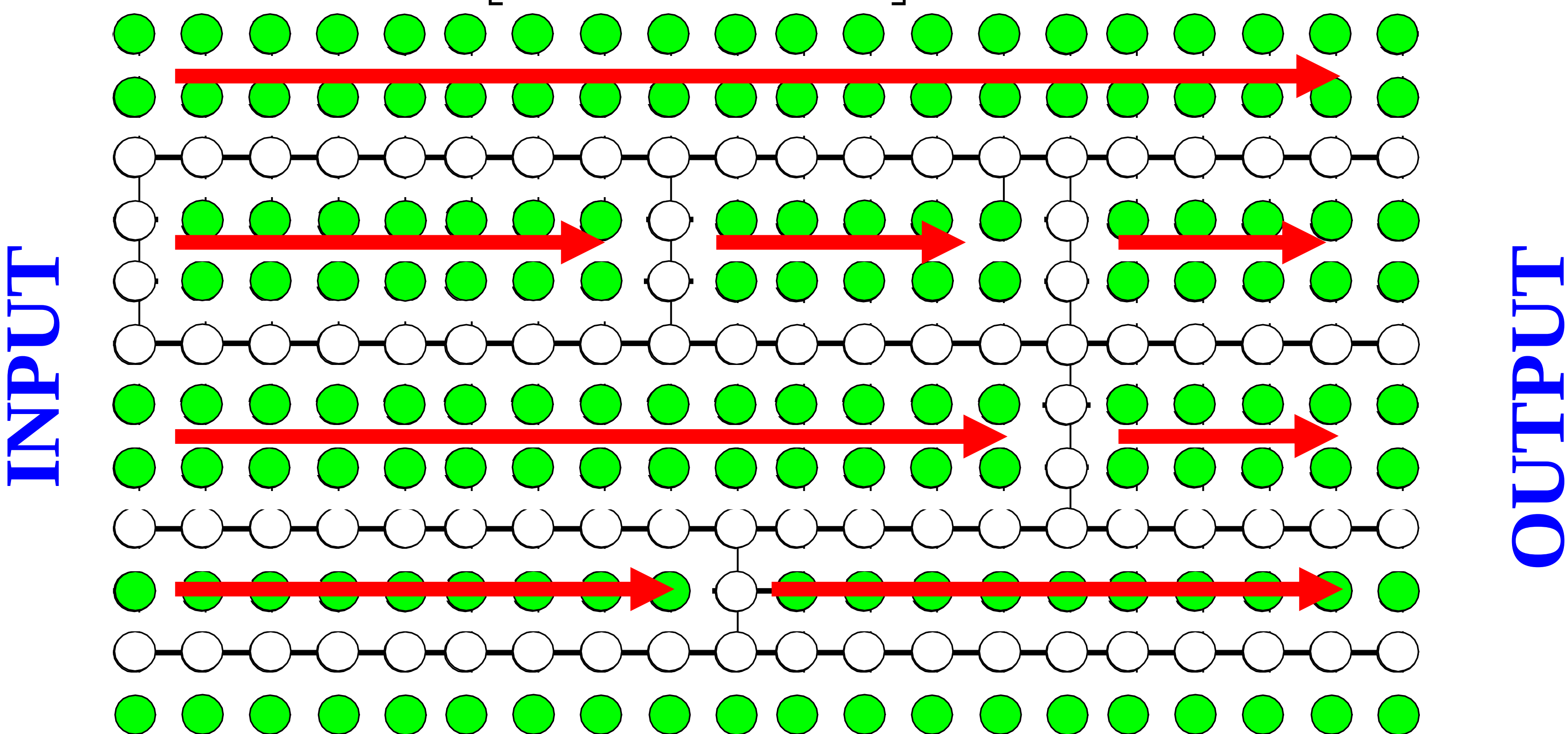
The state is completely defined by the stabilizer eigenvalue equations, all of its properties can be calculated in terms of the stabilisers.

For κ_a=0, we have a special case

For: $\kappa_a = 0, \quad \forall a \in C$

An Ising Hamiltonian will transform a latice (1,2,3D) into a cluster state

$$\exp\left[-i\frac{\pi}{4}\sum_{\langle j,k\rangle}\sigma_z^{(j)}\sigma_z^{(k)}\right]\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)^{\otimes n}$$



Example Cluster States

- For one dim cluster with two qubits

$$\frac{1}{\sqrt{2}}(|00> + |01> + |10> - |11>)$$

- For one dim cluster with three qubits

$$\frac{1}{2\sqrt{2}}(|000> + |001> + |010> - |011> + |100> + |101> - |110> + |111>)$$

- For one dim cluster with four qubits

$$\frac{1}{4}(|0000> + |0001> + |0010> - |0011> + |0100> + |0101> - |0110> + |0111>$$

$$+ |1000> + |1001> + |1010> - |1011> - |1100> - |1101> + |1110> - |1111>)$$

Generating a Cluster State

- First produce the product state

$$|+>_C = \otimes_{a\in C} |+>_a$$

- Then apply the entangling operator

$$S^{(C)} = \prod_{a,b\in C | b-a\in\gamma_d} S^{ab}$$

Where γ_d is the set of positive shifts by one place in one dimension (i.e. for d = 3 $\gamma_3 = \{(1,0,0)^T, (0,1,0)^T, (0,0,1)^T\}$)

And $$S^{ab} = \frac{1}{2}(1 + \sigma_z^{(a)} + \sigma_z^{(b)} + \sigma_z^{(a)} \otimes \sigma_z^{(b)})$$

The resultant state can be shown to satisfy eigenvalue equations

How much entangelment is in there?

- Two measures of entanglement useful in characterizing the properties of a cluster state can be defined on the states of n qubits:
  - A state is ***maximally connected*** if any pair of qubits can be projected, with certainty into a pure Bell state by local measurements on a subset of the other qubits
  - The ***persistency of entanglement*** is the minimum number of local measurements such that, for all measurement outcomes, the state is completely disentangled
- A cluster state of n qubits is maximally connected and has

$$P_e = \max\{p \mid p \le n/2\}$$

Logical and cluster qubits

- A distinction is made between cluster qubits as shown in the diagram and logical qubits which correspond to qubits in a register in a quantum network computation

- The logical qubits can be thought to “flow” during the computation from input clusters qubits 1, 15 to output cluster qubits 7, 21

