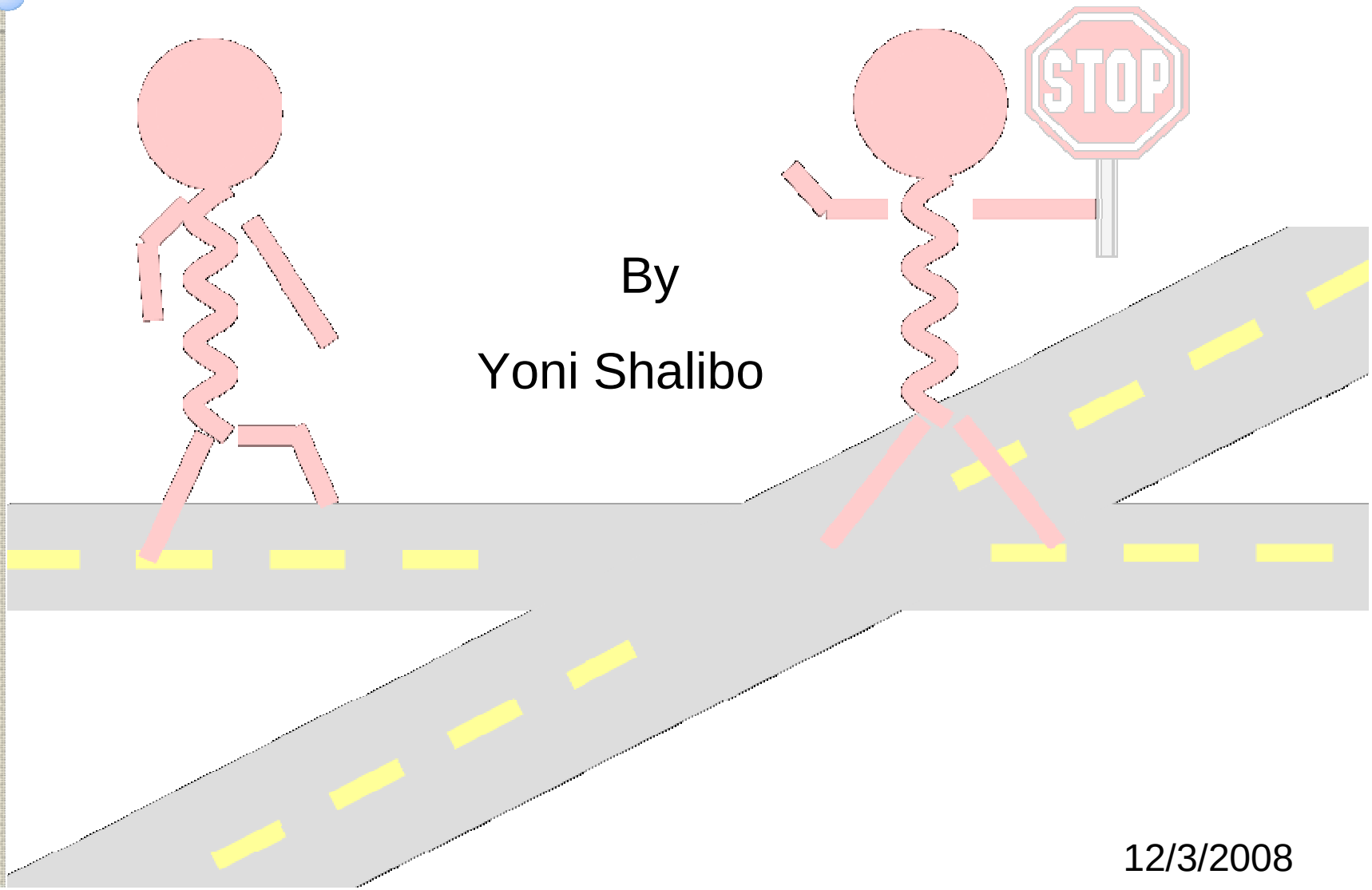
Quantum Logic with Photons



By

Yoni Shalibo

12/3/2008

Outline

- **Introduction to quantum computation**
- Theoretical implementations of quantum gates (KLM)
- Experimental CNOT gates

The Qubit

Classical bits can be in one of two states:

$$|0\rangle \quad \textbf{Or} \quad |1\rangle$$

Quantum bits (Qubits) can be in a *superposition* of states:

$$|\Psi\rangle = a|0\rangle + b|1\rangle, \quad \text{where} \quad |a|^2 + |b|^2 = 1$$

Examples of Qubit states:

Optics:Photon polarization modes, Photon spatial modes

Condensed Matter: States of Josephson junction circuit, two-level atoms (Ion traps), Spins in magnetic field (NMR)

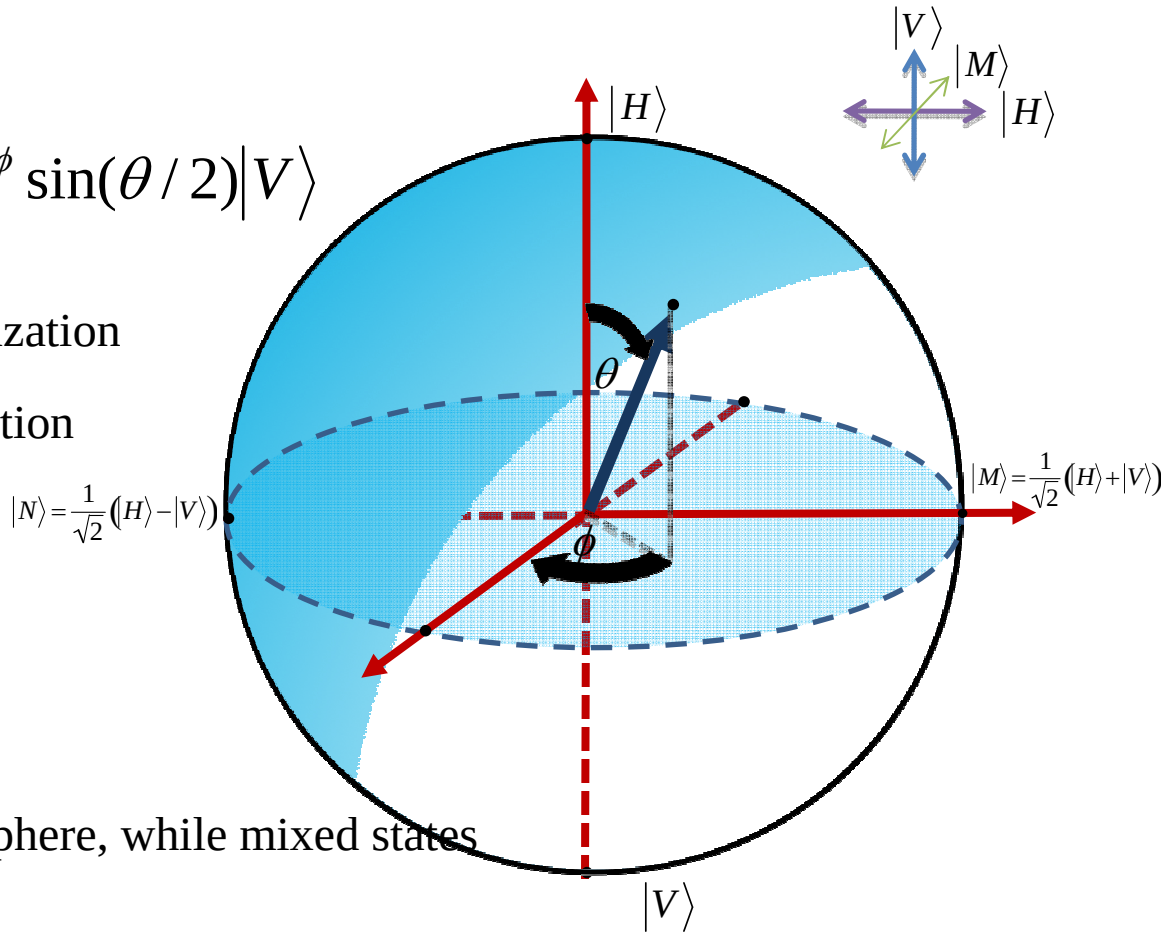
The Poincaré sphere

A two dimensional mapping of the complex amplitudes a,b into a unit sphere, where the states are polarization states of photons:

$$|\Psi\rangle = a|H\rangle + b|V\rangle$$
$$= \cos(\theta/2)|H\rangle + e^{i\phi}\sin(\theta/2)|V\rangle$$

$|H\rangle$ - Horizontal Polarization

$|V\rangle$ - Vertical Polarization



• Pure states lie on the sphere, while mixed states lie within it.

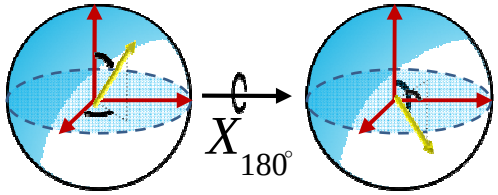
Qubit Gates

- Represented by a *unitary* transformation.
- Are thus *reversible* (unlike some classical gates).

