Synchrony in heterogeneous networks of spiking neurons

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Abstract

The emergence of synchrony in the activity of large heterogeneous networks of spiking neurons is investigated. We define the robustness of synchrony by the critical disorder at which the asynchronous state becomes linearly unstable. We show that at low firing rates, synchrony is more robust in excitatory networks than in inhibitory networks but excitatory networks cannot display any synchrony when the average firing rate becomes too high. We introduce a new regime where all inputs, external and internal are strong, and have opposite effects that cancel each other when averaged. In this regime, the robustness of synchrony is strongly enhanced, and robust synchrony can be achieved at high firing rate in inhibitory networks. On the other hand, in excitatory networks, synchrony remains limited in frequency due to the intrinsic instability of strong recurrent excitation.

1 Introduction

Synchrony in large scale neural systems is evidenced by significant fluctuations in time of the ensemble activity. Complementary experimental techniques, namely EEG, MEG, recording local field potentials, and multi-unit and multi-electrode recordings have shown it to be an ubiquitous feature of neural activity in the central nervous system of vertebrates. It occurs, for instance, in the limbic, motor, olfactory, and visual systems. Different manifestations of synchrony have been shown to be associated with specific physiological states. Synchrony is also observed in pathological conditions (e.g. epileptic seizures, Parkinsonian disease) (Gray 1994, Ritz and Sejnowki 1997).

Recent studies have largely focused on spike to spike synchrony and strong oscillatory activity, but the concept of synchrony actually covers a very large span of phenomena, as is evidenced by a few examples. During epileptic seizures, hippocampal pyramidal cells may exhibit global low frequency synchronous bursting. In the same structure, high frequency episodes, at about $200\ Hz$, are observed during sharp waves events (Buzsáki and Chrobak 1995). Oscillatory synchrony, involving neural assemblies that evolve in time, was demonstrated in the olfactory system of insects (Laurent 1996), and shown to play a role in odor recognition. Indeed, disrupting synchrony in the antennal lobe through $GABA_A$ antagonists impedes the discrimination between similar odors (Stopfer et al. 1997). This remains the most convincing evidence on the function of synchrony in neural systems.

In the last ten years, following the pioneering works of (Gray and Singer 1989) and (Eckhorn et al 1988), many experiments have been devoted to the investigation of stimulus induced oscillatory activity in the visual system, from the retina to the primary visual cortex. It has been suggested that the γ rhythm might play an important function in the processing of visual stimuli by providing a flexible way to implement the concepts of feature binding and segmentation (for a review see (Gray 1994)), but the functional significance of synchrony in this framework remains debated (Gray 1994).

A fundamental issue is to understand when synchrony can emerge, and how cell properties, synaptic interactions and network architecture interact to determine the nature of the synchronous states in a large neural network. Many theoretical works in the last few years have focused on the emergence of synchrony through the *collective* dynamics of neural networks. Then, the correlations of pairs of neurons originate neither from the direct influence of one neuron on another, nor from a common external input to the system. Rather, they stem from the common synaptic input generated by the very activity of the network (Hansel and Sompolinsky 1996). In this mean field framework, the respective roles of synaptic interactions and cell properties in building synchrony was largely unraveled by studying networks of integrate-and-fire neurons (Abbott and van Vreeswijk 1993, ,Gerstner et al. 1996), and by analyzing the dynamics of conductance based models in the weak coupling limit (Hansel et al. 1993a, Hansel et al 1993). In this limit, the collective behavior of the system is determined only by the linear response of the neurons to small depolarizations, and, the nature of the interaction between neurons. It was thus shown, for a large class of homogeneous neural systems, that slow excitatory interactions could not lead to full synchrony while inhibitory interactions did (van Vreeswijk et al. 1994, Hansel et al. 1995). These systems are characterized by the fact that the neurons respond to a brief depolarization by advancing the firing of their next spike, with a phase excitability curve that peaks near the end of the interspike. This type of response to perturbations is exhibited by integrate-and-fire models, as well as many conductance based models, such as the model of hippocampal inhibitory interneuron introduced in (Wang and Buszàki 1996) (see below). It was experimentally demonstrated that regular spikers in slices of neocortex have such response properties (Reyes and Fetz 1993a, Reyes and Fetz 1993b).

On the basis of these results, it was suggested that mutual inhibition between neurons might play an important role in the emergence of synchrony in large neuronal systems (van Vreeswijk et al. 1994, Hansel et al. 1993b, Wang and Buszàki 1996). This theoretical idea is supported by several experimental works. In particular, Whittington et al. (1995) have shown that a collective oscillation around $40\ Hz$ emerges in hippocampal and neocortical slices due to fast inhibition between interneurons, when slow $GABA_B$ inhibition and ionotropic glutamate excitation are pharmacologically blocked. Therefore, the rhythm observed in pyramidal cells could result from their entrainment by the inhibitory network (Cobb et al. 1995).

Most theoretical works on synchrony have focused on homogeneous or mildly heterogeneous neural systems. For heterogeneous systems of weakly coupled phase oscillators (Kuramoto 1984), a transition to the asynchronous state is found at a value of the disorder proportional to the strength of the synaptic coupling. This suggests that increasing the coupling strength might help to synchronize heterogeneous systems. However, increasing the coupling also has adverse effects, such as silencing low frequency neurons in inhibitory networks (suppression), or strongly increasing the firing rates of neurons in excitatory networks. Therefore, understanding how synchrony can be achieved in neural populations, when the spread of the intrinsic properties of neurons is large (see Gustafsson and Pinter 1984a, 1984b) for a thorough study of the spread in the intrinsic properties of lumbar α motoneurons in anesthetized cats) remains an open issue. The dependence of synchrony on the synaptic time course in inhibitory systems was recently investigated by several authors (Golomb and Rinzel 1993, Hansel and Mato 1993, Wang and Busz`ki 1996, White et al. 1998, Chow 1998). When inhibition is fast with respect to the typical firing rate of neurons (i.e., the phasic regime of (White et al. 1998)), the network settles in cluster states, that are easily destabilized when heterogeneities are introduced. On the opposite, when inhibition is slow *homogeneous* systems display stable in-phase synchrony (White et al. 1998). Yet it was shown that when mild heterogeneities in the cell properties throughout the network are taken into account, a transition to asynchrony occurrs in this tonic regime for disorder levels of the order of 5% (White et al 1998). This is easily understood since the modulation of the synaptic input is small in this regime. Thus, it seems a priori difficult to explain synchrony phenomena at high frequency on the basis of mutual inhibition. Accordingly, recent works on hippocampal high frequency ripples have investigated the possible role of gap junctions in the emergence of synchrony (Draguhn et al. 1998).

The goal of the present paper is to investigate the robustness of synchrony, i.e. the maximal spread of cell properties that is compatible with synchrony, in the framework of fully coupled one-population networks. Namely, we define the robustness as the value of the disorder at which the asynchronous state becomes linearly stable. For lesser values of the disorder, the system will then evolve at large times to some synchronous state. This is a conservative definition of the robustness, as synchrony may still be observed at higher disorder if the stable asynchronous state coexists with some synchronous state (multistability). We especially address the question of which interactions, excitatory or inhibitory, are best able to synchronize neurons at a given firing rate. The paper is organized as follows: after reviewing the models of neurons and the methods we use to investigate network dynamics, we study the impact of heterogeneities on synchrony. The question is first addressed in the framework of phase reduction techniques, where analytical results on the onset of synchrony can be derived. These results are extended to finite coupling by performing numerical simulations. We then study a strong coupling regime, which has never been investigated by previous authors, where balancing the synaptic current experienced by the neurons by an external drive of opposite sign prevents the firing rates from drifting. Finally, we discuss our results and their implications in the last section.

