Further response to “Cosmic Rays, Carbon Dioxide and Climate” by Rahmstorf et al.

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Rahmstorf et al. [2004] published a “critique” of the article “Celectial driver of Phanerozoic Climate?” [Shaviv & Veizer, 2003]. Our rebuttal was published in Eos (and an unabridged version can be found at `http://www.phys.huji.ac.il/~shaviv/ClimateDebate`), together with with Rahmstorf et al.’s reply. Since Eos denied us the opportunity to respond to the latest allegations, we bring our response here. We show that Rahmstorf et al.’s claim for statistical insignificance is based on misunderstanding of the underlying assumptions in Bartlett’s formula for the effective number of degrees of freedom when correlating time series. They employed Bartlett’s formula at a limit where it grossly fails to yield a meaningful result. When properly used, the correlation between the reconstructed Phanerozoic temperature and CRF is shown to be statistically significant, conservatively at least at the 99.7% level. It is in fact the most significant correlation between any climate variable and a radiative forcing proxy on a time scale longer than a few million years. Moreover, the CRF data and the ^{18}O data are backed with additional, independent data sets, making the link redundant and robust. It implies, again, that the CRF was the dominant climate driver on the multimillion year time scale.

I. THE EFFECTIVE NUMBER OF DEGREES OF FREEDOM

The main criticism in Rahmstorf et al.’s reply is that the effective number of degrees of freedom in the comparison between the cosmic ray flux history and temperature reconstruction is between 0 and 1. To reach this conclusion, they quote from Quenouille’s [1952] text book the following equation for the effective number of degrees of freedom (d.o.f.):

$$N_{\text{eff}} \approx \frac{N}{1+\sum_{k=1}^{N} r_1(k) r_2(k)}, \qquad (1)$$

where N is the number of data points and $r_{1,2}(k)$ are the autocorrelation functions (with a lag k) of the first and second signals respectively. This formula was first derived by Bartlett [1935].

As an insignificant note, the formula should have a “+” and not a “−” in the denominator as they quote. In their calculation, they do use a “+” (Otherwise they would have obtained a negative effective number of d.o.f.).

Using the nominal CRF and the ^{18}O temperature reconstructions, a linear correlation coefficient of -0.58 is obtained (higher CRF = lower temperature). If the time scale is fine-tuned by 5% (because the CRF time scale and the geological time scale are not known to absolute precision), the linear correlation r increases to -0.75. Rahmstorf et al. then used the above formula to obtain $N_{\text{eff}} = 4.8$, from which they subtract 2 d.o.f. because of the linear fit, and an additional d.o.f. because of fine-tuning. They then subtract another d.o.f. due to their repeated erroneous claim that we arbitrarily shifted one spiral arm passage, while they forgot to subtract a d.o.f. because the data was “de-trended” to remove secular drifts. Thus, according to Rahmstorf et al., we should be left with 0.8 d.o.f. implying that, given any CRF reconstruction, or any random realization of the temperature reconstruction, we would have had about a 50% probability of obtaining a linear correlation this high!

This extremely low statistical significance seems *very* strange, considering that the signals (reproduced in fig. 1) are far from monotonic, with structure on times scales $\lesssim$50 Ma, and a span more than ten times as long.

A careful study of Bartlett [1935] reveals the assumptions upon which this formula is based.

For convenience, Bartlett worked with normalized vectors $x_i \equiv (X_i - \langle X \rangle)/\sigma_X$ and $y_i \equiv (Y_i - \langle Y \rangle)/\sigma_Y$, where $\{X_i\}$ and $\{Y_i\}$ are the two signals that we linearly cor-

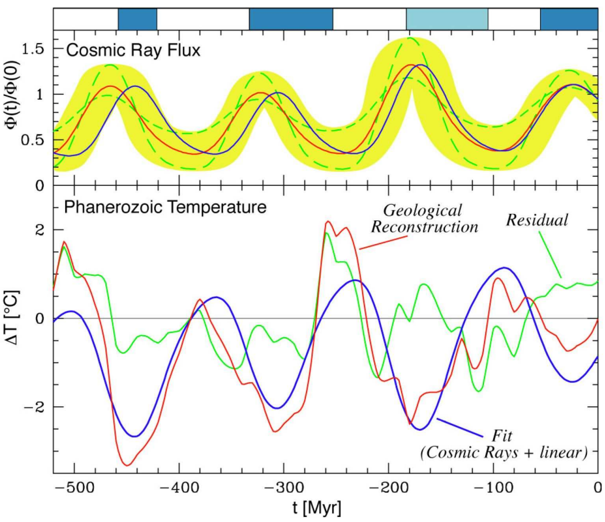

FIG. 1 The CRF and δ^{18}O based paleotemperatures (Shaviv and Veizer, 2003). The paleotemperature data is composed of 58 independent points between 0 and 570 Myr before present.