Examples

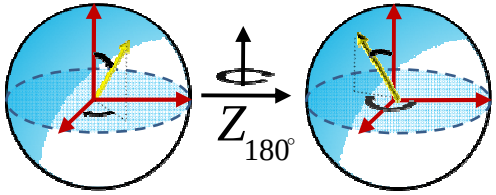
NOT: $a|H\rangle + b|V\rangle \rightarrow b|H\rangle + a|V\rangle$

Pauli σ_X rotation: $\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$



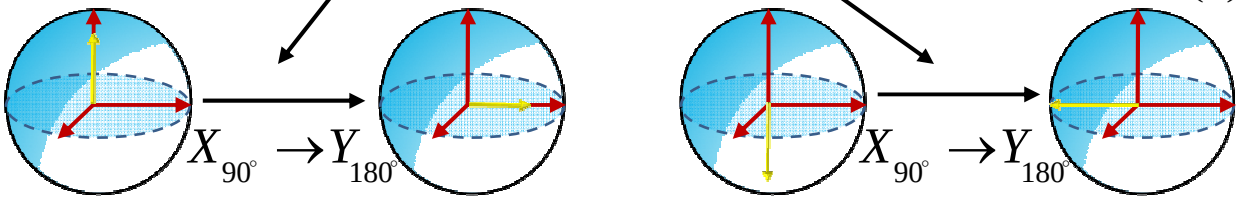
π-Phase shift: $a|H\rangle + b|V\rangle \rightarrow a|H\rangle - b|V\rangle$

Pauli σ_Z rotation: $\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$



Hadamard: $|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ $|V\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$

Hadamard: $\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$



Multiple Qubit Gates

- Represented by $2^N x 2^N$ *unitary* transformation.
- Visualized by higher dimensional Poincare spheres
- Can be constructed from the *universal* C-NOT gate plus single qubit gates

Controlled-NOT (CNOT) Gate

A two-qubit operation on the input state: $|\Psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

"Target" qubit

"Control" qubit

Transformation: $U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

Operation on binary inputs:

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

$$|a,b\rangle \rightarrow |a, a \oplus b\rangle$$

where $a \oplus b \equiv (a+b) \bmod 2$

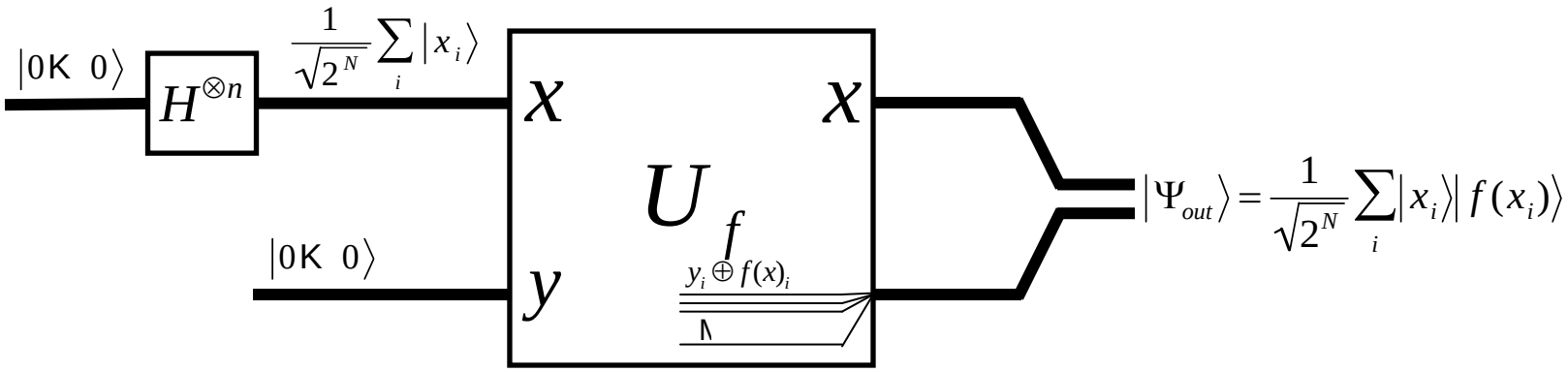
Quantum Circuits

The laws of quantum circuits:

- *Acyclic* (no loops) – one cannot feedback the output into the input
- "Wires" cannot be joined together (reversibility)
- No "cloning" – The circuit cannot make copies of an arbitrary qubit state

Quantum Parallelism?

A quantum circuits can evaluate the outcome of *all possible* input states *simultaneously*:



→ All the values of f(x) are encoded in the output state!

The catch – the output state collapses in the measurement, thus only one outcome can be measured.

One can still interfere different outcomes to exploit this parallelism

Optical Quantum Computation

Why photon qubits?

• Photons exhibit quantum phenomena (as we learned…)

• They do not interact strongly with each other and with matter (bad for coupling between them – photon gates usually work with probability smaller than one)

• They can be guided along great distances with low loss (good for communication)

Qubit state representation

Mode 1
Mode 2
} Such as polarization or spatial modes

The *Dual-rail* representation: $|\Psi\rangle = a|10\rangle + b|01\rangle \equiv a|\mathbf{0}\rangle + b|\mathbf{1}\rangle$

$|10\rangle$ One photon in mode 1, No photon in the mode 2

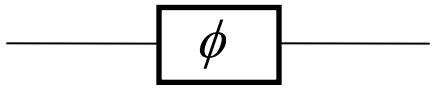
$|01\rangle$ No photon in the mode 1, One photon in mode 2

Optical elements for quantum gates:

Phase shifters

Retard a photon mode by passing it through a medium with a higher index of refraction.

Perform an arbitrary rotation about the Z axis: $R_z(\Delta) = \exp(-i\omega Z\Delta/2) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$



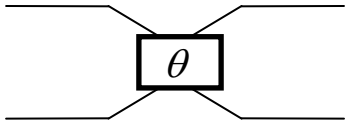
$where: \Delta = (n - n_0)d/c_0$

For polarization encoded qubits, a PS transformation can be done by a λ/2 waveplate, aligned with one of the polarization directions.

Beam splitters (BS)

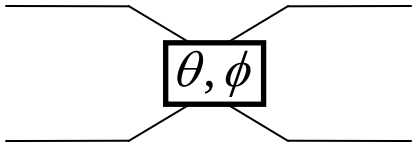
"Half" silvered mirrors. Reflect a fraction R, and transmit (1-R) of the beam. R is usually parameterized by an angle.

Perform an arbitrary rotation about the X axis: $R_X(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$



For polarization encoded qubits, a BS transformation can be done by a λ/2 waveplate.

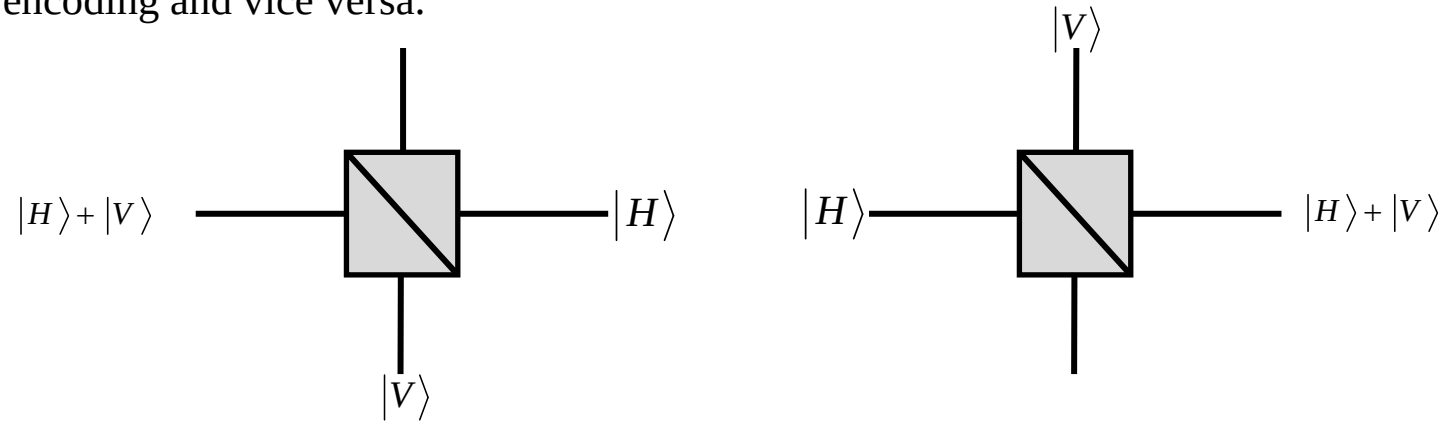
A generalized beam splitter induces an arbitrary rotation (can be accomplished by combining a regular BS with Phase shifters):



$$R(\theta,\phi) = \begin{pmatrix} \cos(\theta) & -e^{i\phi}\sin(\theta) \\ e^{-i\phi}\sin(\theta) & \cos(\theta) \end{pmatrix}$$

Polarizing beam splitters (PBS)

Transmits one mode of polarization and reflects the other. Used to convert from polarization to spatial encoding and vice versa.



Non-linear Kerr media

Induce photon interaction by intensity sensitive change in the refraction index:

$$n(I) = n_0 + n_1 I$$

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- **Theoretical optical implementations of quantum gates (KLM)**
- Experimental optical CNOT gates

E. Knill, R. Laflamme, G.J. Milburn, Nature **409**, 46 (2001)

"A Scheme for efficient quantum computation with linear optics":

Three main results:

- Non-deterministic quantum computing is possible with *linear optics*
- The probability of success can be increased arbitrarily close to one
- A highly efficient resource for accurately encoded qubits can be achieved using quantum coding

The first two results will be briefly explained hereafter.