2 Methods

2.1 Models

We analyze the collective behavior of large networks consisting of only one population of neurons, either excitatory or inhibitory. The neurons are coupled all-to-all with the same coupling strength throughout the network. All the neurons are the same, but we assume that the external input varies from neuron to neuron in such a way that the distribution of the firing rates is either Gaussian or Lorentzian when the neurons are uncoupled.

Firing frequencies of neurons must be positive. Therefore, we use firing rate distributions restricted between 0 and $2\bar{f}$, obtained by truncating the original Gaussian or Lorentzian distributions symmetrically with respect to the average frequency, $\bar{f}$, and modifying accordingly the normalization constant (see Fig. (1)). The standard deviation of the truncated distribution is different from the standard deviation of the original distribution. However, for a Gaussian distribution, the relative difference between these two quantities is less than 1% as long as the standard deviation of the original distribution is less than $0.3\bar{f}$.

Individual neurons are described by a single compartment model, either a simple Lapicque integrate-and-fire model (hereafter IF model) or a Hodgkin-Huxley-like model, namely the Wang-Buszáki model (hereafter WB model), previously introduced in (Wang and Buszàki 1996). This latter model was designed as a minimal model of hippocampal inhibitory interneuron, and presents a wide firing frequency range from 0 to several hundreds Hz. The membrane potential of the IF model evolves in the sub-threshold regime according to the differential equation:

$$C\frac{dV}{dt} = I_\ell + I_{syn}(t) + I \tag{2.1}$$

where $I_\ell = g_\ell(V_\ell - V)$ is a passive leak current of conductance g_ℓ and reversal potential V_ℓ, and

I_{syn} is the synaptic current due to the action of the other neurons of the network. The current I is a constant external drive. The membrane capacitance C is set to $1\ \mu F/cm^2$. Requiring a passive membrane time constant $\tau = C/g_\ell$ of $10\ ms$ leads to a leak conductance $g_\ell = 0.1\ mS/cm^2$. The resting membrane potential $V_{rest} = V_\ell$ is equal to $-60\ mV$ in all our simulations. Whenever the membrane potential, V, reaches the threshold θ ($-40\ mV$ in our simulations), a spike is fired and V is instantaneously reset to the resting level. In the WB model the membrane potential evolves according to

$$C\frac{dV}{dt} = I_{Na}(t) + I_K(t) + I_\ell(t) + I_{syn}(t) + I \tag{2.2}$$

where I_{Na} and I_K are voltage-dependent sodium and potassium currents. Their kinetics as well as the passive parameters of the model are the same as in (Wang and Buszàki 1996) and recalled in Appendix A. In all cases (IF or WB model) the dynamics of the networks are integrated using an adaptive second order Runge-Kutta algorithm as explained in (Hansel et al. 1998).

The synaptic current induced by a given presynaptic neuron into a postsynaptic neuron is modeled as,

$$I_{syn}(t) = g \sum_{spikes} f(t - t_{spike}) \tag{2.3}$$

where summation is performed over all the spikes emitted prior to time t by the presynaptic neuron. Each synaptic activation then induces a single event, described by the function

$$f(t) = \frac{1}{\tau_1 - \tau_2}\left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}}\right)\Theta(t) \tag{2.4}$$

where $\Theta(t)$ is the Heaviside function. The larger of the time constants, τ_1 and τ_2, characterizes the rate of exponential decay of the synaptic potentials, while their time to peak is equal to $t_{peak} = (\tau_1\tau_2/(\tau_1 - \tau_2))\ln(\tau_1/\tau_2)$. The normalization adopted here ensures that the time integral of $f(t)$ is always unity. The coupling strength g is then proportional to the maximal synaptic conductance but not equal to it; it is actually the total charge transferred during a single synaptic event. In a fully connected network of size N, g must scale as $1/N$ so that the total synaptic input remains finite in the limit $N \to +\infty$. Inhibition corresponds to a negative coupling, $g < 0$, and excitation to a positive coupling, $g > 0$. Our description of the synaptic coupling does not take into account the shunting effect of the synapses. However, we have checked that the conclusions of our study still hold when synapses are described in term of conductances.

2.2 Synchrony

We measure the degree of synchrony as it was proposed and developed in (Hansel and Sompolinsky 1992, Golomb and Rinzel 1994, Ginzburg and Sompolinsky 1994), namely by analyzing the temporal fluctuations of the network activity. One evaluates the average membrane potential in the network at a given time t,

$$A_N(t) = \frac{1}{N}\sum_{i=1}^{N} V_i(t) \tag{2.5}$$

Its time fluctuations can be characterized by the variance

$$\Delta_N = \left\langle A_N(t)^2 \right\rangle_t - \langle A_N(t) \rangle_t^2 \tag{2.6}$$

When this variance is normalized to the population averaged variance of single cell activity

$$\Delta = \frac{1}{N}\sum_{i=1}^{N}(\left\langle V_i(t)^2 \right\rangle_t - \langle V_i(t) \rangle_t^2) \tag{2.7}$$

the resulting parameter

$$\Sigma_N = \frac{\Delta_N}{\Delta} \tag{2.8}$$

generally behaves for large N as

$$\Sigma_N = \chi + \frac{a}{N} + \mathcal{O}(\frac{1}{N^2}) \tag{2.9}$$

The constant χ, comprised between 0 and 1, measures the degree of coherence in the system in the infinite size limit, while a yields an estimate of finite size effects. The coherence parameter χ is equal to 1 if the system is totally synchronous (i.e. $V_i(t) = V(t)$ for all i), and vanishes if the state of the system is asynchronous in the large N limit. Note that this definition of coherence encompasses all possible forms of synchrony in the network. In practice, χ was estimated by calculating Σ_N for large networks (N ranging from 128 to 4096) and checking that finite size effects were small (in some cases we have checked for finite size effects for network size up to $N = 65536$).

2.3 Phase models

The dynamics of a conductance-based neuron involves several variables: membrane potential, gating variables, etc. However, if all the neurons in the network fire periodically when they are uncoupled, with their firing rates, f_i, lying in a narrow range, and if they are weakly coupled together, the dynamics of each neuron in the network can be described by a single angle variable, ϕ_i, varying on $[0, 2\pi]$. This variable is the phase of the neuron along its unperturbed limit cycle (i.e., when decoupled from the network). Thanks to this reduction (Hansel et al. 1995), the network dynamics is then described by the simpler differential system,

$$\frac{d\phi_i}{dt} = 2\pi f_i + \frac{g}{N}\sum_{j=1}^{N}\Gamma(\phi_i - \phi_j) \tag{2.10}$$

where g is the coupling strength and Γ is the effective interaction between any two neurons. This interaction function is obtained by convolving the synaptic current generated by the presynaptic neuron, j, with the "response function" Z of the target neuron, i, over one period of the single neuron dynamics,

$$\Gamma(\phi_i - \phi_j) = \frac{1}{2\pi}\int_0^{2\pi} Z(u) I_{syn}(u + \phi_j - \phi_i)du \tag{2.11}$$

The function Z is nothing more than the linear phase response of the neuron to very brief current pulses. It indicates what proportionality exists between the amplitude of a small current pulse and the phase advance (or delay) it induces in the neuron. In the special case of IF neurons, the response function can be analytically derived. It reads,

$$Z(\phi) = \frac{1}{I} e^{\frac{\phi}{2\pi f \tau}} \tag{2.12}$$

For conductance based models Z and its Fourier expansion have to be determined numerically from the single neuron dynamics (Hansel et al 1993a, Ermentrout 1996).

