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The theory of electron current to a spherical probe at intermediate pressures

L Friedland and Yu M Kagan
Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem, Israel

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Abstract. The problem of the calculation of the electron current to a retarding spherical probe at intermediate gas pressures is discussed. The current to the probe is expressed in terms of the probability with which an electron, from the undisturbed plasma, will reach the probe surface. This probability is obtained using the Monte Carlo method. It is shown that the computer simulation has to be performed only for the last few free paths of the electron near the probe surface. An integral equation for the electron energy distribution function in the undisturbed plasma is received from the expression for the electron current.

1. Introduction

At low gas pressures, when

$$\lambda \gg a \tag{1}$$

where λ is the electron mean free path and a is the probe radius, the following expression for the electron current to a retarding convex probe can be written (Kagan *et al* 1963):

$$i_e = \frac{2\pi e}{m^2} S \int_{eV}^{\infty} (\epsilon - eV)\rho(\epsilon)\, d\epsilon \tag{2}$$

where $\rho(\epsilon)$ is the electron energy distribution function in the undisturbed plasma, defined as the number density of electrons in the velocity interval d^3v, S is the area of the probe surface and V $(V > 0)$ is the potential of the plasma relative to the probe. According to equation (2), the electron current to the retarding probe does not depend on the potential distribution around the probe, but only on the potential difference V itself. Differentiating equation (2) twice with respect to V, we obtain the Druyvestein formula

$$\frac{d^2 i_e}{dV^2} = \frac{2\pi e^3}{m^2} \rho(eV) \tag{3}$$

which is frequently used for direct measurements of the distribution functions $\rho(\epsilon)$ at low pressures. In the opposite case, when instead of (1) we have

$$\lambda \lesssim a, \tag{4}$$

no experimental work was done to determine the distribution function, due to the lack of a theory. The main purpose of this work is to find an expression which will replace equation (2) at intermediate pressure.

The continuum and kinetical approaches to this problem and their limitations will be discussed in the next two sections. The probabilistic approach based on the computer simulation will be then described (§§4, 5 and 6).

2. Continuum plus free-fall theory

The only analogue to equation (2) at intermediate pressures was derived by Swift (1962). For completeness, we shall give here a brief description of the assumptions and the results of Swift's work. In order to take into account the disturbance produced in the plasma by the final size of the probe, the space around the probe was divided into two regions: region I, outside a spherical surface of radius $a+\lambda$, concentric with the centre of the probe; and region II, between this surface and the probe. It was supposed also that the probe field does not penetrate into region I and that in this region the electrons move to the probe mainly by free diffusion. The current of electrons, having an energy ϵ, through a spherical surface of radius r can then be expressed in the following form:

$$\mathrm{d}i_{\mathrm{e}}=eD_{\mathrm{e}}\frac{\mathrm{d}}{\mathrm{d}r}(F(\epsilon)\,\mathrm{d}\epsilon)\,4\pi r^2 \tag{5}$$

where

$$D_{\mathrm{e}}=\tfrac{1}{3}\lambda_{\mathrm{e}}v=\tfrac{1}{3}\lambda\,(2\epsilon/m)^{1/2}$$

is the diffusion coefficient of the electrons and

$$F(\epsilon)=(4\pi 2^{1/2}/m^{3/2})\,\epsilon^{1/2}\rho(\epsilon)$$

is the electron energy distribution function. Integrating (5) from infinity to $a+\lambda$, Swift obtained

$$F_1(\epsilon)\,\mathrm{d}\epsilon=F_0(\epsilon_0)\,\mathrm{d}\epsilon-\frac{3}{4\pi}\frac{\mathrm{d}I_{\mathrm{e}}}{\lambda_{\mathrm{e}}v(a+\lambda)} \tag{6}$$

where F_0 and F_1 are the energy distribution functions in the undisturbed plasma and on the surface of radius $a+\lambda$ respectively; i.e., the electrons in this region do not collide with gas atoms and reach the probe by free fall from the surface of radius $a+y$. Then, according to equation (2) we have

$$\mathrm{d}i_{\mathrm{e}}=\frac{2\pi e}{m^2}\,S(\epsilon-eV)\,\rho_1(\epsilon)\,\mathrm{d}\epsilon \tag{7}$$

where $S=4\pi a^2$. From equations (6) and (7) we obtain

$$i_{\mathrm{e}}=\frac{2\pi e}{m^2}\,S\int_{eV}^{\infty}\frac{(\epsilon-eV)\,\rho_0(\epsilon)\,\mathrm{d}\epsilon}{1+\frac{3}{4}\,[(a^2/\lambda(a+\lambda)](\epsilon-eV)/\epsilon}. \tag{8}$$