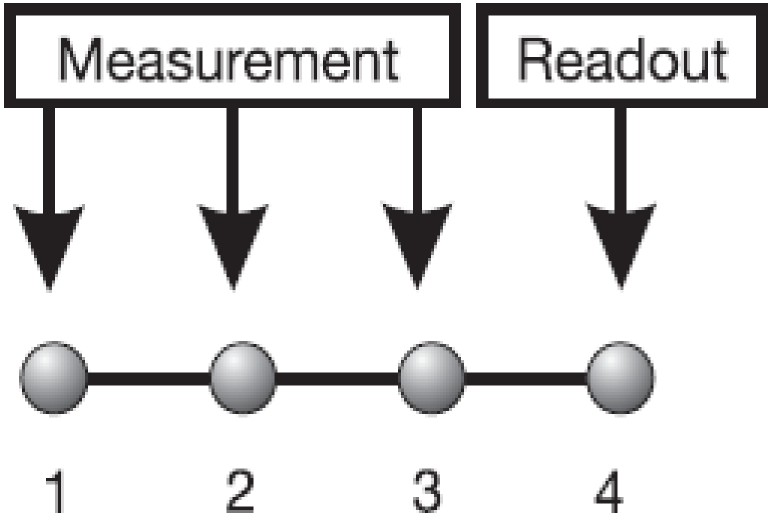
Operations on qubits

- Prepare cluster state

  Measure the state of qubit j in an chosen basis

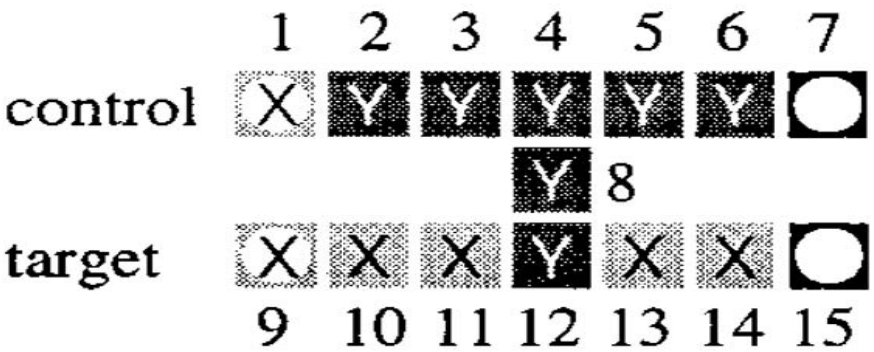
$$B_j(\alpha)=\left\{\left|+\alpha\right\rangle_j,\ \left|-\alpha\right\rangle_j\right\} \quad \text{where} \quad \left|\pm\alpha\right\rangle_j=\frac{1}{\sqrt{2}}\left(\left|0\right\rangle_j \pm e^{i\alpha}\left|1\right\rangle_j\right)$$

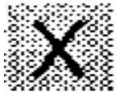
- Consecutive measurements on qubits 1, 2, 3 disentangle the state and completely determine the state of qubit 4.
- The state of „output" qubit 4 is dependant on the chosen bases.
- Classical feedforward makes a OWQC deterministic



Realization of a CNOT gate

- Prepare the state: $|\Psi_{\text{in}}\rangle_{\mathcal{C}_{15}} = |\psi_{\text{in}}\rangle_{1,9} \otimes \left(\bigotimes_{i \in \mathcal{C}_{15 \setminus \{1,9\}}} |+\rangle_i \right)$
- Entangle the 15 qubits of the cluster C_{15} via the unitary operation $S^{(C_{15})}$
- Measure all qubits of C15 except for the outputs (7, 15) as in the following sketch





Measure in σ_x basis

Measure in σ_y basis

Dependent on the measurement results we get the following gate: $U'_{\mathrm{CNOT}} = U_{\Sigma,\mathrm{CNOT}} \mathrm{CNOT}(c,t)$

With the *byproduct* having the form:

$$U_{\Sigma,\mathrm{CNOT}} = {\sigma_x^{(c)}}^{\gamma_x^{(c)}}, {\sigma_x^{(t)}}^{\gamma_x^{(t)}} {\sigma_z^{(c)}}^{\gamma_z^{(c)}}, {\sigma_z^{(t)}}^{\gamma_z^{(t)}}$$

$$\gamma_x^{(c)} = s_2 + s_3 + s_5 + s_6,$$

$$\gamma_x^{(t)} = s_2 + s_3 + s_8 + s_{10} + s_{12} + s_{14},$$

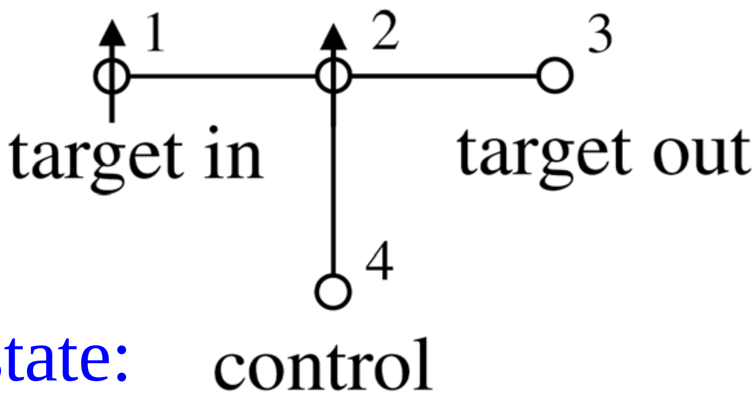
$$\gamma_z^{(c)} = s_1 + s_3 + s_4 + s_5 + s_8 + s_9 + s_{11} + 1$$

$$\gamma_z^{(t)} = s_9 + s_{11} + s_{13}.$$

Measurement s_i on qubit i

Realization of a *4 qubit* CNOT gate

- Prepare the state |ψ>c$_4$ $|i_1\rangle_{z,1} \otimes |i_4\rangle_{z,4} \otimes |+\rangle_2 \otimes |+\rangle_3$
- Entangle the 4 qubits of the cluster C_4 via the unitary operation $S^{(C_4)}$
- Measure σ_x of qubits 1and 2



- You get the following quantum state:

$$|s_1\rangle_{x,1} \otimes |s_2\rangle_{x,2} \otimes U_{\Sigma}^{(34)} |i_4\rangle_{z,4} \otimes |i_1 + i_4 \,\mathrm{mod} 2\rangle_{z,3}$$

$$U_{\Sigma}^{(34)} = \sigma_z^{(3)^{s_1+1}} \sigma_x^{(3)^{s_2}} \sigma_z^{(4)^{s_1}}$$

byproduct

- *You don't keep the control:*

General one qubit SU(2) rotation

Euler Representation

$$U_{Rot}[\xi,\eta,\zeta]=U_x[\zeta]U_z[\eta]U_x[\xi]$$

$$U_x[\alpha]=\exp\left(-i\alpha\frac{\sigma_x}{2}\right)$$

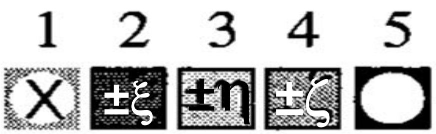
$$U_z[\alpha]=\exp\left(-i\alpha\frac{\sigma_z}{2}\right)$$