relate. The linear correlation coefficient is defined as $r = (\mathbf{x}\cdot\mathbf{y})/n = \sum_{i=1}^{N} x_i y_i/n$. Bartlett then wished to calculate the variance on r. This is because once the variance σ_r^2 on r is known, it is possible to calculate the probability that the $r = 0$ case (i.e., our null hypothesis in which the two signals are statistically uncorrelated) could have a realization with r as large as observed. In particular, if σ_r is known, N is large and the distributions are Gaussian, then this probability is given by

$$P(>r) \approx \operatorname{erfc}\left(\frac{|r|}{\sqrt{2}\sigma_r}\right) \equiv \operatorname{erfc}\left(\frac{|r|\sqrt{N_{\rm eff}}}{\sqrt{2}}\right). \qquad (2)$$

Under the assumption that $r = 0$, its variance was calculated by Bartlett as follows:

If we suppose now, in correlating two series x and y, that each has a serial correlation, ρ_1 and ρ_2 respectively, and the two series are not in fact correlated together, we have

$$E\left\{\frac{1}{n^2}(\Sigma x_r y_r)^2\right\} = \frac{1}{n^2}E\{\Sigma x_r^2 y_r^2 + 2\Sigma x_r y_r \,.\, x_s y_s \ (r \neq s)\}$$

$$= \frac{1}{n^2}\cdot n\sigma_1^2\sigma_2^2 + \frac{2}{n^2}\{\Sigma x_r x_{r+1}\cdot y_r y_{r+1} + \Sigma x_r x_{r+2}\cdot y_r y_{r+2} + \dots\}$$

$$= \frac{1}{n}\sigma_1^2\sigma_2^2 + \frac{2\sigma_1^2\sigma_2^2}{n^2}\{(n-1)\rho_1\rho_2 + (n-2)\rho_1^2\rho_2^2 + \dots\}$$

$$= \frac{1}{n}\sigma_1^2\sigma_2^2\left(1 + \frac{2\rho_1\rho_2}{1-\rho_1\rho_2}\right)$$ approximately, whence the

The difference between the above formula and eq. 1 is that Bartlett further assumed that the two processes are Markovian with serial correlations $\rho_{1,2}$. In the more general case, instead of ρ_i^k we are left with $r_i(k)$ – the autocorrelation function with lag k.

There are two key points in this derivation. First, as explicitly stated, the null hypothesis which we wish to rule out assumes that x_i and y_i are uncorrelated.

The second assumption is implicit, but it is paramount to the derivation. The sum $\sum x_r x_{r+k} y_r y_{r+k}$ can indeed be written as $\left(\sum x_r x_{r+k}\right)\left(\sum y_r y_{r+k}\right)/(n-k)$ as Bartlett did (it is an intermediate step between his second and third lines), since x_i and y_i are uncorrelated under the null hypothesis which we wish to rule out. Each sum can then be written as the correlation function $r_i(k)$ (times $(n-k)/n$ to be exact). The catch, however, is the assumed autocorrelation statistics of x_i and y_i. Without anything better, Bartlett implicitly assumed that the autocorrelation function of the random "noise" in the uncorrelated signals is the same as that of the two actual measured signals. For many cases, it is a fair assumption. This is especially true, for example, when the correlation between the signals is low to begin with, in which case most of the autocorrelation function arises from the underlying independent processes of the signals. This assumption is also valid when analyzing the statistical significance of signals which only have a short autocorrelation. However, it grossly fails when the measured signals are periodic and gets even worse when the S/N ratio is high.

For our signals, for example, the fact that they are nearly periodic oscillations implies that their correlation functions will be oscillating with the same period and will only decay over a long range. This can be seen in figure 2, where the autocorrelation functions of the CRF and temperature are plotted.