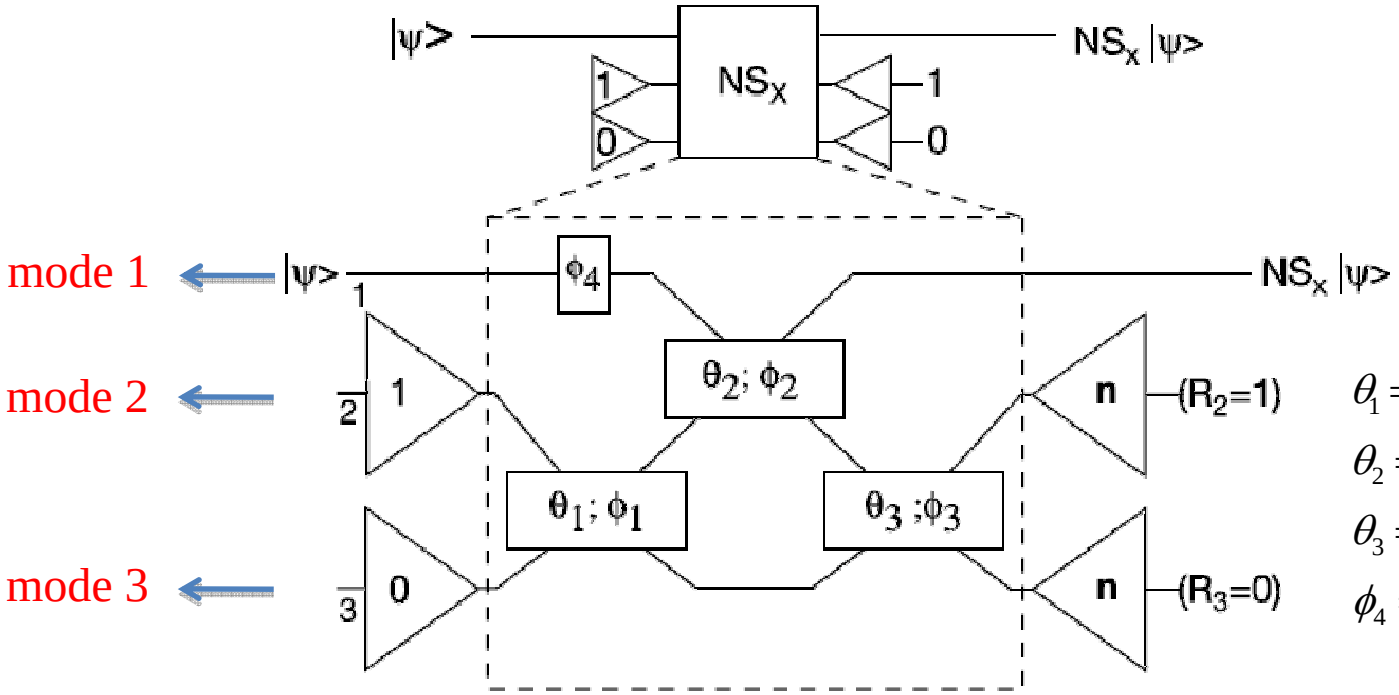
Non-deterministic Nonlinear Sign flip (with linear optics)

Performs the following operation :

Single mode state $\leftarrow$ $|\Psi\rangle = a|0\rangle + b|1\rangle + c|2\rangle \rightarrow a|0\rangle + b|1\rangle - c|2\rangle$

The gate works *if and only if* detector 2 counts one photon and detector 3 counts no photons. This happens with probability 1/4

The quantum circuit:



$\theta_1 = 22.5^o; \phi_1 = 0^o;$

$\theta_2 = 65.5302^o; \phi_2 = 0^o;$

$\theta_3 = -22.5^o; \phi_3 = 0^o;$

$\phi_4 = 180^o;$

Construction of unitary transformation of the NS gate:

$$u=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta_3) & -\sin(\theta_3)\\ 0 & \sin(\theta_3) & \cos(\theta_3)\end{pmatrix}\begin{pmatrix}\cos(\theta_2) & -\sin(\theta_2) & 0\\ \sin(\theta_2) & \cos(\theta_2) & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\phi_4} & 0 & 0\\ 0 & \cos(\theta_1) & -\sin(\theta_1)\\ 0 & \sin(\theta_1) & \cos(\theta_1)\end{pmatrix}$$

$$=\begin{pmatrix}1-\sqrt{2} & 2^{-1/4} & \sqrt{\sqrt{3/2}-2}\\ 2^{-1/4} & 1/2 & 1/2-\sqrt{1/2}\\ \sqrt{\sqrt{3/2}-2} & 1/2-\sqrt{1/2} & \sqrt{1/2}-1/2\end{pmatrix}$$

U operates on "tri-rail" states (Fock states of 3 modes occupying 1 photon)!

Input state representation:

$$\left|n,1,0\right\rangle=\left(a_1^{+}\right)^n a_2^{+}\left|0,0,0\right\rangle\equiv\left|0,1,0\right\rangle\otimes\underbrace{\left|1,0,0\right\rangle\otimes\ldots\otimes\left|1,0,0\right\rangle}_{n}\equiv\begin{pmatrix}0\\1\\0\end{pmatrix}\otimes\underbrace{\begin{pmatrix}1\\0\\0\end{pmatrix}\otimes\ldots\otimes\begin{pmatrix}1\\0\\0\end{pmatrix}}_{n}$$

By linearity of the BSs, the U transformation operates on each component:

$$\begin{pmatrix}0\\1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\end{pmatrix}\rightarrow u\begin{pmatrix}0\\1\\0\end{pmatrix}\otimes u\begin{pmatrix}1\\0\\0\end{pmatrix}\otimes u\begin{pmatrix}1\\0\\0\end{pmatrix}$$

Case 1: 0 photons are in mode 1

$$\begin{pmatrix}0\\1\\0\end{pmatrix} \to u\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}2^{-1/4}\\1/2\\1/2-\sqrt{1/2}\end{pmatrix} \Rightarrow A(|0\rangle \to |0\rangle) = 1/2$$

Case 2: 1 photons are in mode 1

$$\begin{pmatrix}0\\1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\end{pmatrix} \to u\begin{pmatrix}0\\1\\0\end{pmatrix} \otimes u\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2^{-1/4}\\1/2\\1/2-\sqrt{1/2}\end{pmatrix} \otimes \begin{pmatrix}1-\sqrt{2}\\2^{-1/4}\\\sqrt{\sqrt{3/2}-2}\end{pmatrix}$$

$$A(|1\rangle \to |1\rangle) = 1/2 \times \left(1-\sqrt{2}\right) + 2^{-1/4} \times 2^{-1/4} = 1/2$$

Case 3: 2 photons are in mode 1

$$\begin{pmatrix}0\\1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\end{pmatrix} \to u\begin{pmatrix}0\\1\\0\end{pmatrix} \otimes u\begin{pmatrix}1\\0\\0\end{pmatrix} \otimes u\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2^{-1/4}\\1/2\\1/2-\sqrt{1/2}\end{pmatrix} \otimes \begin{pmatrix}1-\sqrt{2}\\2^{-1/4}\\\sqrt{\sqrt{3/2}-2}\end{pmatrix} \otimes \begin{pmatrix}1-\sqrt{2}\\2^{-1/4}\\\sqrt{\sqrt{3/2}-2}\end{pmatrix}$$

$$A(|2\rangle \to |2\rangle) = 1/2 \times \left(1-\sqrt{2}\right)^2 + 2^{-1/4} \times 2^{-1/4} \times \left(1-\sqrt{2}\right) + 2^{-1/4} \times \left(1-\sqrt{2}\right) \times 2^{-1/4} = -1/2$$