In the limiting case of low pressures, where $\lambda\gg a$ this equation reduces to equation (2). The free-fall assumption in region II seems to be the weakest point of Swift's theory. It is quite obvious that a considerable part of the electrons, crossing the surface of radius $a+\lambda$, overgo at least one collision before reaching the probe.

3. Kinetic approach

An equation similar to (8) can be received without dividing the space around the probe into two regions. Such a method is based on the idea proposed in papers by Kagan and

Perel (1954) and Mott-Smith (1954) in which the equations for the moments are solved instead of the kinetic equation itself. In order to take into account the anisotropy of the distribution function, the *velocity* space of the electrons is divided into two regions. It is assumed that the distribution functions in those two regions are different but isotropic.

This method was used by Kagan *et al* (1965), Wasserstrom *et al* (1965), Chou *et al* (1966), Bienkowski and Chang (1968) and Self and Shin (1968) in order to find the electron current to the retarding probe. In all these papers, however, a maxwellian electron energy distribution is assumed. Let us show now that the same approach can be also used in the case of an arbitrary electron energy distribution. With this goal in view we shall use some results derived by Kagan and Perel (1965), where the following expression for the current to the spherical probe is given:

$$i_e = 4\pi \int_{[2e\phi(a)/m]^{1/2}}^{\infty} F(u)u^2\,du \tag{9}$$

where u is the electron velocity in the undisturbed plasma far from the probe, $\phi(a)$ is the probe potential and

$$F(u) = \frac{4\pi x_a^2 u\rho(u)}{4} \frac{1}{1+\frac{3}{4}x_a^2 \int_{x_a}^{\infty} dx/gx}. \tag{10}$$

Here

$$x = \frac{[u^2+(2e/m)\,\phi(r)]^{1/2}}{u} r \qquad x_a = \frac{[u^2+(2e/m)\,\phi(a)]^{1/2}}{u} a$$

$\phi(r)$ is the potential at a distance r from the probe and

$$g(u, r) = \lambda(v) \frac{u^2+(2e/m)\phi(r)+(e/m)r(d\phi/dr)}{u[u^2+(2e/m)\phi(r)]^{1/2}} \tag{11}$$

with $v = [u^2+(2e/m)\phi(r)]^{1/2}$. Substituting equation (10) into equation (9) and using the new variable $z = r/a$ we obtain

$$i_e = \frac{4\pi e}{m^2} 4\pi a^2 \int_{eV}^{\infty} \frac{(\epsilon - eV)\rho(\epsilon)\,d\epsilon}{1+\frac{3}{4}a(\epsilon - eV)\int_1^{\infty} dz/\lambda(v)z^2[\epsilon + e\phi(z)]} \tag{12}$$

where $V = -\phi(a)$. The integral in the denominator in equation (12) can be divided into two parts: (*a*) the integral across the thin space-charge layer around the probe, and (*b*) the integral outside the layer. If the width of the layer is assumed to be much less than the mean free path λ, the integral (*a*) can be neglected. The next step is to assume that the probe field does not penetrate into plasma outside the space charge layer and thus $\phi(z) = 0$ in the integral (*b*). Therefore, equation (12) can be rewritten in the following form:

$$i_e = \frac{2\pi e}{m^2} 4\pi a^2 \int_{eV}^{\infty} \frac{(\epsilon - eV)\rho(\epsilon)\,d\epsilon}{1+\frac{3}{4}[a/\lambda(\epsilon)]/(\epsilon - eV)/\epsilon}. \tag{13}$$

This is in agreement with equation (8), if $a \gg \lambda$. Although the space around the probe in the kinetical approach was not divided into two regions, as it was done by Swift (1962), the division of the velocity space into two isotropic parts seems to be doubtful close to the probe surface.