Large networks may display a transition from the asynchronous state to some synchronous state when varying control parameters. In particular, such a transition may occur as the level of disorder is decreased or the coupling strength is changed. This can be analytically investigated in networks of phase oscillators, as shown in Appendix B. Let, $h(\omega)$ be the distribution of the natural angular frequencies of the phase oscillators, $\omega = 2\pi f$. Let the Fourier expansion of $\Gamma(\phi)$ be :

$$\Gamma(\phi) = \sum_{n=0}^{+\infty} a_n \sin(n\phi) + b_n \cos(n\phi) \tag{2.13}$$

For a fixed value of the disorder, the asynchronous state becomes linearly unstable, when the coupling strength, g, is changed, as soon as one of the Fourier modes, n, of the interaction function, Γ

satisfies the two conditions

$$h(-gb_0 - \Omega_n) = -\frac{2}{\pi g} \frac{a_n}{{a_n}^2 + {b_n}^2} \tag{2.14}$$

$$\lim_{\epsilon \to 0} \int_{\epsilon}^{\infty} \frac{h(\omega - gb_0 - \Omega_n) - h(-\omega - gb_0 - \Omega_n)}{\omega} d\omega = \frac{2}{g} \frac{b_n}{{a_n}^2 + {b_n}^2} \tag{2.15}$$

These equations determine the critical value of the disorder, σ_c, at which the mode n is marginally stable, and the corresponding purely imaginary eigenvalue $in\Omega_n$. The mode n is then linearly unstable for all level of disorder larger than the critical value ($\sigma > \sigma_c$) (see Appendix B). Alternatively equations (2.14) and (2.15) give the critical value, g_c, of the disorder at which the mode n becomes unstable, when the level of disorder σ is kept fixed. Note that condition (2.14) can be fulfilled only if a_n and g have opposite signs. This means that the mode n is always linearly stable in an inhibitory network if $a_n < 0$, and cannot be involved in the destabilization of the asynchronous state. Conversely the mode n is linearly stable in an excitatory network if $a_n > 0$. Note that the functions, Z and I_{syn} depend on the intrinsic firing rates of presynaptic and postsynaptic neurons. In Eq(2.14, 2.15) the fourier coefficients are computed for the effective interaction of neurons with intrinsic angular frequency $\bar{\omega} = 2\pi\bar{f}$. Taking into account the spread of freqencies would only result in higher order corrections.

2.4 Robustness of synchrony

For phase models, σ_c is strictly proportional to g. Beyond the weak coupling limit the critical disorder also depends on the coupling strength, albeit in a more complex way. In all cases we define, for any given value of the coupling strength, g, the *robustness* of synchrony, $R(g)$, as the critical disorder at which the *asynchronous* state becomes linearly stable. This critical disorder is always expressed in what follows as a fraction of the average angular frequency, $\bar{f}$, in the uncoupled network. Meaningful comparisons of the robustness of the synchronous state for two neuron models require that numerical simulations be performed in equivalent conditions. In such a situation we do not choose the same value of the coupling strength for both models, as the effect of a given synaptic current will largely depend on the membrane properties of the post-synaptic neuron. We set the coupling strength at different values for the two models, chosen so that a single synaptic event leads in both cases to a post-synaptic potential of $0.5\ mV$ in a neuron at rest. This guarantees that the effects of synaptic interactions between neurons are similar in the two models.

3 Results

3.1 Weak coupling

Figures 2 A and B, respectively, display the robustness of synchrony in an excitatory network (R_E) of IF neurons, and in an inhibitory network (R_I) of IF neurons, as a function of the average firing frequency, $\bar{f}$. These results were obtained in the weak coupling limit and for a Gaussian distribution of firing rates. Both R_E and R_I go to infinity as the firing rate goes to zero, which means that synchrony can always be achieved at very low rates in a network of IF neurons, even if heterogeneities are strong. This result is also valid for a Lorentzian distribution of frequencies. In this simpler case, this can be analytically derived as shown in Appendix C.

In the excitatory case, the robustness of synchrony, R_E, monotonically decreases with increasing frequency (see Figure 2 A), and eventually vanishes at a *finite* frequency. This means that no synchrony is possible at high firing rate, even in an homogeneous network. On the contrary, in the inhibitory case R_I is a non-monotonic function of the firing frequency and never vanishes as can be seen on Figure 2 B. This entails that a mildly heterogeneous inhibitory network always displays some degree of synchrony. The discontinuities in the derivative visible on all the curves correspond to the frequencies at which changes in the order of the critical mode, n_c, occur. As can be seen on Figures 3 A and B, the order of the critical mode monotonically decays with increasing firing rate, both in the excitatory case and in the inhibitory case, and eventually becomes 1. By contrast, it diverges at low frequency as shown above. This means that, at low frequency, destabilization of the asynchronous state will lead to cluster states where the system splits into several groups of in-phase locked neurons (Golomb et al. 1992).

To compare the efficiency of inhibitory and excitatory couplings at synchronizing neuronal activity in IF networks for given values of the synaptic time constants, we have computed the ratio R_E/R_I. In Figure 4, this ratio is plotted as a function of the average firing rate, $\bar{f}$. At high frequency, R_E/R_I vanishes because the asynchronous state is stable for excitatory interactions, but not for inhibitory ones. When the frequency is decreased, R_E/R_I rapidly becomes larger than 1. This means that at low firing rate, excitation is much more efficient than inhibition at synchronizing neural activity in heterogeneous IF networks. Note that R_E/R_I remains finite in the limit of vanishing firing rates, in spite of the low-frequency divergence of both R_E and R_I.

We now analyze how synchrony depends on synaptic time course. For the sake of simplicity, we assume that $\tau_1 = \tau_2 = \tau_{syn}$. The synaptic interaction is then described by an "α function",

$$f(t) = \frac{t}{\tau_{syn}^2} e^{-t/\tau_{syn}} \Theta(t) \quad (3.16)$$

Comparing Figure 4 and Figure 5, we see that the ratio R_E/R_I diverges when $\bar{f}\tau_{syn}$ goes to 0 at fixed $\bar{f}$, while it tends to a constant value when $\bar{f}\tau_{syn}$ goes to 0 at fixed τ_{syn}. Thus, $\bar{f}\tau_{syn}$ is not a scaling variable for R_E/R_I. This is because R_E/R_I depends not only on the temporal modulation of the synaptic current (controlled by $\bar{f}\tau_{syn}$), but also on the phase response of neurons, which explicitly depends on $\bar{f}$. This can be analytically shown for a Lorentzian distribution (see Appendix C).

Are the above results modified when active properties of neurons are taken into account? To investigate this issue we have studied networks of WB neurons. We have numerically computed the response function for various values of the external drive, I, leading to different firing rates of the neurons. Using the Fourier expansion of the response function, we have analytically computed the robustness of synchrony. The results are shown for excitatory and inhibitory couplings respectively in Figures 6 A and B. Qualitatively these results are similar to those obtained for the IF model, except that both R_E and R_I remain finite in the limit of vanishing firing rates for the WB model. In particular, R_I never vanishes, R_E is zero above some critical frequency, f_c, and R_E is larger than R_I at low frequency (compare Figures 6A and B).

3.2 Beyond weak coupling

The results of the previous section were derived under the assumption of weak coupling. To assess what is their range of validity we numerically compute in the present section R_E and R_I as a function of the coupling strength for both IF or WB neurons, and compare those results to the predictions obtained using the phase reduction technique. Figure 7 displays the coherence, χ, versus the spread, σ, of firing rates for excitatory and inhibitory networks of IF neurons. The average firing rate was kept fixed at $80\ Hz$. For that value, partial synchrony ($\chi \approx 0.36$) is achieved in the homogeneous ($\sigma = 0$) excitatory case, and the fully synchronous state ($\chi = 1$) is stable in the homogeneous inhibitory case. Remarkably, χ decays to 0 much faster for inhibition than for excitation when disorder is increased, although it starts from much higher values. Networks of WB neurons display a qualitatively similar behavior, although the range over which χ decays to zero is larger by one order of magnitude.