Measurement basis:

$$\mathcal{B}_j(\varphi_j)=\left\{\frac{|0\rangle_j+e^{i\varphi_j}|1\rangle_j}{\sqrt{2}},\frac{|0\rangle_j-e^{i\varphi_j}|1\rangle_j}{\sqrt{2}}\right\}$$

General one qubit SU(2) rotation

- Prepare the state: $$|\Psi_{\text{in}}\rangle_{C_5} = |\psi_{\text{in}}\rangle_1 \otimes \left(\bigotimes_{i=2}^{5} |+\rangle_i \right)$$
- Entangle the 5 qubits of the cluster C_5 via the unitary operation $S^{(C_5)}$



Measure qubits 1–4 in the following order and basis:

measure qubit 1 $\mathcal{B}_1(0)$,

measure qubit 2 in $\mathcal{B}_2(-\xi(-1)^{s_1}t)$,

measure qubit 3 in $\mathcal{B}_3(-\eta(-1)^{s_2})$,

measure qubit 4 in $\mathcal{B}_4(-\zeta(-1)^{s_1+s_3})$

General one qubit SU(2) rotation

Dependent on the measurement results we get the following gate:

$$U'_{Rot}[\xi,\eta,\zeta] = U_{\Sigma,Rot} U_{Rot}[\xi,\eta,\zeta]$$

With the *byproduct* having the form:

$$U_{\Sigma,Rot} = \sigma_x^{s_2+s_4} \sigma_z^{s_1+s_3}$$

Measurement s_i on qubit *i*

Qustion: What do we do with the byproduct U_Σ?

Answer: propagate it forward using classical communication and re-interpret the final answer at according to the measurement results.

Generaly: $$|\psi_{\text{out}}\rangle = \left(\prod_{i=1}^{|\mathcal{N}|} U_{\Sigma,g_i} U_{g_i} \right) |\psi_{\text{in}}\rangle$$

We use the following propagation relations:

$$\text{CNOT}(c,t)\sigma_x^{(t)} = \sigma_x^{(t)}\text{CNOT}(c,t),$$
$$\text{CNOT}(c,t)\sigma_x^{(c)} = \sigma_x^{(c)}\sigma_x^{(t)}\text{CNOT}(c,t),$$
$$\text{CNOT}(c,t)\sigma_z^{(t)} = \sigma_z^{(c)}\sigma_z^{(t)}\text{CNOT}(c,t),$$
$$\text{CNOT}(c,t)\sigma_z^{(c)} = \sigma_z^{(c)}\text{CNOT}(c,t),$$

for CNOT gates:

and

$$U_{Rot}[\xi,\eta,\zeta]\sigma_x = \sigma_x U_{Rot}[\xi,-\eta,\zeta],$$
$$U_{Rot}[\xi,\eta,\zeta]\sigma_z = \sigma_z U_{Rot}[-\xi,\eta,-\zeta],$$

for arbitrary rotation

$$H\sigma_x = \sigma_z H, \qquad U_z[\pi/2]\sigma_x = \sigma_y U_z[\pi/2],$$
$$H\sigma_z = \sigma_x H, \qquad U_z[\pi/2]\sigma_z = \sigma_z U_z[\pi/2],$$

for Hadamard and p/2 phase gates

As a result:

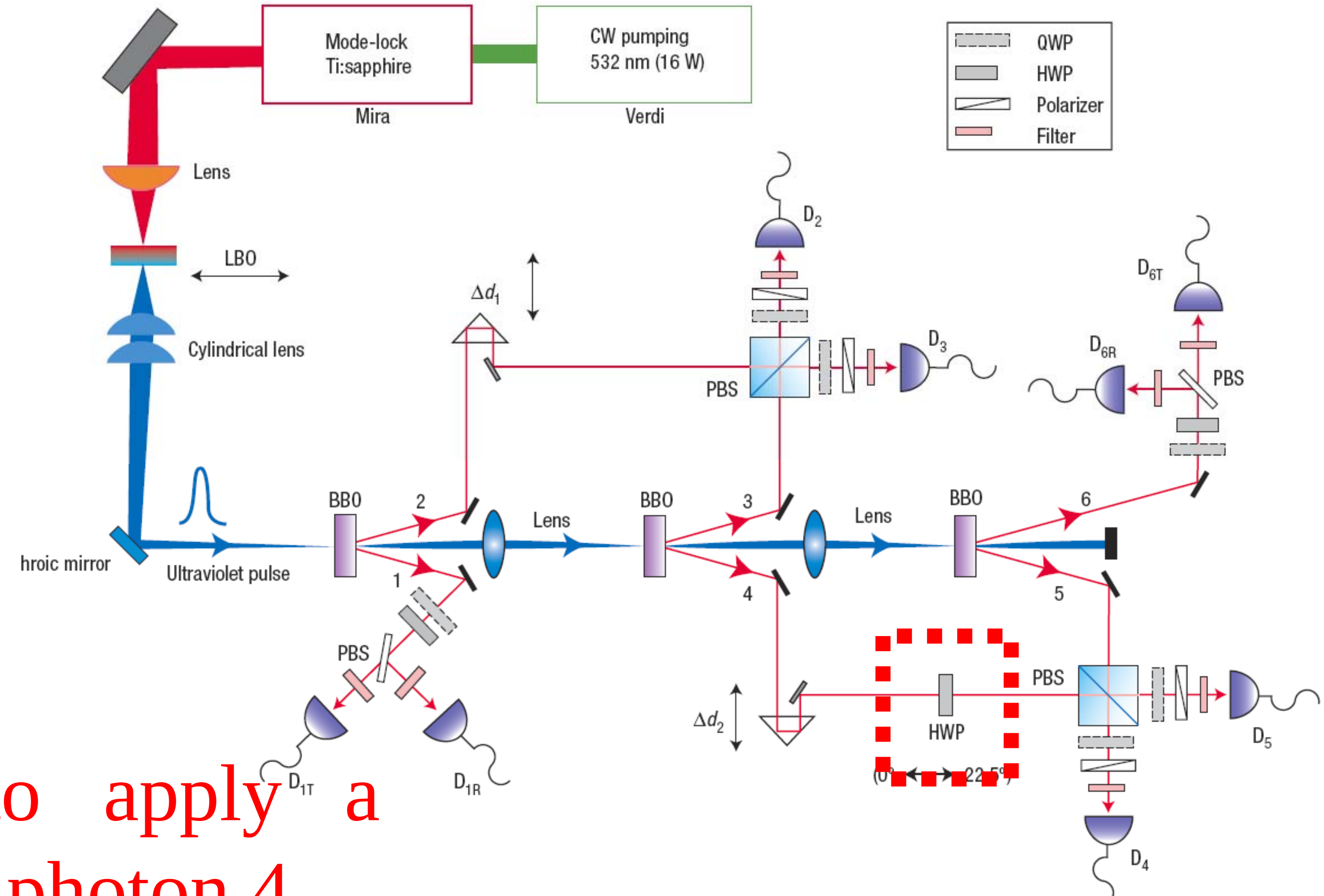
$$|\psi_{\text{out}}\rangle = \left(\prod_{i=1}^{|\mathcal{N}|} U_{\Sigma,g_i} U_{g_i}\right)|\psi_{\text{in}}\rangle \quad \Rightarrow \quad |\psi_{\text{out}}\rangle = \left(\prod_{i=1}^{|\mathcal{N}|} U_{\Sigma,g_i}|_{\Omega}\right)\left(\prod_{i=1}^{|\mathcal{N}|} U'_{g_i}\right)|\psi_{\text{in}}\rangle$$