The comparison of equations (8), (12) and (13) shows that the electron current to the retarding spherical probe can be expressed in the following form:

$$i_e = \frac{2\pi e}{m^2} 4\pi a^2 \int_{Ve}^{\infty} \alpha(\epsilon, V)\epsilon\rho(\epsilon)\,\mathrm{d}\epsilon \tag{14}$$

where

$$\alpha(\epsilon, V) = \frac{(\epsilon - eV)/\epsilon}{1 + D(\epsilon - eV)/\epsilon} \tag{15}$$

with $D = \frac{3}{4}(a/\lambda)[a/(a+\lambda)]$ according to the continuum theory and $D = \frac{3}{4}a/\lambda$ in the kinetical approach. It will be shown in the next section that equation (14) represents a general form for the electron current to the probe and that the function in this equation has a deep probabilistic nature.

4. Probabilistic interpretation of $\alpha(\epsilon, V)$

In this section we shall show for the case of a spherical probe in an infinite plasma that the function $\alpha(\epsilon, V)$ in equation (15) can be expressed in terms of probabilities. This can be done in the following way. Let R be the radius of such a sphere, with its centre in the centre of the probe, such that the surface plasma can be considered as undisturbed by the presence of the probe. Let us assume also that the electron energy distribution function $\rho(\epsilon)$ on this surface is isotropic. Then in each point A on the undisturbed spherical surface, the number density of electrons having energies in the interval $[\epsilon, \epsilon + \mathrm{d}\epsilon]$ and velocities directed into the solid angle $\mathrm{d}\Omega = 2\pi \sin\theta\,\mathrm{d}\theta$, where θ is the angle between the velocity vector and the normal to the surface in this point, can be written as

$$\tfrac{1}{2}F(\epsilon)\,\mathrm{d}\epsilon \sin\theta\,\mathrm{d}\theta = \tfrac{1}{2}\frac{4\pi 2^{1/2}}{m^{3/2}}\rho(\epsilon)\,\epsilon^{1/2}\,\mathrm{d}\epsilon \sin\theta\,\mathrm{d}\theta.$$

The distribution function $\rho(\epsilon)$ here is normalised so that

$$\int_0^{\infty} F(\epsilon)\,\mathrm{d}\epsilon = \frac{4\pi 2^{1/2}}{m^{3/2}} \int_0^{\infty} \rho(\epsilon)\,\epsilon^{1/2}\,\mathrm{d}\epsilon = n$$

where n is the electron concentration at point A. The current $\mathrm{d}I_R$ through the spherical surface of radius R due to electrons with energy ϵ and velocities directed into $\mathrm{d}\Omega$ can now be expressed in the following way:

$$\mathrm{d}i_R = \tfrac{1}{2}\,4\pi R^2\,e\,\frac{4\pi 2^{1/2}}{m^{3/2}}\rho(\epsilon)\,\epsilon^{1/2}\,\mathrm{d}\epsilon \sin\theta\,\mathrm{d}\theta \left(\frac{2\epsilon}{m}\right)^{1/2}\cos\theta. \tag{16}$$

A certain part of this current arrives to the probe, and the value of this part depends on the probability $P_R(\epsilon, V, \theta)$ with which an electron, starting on the undisturbed surface, reaches the probe. Therefore, in the stationary conditions, the total current to the probe can be written in the form

$$i_e = \int_{\theta}\int_{\epsilon} P_R\,\mathrm{d}i_R = \frac{2\pi e}{m^2} 4\pi a^2 \int_{eV}^{\infty} \rho(\epsilon)\,\epsilon\,\mathrm{d}(\epsilon, V)\,\mathrm{d}\epsilon \tag{17}$$

where

$$\alpha(\epsilon, V) = 2\frac{R^2}{a^2}\int_0^{\pi} \cos\theta \sin\theta . P_R(\epsilon, V, \theta)\,\mathrm{d}\theta. \tag{18}$$