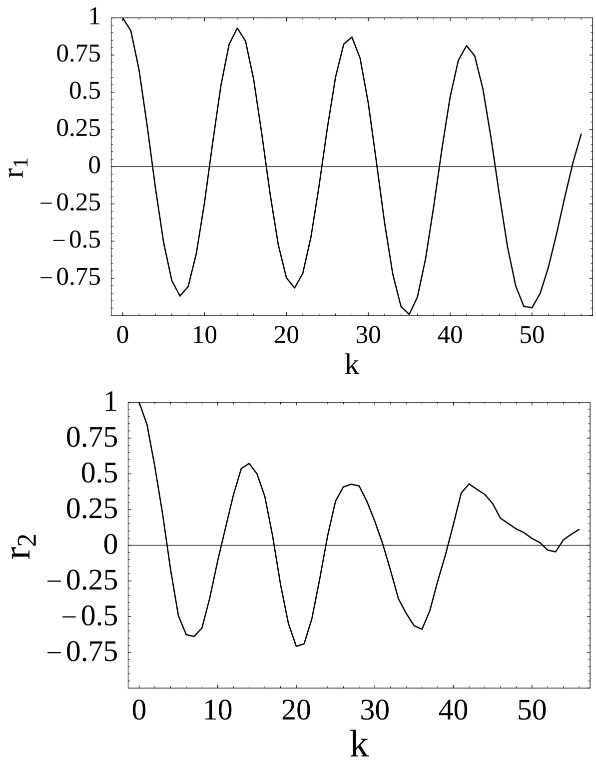

FIG. 2 The autocorrelation function $r_{1,2}$ of the CRF and δ^{18}O temperature reconstruction, respectively. The CRF is derived from a nearly periodic model. Consequently, the autocorrelation is a near perfect cosine. Because the temperature reconstruction follows the CRF with a high correlation, its autocorrelation function is similar to that of the CRF, but since it has noise and other components driving climate change, the amplitude of the cosine decreases with the lag.

Because Bartlett's formula implicitly assumes that the autocorrelation of the null-hypothesis signal is the same as the measured signal, all random realizations implicitly assumed in Bartlett's formula will have the same autocorrelation—a slowly decaying cosine, with a period of 145 Ma. Such signals will look like a harmonic wave, with a period of 145 Ma and with a slowly changing amplitude and phase. 16 random realizations satisfying this constraint on the autocorrelation are depicted in figure 3. A quick glance reveals that while this type of realizations cannot be ruled out statistically, as Rahmstorf et al. found out, such realizations appear highly contrived. This is because if we assume the temperature and CRF to be uncorrelated, there is a priori no reason for the temperature to be oscillating with a 145 Ma period. The origin of this type of unrealistic realizations is the assumption that the autocorrelation function is the oscillating autocorrelation function of the signal.