The byproduct is propagated to the end state

6 photon *Cluster* State

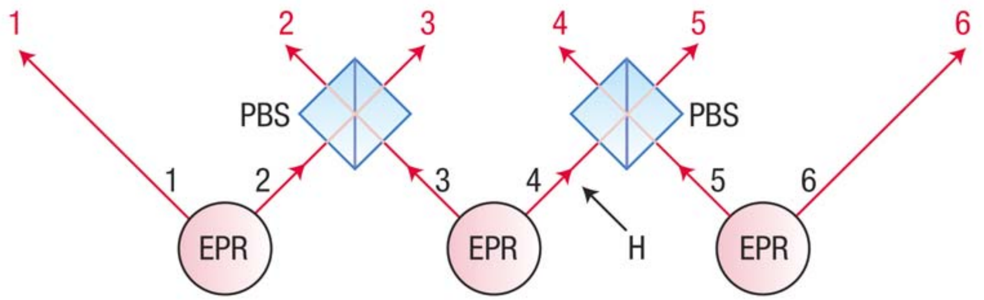
Experimental entanglement of six photons in graph states

CHAO-YANG LU[1]*, XIAO-QI ZHOU[1], OTFRIED GÜHNE[2], WEI-BO GAO[1], JIN ZHANG[1], ZHEN-SHENG YUAN[1], ALEXANDER GOEBEL[3], TAO YANG[1] AND JIAN-WEI PAN[1,3]*



If one is to apply a Hadamard to photon 4

6 photon *Cluster* State



Lets' do it in two steps

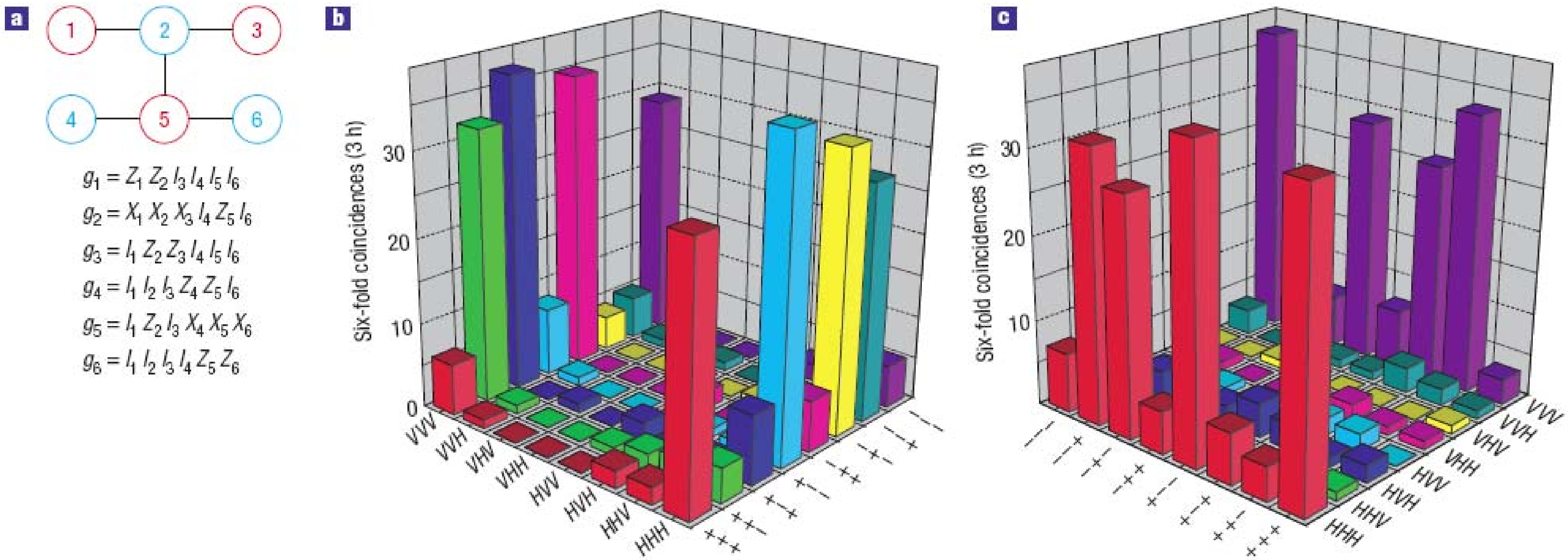
1: Combine 3 and 2

$$(1/\sqrt{2})(|H\rangle_1|H\rangle_2|H\rangle_3|+\rangle_4 + |V\rangle_1|V\rangle_2|V\rangle_3|-\rangle_4),$$