The last equation can be used in order to obtain $\alpha(\epsilon, V)$ if the probability $P_R(\epsilon, V, \theta)$

is known. Let us find this probability in the case of low pressure, when inequality (1) is fulfilled. We consider a particle, crossing the undisturbed spherical surface in a certain point A at an angle θ to the normal in this point. The electron energy ϵ and angular momentum J at point A are equal to $\epsilon = mv^2/2 = mv_\perp^2/2 + mv_\parallel^2/2$ and $J = mv_\parallel R$ respectively, where $v_\perp = v\cos\theta$ and $v_\parallel = v\sin\theta$ are the normal and tangential components of the velocity. In the central force field around the probe both the quantities ϵ and J are conserved. Therefore, at an arbitrary distance r from the centre of the probe

$$\epsilon = \frac{mv_\perp^2(2)}{2} + \frac{mv_\parallel^2(r)}{2} + eV(r)$$

and

$$J = mv_\perp(r)r.$$

The electron will reach the probe surface only if

$$\frac{mv_\perp^2(a)}{2} \geqslant 0$$

$$\epsilon - \frac{J}{2ma^2} - eV \geqslant 0$$

where V is the potential of the probe. The last inequality can be rewritten in the form

$$\epsilon - eV - \epsilon\frac{R^2}{a^2}\sin^2\theta \geqslant 0$$

or

$$\cos\theta \geqslant \left(1 - \frac{a^2}{R^2}\frac{\epsilon - eV}{\epsilon}\right)^{1/2}.$$

Thus

$$P_R(\epsilon, V, \theta) = \begin{cases} 1 & \cos\theta \geqslant \left(1 - \dfrac{a^2}{R^2}\dfrac{\epsilon - eV}{\epsilon}\right)^{1/2} \\ 0 & \cos\theta < \left(1 - \dfrac{a^2}{R^2}\dfrac{\epsilon - eV}{\epsilon}\right)^{1/2}. \end{cases} \qquad (19)$$

Then integrating in equation (18) we get $\alpha(\epsilon, V) = (\epsilon - eV)/\epsilon$ in accordance with equation (2).

Let us consider now the case of high pressures. As it was mentioned previously, the number of difficulties in this case make the use of the kinetic or diffusion approaches too complicated. In contrast to these methods, equation (18) enables to determine (at least in principle) the function $\alpha(\epsilon, V)$ by computer simulation (Monte Carlo method). In this approach we follow the history of a large number of electrons with given initial ϵ and θ in order to find the probability $P_R(\epsilon, V, \theta)$. We cannot, however, apply directly the Monte Carlo method in our problem for the case of high pressures. The reason is that when $\lambda_e \ll a$, the electrons, on their way from the undisturbed region to the probe, collide with the atoms so many times, that the usual limits on computing time make it impossible to compute $P_R(\epsilon, V, \theta)$ accurately. In the following part of this section we shall show, however, that equation (18) for $\alpha(\epsilon, V)$, can be transformed in such a way that the computer simulation has to be applied only to few electron free paths from the probe surface.

We assume, as before, that the negative potential on the probe is shielded by a thin sheath, whose dimensions are of the order of the Debye length λ_D, and that the field outside the sheath is negligible. We shall assume also that only elastic collisions between electrons and atoms are important and that the scattering is isotropic. Let us consider an electron crossing a certain point A of the undisturbed spherical surface and let $\boldsymbol{R}$ denote the radius-vector of this point with the origin at $\boldsymbol{R}$ in the centre of the probe. At the moment of its first collision with an atom of the gas, the electron radius-vector will be $\boldsymbol{r}=\boldsymbol{R}+\boldsymbol{x}$, where $\boldsymbol{x}$ is the free motion displacement vector. We define now a new function $\beta(r, \epsilon, V)$, as the probability that the electron, which is isotropically scattered in the point $\boldsymbol{r}$, reaches at some time the probe surface. Then it is obvious that

$$P_{\mathrm{R}}(\epsilon, V, \theta)=\int_0^\infty \frac{\mathrm{d}x}{\lambda} \exp\left(-\frac{x}{\lambda}\right) \beta(r, \epsilon, V) \tag{20}$$

here $r^2=R^2+x^2-2Rx\cos\theta$ and $\lambda \ll R$ and, therefore, the most important contribution to the integral (20) is when $x \ll R$. According to that, the second-order expansion of β in powers of x will be used in equation (20):

$$\beta(r, \epsilon, V)=\beta(R, \epsilon, V)+(x^2-2Rx\cos\theta)\frac{\partial\beta}{\partial R^2}+2R^2x^2\cos^2\theta\frac{\partial^2\beta}{\partial(R^2)^2}. \tag{21}$$