Theoretically, the best estimate for $N_{\rm eff}$ would have

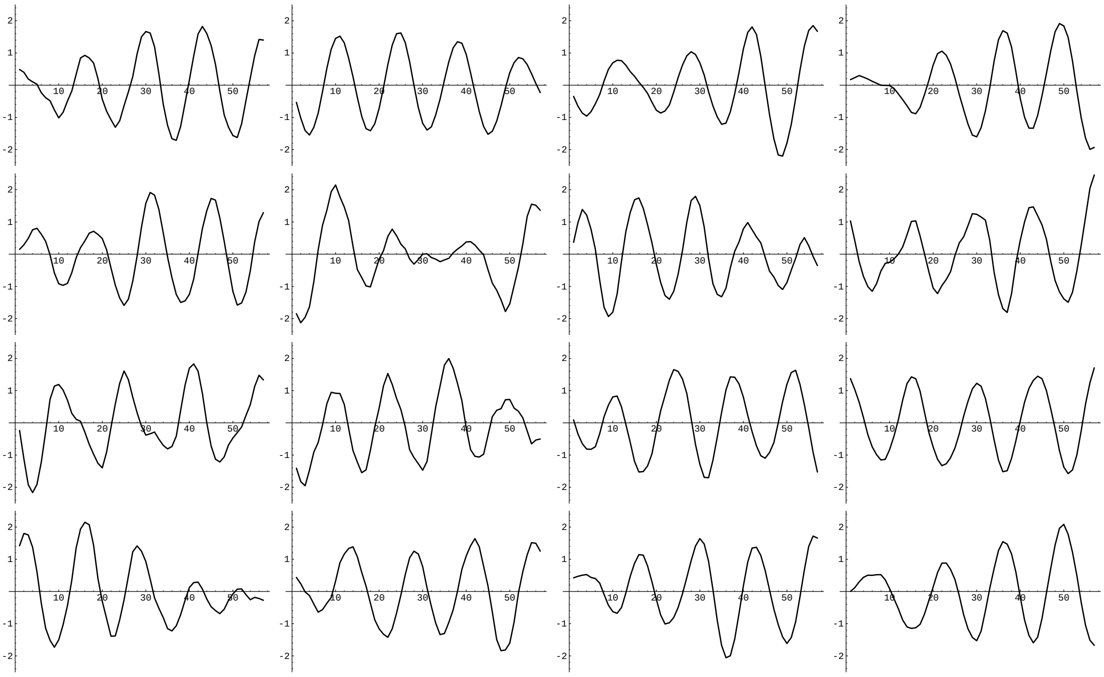

FIG. 3 16 random realizations for the normalized temperature $(\delta T_i - \langle \delta T \rangle)/\sigma_T$ as a function of measurement number, assuming the temperature fluctuations follow Gaussian statistics with the same autocorrelation function as that observed. Because this function is a slowly decaying cosine, random "noise" includes sine waves with a periodicity of 145 Ma, with some freedom in a slowly changing phase and amplitude. The formula for $N_{\rm eff}$ used by Rahmstorf et al. implicitly assumes this autocorrelation function for the null hypothesis. While this kind of random realizations cannot be ruled out, they are highly contrived and therefore meaningless.

been through the measurement of the temperature autocorrelation function when the CRF is switched off. However, since this is not a lab experiment, this possibility is not available to us.

To estimate $N_{\rm eff}$ we therefore require a more realistic autocorrelation function for the temperature reconstruction. One possibility is to truncate the autocorrelation function at its first null. This implies that the effective width of a measurement is comparable to the half width of a "hump". By truncating it at larger k we would be biasing our noise towards an oscillating signal with a 145 Ma periodicity. Truncating it at a smaller k would imply that we know that the underlying temperature processes have a finer resolution than the humps, which need not be the case.

We also need to estimate the autocorrelation function for the CRF reconstruction. However, since here we do have a bias that a 145 Ma cycle should arise, taking the autocorrelation function of the actual CRF reconstruction is legitimate. Irrespectively, if we truncate the temperature autocorrelation function at its null (at $k = 4$ or equivalently at 40 Ma), then it does not matter whether we truncate or not the CRF autocorrelation function.

Random realizations for the temperature based on this truncated autocorrelation function appear in figure 4. These realizations are much more realistic, because their structure is smoothed on the 40 Ma time scale (we do not know if the underlying processes determining the temperature have a finer structure than this scale) and because over longer time scales the signals are indeed random.

If we take the truncated correlation function, we find $N_{\rm eff} = 17.3 - 4 = 13.3$, where we subtracted 4 d.o.f. (2 for the fit, 2 because of the de-trending and the fine-tuning). The linear correlation coefficient is $r = 0.82$ for the correlation between the expected density of cloud condensation nuclei (proportional to the square root of the cosmic ray flux) and the reconstructed temperature. Thus, the statistical significance, namely the probability that random realizations of the temperature using a realistic autocorrelation function (e.g., figure 4) will yield such a high r is only 0.3%.

A second possibility for estimating the correlation function is using that part of the temperature reconstruction which is orthogonal to the CRF. This ensures that we are looking at the autocorrelation function of the temperature processes unrelated to the CRF. In other words, we can look at

$$\tilde{\mathbf{y}} = \frac{\mathbf{y} - (\mathbf{x} \cdot \mathbf{y})\mathbf{x}}{1 - (\mathbf{x} \cdot \mathbf{y})^2}. \tag{3}$$

This choice is probably even better than the previous one, because it allows for the possibility that the actual processes determining the temperature variations have a better resolution than 40 Ma. These, in turn, would manifest themselves in the residual noise. If we use the autocorrelation of $\tilde{y}$, we find $N_{\rm eff} = 22.3 - 4$, yielding