2: Combine 5 and 4

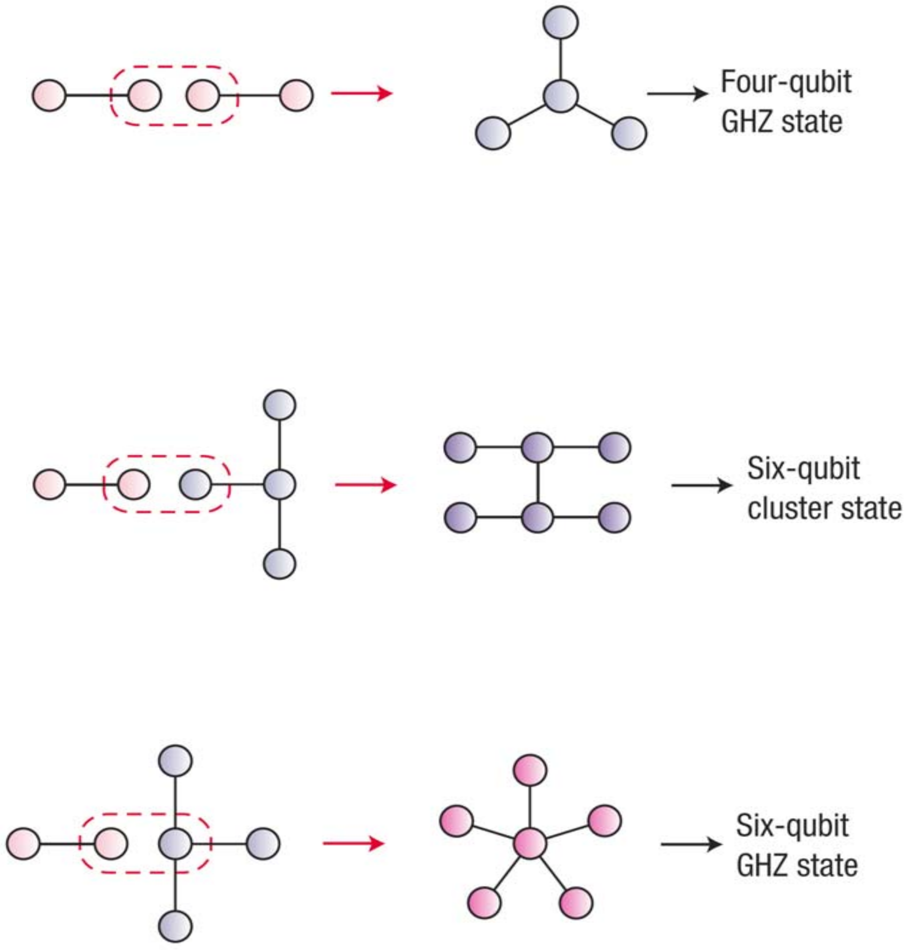
$$\begin{aligned}|C_6\rangle = \frac{1}{2}(&|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4|H\rangle_5|H\rangle_6\\ &+|H\rangle_1|H\rangle_2|H\rangle_3|V\rangle_4|V\rangle_5|V\rangle_6\\ &+|V\rangle_1|V\rangle_2|V\rangle_3|H\rangle_4|H\rangle_5|H\rangle_6\\ &-|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4|V\rangle_5|V\rangle_6)\end{aligned}$$

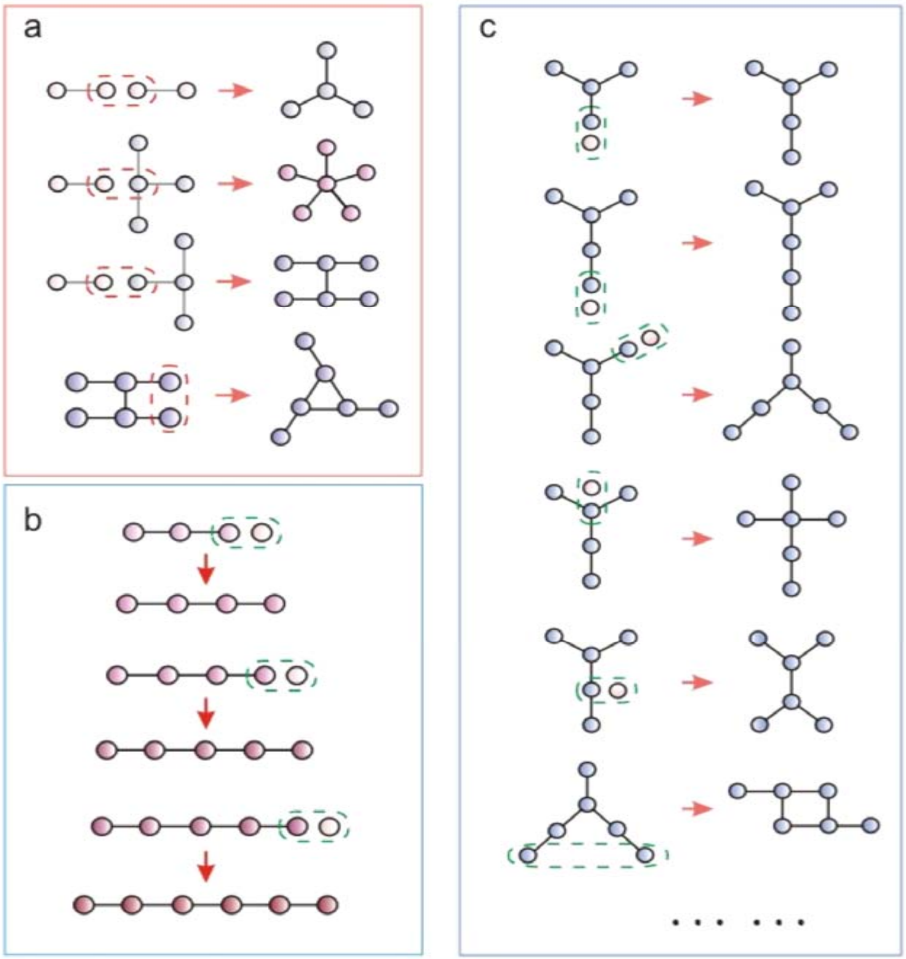
For the six-photon Cluster state a different witness is used:



$$\mathrm{Tr}(W_{\mathrm{C}}\rho_{\mathrm{exp}}) = -0.095 \pm 0.036$$

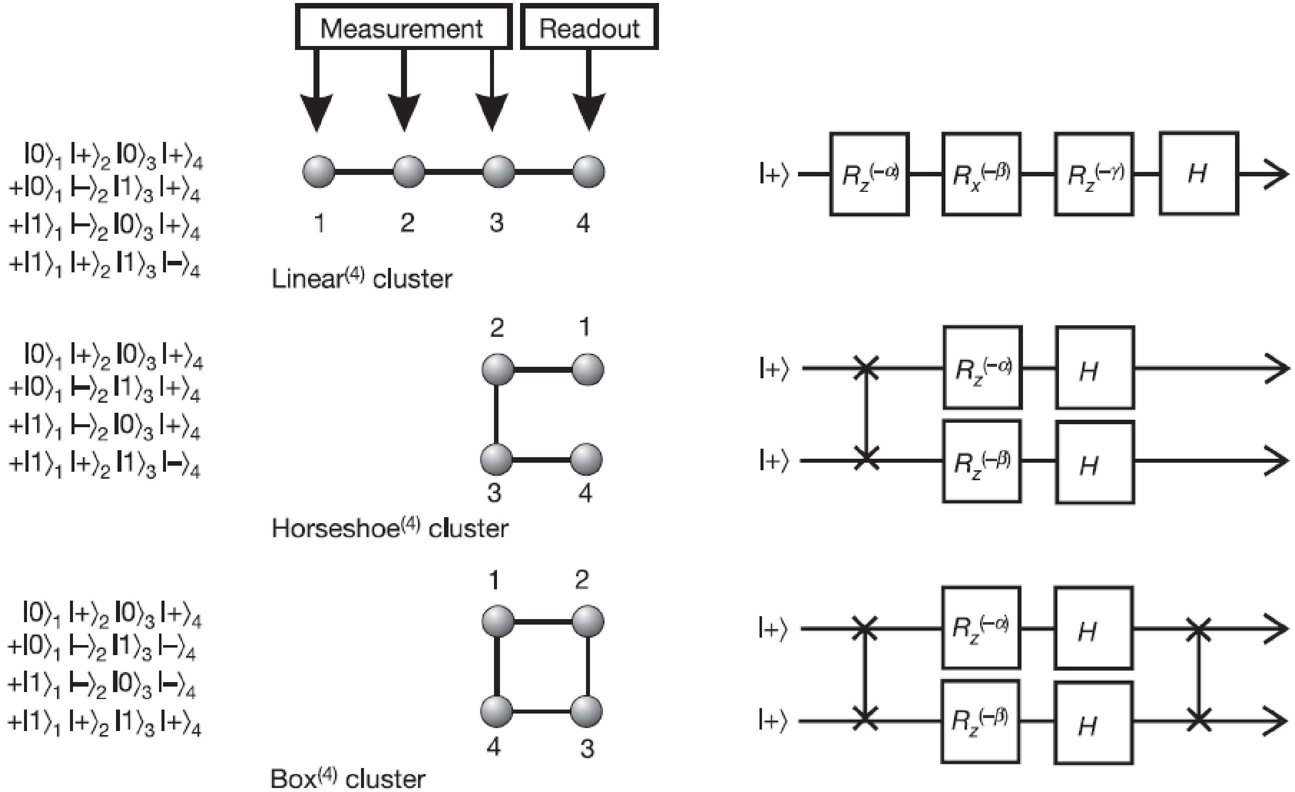
Scheme to construct various six-photon 'graph' states