Then we have

$$P_{\mathrm{R}}(\epsilon, V, \theta)=\beta(R, \epsilon, V)+(2\lambda^2-2R\lambda\cos\theta)\frac{\partial\beta}{\partial R^2}+4R^2\lambda^2\cos^2\theta\frac{\partial^2\beta}{\partial(R^2)^2}. \tag{22}$$

Substituting this expression into equation (18) for $\alpha(\epsilon, V)$ we obtain

$$\alpha(\epsilon, V)=-\frac{8}{3}\frac{R^3\lambda}{a^2}\frac{\partial\beta}{\partial R^2}. \tag{23}$$

Let us now find $\partial\beta/\partial(R^2)$. It is easy to see that

$$\beta(R, \epsilon, V)=\int_0^\infty \frac{\mathrm{d}x}{\lambda}\exp\left(-\frac{x}{\lambda}\right)\int_0^\pi \frac{\sin\theta\,\mathrm{d}\theta}{2}\beta(r, \epsilon, V). \tag{24}$$

Then according to equation (21) we receive the following differential equation for $\beta(R, \epsilon, V)$:

$$\frac{\partial^2\beta}{\partial(R^2)^2}=-\frac{3}{2}\frac{1}{R^2}\frac{\partial\beta}{\partial(R^2)}. \tag{25}$$

The solution of this equation, with the boundary condition $\beta|_{R\to\infty}=0$, is

$$\beta(R, \epsilon, V)=A(\epsilon, V)/R. \tag{26}$$

Substituting this solution into equation (23) we finally receive

$$\alpha(\epsilon, V)=\frac{4}{3}\frac{\lambda}{a^2}A(\epsilon, V). \tag{27}$$

We obtained a very simple form for $\alpha(\epsilon, V)$ and that this function does not depend on R, as could be expected. In order to find the function $A(\epsilon, V)$ in (27), the following considerations can be used. According to the way equation (25) was obtained, it is clear that its solution (26) is correct for all values of R provided (*a*) $R \gg \lambda$, (*b*) $R > a+3\lambda$. The first of these conditions guarantees the validity of the expansion (21) and the second ensures that an electron starting at point $\boldsymbol{R}$ does not practically reach the probe, before it collides at least once with an atom of the gas. Thus $A(\epsilon, V)$ can be obtained by calculating the probability $\beta(R_0, \epsilon, V)$ with the help of the Monte Carlo method there:

$$R_0 = \max\{10\lambda, a+3\lambda\}. \tag{28}$$

Then

$$A(\epsilon, V) = R_0\beta(R_0, \epsilon, V).$$

In the next section we shall describe the simulation procedure.

5. The simulation method

In order to find the probability $\beta(R_0, \epsilon, V)$ we divide it into two parts. Let $\beta_1(R_0, \epsilon, V)$ be the probability that the electron scattered isotropically in the point $\boldsymbol{R}_0$ reaches the probe surface without leaving the sphere of radius R_0. Then

$$\beta(R_0, \epsilon, V) = \beta_1(R_0, \epsilon, V) + \beta_2(R_0, \epsilon, V) \tag{29}$$

where $\beta_2(R_0, \epsilon, V)$ denotes the probability that the electron collides with an atom at least once outside the sphere of radius R_0, before reaching the probe. Let us define now the new probability μ_R that the first collision of the electron outside the sphere of radius R_0 occurs at distance R from the centre of the probe. We shall assume also that according to (28), for $R > R_0$, equation (26) for $\beta(R, \epsilon, V)$ can be used. Then

$$\beta_2(R_0, \epsilon, V) = \sum_{R>R_0} \frac{A(\epsilon, V)}{R} \mu_\mathrm{R} \tag{30}$$

and therefore

$$\beta(R_0, \epsilon, V) = \frac{A(\epsilon, V)}{R_0} = \beta_1 + A(\epsilon, V) \sum_{R>R_0} \frac{\mu_\mathrm{R}}{R}. \tag{31}$$

The last equation gives

$$A(\epsilon, V) = \frac{R_0\beta_1}{1 - R_0 \sum_{R>R_0} \mu_\mathrm{R}/R}. \tag{32}$$

Thus, in order to find $A(\epsilon, V)$ it is enough to know β_1 and $\sum_{R>R_0} \mu_\mathrm{R}/R$. These quantities can be calculated with the help of the Monte Carlo method. The simulation in this case has to be performed only in a comparatively small space region between the probe surface and the sphere of radius $\sim R_0 + \lambda$, and this is the main advantage of the method suggested here for the determination of $\alpha(\epsilon, V)$.