Figures 8A and B, respectively, display R_E and R_I as a function of the coupling strength for IF networks and a fixed value of the average firing rate in the uncoupled network. Note on Figures 8B the multistability present in the inhibitory case, as the transition to the asynchronous state occurs at different values of the disorder, when starting from an initial condition, that is close to the fully synchronous state, and from an initial condition, where the membrane voltages of neurons are widely spread. For small coupling strength, g, the robustness linearly varies with g with a slope which is correctly predicted by the phase model limit. For larger values of g R_E and R_I monotonically decreases with increasing g. In excitatory networks this stems from the increase of the firing rate with the coupling, while in inhibitory networks this is related to clustering (see Appendix D). However, in the inhibitory case, the robustness of synchrony, as defined in this paper, may eventually increase again when the coupling strength is still further increased. This is because low frequency neurons are progressively suppressed when the coupling strength is increased, which decreases the frequency spread of the spiking population and increases the maximum level of disorder required to stabilize the asynchronous state. WB networks behave in a similar way, although synchrony can be achieved in heterogeneous excitatory networks only in a much more restricted range of firing rates (see Figures 9A and B).

The results of this section show that for both models and both types of interactions, excitatory and inhibitory, when the external input is kept fixed and the strength of the interaction is varied, the conclusions derived using the phase reduction method remain qualitatively valid on a finite range of coupling strength. In this range, excitation is more efficient than inhibition at synchronizing heterogeneous networks at low firing rates. The opposite is true at large firing rates where only inhibition may synchronize heterogeneous networks.

3.3 Synchrony at strong coupling

In the weak coupling limit, the interactions between neurons have a negligible effect on the average firing rate. This is not so at strong couplings. This is particularly obvious in excitatory networks where recurrent excitation strongly increases the average firing rate with respect to its value in the uncoupled network. Consequently, the synaptic interaction becomes closer and closer to a constant as the oscillation period becomes much smaller than the synaptic time constants, and it can no longer destabilize the asynchronous state. This is one reason why no synchrony can be achieved at large coupling strengths in an excitatory network. In inhibitory networks, increasing the coupling strength decreases the firing rate and would seem to favor synchrony. However, this also leads to suppression of low frequency neurons. Moreover, the states of partial synchrony exhibited by inhibitory networks depend on both the firing rate and the coupling strength, and synchrony may also be reduced due to a higher degree of clustering when the coupling strength becomes larger.

To analyze the *sole* effect of the coupling strength, g, on the robustness of synchrony in excitatory and inhibitory networks, *independently* of the change of firing rate it provokes, we shall simultaneously change g, and the external drive, I, acting on neurons so that the average frequency in the network remains constant, whatever the value of g. This can be achieved by making the drive I_i on neuron i depend on g as

$$I_i(f, g) = \hat{I}(f) - g\bar{f} \tag{3.17}$$

where $\hat{I}(f)$ is the constant drive required for neuron i to fire at frequency f, and $\bar{f}$ is the average frequency of neurons across the uncoupled network. In the asynchronous state the total synaptic input on a neuron due to the rest of the network is then exactly balanced by the coupling dependent term of the applied current, so that the frequency remains that of the single neuron.

The results then obtained for excitatory and inhibitory WB networks under the above constraint (3.17) are shown in Figure 10 for a constant low firing rate, $\bar{f} = 3\ Hz$. Increasing the synaptic coupling allows us to substantially improve the robustness for both excitation and inhibition within certain limits, which will be elaborated on in the Discussion. Also note that a larger robustness can be achieved in the excitatory case than in the inhibitory case. Similar, though quantitatively different, results are obtained for IF networks (results not shown).

For large enough values of $\bar{f}$ the situation changes. Indeed, for $\bar{f}$ larger than a critical value that depends on the synaptic time constants, the asynchronous state is the only stable active state of the excitatory WB network, even in the homogeneous case (it coexists with the stable quiescent state of the network). In other words, $R_E = 0$. By contrast, partial synchrony is achieved in an inhibitory network, even at high rate : R_I monotonically increases with g up to values of more than 20% when $\bar{f} = 50\ Hz$ (results not shown).

Thus, the conclusions previously established for weak couplings generalize at all coupling strength, when the average frequency is kept fixed by appropriately tuning the external drive. Excitation synchronizes neurons more efficiently at low firing rates, but above some threshold frequency only inhibition can synchronize neural activity.

3.4 The balanced synchronous state

The previous section might give the feeling that a precise tuning of the external drive, so that it exactly compensates for the shift of the average frequency in the asynchronous state, that is induced by the network dynamics, is required to achieve synchrony at large coupling. In the present section, we show that when both external drive input and the coupling strength are strong enough, and, under some restrictions that are elaborated upon in Appendix D, this balancing automatically takes place in the asynchronous state, without any need to finely tune the parameters.

For simplicity, we define all currents relative to the current threshold, I_c, so that any current above (respectively below) this threshold is positive (respectively negative). We shall assume that both the coupling strength, g, and the average external drive, I, linearly depend on a scaling parameter, K, and we shall investigate the large K regime where both coupling and average drive are strong. More specifically, we shall assume that,

$$g = -KJ \tag{3.18}$$

where J is some positive constant, and that the drive I_i on neuron i satisfies,

$$I_i = KI_0 + \delta_i \tag{3.19}$$

where I_0 is some constant current, $I_0 > 0$, and the δ_i are equally distributed independent random variables with zero mean. For an excitatory network, K is negative, so that the external drive is below the current threshold and opposes the frequency increase due to recurrent excitation. In inhibitory networks K is positive. The external drive is then positive and opposes the frequency decrease due to inhibitory synaptic interactions. In both cases, when the network is in the asynchronous state with an average firing rate $\bar{f}$, the total current input on neuron i is equal to,

$$I_i + g\bar{f} = K(I_0 - J\bar{f}) + \delta_i \tag{3.20}$$

Both the firing rate, f_i, of neuron i and the average firing rate, $\bar{f}$, are then determined by the self-consistent equations,

$$\begin{aligned} f_i &= G\left[K(I_0 - J\bar{f}) + \delta_i\right] && (3.21)\\ \bar{f} &= \int P(\delta) G\left[K(I_0 - J\bar{f}) + \delta\right] d\delta && (3.22) \end{aligned}$$

where $f = G[I]$ is the frequency-current relation of the neurons and P is the probability density of the random variable δ. In the large K limit, the average firing rate of the neurons, $\bar{f}$, remains finite and is equal to

$$\bar{f} = \frac{I_0}{J} + \mathcal{O}(1/K) \tag{3.23}$$

The correction of order $1/K$ to $\bar{f}$, $-A/K$, is determined by the non-linear equation,

$$\frac{I_0}{J} = \int P(\delta) G[JA + \delta]\, d\delta \tag{3.24}$$

This correction to the average frequency is positive for excitatory networks ($K < 0$), and negative for inhibitory networks ($K > 0$). Substituting the order $1/K$ approximation of $\bar{f}$ thus obtained in Eq. (3.21), gives the firing rates, f_i, of the neurons up to order $1/K$,

$$f_i = G[JA + \delta_i] \tag{3.25}$$

Note that the firing rates depend on the single neuron dynamics only through the frequency-current relation. Also note that in the asynchronous state the width of the frequency distribution only depends on the probability density, P, and on the frequency-current relation, G. It is *independent* of the average drive on neurons.

Equation (3.23) implies that, although both the external drive on neurons and the synaptic couplings are large, the net current input on neurons and their firing rates remain finite in the limit of large K. Moreover, in this limit, the average firing rate no longer depends on K. Therefore, if the external drive and the coupling strength are sufficiently strong and have opposite effects on the firing rate, the system settles into a "balanced state" where the average firing rate only depends on the ratio of these two quantities (up to small corrections). The equations satisfied by the average drive and the average firing rate in this state are the same as in the "chaotic balanced state" studied in (van Vreeswijk and Sompolinsky 1996b, van Vreeswijk and Sompolinsky 1998). However, at variance with the chaotic balanced state, the neurons fire *periodically* in the asynchronous balanced state we consider here. This is because the network is fully connected in the present study (see also (Golomb and Hansel 1998).