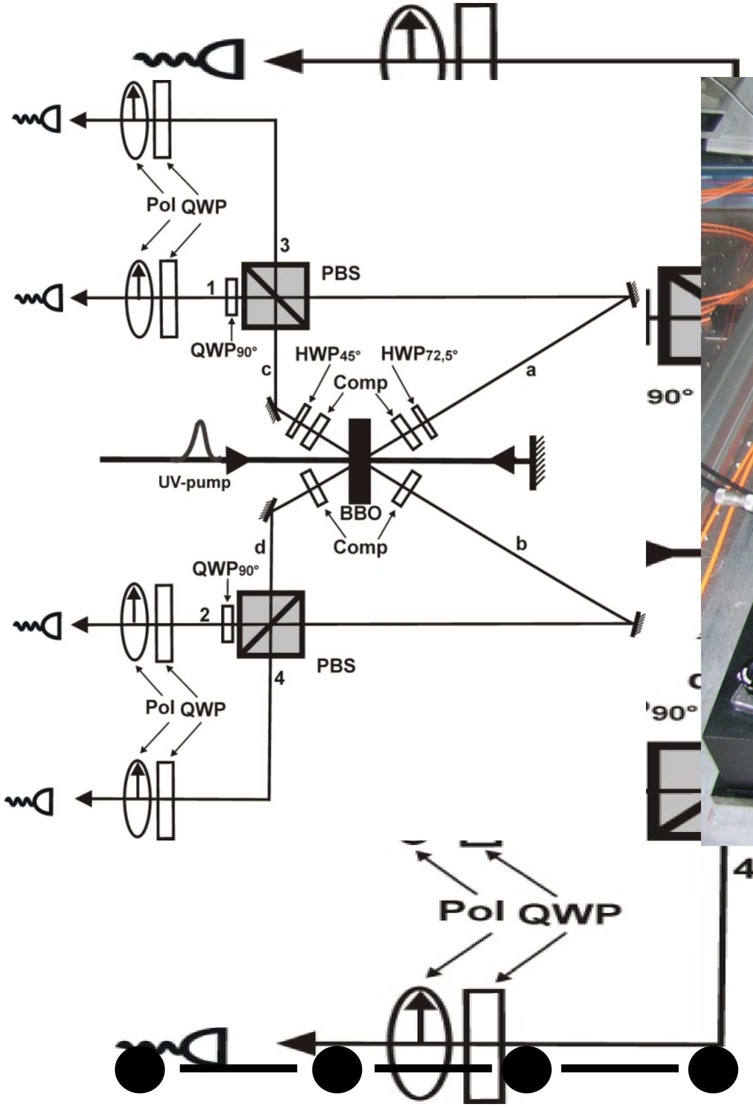
SU(2) rotation & gates (*Zeilinger)*

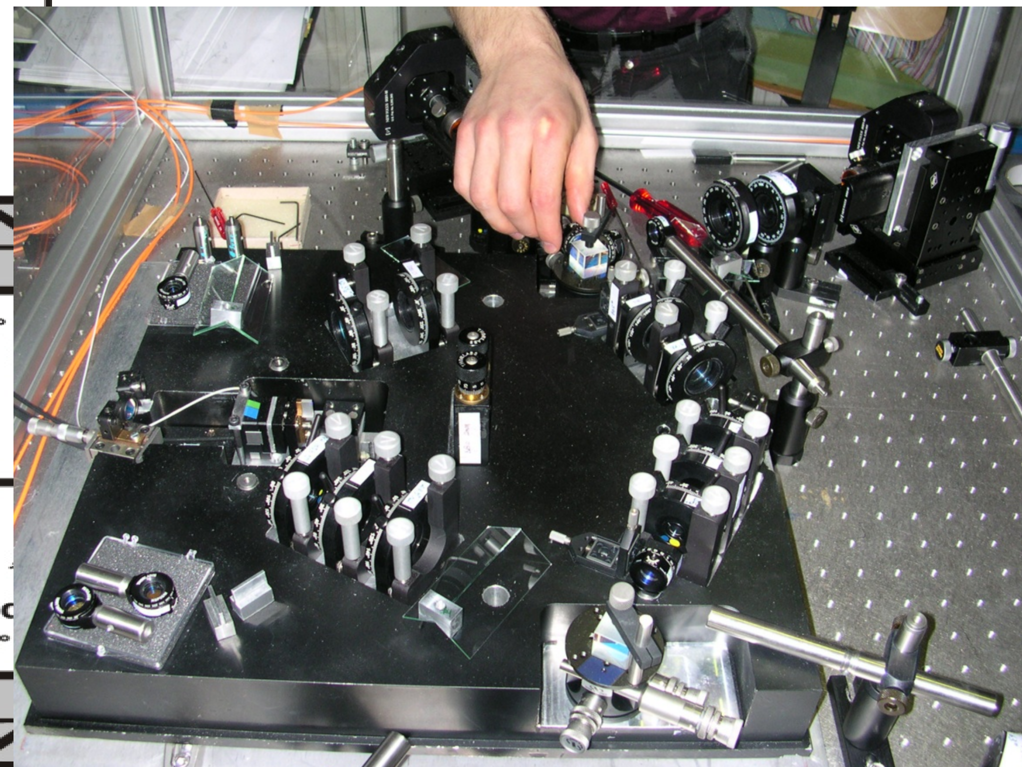
- A general SU(2) rotation and 2-qubit gates



- CPhase operations + single qubit rotations = universal quantum computer!