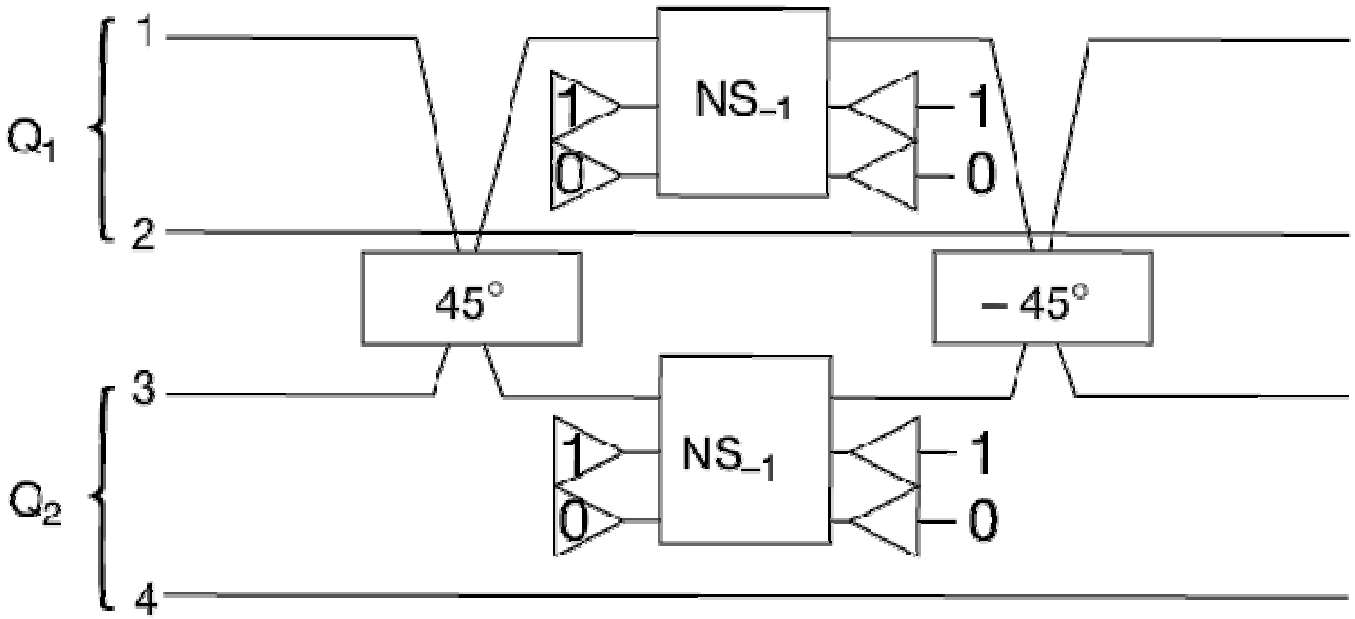
Conditional Sign flip: $|ab\rangle \rightarrow (-1)^{ab}|ab\rangle$

1. Assume qubits Q_1 and Q_2 are both in state $|1\rangle$. Modes 1,3 will therefore be in state $|11\rangle_{13}$.

2. The two photons emerge "bunched" from BS_1, i.e. : $|\Psi\rangle_{BS_1} = |20\rangle_{13} - |02\rangle_{13}$

3. The NS gates will induce a phase shift to the above state with probability 1/16.

4. $-|11\rangle_{13}$ will emerge from BS_2. In all other cases NS do nothing, and the state will not change.



Bunching in BSs

$$BS(45^{o}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

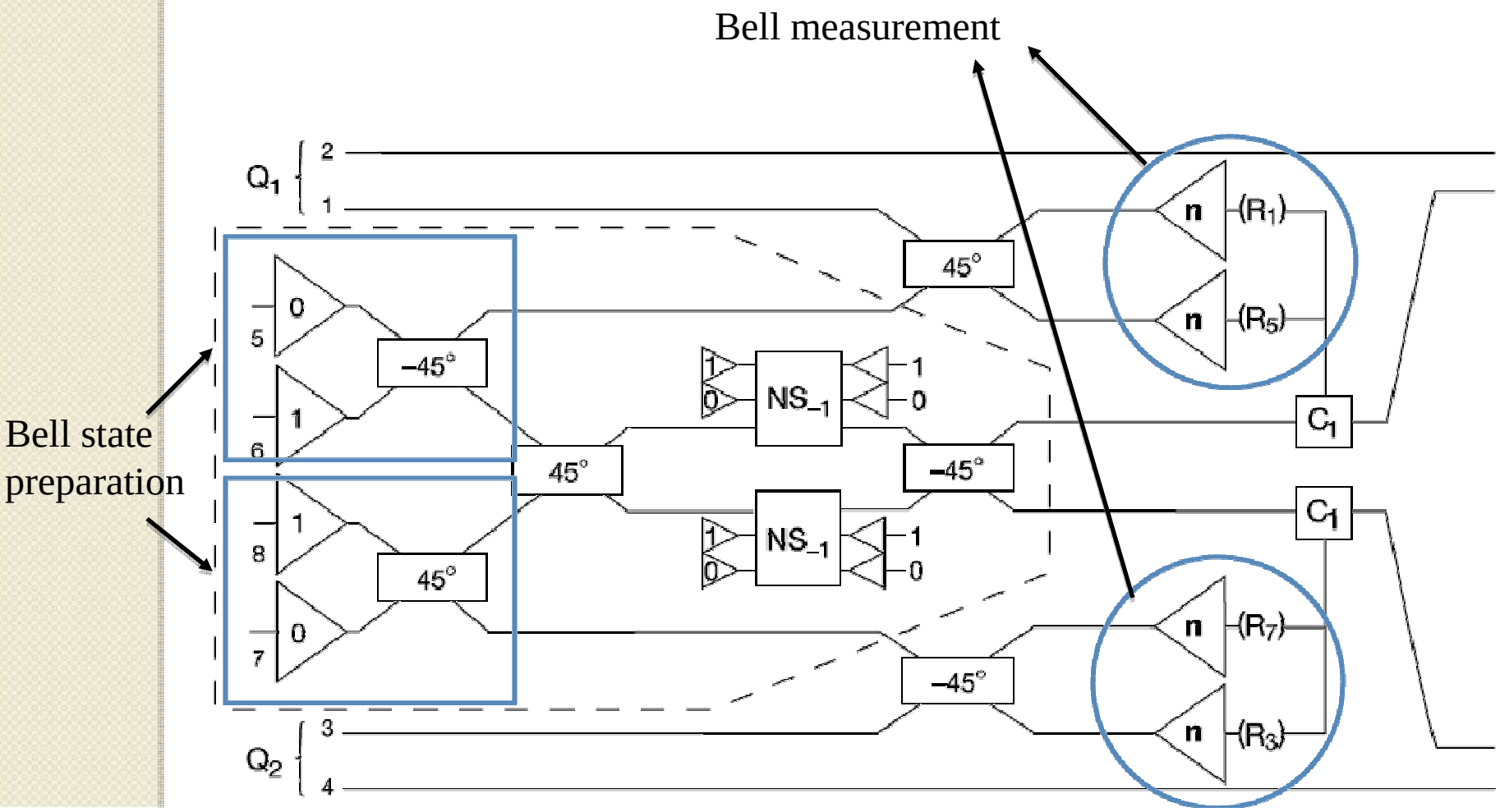
$$|11\rangle_{ab} \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}_a \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_b \rightarrow u_{BS}\begin{pmatrix} 1 \\ 0 \end{pmatrix}_a \otimes u_{BS}\begin{pmatrix} 0 \\ 1 \end{pmatrix}_b = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}_a \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}_b$$

$$\equiv \sqrt{2} \times \frac{1}{2}\left(|20\rangle_{ab} - |02\rangle_{ab}\right) + \frac{1}{2}\left(|11\rangle_{ab} - |11\rangle_{ab}\right) = \frac{1}{\sqrt{2}}\left(|20\rangle_{ab} - |02\rangle_{ab}\right)$$

Bunching!

Near-deterministic conditional sign flip via Teleportation

- The NS gates operate on qubits state only if it works.
- Conditional sign flip will work only if teleportation works
- Teleportation of each qubit state works with probability 1/2



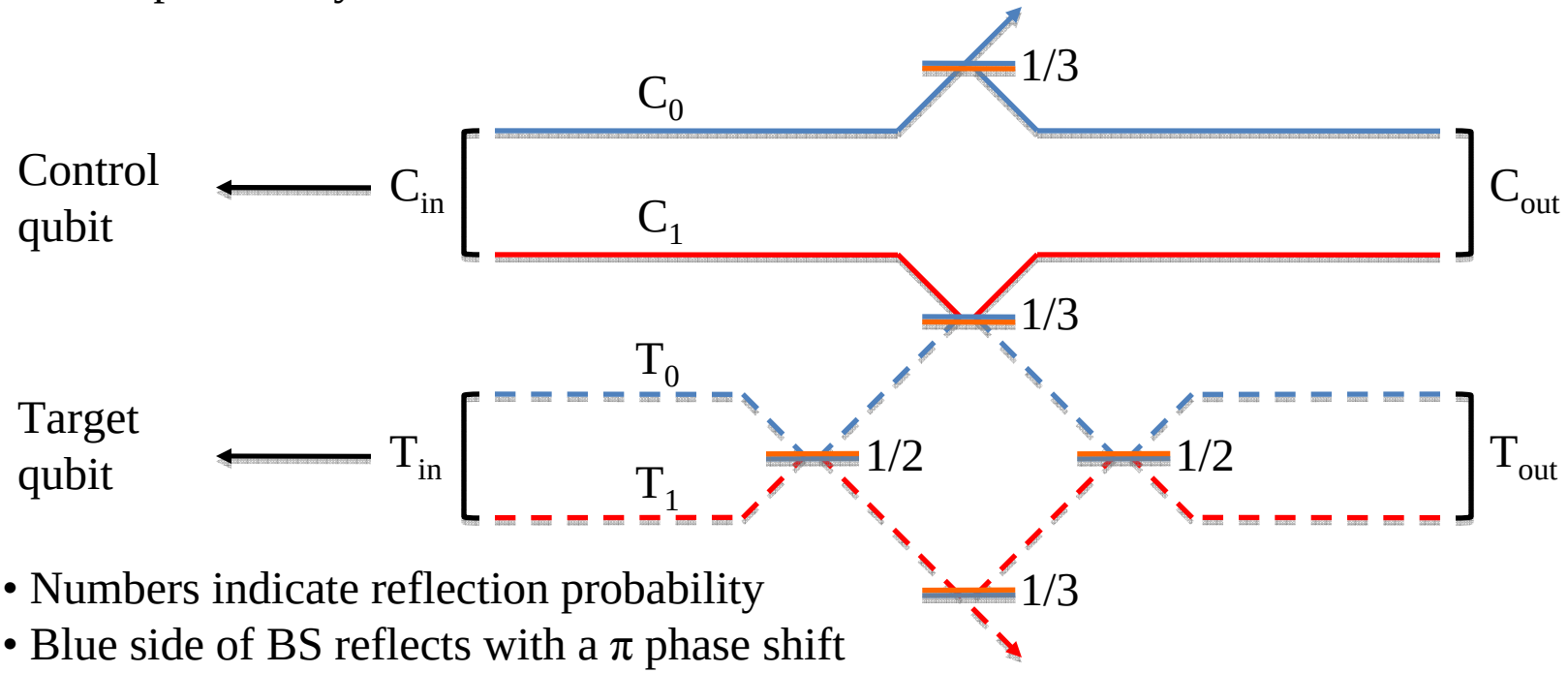
Outline

- Introduction to quantum computation
- Theoretical optical implementations of quantum gates (KLM)
- **Experimental optical CNOT gates**

