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Ionisation growth in gases in strong uniform electric fields

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Abstract. The theory of electron multiplication in gases at large values of E/P is extended to cases where the characteristic energies of individual electrons are of the order of 10^2 eV or more. The first Townsend ionisation coefficient is found analytically and is compared with existing experimental data.

Electron multiplication in gases in uniform electric fields has been studied experimentally for a wide range of values of E/P. Nevertheless, the usual theoretical approach to the problem (Loeb 1955), based on the use of the electron velocity distribution function found from the Boltzmann equation, is valid only for small values of E/P. This limitation of the theory can be explained by two reasons. For large enough values of E/P the distribution function becomes strongly anisotropic and the usual perturbation method of the solution of the Boltzmann equation becomes invalid. In addition, at large E/P, the distribution function loses its local character. This is due to the increasing number of ionisations, so that the distribution at a given point x becomes affected by the electrons which have been created at points $x' < x$ and which have not yet reached their equilibrium distribution when they pass the point x. Moreover, such an equilibrium distribution may not exist at all for very strong electric fields. Thus, a different method has to be applied to describe the current growth in strong uniform electric fields. Such a method was suggested by Friedland (1974). Although the method is general, several assumptions on the functional dependence of the inelastic cross-sections on the electron energy limited the analysis to moderately large values of E/P. In this paper we extend the application of the method to larger values of E/P, thus completing the previous analysis.

Consider the conventional system of two plane-parallel electrodes in a gas with a strong uniform electric field applied in the gap between the planes. Assume a stationary current flow in the discharge and let j_0 be the electron current density on the cathode (at $x = 0$). As a result of ionisations, the current density at a given point $x > 0$ will be

$$j(x) = j_0 P(x). \tag{1}$$

Let us find $P(x)$ by assuming that the electric field is so strong that (a) all the electrons in the discharge are moving along the field lines of the electric field, (b) new electrons are created in ionising collisions and initially have zero energy and (c) the energy losses by the ionising electron can be neglected.

Consider an interval $\mathrm{d}x$ at a distance x from the cathode and define $n(x)\,\mathrm{d}x$ as the number of ionisations in $\mathrm{d}x$ per unit time. Then the number of ionisations in the interval $\mathrm{d}x$ which is caused by the electrons created in an interval $\mathrm{d}x'(x' < x)$ will be $n(x')Nq(\varepsilon_{x'x})\,\mathrm{d}x'\,\mathrm{d}x$, where N is the concentration of the gas atoms and $q(\varepsilon_{x'x})$ is the ionisation cross-section evaluated at the energy $eE(x - x')$. Thus the total number of ionisations in $\mathrm{d}x$ or, by definition, $n(x)\,\mathrm{d}x$ can be found by adding the contribution from all the electrons created in the interval $(0, x)$:

$$n(x)\,\mathrm{d}x = \mathrm{d}x \int_0^x n(x')\,Nq(\varepsilon_{x'x})\,\mathrm{d}x' + j_0 Nq(\varepsilon_{0x})\,\mathrm{d}x. \tag{2}$$

On employing the continuity equation $n(x) = \mathrm{d}j(x)/\mathrm{d}x$, defining $Q(\varepsilon) = Nq(\varepsilon)$ and using (1), one can rewrite (2) as

$$\frac{\mathrm{d}P(x)}{\mathrm{d}x} = \int_0^x \frac{\mathrm{d}P(x')}{\mathrm{d}x} Q(\varepsilon_{x'x})\,\mathrm{d}x' + Q(\varepsilon_{0x}). \tag{3}$$

Then, integrating (3) by parts and using $P(0) = 1$ and $Q(\varepsilon_{xx}) = 0$, we finally get the following equation for $P(x)$:

$$\frac{\mathrm{d}P(x)}{\mathrm{d}x} = -\int_0^x P(x')\frac{\partial Q(\varepsilon_{x'x})}{\partial x'}\,\mathrm{d}x'. \tag{4}$$

Let us seek an asymptotic solution of (4) in the form

$$P(x) = j(x)/j_0 = \mathrm{e}^{\alpha x} \qquad \alpha > 0 \tag{5}$$

where α is the first Townsend ionisation coefficient. On substituting (5) into (4), one then gets

$$\alpha = \lim_{x\to\infty} \int_0^x \exp[-\alpha(x - x')]\,\frac{\partial Q}{\partial x'}\,\mathrm{d}x'. \tag{6}$$

In order to further simplify the problem we will approximate the ionisation cross-section in (6) by

$$Q(\varepsilon) = D\varepsilon^n\,\mathrm{e}^{-\gamma\varepsilon} \tag{7}$$

where D, n and γ are constants characterising the gas. Then

$$\partial Q(\varepsilon_{x'x})/\partial x' = -An(x - x')^{n-1}\exp[-B(x - x')] + AB(x - x')^n \exp[-B(x - x')] \tag{8}$$

where

$$A = D(eE)^n \qquad B = \gamma eE. \tag{9}$$

Substituting (8) into (6), defining $z = \alpha + B$ and introducing a new variable $t = z(x - x')$, we now have

$$z - B = Anz^{-n}\int_0^\infty t^{n-1}\,\mathrm{e}^{-t}\,\mathrm{d}t - ABz^{-(n+1)}\int_0^\infty t^n\,\mathrm{e}^{-t}\,\mathrm{d}t. \tag{10}$$