Doing the experiment (Zielinger of course)

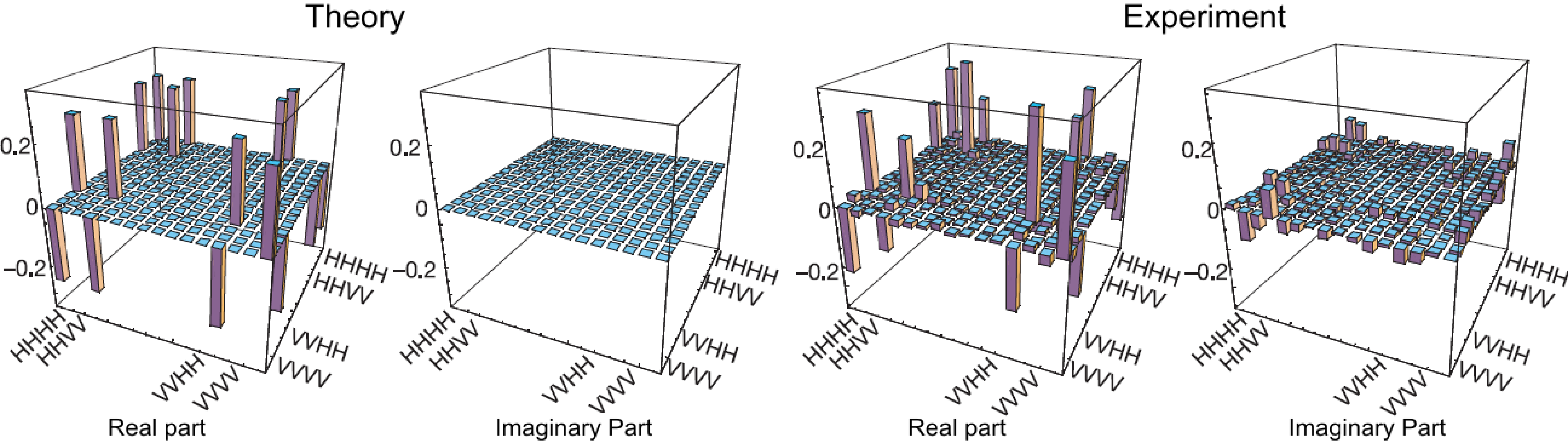




$$|\psi\rangle = |HHHH\rangle + |HHVV\rangle + |VVHH\rangle - |VVVV\rangle$$

Quantum state tomography Reconstructed density matrix

$$\left|\langle \phi_{Cluster} | \left(|A\rangle \otimes |B\rangle \otimes |C\rangle \otimes |D\rangle\right)\right|^2 \quad \text{with } |A\rangle, |B\rangle, |C\rangle, |D\rangle \in \left\{ \begin{array}{c} |H\rangle, \\ |V\rangle, \\ \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right), \\ \frac{1}{\sqrt{2}}\left(|H\rangle - i|V\rangle\right) \end{array} \right\}$$



Fidelity $\quad F = \langle \phi_{Cluster} | \rho | \phi_{Cluster} \rangle = (0.63 \pm 0.02)$

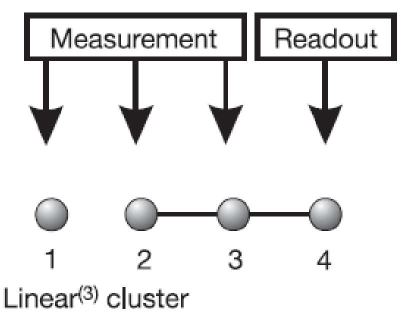
Rotation

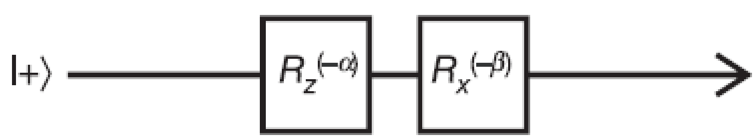
- Disentangle qubit 1 from qubits 2, 3, 4

$$|0\rangle_1 \otimes \begin{pmatrix} |+\rangle_2 |0\rangle_3 |+\rangle_4 \\ +|-\rangle_2 |1\rangle_3 |-\rangle_4 \end{pmatrix} = |0\rangle_1 \otimes \left\{ \begin{array}{l} |+\alpha\rangle_2 |+\beta\rangle_3 \otimes \left(e^{+i\frac{\beta}{2}} \cos\frac{\alpha}{2} |+\rangle_4 \ + \ e^{-i\frac{\beta}{2}} \cdot i \ \sin\frac{\alpha}{2} |-\rangle \right) \\ + \ |+\alpha\rangle_2 |-\beta\rangle_3 \otimes \left(e^{+i\frac{\beta}{2}} \cos\frac{\alpha}{2} |+\rangle_4 \ - \ e^{-i\frac{\beta}{2}} \cdot i \ \sin\frac{\alpha}{2} |-\rangle \right) \\ + \ |-\alpha\rangle_2 |+\beta\rangle_3 \otimes \left(e^{+i\frac{\beta}{2}} \cdot i \ \sin\frac{\alpha}{2} |+\rangle_4 \ + \ e^{-i\frac{\beta}{2}} \cos\frac{\alpha}{2} |-\rangle \right) \\ + \ |-\alpha\rangle_2 |-\beta\rangle_3 \otimes \left(e^{+i\frac{\beta}{2}} \cdot i \ \sin\frac{\alpha}{2} |+\rangle_4 \ + \ e^{-i\frac{\beta}{2}} \cos\frac{\alpha}{2} |-\rangle \right) \end{array} \right\}$$

$$= \ |0\rangle_1 |+\alpha\rangle_2 |+\beta\rangle_3 \otimes \left(\mathrm{R}_x^{(-\beta)} \ \mathrm{R}_z^{(-\alpha)} |+\rangle_4 \right) \ + \ \text{other 3 terms}$$

- and project the state on $|+\alpha\rangle_2 |+\beta\rangle_3$ => post selection