The robustness, R_I, of an inhibitory WB network in the balanced state is shown on Figures 11 A and B as a function of the scaling parameter K for two values of the average frequency in the large K limit, $f = 50\ Hz$ (A) and $f = 200\ Hz$ (B). In both cases, a much larger robustness than at weak coupling can be achieved by sufficiently increasing K. This is because, in spite of the averaging effect of the high firing rate, the synaptic current remains strongly modulated as the coupling strength is large. One can even show analytically that, as the firing rate increases, the index of the mode that destabilizes the asynchronous state decreases and synchrony becomes more robust. The behavior of excitatory networks is different, as the firing rate can no longer be controlled when the coupling gets too large. This can be analytically proven for an homogeneous IF network (see Appendix D). As a consequence, there is a limiting value of the firing rate beyond which a purely excitatory network cannot exhibit synchrony. Therefore we are led in the strong coupling regime to the same conclusions as before: excitatory couplings give rise to more robust synchrony at low firing rates while only inhibition can synchronize networks at high rates.

4 Discussion

The present study has addressed the issue of synchrony in heterogeneous neural networks, either made up of only excitatory spiking neurons or made up of only inhibitory spikers. Previous works have stressed the role of inhibitory interactions, showing that mutually inhibiting neurons can synchronize (Wang and Buszàki 1996) even in the presence of mild heterogeneities (White et al. 1998). We have shown here, in contrast with the results of White et al. (White et al. 1998), that synchrony is also possible in heterogeneous inhibitory networks, when the spread in the firing rates of neurons is a substantial fraction of the average firing rate, a situation that is probably frequent in biological preparations. Synchrony is even possible at high firing rates provided that the average synaptic current is balanced by a constant external input of the same order of magnitude. In this way, the coupling strength can be strongly increased, and modulation of the synaptic current maintained, while suppression is avoided. This situation, which can be achieved without finely tuning the parameters, is different from the asymptotic regimes defined in (Chow et al. 1998), except at very low frequency where it corresponds to the *fast* regime. It is neither the *phasic* regime because the membrane time constant and the inhibition decay time are of the same order, nor the *tonic* regime because the oscillation period is not much higher than the inhibition decay time. By contrast balancing cannot control the firing rate in excitatory networks when the average firing rate gets too high. As a consequence, synchrony at high firing rate cannot be achieved in excitatory networks. Still, excitation is more efficient than inhibition at synchronizing neurons at sufficiently low firing rates. Note that this robustness of synchrony in excitatory networks relatively to the inhibitory situation, does not entail that excitatory networks can display synchrony at low firing rates in spite of a large spread of parameters. Indeed, near the bifurcation point, the firing rates of the neurons is strongly sensitive to intrinsic neuronal properties (e.g., the leak conductance) and to the value of the external drive.

We have reached the above conclusions by analyzing under which conditions the asynchronous state is unstable in large networks, and the system therefore settles in some synchronous state. This is in contrast with most previous theoretical works (see however (Abbott and van Vreeswijk 1993, van Vreeswijk 1996), that mainly focused on synchronous states in homogeneous networks, in particular on the fully synchronous state, and on what these states became when heterogeneities were introduced. More specifically, we have studied the destabilization of the asynchronous state to the benefit of some coherent state when network parameters are varied. To address this question, we have first used the weak coupling limit in which the stability of the asynchronous state and its dependence on synaptic parameters and on the average firing frequency can be analytically investigated. Thus, we could assess the robustness of synchrony, defined as the relative spread in the natural frequencies of neurons beyond which the incoherent state becomes stable, for excitatory and inhibitory networks. This procedure was performed for both the IF model, in which the inputs are passively integrated below the firing threshold, and the WB model, which incorporates voltage dependent sodium and potassium currents. We reached the same qualitative conclusions for both models. At sufficiently low frequency, synchrony can be achieved in both excitatory and inhibitory networks but is more robust in the excitatory case. At high frequency, only inhibition can lead to synchrony, providing that the degree of heterogeneity is sufficiently small. The critical frequency at which excitatory couplings can no longer synchronize neural activity depends on the synaptic time constants and on the neuron model.

Quantitatively, the IF model behaves nonetheless differently from the WB model in several important aspects. First, for excitatory couplings and given synaptic time constants, the critical frequency is larger, by one order of magnitude, in IF networks. Second, the robustness of synchrony is one order of magnitude smaller in inhibitory IF networks than in inhibitory WB networks. Third, the robustness of synchrony diverges as the average firing rate goes to zero in IF networks while it decreases in WB networks. Many conductance based models start to fire at vanishingly low frequency at the current threshold, like the WB model. All such models that we have studied so far behave qualitatively like the WB model, but the value of the critical frequency may strongly change with the model. The atypical behavior of the IF model stresses the importance of active properties for the emergence of synchrony in networks. A similar conclusion was reached by Golomb and Hansel in a study of synchrony in sparsely connected neuronal networks (Golomb and Hansel 1998).

The small robustness of synchrony at low firing rates in inhibitory WB networks is due to the fact that the order of the Fourier mode responsible for the linear instability of the asynchronous state increases with decreasing firing rate. Robustness then decreases with the amplitude of the corresponding component in the Fourier spectrum of the interaction function. In addition, for the WB model (and more generally for neurons with similar frequency-current relations), all non constant Fourier components of the response function vanish with the firing rate (Ermentrout 1996). On the other hand, the modulation of the synaptic current becomes too small at high frequency to efficiently entrain neurons and robustness becomes low. Between these two regimes, an optimal value of the average firing rate can be found where robustness is maximal. Similarly, for a given firing frequency, synaptic time constants can be found that maximize the robustness of synchrony in inhibitory networks. Note, however, that robustness is not determined by the sole ratios $\bar{f}\tau_1$ and $\bar{f}\tau_2$ (see Figures 4 and 5.)

Using numerical simulations of large networks we have shown that the picture that emerges from the weak coupling limit remains valid in a finite range of coupling strength. Moreover, for a given external drive the robustness linearly increases with the coupling strength in this range. However, when the coupling strength becomes too large, robustness again decreases in both excitatory and inhibitory networks, albeit for different reasons. In excitatory networks, the firing rate dramatically increases with the coupling due to recurrent excitation between neurons and the synaptic current is no longer modulated in time. By contrast, the firing rate decreases with increasing coupling strength in inhibitory networks, which favors cluster states. The higher the order of these cluster states, the more easily they are destabilized and replaced by the asynchronous state when heterogeneities are introduced. These desynchronizing effects can be prevented by introducing an additional current proportional to the coupling and of opposite sign. When both the synaptic interactions and the external drive are strong the synaptic current on neurons automatically adjust: so that their firing rate remains nearly constant when the coupling strength is varied. This leads to robust synchrony in inhibitory networks, even at high firing rate. By contrast, it is unable to control the firing frequency in excitatory networks beyond some limiting value of the coupling strength, due to an instability arising from recurrent excitation (see Appendix C).

The behavior of the networks in the regime where both synaptic currents and external drive are strong still depends on the coupling strength. For instance, the frequency at which the maximum robustness is achieved in the inhibitory case shifts to higher frequencies when the scaling factor, K, increases. For example, for synaptic times $\tau_1 = 3\ ms$, $\tau_1 = 1\ ms$, the maximum robustness for the WB model is observed around $40\ Hz$ for $K = 16$, and around $60\ Hz$ for $K = 32$ (for a reference value of the coupling constant, J, always set to unity). Moreover, the transients become shorter with increasing K. For a WB network, firing at $50\ Hz$, around one hundred cycles are required to reach the asymptotic state when $K = 1$, whereas only four or five are required when $K = 32$.