Thus, on using the definition of the gamma function $\Gamma(n)$ and the relation $n\Gamma(n) = \Gamma(n + 1)$, one finally gets an equation for z:

$$A\Gamma(n+1)z + Bz^{n+1} - z^{n+2} = AB\Gamma(n+1). \tag{11}$$

In order to estimate the distance from the cathode for which our asymptotic analysis is valid, one can consider the asymptotic expansion of incomplete gamma function (Abramowtiz and Stegun 1968):

$$\Gamma(n+1) - \int_0^{zx} t^n \, e^{-t} \, dt \simeq (zx)^n \, e^{-zx}. \tag{12}$$

It can be checked that for values of n from table 1 this expression will be less than 0.03 if $zx > 4$. Thus one can conclude that for distances x satisfying this condition, the asymptotic solution (5) makes sense and one can use the concept of the first Townsend ionisation coefficient.

Table 1.

	n	$\gamma\,(\mathrm{eV}^{-1})$	$D_0\,(\mathrm{cm}^{-1}\,\mathrm{Torr}^{-1}\,\mathrm{eV}^{-n})$
He	0.2	2×10^{-3}	0.58
Ne	0.41	2.3×10^{-3}	0.54
Ar	0.4	4×10^{-3}	3
Hg	0.24	3.2×10^{-3}	11.3
N_2	0.23	2.3×10^{-3}	4.43

Equation (11) has two solutions, $z_1 = B$ and $z_2^{n-1} = A\Gamma(n+1)$, and therefore one gets two values for α, $\alpha = 0$ (this solution must be excluded) and

$$\alpha = (A\Gamma(n+1))^{1/(n+1)} - B. \tag{13}$$

On writing $D = D_0 P$, where D_0 defines the cross-section under normal conditions, we can rewrite (9) as

$$A = D_0(eE/P)^n P^{n+1} \qquad B = \gamma(eE/P)P \tag{14}$$

and thus (13) yields

$$\frac{\alpha}{P} = \left[D_0\left(\frac{eE}{P}\right)^n \Gamma(n+1)\right]^{1/(n+1)} - \gamma\frac{eE}{P}. \tag{15}$$

Let us apply (15) to various gases. First we find the constants D_0, n and γ from experimental data on ionisation cross-sections (Brown 1959). In approximating the experimental curves by equation (7), we demand that the maximum of the cross-section $Q_{\max}$ and the electron energy $\varepsilon_{\max}$ at the maximum should be the same when found from experiment and by using (7). In addition, we require a good approximation for energies $\varepsilon > \varepsilon_{\max}$. With these constraints the ionisation cross-section at low energies ($\varepsilon \ll \varepsilon_{\max}$) is not described correctly by equation (7). Nevertheless, in strong electric fields, when the electrons gain energy very fast, this inaccuracy does not influence our asymptotic analysis significantly. The interpolation of the experimental data was carried out for He, Ne, Ar, Hg and N_2 (in Ar and Hg the possibility of multiple ionisation was taken into account). The results are summarised in table 1, where the values of n, γ and D_0 are given for different gases. The dependence of α/P on E/P for different gases obtained from (14) using the data from table 1 is shown in figure 1 (broken curve). In the same figure we also present for comparison experimental results for α/P (full curves) available in the literature (Brown 1959). One can see that the calculated and experimental curves are in good agreement at large values of E/P. Note that it follows from (3) that

$$\alpha/P < Q(\varepsilon_{\max}). \tag{16}$$

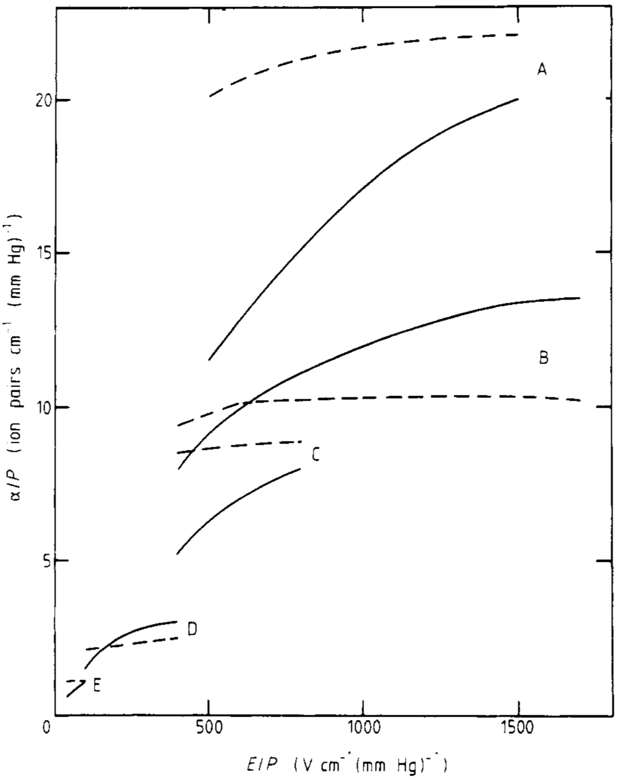

Figure 1. First Townsend ionisation coefficient. A, Hg; B, Ar; C, N_2; D, Ne; E, He. Broken curves, calculations; full curves, experimental results.

Comparison of ionisation curves with the data for α/P (Brown 1959) shows that this inequality is usually satisfied.

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