The simulation procedure is as follows. We start with a single electron at the point $\boldsymbol{R}_0$. At first, the value of $S = \cos\theta$, where θ is the angle between the electron velocity vector and $\boldsymbol{R}_0$, is determined. As it is well known (Friedland 1977), for isotropic scattering of the electron in the point $\boldsymbol{R}_0$ the simulation formula for the random quantity s is given by

$$s = 2\gamma - 1 \tag{33}$$

where γ are pseudo-random numbers uniformly distributed in the interval [0, 1], generated by the computer. Once the scattering angle θ is identified, the free path x of the electron is simulated, using the next pseudo-random number γ:

$$x = \lambda \ln \gamma. \tag{34}$$

If during this free path the electron does not cross the space charge layer surface around the probe (the radius of this surface is assumed to be equal to the radius of the probe), the collision will occur at the distance

$$r=(R_0^2+x^2-2R_0xS)^{1/2} \tag{35}$$

from the centre of the probe. For the case $r<R_0$ the simulation cycle is repeated (new s and x are found) until either (*a*) the successive r exceed R_0, or (*b*) the electron crosses the space charge layer surface. In case (*a*) we memorise the value of r, stop the simulation of the motion of the electron and start the simulation process for new electron from the point $\boldsymbol{R}_0$. In case (*b*) the possibility of the absorption of the electron at the probe surface is checked. In the noncollisional layer case, the electron will reach the probe if

$$\epsilon \cos^2 \chi > eV \tag{36}$$

where χ is the angle at which the electron crosses the boundary of the layer. If the electron is absorbed at the probe, we start the simulation procedure for new electron. In the opposite case, the electron is rejected from the probe and its new distance from the probe centre is

$$r_{i+1}=(r_i^2+l^2-2r_ils_i)^{1/2} \tag{37}$$

where r_i and s_i are the electron parameters at the last collision before the rejection and $l=2L-x_i$, where L is the part of the free path x; which the electron passes until it enters the space charge layer.

The simulation procedure is repeated for a large number N_0 of primary electrons. Then, according to the definitions of the values of β_1, and $\sum_{R>R_0} \mu_R/R$ we have

$$\beta_1=\Delta N/N_0 \tag{38}$$

and

$$\sum_{R>R_0} \mu_R/R=1/N_0 \sum_{n=1}^{N-\Delta N} 1/r_n{}^* \tag{39}$$

where ΔN is the number of the primary electrons reaching the probe without leaving, during their motion to the probe, the sphere of radius R_0. The summation in (39) is performed for $N_0-\Delta N$ electrons leaving this sphere. The values of $r_n{}^*$ here denote the distances from the centre of the probe at which the electrons collide for the first time after they leave the sphere of radius R_0.

6. Results and conclusions

The simulation procedure, described in the previous section, shows that the number of parameters, defining the function $\alpha(\epsilon, V)$ can be considerably reduced. In fact, it follows from equations (34)–(37) that only two quantities r/λ and eV/ϵ can be used during the simulation. Thus, it is clear that $\alpha(\epsilon, V)$ depends only on a/λ and eV/ϵ. It follows also directly from a dimensional argument without recourse to the simulation procedure. This conclusion agrees also with equation (15), which was derived using different theories. Let us now describe the results of the simulation. The dependence of $1/\alpha(\epsilon, V)$ on $1/y$, where $y=1-eV/\epsilon$ is shown in figure 1 for various values of a/λ. In order to determine the statistical error in the calculation five values for $1/\alpha$ were found for five statistically independent groups of test electrons, with 10^4 electrons in each one. The analysis of the