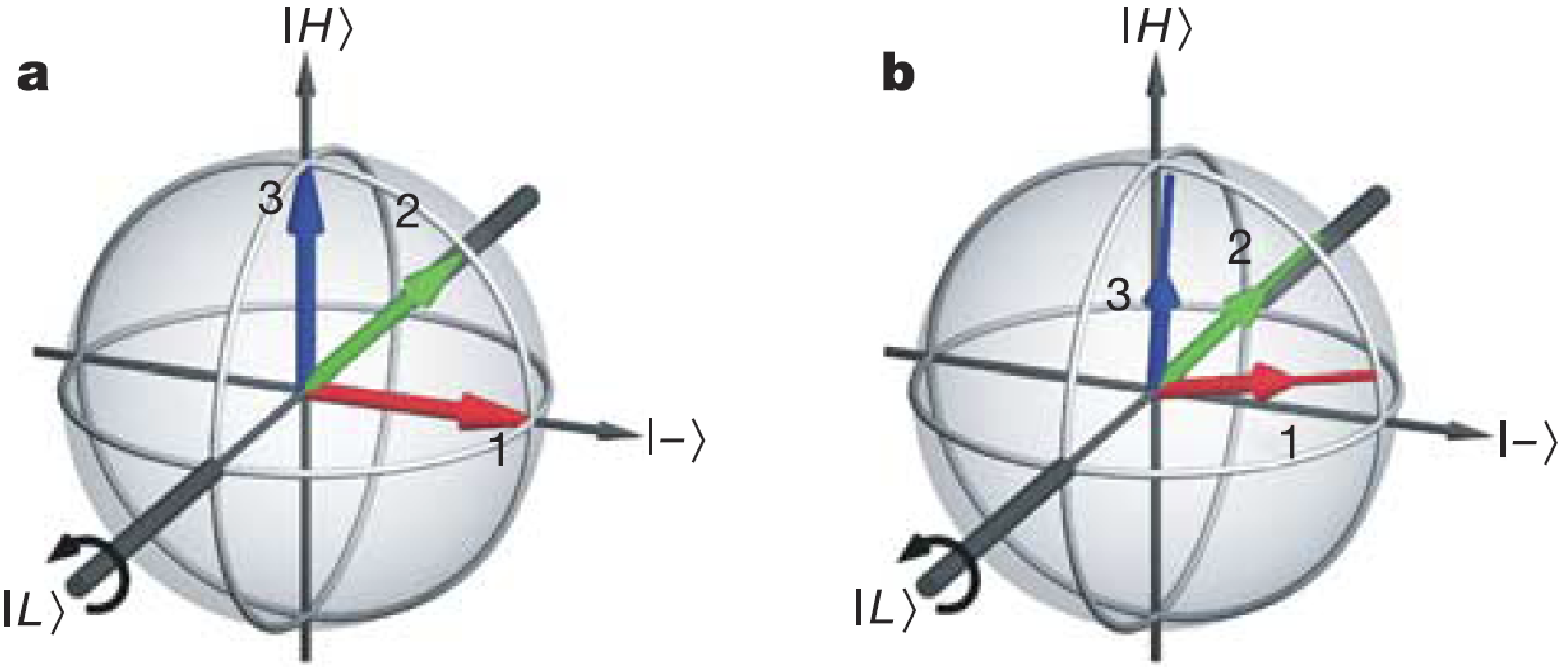




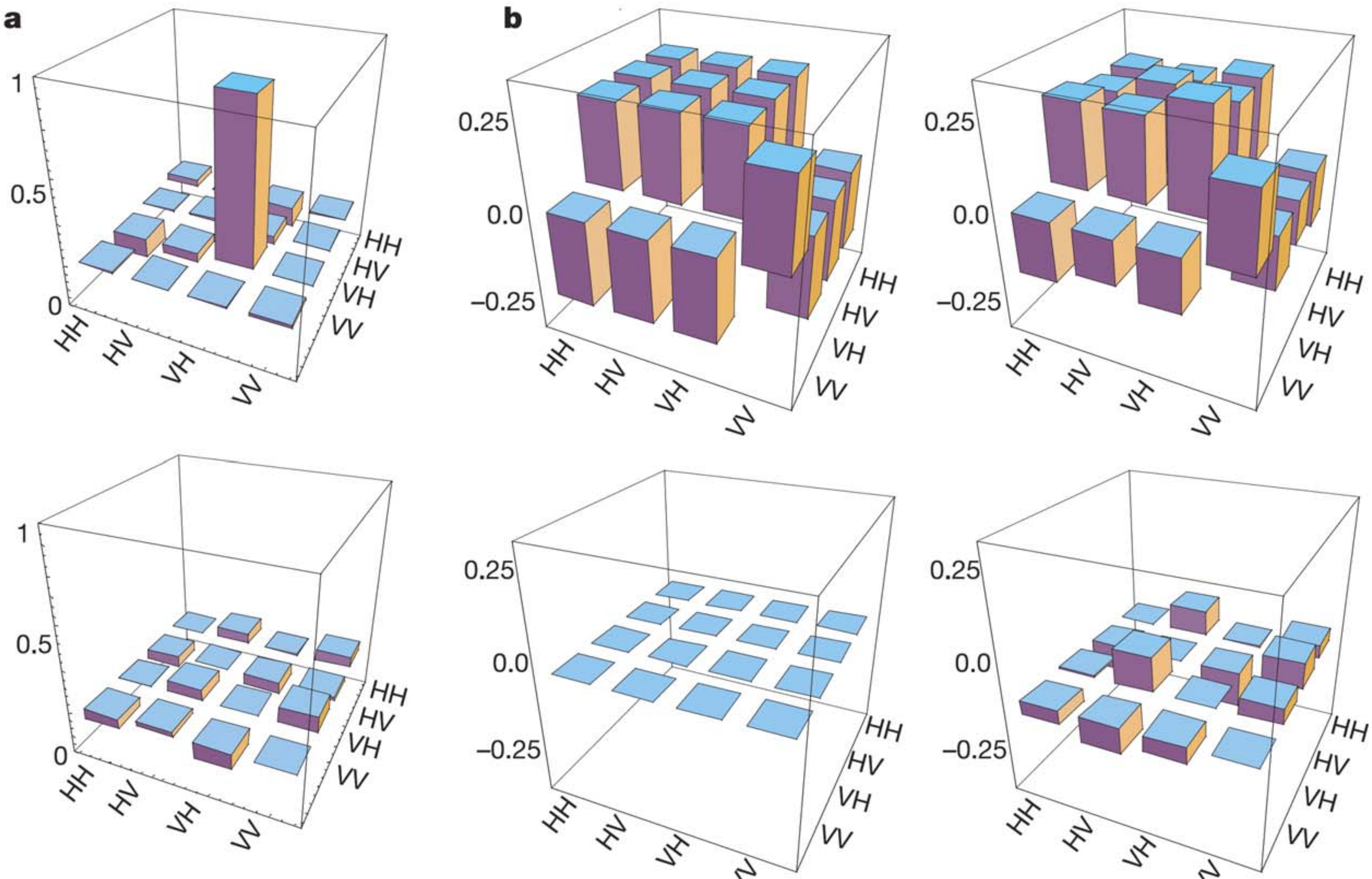
Single qubit rotation

Single-qubit rotations

$$\alpha = \begin{cases} \pi/2 \\ \pi/4 \\ 0 \end{cases} \quad \beta = \pi/2 \quad F = \begin{cases} 0.86 \pm 0.03 \\ 0.85 \pm 0.04 \\ 0.83 \pm 0.03 \end{cases}$$



Two-qubit gates



Thank

You