In conclusion, we have brought to light a strong coupling regime where the robustness of synchrony is strongly enhanced. In particular we have shown that significant synchrony could then be achieved in heterogeneous inhibitory networks, even at high firing rate, thanks to the balancing mechanism that occurs when the external drive also is strong and scales as the coupling strength. However, the physiological relevance of this regime remains to be established. The values of the coupling strength we use in our numerical studies of fully connected networks correspond to relevant values of the coupling strength in a network with more realistic connectivity. Consider, for instance, a network of WB neurons, where each neuron is contacted by $K = 100 - 1000$ other neurons firing incoherently with an average frequency of $f\ Hz$. Assuming that each synapse evokes post-synaptic potentials of the order of $0.2\ mV$. This corresponds, in the WB model, to synaptic strength between two neurons on the order of $g_s = 0.4 nC/cm^2$. In a state in which the neurons fire incoherently with an average frequency $f\ Hz$ the total synaptic input received by any neuron is the order of $K \times g_s \times f$. To obtain a total synaptic input in the fully connected WB networks we have investigated, we must set the coupling parameter g such that: $g \sim K g_s \sim 40 - 400\ nC/cm^2$. This range of coupling corresponds to the strong coupling regime we have studied in this paper. Note, however, that if K is too small compared to the total number of neurons in the in the system, the *stability* of the asynchronous state can be significantly affected by the sparseness of the connectivity pattern (see (Golomb and Hansel 1998) for a discussion of this issue). In that case, asynchronous state can be stable even in the strong coupling regime we have studied (Golomb and Hansel 1998). Finally, we also stress that our work did not cover more realistic situations where two populations, one excitatory and the other inhibitory, interact. Our preliminary results (not shown here) on heterogeneous two populations systems show that the loop between excitatory and inhibitory neurons can lead to an astonishingly high robustness of synchrony.

A The Wang-Buzsáki model

Parameters of the Wang-Buzsáki model are as follows. The capacitance C is equal to $1\ \mu F/cm^2$. The leak current is given by $I_\ell = g_\ell(V_\ell - V)$ where $g_\ell = 0.1\ mS/cm^2$ and $V_\ell = -65\ mV$. The transient sodium current is given by $I_{Na} = g_{Na}{m_\infty}^3 h(V_{Na} - V)$ where $g_{Na} = 35\ mS/cm^2$ and $V_{Na} = 55\ mV$. The activation variable m instantaneously adjusts to its steady-state value $m_\infty = \alpha_m/(\alpha_m + \beta_m)$ where $\alpha_m(V) = -0.1(V + 35)/(\exp(-0.1(V + 35)) - 1)$ and $\beta_m(V) = 4\exp(-(V+60)/18)$. The inactivation variable h evolves according to $\frac{dh}{dt} = \alpha_h(1-h) - \beta_h h$ where $\alpha_h(V) = 0.35\exp(-(V + 58)/20)$, $\beta_h(V) = 5/(exp(-0.1(V + 28)) + 1)$. The delayed rectifier current is given by $I_K = g_K n^4(V_K - V)$ where $g_K = 9\ mS/cm^2$, $V_K = -90\ mV$. The activation variable n evolves according to $\frac{dn}{dt} = \alpha_n(1-n) - \beta_n n$ with $\alpha_n(V) = -0.05(V+34)/(exp(-0.1(V+34)) - 1)$ and $\beta_n(V) = 0.625exp(-(V + 44)/80)$.

B Instability of the asynchronous state in heterogeneous networks of phase oscillators

We consider a population of neurons in which the intrinsic frequencies are distributed according to $h(\omega)$. The phases of the neurons evolve according to

$$\dot{\phi}_i = \omega_i + \frac{g}{N}\sum_{j=1}^{N}\Gamma(\phi_i - \phi_j) \tag{B.1}$$

The evolution of the one phase probability density, $\rho(\phi, t, \omega)$, is then given by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \phi}(\rho v) = 0 \tag{B.2}$$

where

$$v = \omega + g\sum_{n=0}^{+\infty} r_n \sin(n(\phi - \psi_n)) + s_n \cos(n(\phi - \psi_n)) \tag{B.3}$$

and the coefficients r_n, s_n, and ψ_n can be expressed in terms of the Fourier coefficients, a_n and b_n, of the interaction function,

$$r_n e^{in\psi_n} = a_n \int_0^{2\pi}\int_{-\infty}^{+\infty} \rho(\phi, t, \omega) h(\omega) e^{in\phi} d\omega d\phi \tag{B.4}$$

$$s_n e^{in\psi_n} = b_n \int_0^{2\pi} \int_{-\infty}^{+\infty} \rho(\phi,t,\omega) h(\omega) e^{in\phi} d\omega d\phi \tag{B.5}$$

To analyze the linear stability of the incoherent state, $\rho_0(\phi,t,\omega) = \frac{1}{2\pi}$, we write $\rho(\phi,t,\omega)$ as

$$\rho(\phi,t,\omega) = \rho_0(\phi,t,\omega) + \epsilon\eta(\phi,t,\omega) \tag{B.6}$$

where

$$\eta(\phi,t,\omega) = \sum_{n=-\infty}^{n=+\infty} c_n(t,\omega) e^{in\phi} \tag{B.7}$$

and linearize the continuity equation around ρ_0. Thus, we get

$$\frac{\partial c_n}{\partial t} = -\{i(\omega + gb_0)n\}c_n - g\frac{na_n}{2}\int_{-\infty}^{+\infty} h(\omega)c_n(t,\omega)d\omega + g\frac{nb_n i}{2}\int_{-\infty}^{+\infty} h(\omega)c_n(t,\omega)d\omega \tag{B.8}$$

We then seek for eigenvectors of the form

$$c_n(t,\omega) = d_n(\omega)e^{\lambda_n t} \tag{B.9}$$

where $\lambda_n = \mu_n + in\Omega_n$. Equation (B.8) then leads to

$$1 = g\frac{n}{2}(-a_n + ib_n)\int_{-\infty}^{+\infty} \frac{h(\omega)}{\lambda_n + i(\omega + gb_0)n} d\omega \tag{B.10}$$

This equation determines the eigenvalues λ_n for $n \geq 1$. Separating the real and imaginary parts of equation (B.10) we then obtain two equations that determine the bifurcation point and the value of Ω_n

$$\int_{-\infty}^{+\infty} h(\omega)\frac{\mu_n}{\mu_n^2 + n^2(\omega + gb_0 + \Omega_n)^2} d\omega = -\frac{2}{gn}\frac{a_n}{{a_n}^2 + {b_n}^2} \tag{B.11}$$

$$\int_{-\infty}^{+\infty} h(\omega)\frac{n(\omega + gb_0 + \Omega_n)}{\mu_n^2 + n^2(\omega + gb_0 + \Omega_n)^2} d\omega = \frac{2}{gn}\frac{b_n}{{a_n}^2 + {b_n}^2} \tag{B.12}$$

When the stability of the mode n changes, the associated eigenvalue is purely imaginary: $\lambda_n = in\Omega_n$. Taking the corresponding limit, $\mu_n \to 0$ in Eqs. (B.11,B.12) we get:

$$h(-gb_0 - \Omega_n) = -\frac{2}{\pi g}\frac{a_n}{{a_n}^2 + {b_n}^2} \tag{B.13}$$

$$\lim_{\epsilon \to 0}\int_{\epsilon}^{\infty} \frac{h(\omega - gb_0 - \Omega_n) - h(-\omega - gb_0 - \Omega_n)}{\omega} d\omega = \frac{2}{g}\frac{b_n}{{a_n}^2 + {b_n}^2} \tag{B.14}$$

In a case of a Gaussian distribution of frequencies

$$h(\omega) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(\omega - \bar{\omega})^2}{2\sigma^2}) \tag{B.15}$$

equationsB.13) and (B.14) lead to

$$\sqrt{\frac{2}{\pi}} \int_0^{\frac{\Omega_n + gb_0 + \bar{\omega}}{\sigma}} \exp(-\frac{x^2}{2}) dx = -\frac{b_n}{a_n} \tag{B.16}$$

$$\sigma_c = -g\sqrt{\frac{\pi}{8}} \frac{{a_n}^2 + {b_n}^2}{a_n} \exp(-\frac{(\Omega_n + gb_0 + \bar{\omega})^2}{2{\sigma_c}^2}) \tag{B.17}$$

where σ_c is the critical value of the disorder at which the mode n becomes unstable.