J.L O'Brien, G.J. Pride, A.G. White, T.C. Ralph, D.Branning, Nature **426**, 264 (2003)

"Demonstration of an all-optical quantum controlled-NOT gate"

The following circuit performs a CNOT gate on qubits C, T using linear optics with success probability 1/9:



- Numbers indicate reflection probability
- Blue side of BS reflects with a π phase shift

- The gate works only when a single photon is detected on each qubit.
- When one photon exists in the upper mode and zero photons in the lower mode, the qubit is in the state $|0\rangle$. Otherwise it is in state $|1\rangle$.

Calculation of the CNOT gate operation.

Output modes operators:

$$C_{0,out}^{+} = \frac{1}{\sqrt{3}} C_{0,in}^{+} \qquad T_{0,out}^{+} = \frac{1}{\sqrt{3}} \left(C_{1,in}^{+} + T_{0,in}^{+} \right)$$

$$C_{1,out}^{+} = \frac{1}{\sqrt{3}} \left(-C_{1,in}^{+} + T_{0,in}^{+} + T_{1,in}^{+} \right) \qquad T_{1,out}^{+} = \frac{1}{\sqrt{3}} \left(C_{1,in}^{+} + T_{1,in}^{+} \right)$$

Input state:

C T

$$\left|\Psi\right\rangle_{in} = a\left|1010\right\rangle + b\left|1001\right\rangle + c\left|0101\right\rangle + d\left|0110\right\rangle$$

Fock basis

$$\equiv \left(aC_{0,in}^{+}T_{0,in}^{+} + bC_{0,in}^{+}T_{1,in}^{+} + cC_{1,in}^{+}T_{0,in}^{+} + dC_{1,in}^{+}T_{1,in}^{+} \right)\left|0000\right\rangle$$

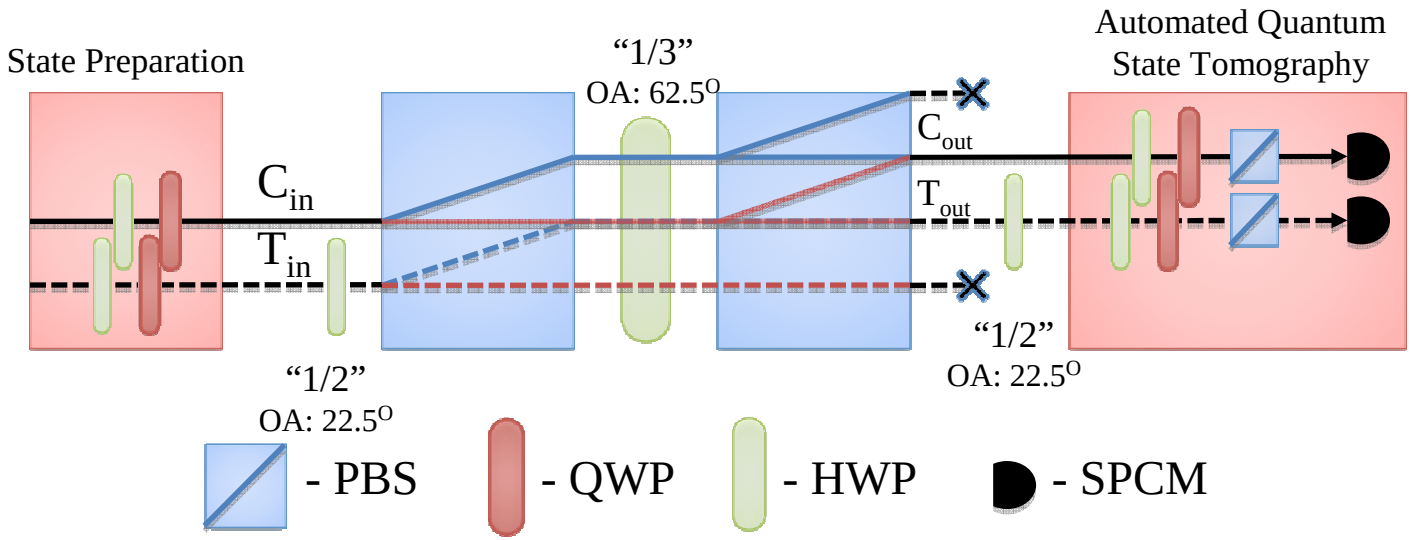
Output state:

$$\left|\Psi\right\rangle_{out} = \left(aC_{0,out}^{+}T_{0,out}^{+} + bC_{0,out}^{+}T_{1,out}^{+} + cC_{1,out}^{+}T_{0,out}^{+} + dC_{1,out}^{+}T_{1,out}^{+} \right)\left|0000\right\rangle$$

Plugging output operators and taking only terms with single photon in each qubit:

$$\left|\Psi\right\rangle_{out} = a\left|1010\right\rangle + b\left|1001\right\rangle + c\left|0110\right\rangle + d\left|0101\right\rangle$$

Experimental setup for equivalent CNOT circuit



1. The qubits are prepared in the polarization basis.

2. First and last HWPs perform Hadamard transformation, which is equivalent to a balanced BS.

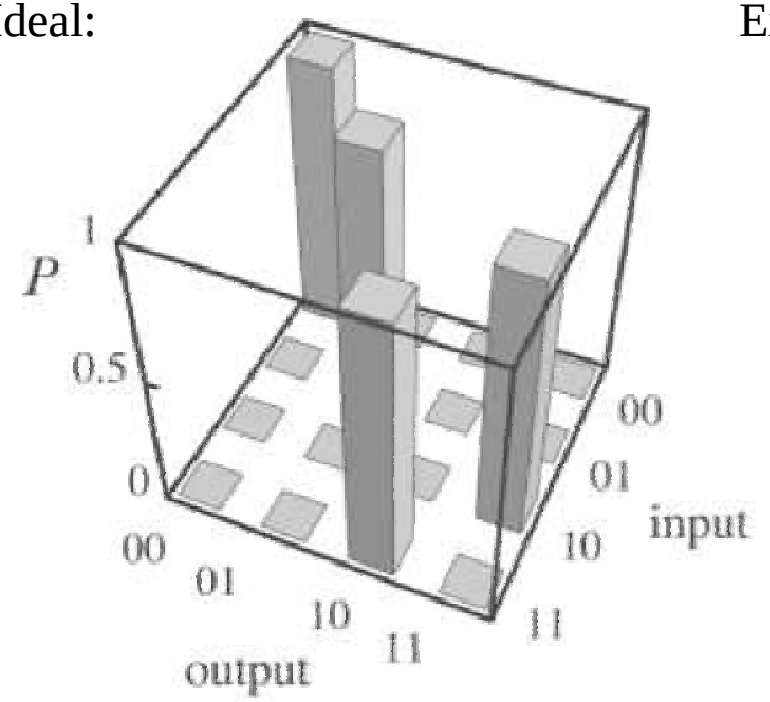
3. The 1st PBS separates the two polarization modes into two spatial modes, thus allowing modes from the two qubits to interfere non classically at the 1/3 BS.

4. The 2nd HWP performs a polarization rotation, which is equivalent to a 1/3 BS in the polarization basis.

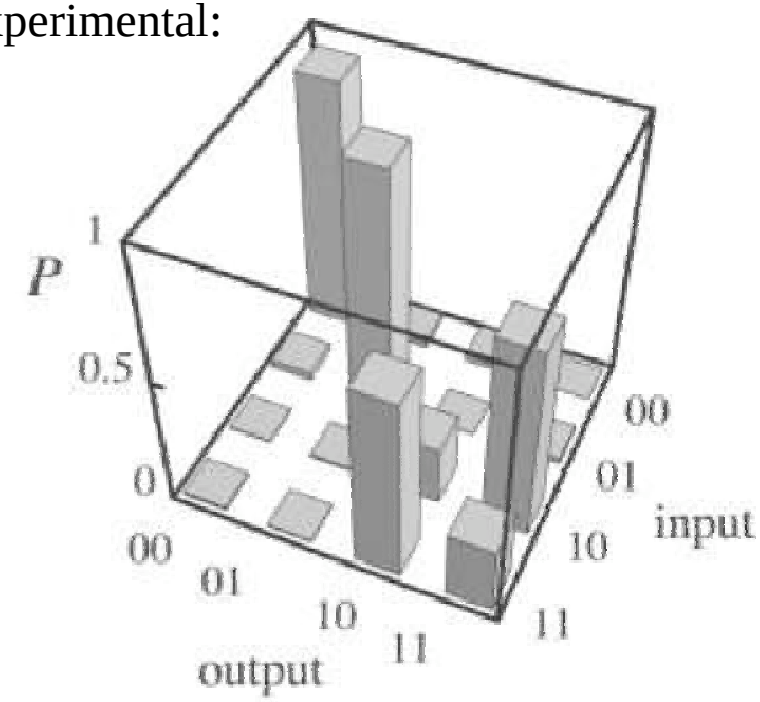
5. The 2nd PBS recombines different spatial modes into the same spatial mode.