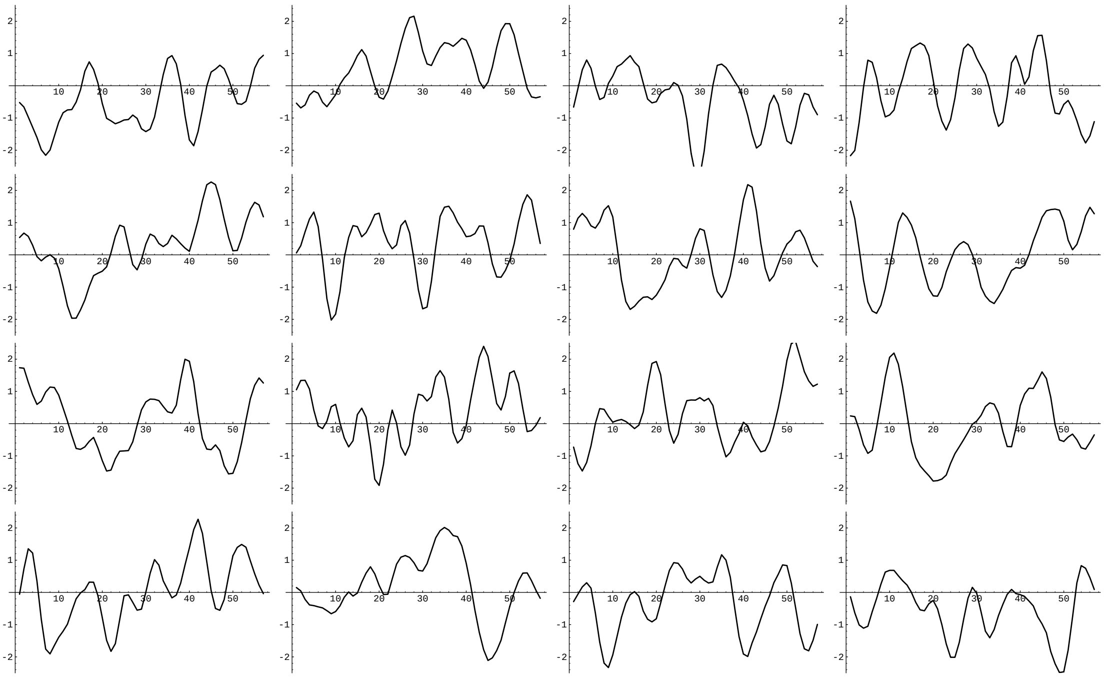

FIG. 4 The same as the previous realizations, except that the autocorrelation function is truncated at its first null (at a lag of $k = 4$ or 40 Ma equivalently). This assumes a much more reasonable autocorrelation for the temperature realizations in our null hypothesis. This type of random realizations can be ruled out at the 0.3% level.

a statistical significance of 0.05%. This means that the the noise, the residual between the empirical temperature and the CRF fit, has a finer structure than the temperature signal itself. The latter is dominated by the periodic oscillations correlated with the CRF and does not have as fine a structure as the residual.

Conservatively, we accept the less significant result, that the correlation between the CRF and temperature reconstruction is statistically significant to better than 99.7%. Counter to the claims of Rahmstorf et al. this is very significant. We stress, however, that Rahmstorf et al. were not formally wrong, but the model they applied is simply unrealistic.

As an example, one can note the following. Suppose we observe $M \gg 1$ cycles of two harmonic signals with near perfect correlation, measured using $N \gg M$ measurements. Using Bartlett's formula for N_{eff} would yield $N_{\text{eff}} \approx 3$! In other words, irrespective of how large M or N are, we could never measure a statistically significant correlation. Of course, this reductio ad absurdum arises because the autocorrelation function assumed for the null-hypothesis implies random realizations which are composed entirely of sine waves with exactly the same period as the signal, such that the phase and amplitude are the only free parameters.

II. OTHER POINTS

The issue of caveats (aerosols, GHG, lags, etc.) was injected into the debate by Rahmstorf et al. in order to bolster the claim for a CO_2⇒climate link in the Vostok cores, despite empirical observations showing that rises in temperature precede those of CO_2 by centuries. For a start, we repeat that SVO3 dealt only with the multimillion year Phanerozoic time scale and not with the multimillenial one of the ice cores. We also could point out to Sharma [2002] for an alternative interpretation to Rahmstorf et al. scenario, but that would hardly help to settle the argument. Considering the complexity of climate models, with their multitude of "tunable" and poorly known parameters and taking into account the skills of the practitioners, the models embody a remarkable capacity for the accommodation of special pleadings (or caveats), thus "explaining" away any uncomfortable empirical observations.

In our printed response we explained in detail how we dealt with the CRF/CO_2 regression. Note that our approach maximizes the climate sensitivity to CO_2, because it subsumes all possible forcings into CO_2.

Finally, except as a contrived justification for public attacks, we fail to see the need for public clarification of the time scales, since the SV03 clearly stated that it was the multimillion year one.

References

Bartlett, M. S., Some Aspects of the time-Correlation Problem in Regard to Tests of Significance, *J. R. Stat. Soc.*, **98**, 536–543,1935.

Quenouille, M.H., Associated Measurements, 242 pp., Butterworth Scientific Publications, London, 1952.

Rahmstorf, S., D. Archer, D. S. Ebel, O. Eugster, J. Jouzel, D. Maraun, U. Neu, G. A. Schmidt, J. P. Severinghaus, A. J. Weaver, and J. Zachos, Cosmic rays, carbon dioxide and climate, Eos, **85** (4), 38, 2004.

Sharma, M., Variations in the solar magnetic activity over the last 200,000 years: Is there a sun-climate connection? Earth Planet. Sci. Lett., **199**, 459–472, 2002.

Shaviv, N. J., and J. Veizer, 2003, Celestial driver of Phanerozoic climate? *GSA Today*, **13**, July, 4–10.