For a Lorentzian distribution,

$$h(\omega) = \frac{\sigma}{\pi} \frac{1}{\sigma^2 + (\omega - \bar{\omega})^2} \tag{B.18}$$

equations Eq.(B.13, B.14) take the simpler form:

$$\frac{\sigma}{\sigma^2 + (\Omega_n + gb_0)^2} = -\frac{2}{g} \frac{a_n}{a_n^2 + b_n^2} \tag{B.19}$$

$$\frac{\Omega_n + gb_0}{\sigma^2 + (\Omega_n + gb_0)^2} = \frac{2}{g} \frac{b_n}{a_n^2 + b_n^2} \tag{B.20}$$

$$\tag{B.21}$$

Therefore:

$$\sigma_c = -g\frac{a_n}{2} \tag{B.22}$$

$$\Omega_n + gb_0 = \pm\frac{b_n}{2} \tag{B.23}$$

Note that in that case, the critical level of disorder, σ_c, depends on the sole Fourier coefficient, a_n.

C Lorentzian distribution of intrinsic frequencies

In this appendix we calculate the robustness at low frequency or for fast synaptic interactions for excitatory or inhibitory IF networks and a Lorentzian distribution of heterogeneities.

Let us investigate first the robustness in the low frequency limit. We write the the Fourier expansion of the response function as:

$$Z(\phi) = \sum_{n=0}^{+\infty} A_n \sin(n\phi) + B_n \cos(n\phi) \tag{C.1}$$

A straightforward calculation using Eq. (2.4,2.11) shows that the Fourier coefficients of the phase interaction are given by:

$$a_n = \frac{A_n(1 - n^2\omega^2\tau_1\tau_2) - B_n n\omega(\tau_1 + \tau_2)}{(1 + n^2\omega^2\tau_1^2)(1 + n^2\omega^2\tau_2^2)} \tag{C.2}$$

$$b_n = \frac{A_n n\omega(\tau_1 + \tau_2) + B_n(1 - n^2\omega^2\tau_1\tau_2)}{(1 + n^2\omega^2\tau_1^2)(1 + n^2\omega^2\tau_2^2)} \tag{C.3}$$

For the IF model, the coefficents A_n and B_n are

$$B_n = \frac{\omega^2\tau}{2\pi I}\frac{1}{1 + (n\omega\tau)^2}\left(e^{2\pi/\omega\tau} - 1\right) \tag{C.4}$$

$$A_n = -n\omega\tau B_n \tag{C.5}$$

Using equation (**??**) the Fourier coefficient a_n of Γ can be written under the form $a_n = K(n\omega, \tau, \tau_1, \tau_2)L(\omega, \tau)$ where,

$$K(x, \tau, \tau_1, \tau_2) = \frac{x^3(\tau\tau_1\tau_2) - x(\tau + \tau_1 + \tau_2)}{(1 + x^2\tau_1^2)\,(1 + x^2\tau_2^2)\,(1 + x^2\tau^2)} \tag{C.6}$$

and

$$L(\omega, \tau) = \frac{\omega^2\tau}{2\pi I}\left(e^{2\pi/\omega\tau} - 1\right) \tag{C.7}$$

The function $K(x)$ reaches its (negative) minimum, K_{min}, at $x = x_{min}$, and its (positive) maximum, K_{max}, at $x = x_{max}$, while L keeps a constant sign.

For a Lorentzian distribution of angular frequencies, the critical level of disorder depends only on a_n, see Eq.(B.23). For excitatory coupling ($g > 0$) the first mode, n_c, that destabilizes the asynchronous state is then given by the best integer approximation of $-x_{min}/\bar{\omega}$. Indeed, K is then closest to K_{min} and a_n takes its most negative value there. When the average frequency goes to zero, $K(n_c\bar{\omega}, \tau, \tau_1, \tau_2)$ goes to K_{min} and the instability condition becomes $\sigma_c = -gK_{min}L(\bar{\omega}\tau)/2$. The quantity $L(\bar{\omega}\tau)$ diverges in this limit and so does the critical disorder, σ_c. By contrast, for inhibitory coupling ($g < 0$) the critical mode, n_c, is given by the best integer approximation of $x_{max}/\bar{\omega}$. The critical value of the disorder, σ_c, then diverges at low firing rate as $gK_{max}L(\bar{\omega}\tau)/2$. In both cases, excitatory and inhibitory, the critical mode, n_c, goes to infinity at low frequency as $1/\bar{\omega}$. This stems from the fact that, in that limit, the membrane potential of an IF neuron remains exponentially close to the threshold over a large part of the period so that small depolarizations can induce large changes in the phase of the neuron. This is a particular feature of IF neurons, which passively integrate their inputs over the whole subthreshold potential range.

To investigate the limit of fast synaptic interactions, we proceed in a similar way. For simplicity, we assume that $\tau_1 = \tau_2 \equiv \tau_{syn}$ The Fourier coefficient a_n can then be written as $a_n = K(n\omega, \tau, \tau_{syn})L(\omega, \tau)$ where,

$$K(x, \tau, \tau_{syn}) = \frac{x^3 \tau \tau_{syn}^2 - x\left(\tau + 2\tau_{syn}\right)}{\left(1 + x^2\tau^2\right)\left(1 + x^2\tau_{syn}^2\right)^2} \tag{C.8}$$

When τ_{syn} is much smaller than τ, K can be rewritten as $K_1(x\tau)K_2(x\tau_{syn})$ where

$$K_1(y) = \frac{y}{1 + y^2} \tag{C.9}$$

$$K_2(y) = \frac{y^2 - 1}{\left(1 + y^2\right)^2} \tag{C.10}$$

Consequently, the critical mode, n_c, that destabilizes the incoherent state, satisfies $n_c\omega\tau \sim 1$ in the excitatory case. It keeps a constant value, independent of τ_{syn}, when τ_{syn} goes to 0. On the contrary, the critical mode is larger than $1/(\omega\tau_{syn})$ for fast inhibitory interactions, so that it diverges when τ_{syn} goes to 0. As a result, R_E/R_I then scales like $\frac{\tau}{\tau_{syn}}$ for fixed firing rate $\bar{f}$.