Experimental VS Ideal output:

Ideal:



Experimental:



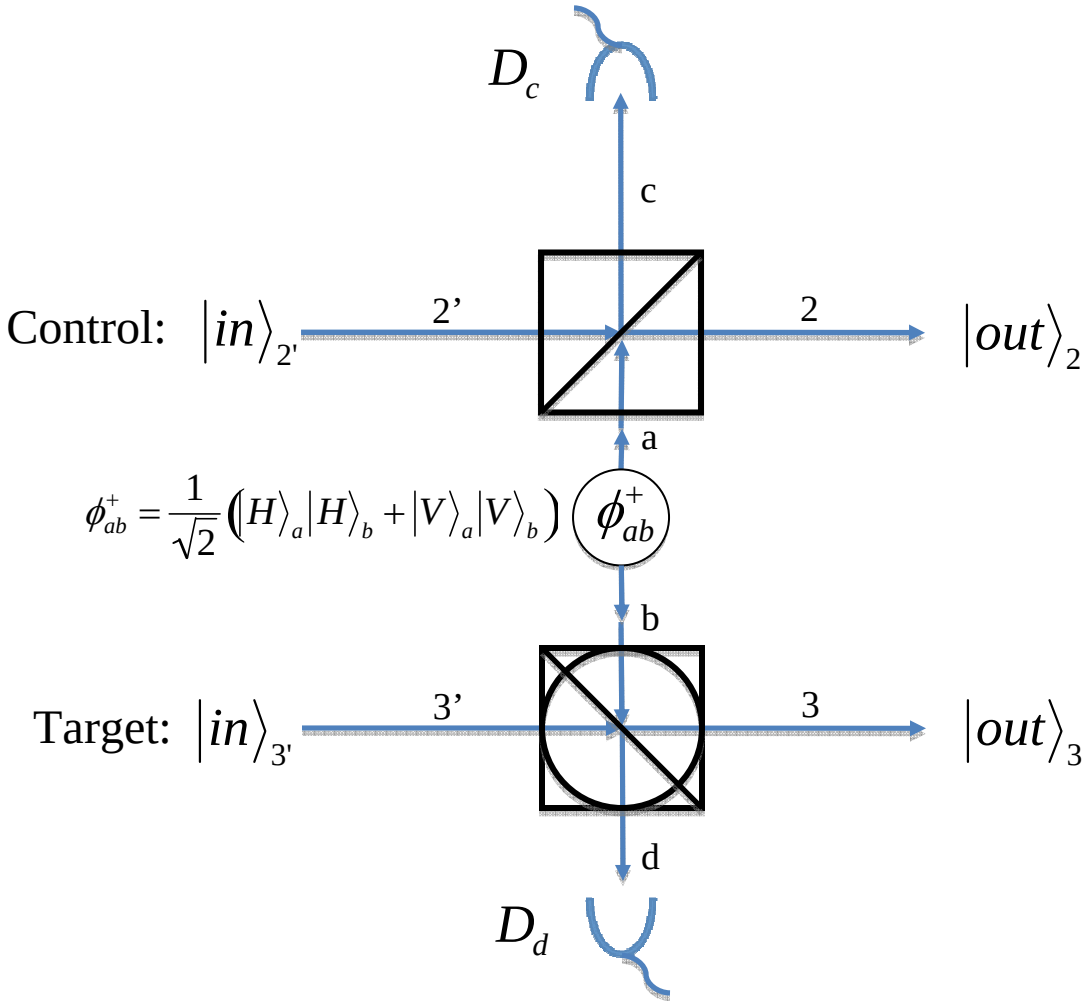
The probability of getting the correct result is low in the case of non-classical interference (i.e., more than one photon): P≈75%

In the case of single photon, the probability of success is ≈95%

T.B. Pittman, B.C. Jacobs, J.D. Franson, Phys. Rev. A **64**, 062311 (2001)

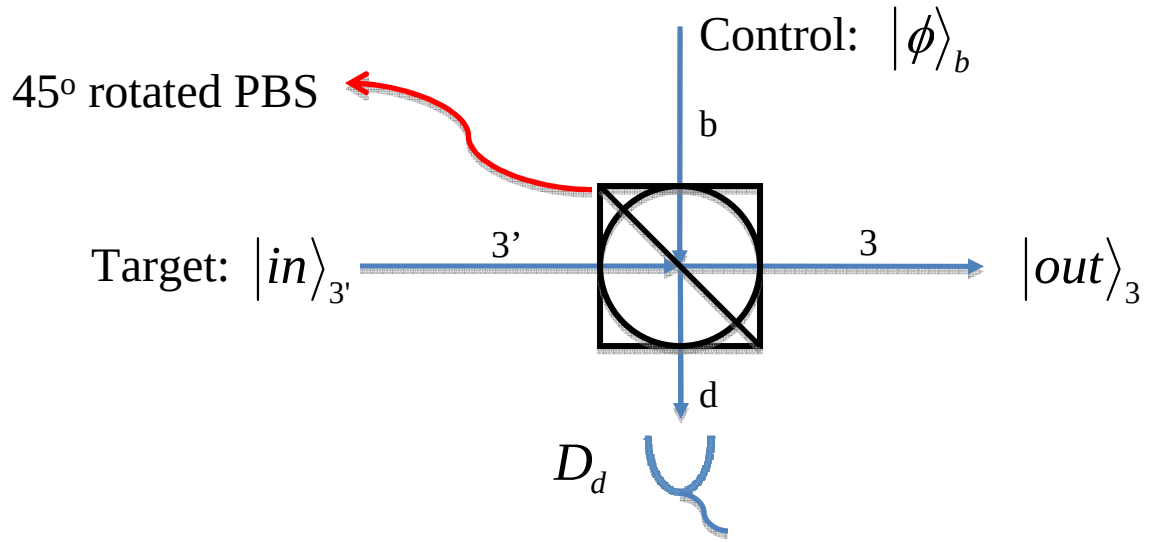
"Probabilistic quantum logic using polarizing beam splitters"

The following gate performs CNOT with probability ¼:

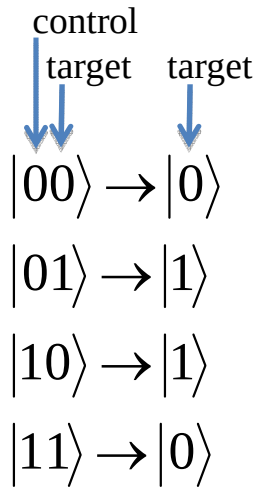


destructive CNOT + Quantum Encoder = non-destructive CNOT

Destructive CNOT gate

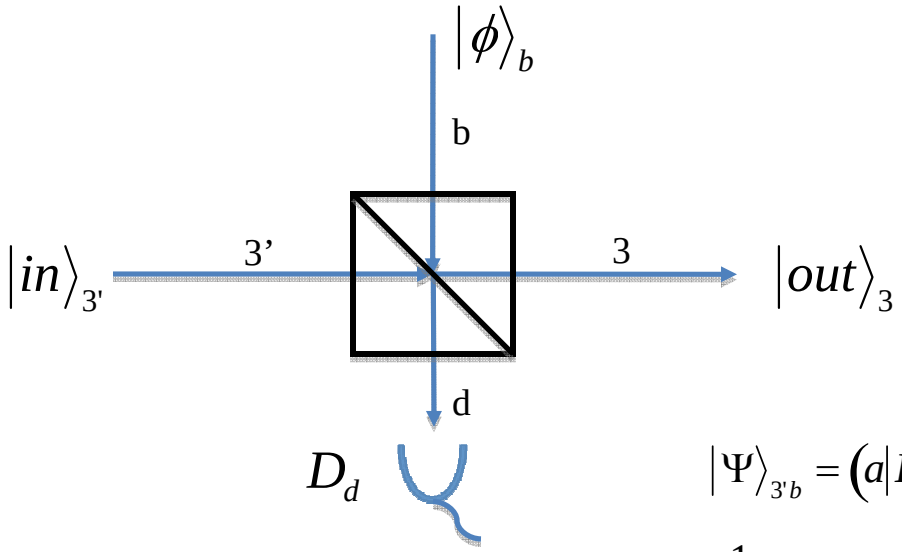


Performs the following operation with a probability of ½:



The gate works if D_d detects 1AO1 photon!