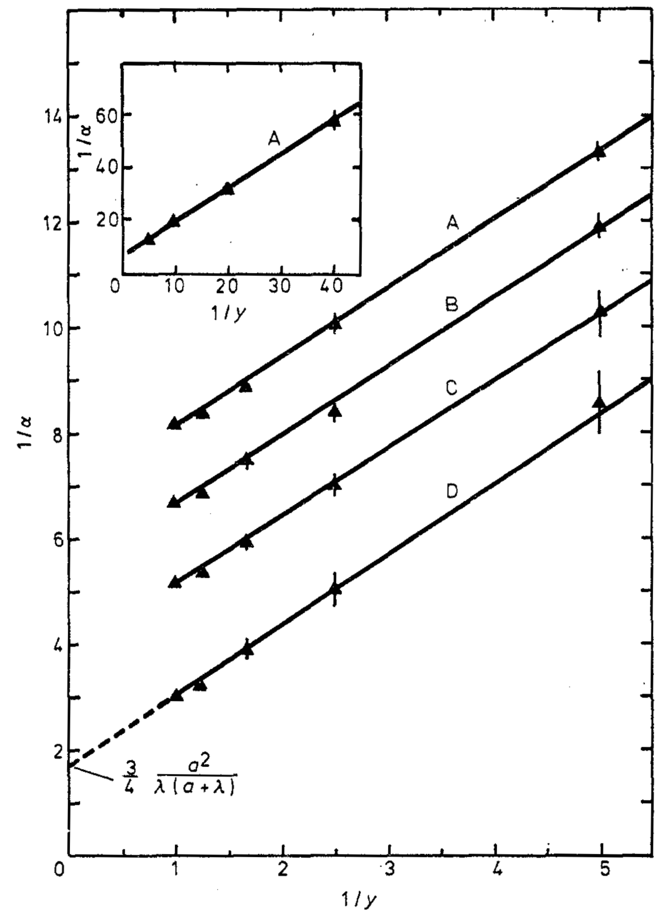

Figure 1. The dependence of $1/\alpha$ on $1/y$: (A) $a/\lambda=10$; (B) $a/\lambda=8$; (C) $a/\lambda=6$; (D) $a/\lambda=3$.

curves in figure 1 shows that for all examined values of a/λ, we received

$$\frac{1}{\alpha}=D+C\frac{1}{y} \tag{40}$$

with $C=1{\cdot}3$ and $D=\frac{3}{4}(a/\lambda)\,a/(a+\lambda)$. The coincidence in values of D in this equation as compared with results of the continuum theory, as well as the difference in values of C is not accidental. The careful examination of the way equation (15) was derived (§2) shows that the value $D=\frac{3}{4}(a/\lambda)\,a/(a+\lambda)$ appears in this formula as a result of the assumption that the continuum theory can be applied, starting from the distance of one mean free path λ from the surface of the probe. Thus the results of the simulation approve this assumption. On the other hand, the value $C=1$ in the continuum theory comes as a result of the assumption of free fall of the electrons from the spherical surface of radius $a+\lambda$ to the probe. In the real situation, the electron current from this surface to the probe is reduced due to the collisions and therefore equations (6) and (7) give the value of C greater than one. The simulation method gives $C=1{\cdot}3$ and thus the free fall assumption is quite poor for $a/\lambda>2$.

A knowledge of the function $\alpha(a/\lambda, y)$ enables one in principle, to find the electron energy distribution function from measurements of the probe current $i=i(V)$. In fact, equation (14) can be interpreted as an integral equation for $\rho(\epsilon)$. Experimentally measured current to the probe, however, always includes also the ion component. This difficulty can be eliminated, when the second derivative $\mathrm{d}^2 i/\mathrm{d}V^2$ of the current is measured. Since $\mathrm{d}^2 i_e/\mathrm{d}V^2 \gg \mathrm{d}^2 i_p/\mathrm{d}V^2$ if V is not very large, we can assume that $\mathrm{d}^2 i/\mathrm{d}V^2 \approx \mathrm{d}^2 i_e/\mathrm{d}V^2$. Differentiating equation (15) twice and remembering that $\alpha(V, V)=0$ we obtain

$$\frac{\mathrm{d}^2 i_e}{\mathrm{d}V^2}=\frac{2\pi e}{m^2}\, 4\pi a^2 \int_{eV}^{\infty} \frac{\partial^2 \alpha}{\partial V^2}\, \rho(\epsilon)\epsilon\, \mathrm{d}\epsilon - \frac{2\pi e^3}{m^2}\, 4\pi a^2\, \frac{\partial \alpha}{\partial V}\, V\rho(V). \tag{41}$$

This integral equation for $\rho(\epsilon)$ can be solved numerically using an iteration method.

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