D Stability of the asynchronous state in IF networks

The stability of the asynchronous state of a fully connected homogeneous ($\delta_i = 0$) IF network was studied by van Vreeswijk and Abbott (Abbott and van Vreeswijk 1993). They showed that the eigenvalues, λ, that determine the linear stability of the asynchronous state are the solutions of the equation

$$f_0\left(e^{\lambda/f_0} - 1\right)(\lambda + \alpha_1)(\lambda + \alpha_2) = \lambda\alpha_1\alpha_2 \int_0^1 dy F(y) e^{\lambda y/f_0} \tag{D.1}$$

where f_0 is the firing rate of neurons in units of $1/\tau$, $\alpha_1 = \tau/\tau_1$, $\alpha_2 = \tau/\tau_2$ and

$$F(y) = \frac{g f_0}{g f_0 + I} \exp^{y/f_0} . \tag{D.2}$$

In the balanced state (see section 3.4) any neuron in the network is submitted to the external drive KI_0 and to the synaptic drive $-KJf_0$, so that it receives the total current $K(I_0 - Jf_0)$. The interaction function then reads,

$$F(y) = \frac{f_0 J}{I_0 - f_0 J} e^{y/f_0} \tag{D.3}$$

Integrating over one period the evolution equation of any neuron of the network, one also derives the equation

$$\frac{1}{K\left(I_0 - Jf_0\right)} = 1 - e^{-1/f_0} \tag{D.4}$$

that enables us to express I_0 as a function of f_0. The eigenvalue equation can then be rewritten as,

$$\left(e^{\lambda/f_0} - 1\right)(\lambda + \alpha_1)(\lambda + \alpha_2) = -\alpha_1\alpha_2 f_0 KJ \frac{\lambda}{\lambda + 1}\left(1 - e^{-1/f_0}\right)\left(e^{(\lambda+1)/f_0} - 1\right) \tag{D.5}$$

Let us investigate the high frequency limit $f_0 \gg 1$. Considering the complex part of the spectrum and looking for solutions of equation (D.5) of the form

$$\lambda_n = 2\pi i n f_0 + \eta_n \tag{D.6}$$

where $\eta_n \ll f_0$, one obtains

$$\eta_n = \frac{1}{\frac{4\pi^2 n^2 f_0^2}{\alpha_1\alpha_2 KJ} - 1} \tag{D.7}$$

The ansatz (D.6) is valid when $K = af_0^2$, with $a \neq 4\pi^2 n^2/\alpha_1\alpha_2 J$. Then the bounded quantity η_n is just the real part of the n-th eigenvalue and determines the linear stability of the asynchronous state with respect to the Fourier mode of order n.

For inhibitory interactions ($K > 0$) η_n is positive for modes of order, n, larger than,

$$n_c = \frac{\sqrt{\alpha_1\alpha_2 aJ}}{2\pi} \tag{D.8}$$

while low order Fourier modes are stable. Accordingly, the homogeneous network settles in cluster states. Let us also note that the coupling strength at which the mode n becomes stable scales as n^2, so that it is difficult to stabilize high order Fourier modes by increasing the coupling. Higher order cluster states are easily destabilized in favor of the asynchronous state, when the disorder is increased. This is why the robustness of synchrony, R_I, in heterogeneous inhibitory networks may be small, although the asynchronous state is never stable in homogeneous networks.

The behavior of excitatory networks ($K < 0$) is quite different. Indeed, it is not possible to increase the coupling strength to compensate the decrease in the modulation of the synaptic current, that occurs at high frequency. If we consider the real part of the spectrum, the eigenvalues satisfy

$$\lambda^2 + \delta\left(\alpha_1 + \alpha_2\right) + \alpha_1\alpha_2\left(1 + KJ\right) = 0 \tag{D.9}$$

in the high frequency limit. When the actual coupling strength, KJ, becomes smaller than -1, the larger real eigenvalue becomes positive, and an instability develops. Balancing is then impossible as the inhibitory external drive cannot oppose recurrent excitation. For this reason, excitatory interactions cannot synchronize neuronal systems at high firing rates.

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Figure Captions

Figure 1. Probability density of the firing frequency f. Symmetrically truncated Gaussian distribution: $\sigma = 0.3\bar{f}$ (full line) and $\sigma = 0.5\bar{f}$ (dashed line). Symmetrically truncated Lorentzian distribution of half-width $0.5\bar{f}$ (dotted line).

Figure 2. Weakly coupled IF neurons: Robustness of synchrony, R vs. the average firing frequency $\bar{f}$. Firing rates in the uncoupled network are spread according to a symmetrically truncated Gaussian distribution (see Methods). Results are displayed for three sets of synaptic time constants: $\tau_1 = 1\ ms$, $\tau_2 = 3\ ms$ (full line); $\tau_1 = 1\ ms$, $\tau_2 = 7\ ms$ (dashed line); $\tau_1 = 6\ ms$, $\tau_2 = 7\ ms$ (dotted line). **A:** Excitatory network. **B:** Inhibitory network. Results obtained for a symmetrically truncated Lorentzian distribution are also plotted in the inhibitory case ($\tau_1 = 6\ ms$, $\tau_2 = 7\ ms$, empty squares).

Figure 3 Weakly coupled IF neurons: order of the critical mode vs. the average firing frequency $\bar{f}$ in an excitatory network.**A:** Excitatory network. **B:** Inhibitory network. Same parameters and conventions as in figure 2.

Figure 4 Weakly coupled IF neurons: Relative robustness R_E/R_I vs. the average firing frequency $\bar{f}$. Same parameters and conventions as in figure 2.

Figure 5 Weakly coupled IF neurons: Relative robustness R_E/R_I vs. the synaptic time τ_{syn} for a fixed averaged firing frequency $\bar{f} = 20\ Hz$ and an "alpha function" synaptic interaction.

Figure 6 Weakly coupled WB neurons: Robustness R of synchrony vs. the average firing frequency $\bar{f}$. **A:** Excitatory network. **B:** Inhibitory network. Same parameters and conventions as in figure 2.

Figure 7 IF neurons; coherence χ as a function of the relative frequency spread $\sigma/\bar{f}$ (Gaussian disorder) for excitatory interactions (filled squares) and inhibitory interactions (empty squares and full line)); $\tau_1 = 3\ ms$, $\tau_2 = 1\ ms$, $g = 0.05 nC/cm^2$, $\bar{f} = 80\ Hz$.

Figure 8 IF neurons. The robustness vs. the coupling strength g (in nC/cm^2). The average free firing rate is $\bar{f} = 50\ Hz$. Filled squares: numerical simulations. Full line: results from phase model. **A**Excitatory interactions, $\tau_1 = 3\ ms$, $\tau_2 = 1\ ms$. **B** Inhibitory interactions $\tau_1 = 7\ ms$, $\tau_2 = 1\ ms$. Results are shown in that latter case for two initial conditions differing by the spread of the the initial voltage in the neuronal population equal. Filled square: spread of 50% of the subtreshold voltage range. Empty squares: 1% (empty squares) of the subtreshold voltage range.

Figure 9 WB neurons. Robustness of synchrony vs. the coupling strength, g (in nC/cm^2). $\tau_1 = 3\ ms$, $\tau_2 = 1\ ms$, and $\bar{f} = 3\ Hz$. **A** Excitatory interactions. Only the increase of R_E with g is displayed; at higher values of g (outside the range of the figure) R_E actually decreases, as in figure 8A for IF neurons, and eventually vanishes. **B** Inhibitory interactions. R_I again increases at higher values of g like in Figure 8B (not shown). Same conventions as in Figure 8.

Figure 10 Robustness of synchrony vs. the coupling strength, g (in nC/cm^2), in a WB network for a constant average firing rate $\bar{f} = 3\ Hz$ in the coupled network. $\tau_1 = 3\ ms$, $\tau_2 = 1\ ms$. **A**: Excitatory interactions. Here again, as in Figure 8A, only with the increase of R_E with g is shown. **B**: Inhibitory interactions. Results are shown in that latter case for two initial conditions differing by the (uniform) spread of the initial voltage in the neuronal population. Filled squares: spread of $\pm 10\ mV$. Empty squares: spread of $\pm 0.1\ mV$. Same conventions as in Figure 8.

Figure 11 Inhibitory network of WB neurons. Robustness, R_I vs. the scaling factor K (see text). $\tau_1 = 7\ ms$, $\tau_2 = 1\ ms$, $\bar{f} = 50\ Hz$ (A) and $200\ Hz$ (B). In both cases R_I saturates with increasing K, as is slighly visible in A. Same conventions as in Figure 8.