PBS as two-qubit gate:



$$|\Psi\rangle_{3'b} = \left(a|H\rangle_{3'} + b|V\rangle_{3'}\right) \otimes \frac{1}{\sqrt{2}}\left(|H\rangle_b + |V\rangle_b\right) =$$

$$\frac{1}{\sqrt{2}}\left(a|H\rangle_{3'}|H\rangle_b + a|H\rangle_{3'}|V\rangle_b + b|V\rangle_{3'}|H\rangle_b + b|V\rangle_{3'}|V\rangle_b\right)$$

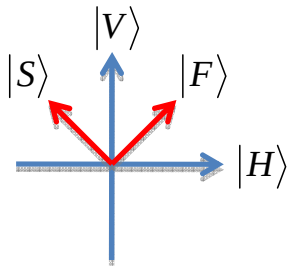
$$\rightarrow PBS \rightarrow$$

$$\frac{1}{\sqrt{2}}\left(a|H\rangle_3|H\rangle_d + a|H\rangle_3|V\rangle_3 + b|V\rangle_d|H\rangle_d + b|V\rangle_3|V\rangle_d\right)$$

$$\equiv \frac{1}{\sqrt{2}}\left(a|H\rangle_3|H\rangle_d + b|V\rangle_3|V\rangle_d + \Psi_{II}\right)$$

Note: given that D_d detects 1AO1 photon, if the two inputs are the same, modes d and 3 will be the same (called Quantum Parity Check)

Case 1:



H-V Basis:

$$|\Psi\rangle_{3'b} = \left(a|H\rangle_{3'} + b|V\rangle_{3'}\right) \otimes H_b$$

F-S Basis:

$$\left[\frac{a}{\sqrt{2}}\left(|F\rangle_{3'} - |S\rangle_{3'}\right) + \frac{b}{\sqrt{2}}\left(|F\rangle_{3'} + |S\rangle_{3'}\right)\right] \otimes \frac{1}{\sqrt{2}}\left(|F\rangle_b - |S\rangle_b\right)$$

PBS

PBS

$$\frac{1}{2}\left[|H\rangle_d\left(a|H\rangle_3 + b|V\rangle_3\right) + |V\rangle_d\left(a|V\rangle_3 + b|H\rangle_3\right)\right] + \frac{1}{\sqrt{2}}\Psi_{II} \leftarrow \left[\frac{a}{2}\left(|F\rangle_d|F\rangle_3 + |S\rangle_d|S\rangle_3\right) + \frac{b}{2}\left(|F\rangle_d|F\rangle_3 - |S\rangle_d|S\rangle_3\right)\right] + \frac{1}{\sqrt{2}}\Psi_{II}$$

Case 2:

H-V Basis:

$$|\Psi\rangle_{3'b} = \left(a|H\rangle_{3'} + b|V\rangle_{3'}\right) \otimes V_b$$

F-S Basis:

$$\left[\frac{a}{\sqrt{2}}\left(|F\rangle_{3'} - |S\rangle_{3'}\right) + \frac{b}{\sqrt{2}}\left(|F\rangle_{3'} + |S\rangle_{3'}\right)\right] \otimes \frac{1}{\sqrt{2}}\left(|F\rangle_b + |S\rangle_b\right)$$

PBS

PBS

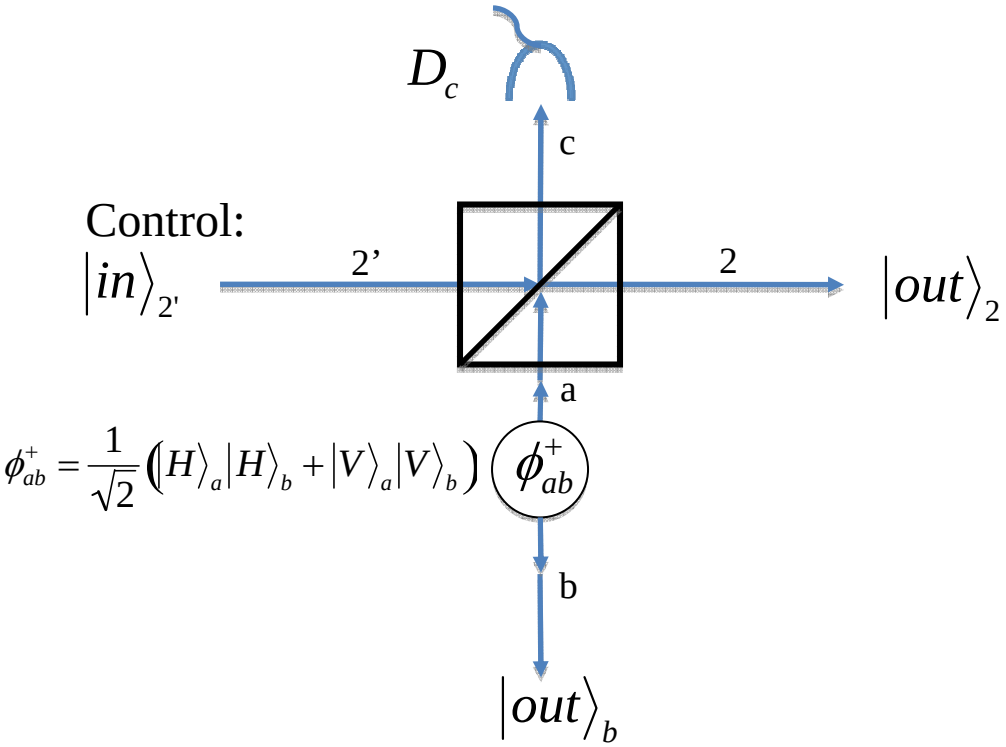
$$\frac{1}{2}\left[|H\rangle_d\left(a|V\rangle_3 + b|H\rangle_3\right) + |V\rangle_d\left(a|H\rangle_3 + b|V\rangle_3\right)\right] + \frac{1}{\sqrt{2}}\Psi_{II} \leftarrow \left[\frac{a}{2}\left(|F\rangle_d|F\rangle_3 - |S\rangle_d|S\rangle_3\right) + \frac{b}{2}\left(|F\rangle_d|F\rangle_3 + |S\rangle_d|S\rangle_3\right)\right] + \frac{1}{\sqrt{2}}\Psi_{II}$$

Up to a single qubit NOT gate (case D_d measures a V photon), the gate works with a probability of ½.

Quantum Encoder

Goal: encode the state of the control into an additional photon (no cloning going on here)

The following gate performs encoding (via quantum parity check) with probability 1/2:

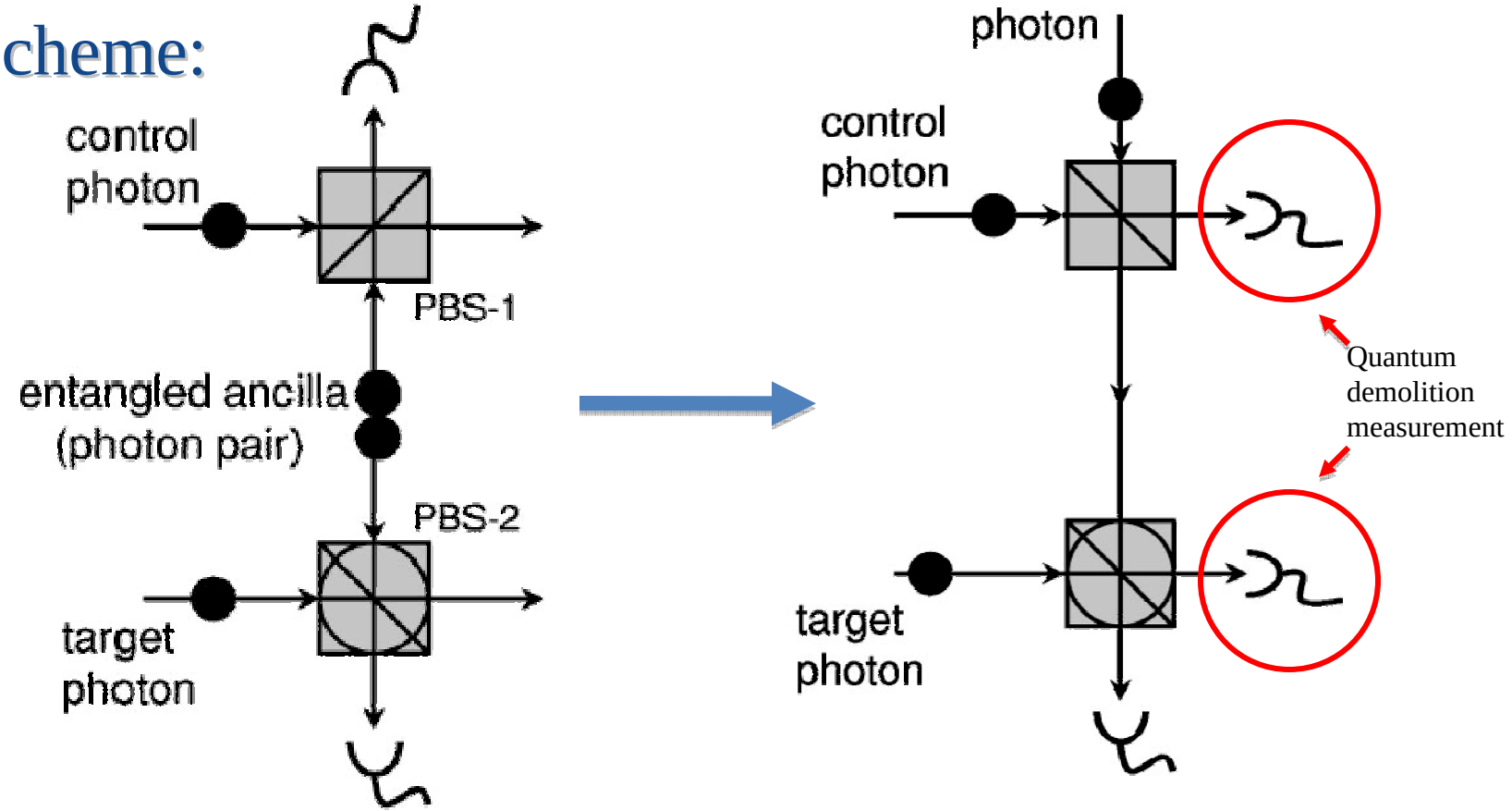


$|\Psi\rangle_{2'b} = a|H\rangle_{2'} + b|V\rangle_{2'} \rightarrow a|H\rangle_2|H\rangle_b + b|V\rangle_2|V\rangle_b$ (After 1AO1 photon post selection)

T.B. Pittman, M.J. Fitch, B.C. Jacobs, J.D. Franson, Phys. Rev. A **68**, 032316 (2003)

"Experimental controlled-NOT logic gate for single photons in the coincidence basis"

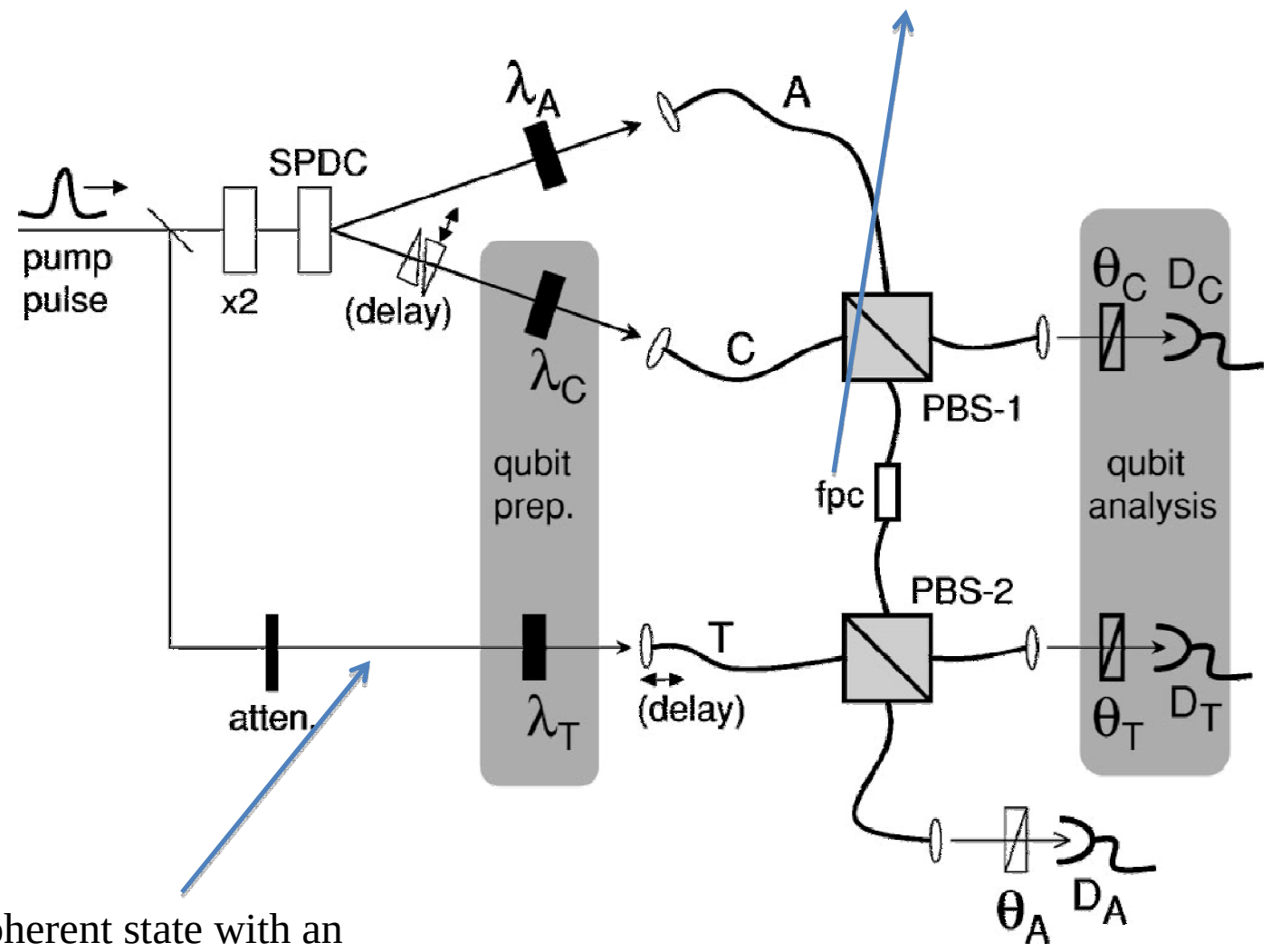
Scheme:



• It is very difficult to produce heralded (on demand) entangled pairs, but easy to produce heralded single photons (parametric down conversion)

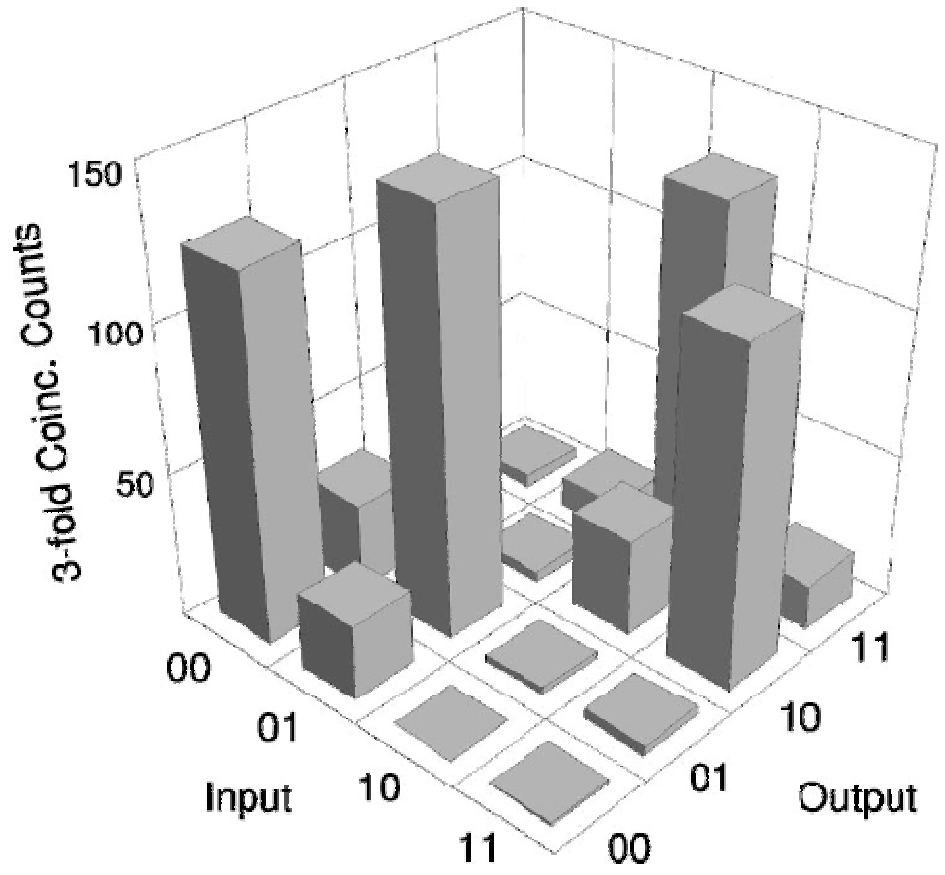
• The new scheme also contains a quantum encoder, but there is a drawback: In order for the encoder to work, 1AO1 photon must be detected in each output – demolition measurement.

Experimental setup:



Coherent state with an average of zero photons (single photon source)

Results



The counts were acquired during 1500s.

Bibliography

M.A. Nielsen, I.L Chuang, “Quantum computation and quantum information”, Cambridge university press, Cambridge (2000)

E. Knill, R. Laflamme, G.J. Milburn, “A scheme for efficient quantum computation with linear optics”, Nature **409**, 46 (2001)

J.L O’Brien, G.J. Pride, A.G. White, T.C. Ralph, D.Branning, “Demonstration of an all-optical quantum controlled-NOT gate”, Nature **426**, 264 (2003)

T.B. Pittman, M.J. Fitch, B.C. Jacobs, J.D. Franson, “Experimental controlled-NOT logic gate for single photons in the coincidence basis”, Phys. Rev. A **68**, 032316 (2003)

T.B. Pittman, B.C. Jacobs, J.D. Franson, “Probabilistic quantum logic using polarizing beam splitters”, Phys. Rev. A **64**, 062311 (2001)