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Super Eddington Objects and their Winds

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Abstract. We discuss the occurrence of super-Eddington atmospheres, how such atmospheres can exist, the winds that they generate, and how their understanding can be applied to various luminous objects. In particular, we study them in the context of classical novae. We show that inhomogeneities are the key to understanding these atmospheres and the winds that they generate are the key to understanding these objects as a whole.

1. Introduction

The Eddington limit has often served as the absolute upper luminosity that objects in steady state can achieve. As we shall see, this conjecture deserves proper analysis. We begin by raising several basic questions. We continue by showing how super-Eddington atmospheres can exist and follow with the implications that the super-Eddington states have on novae. The latter are a particularly interesting example of super-Eddington objects from which much can be learned.

The results described here are a summary of previously published results (Shaviv 1998, 2000, 2001a, 2001b) as well as work in preparation. The reader is advised to follow these references for the unabridged version.

Can super-Eddington atmospheres exist? The common wisdom invoked when studying luminous objects in general and classical novae in particular, is that objects cannot shine beyond their classical Eddington limit $\mathcal{L}_{Edd}$, since no hydrostatic solution exists. In other words, if objects do pass $\mathcal{L}_{Edd}$, they are highly dynamic. They have no steady state, and a huge mass loss should occur since atmospheres are then gravitationally unbound and they should therefore be expelled. Thus, classical novae according to this picture, can pass $\mathcal{L}_{Edd}$ but only for a short duration corresponding to the time it takes them to dynamically stabilize after the onset of the thermonuclear runaway (TNR). This is indeed seen in detailed 1D numerical simulations of nova TNRs, where novae can be super-Eddington but only for several thousand seconds (e.g., Starrfield, 1989). However, once they do stabilize, they are expected and indeed do reach in the simulations a state given by the Core-Mass–Luminosity relation (CMLR) which describes the Hydrogen shell burning state. Namely, we naively expect to find no steady state super-Eddington atmospheres.

Do steady state super-Eddington atmospheres actually exist? As discussed, steady state super-Eddington atmospheres are not expected to exist. Apparently however, nature and common wisdom, do not always go hand in hand since steady state super-Eddington atmospheres do actually occur. The best example is the massive star η Carinae which was shining at roughly 5 times its Eddington luminosity for 20 years during its great eruption 150 years ago (Davidson 1999, Shaviv 2000). Classical novae during there their post-maximum decline are yet another good example.

One one hand, all cases where absolute maximum visual magnitudes where measured in nova eruptions (i.e., there were reasonable distance determinations), it was found that classical novae (at least those with a core mass larger than typically 0.5 $M_{\odot}$) peak at a super-Eddington luminosity (Livio 1992). On the other hand, in all cases where UV measurements were taken, it was found that the bolometric luminosity of classical novae decays only slowly with time. One can conclude from the two facts combined that most if not all classical novae are super-Eddington objects in a quasi steady state. In a few cases where both a bolometric luminosity and a reliable distance measurement exist, the novae are clearly super-Eddington for long durations—Both Nova FH Serpentis and Nova LMC 1988#1 were a few times super-Eddington for a month or two! (e.g., Friedjung 1987, Schwarz et al. 2001, Shaviv 2001b). Clearly, a steady state super-Eddington configuration is realized in nature.

The existence of this super-Eddington state poses an interesting theoretical problem. First, the basic common wisdom prohibits such configurations. More specifically, if one does want to construct for example a steady state solution to the eruption of η-Car, then one would find it impossible to do so. That is, in steady state, the sonic point of the wind should coincide with the critical point (where the net force vanishes). For a super-Eddington atmosphere, the critical point should on one hand reside where the most efficient convection cannot carry enough flux to keep the luminosity sub-Eddington. This takes place deep in the atmosphere where the density is high enough. On the other hand, the relatively low(!) mass loss observed from η-Carinae implies that the sonic point (and therefore the critical point at steady state) has to reside high enough in the atmosphere where the density is low. Namely, with standard means, no solution exists to bridge this gap (Shaviv 2000). To solve the problem, the critical point should be moved much higher than the top of the convection zone. One therefore concludes that a region exists above the convection zone, which is effectively sub-Eddington even though the flux itself is super-Eddington.

When one studies the eruption of novae, the apparent super-Eddington state raises a second problem since stellar structure implies that the steady state burning of a Hydrogen shell should be given by the CMLR (Paczynski 1970). The luminosity obtained from this relation is not a function of the envelope mass but it is an increasing function of the core mass and for large masses it saturates at the Eddington luminosity, not above it. However, the steady state of novae appears to be super-Eddington, counter to the theoretical expectation that the steady state should be given by the CMLR.

2. Required Background

The solution to the above problems requires two crucial background ingredients: (1) The fact that inhomogeneous media generally have a reduced effective opacity and (2) The fact that atmospheres, and Thomson scattering atmospheres in particular, are unstable as the Eddington luminosity is approached.

In an inhomogeneous system, the effective opacity, as defined through the ratio between the radiative flux through the system and the average radiative force on it is not the simple microscopic opacity. Instead, it is

$$\kappa_V^{\mathrm{eff}} \equiv \frac{\left\langle F \kappa_V \right\rangle_V}{\left\langle F \right\rangle_V} \lesssim \kappa_{V,0}, \tag{1}$$

(The average is a volume average). This reduction in opacity (per unit volume) is general and takes place for most opacity laws, with the Thomson opacity being no exception.

For several cases, the reduction in opacity can be calculated accurately. In particular, the effective opacity can be calculated for small isotropic and homogeneous perturbations in the top part of the atmospheres of luminous objects, where the diffusion time scale is much shorter than any dynamical time scale: $\kappa_V^{\mathrm{eff}} = \kappa_{V,0} \left\{ 1 - [(d-1)/d](\sigma_p^2/2) \right\}$. $d = 3$ is the dimension of the system and σ_ρ^2 is the variance of the density perturbations in units of the average density (Shaviv, 1998). Interestingly, this expression was obtained by utilizing results for the porosity of the ground to water. The result also shows that the effective opacity does not change for radial perturbations ($d = 1$).

To conclude, *if* a system is inhomogeneous (with non-radial perturbations), its opacity will generally be reduced. This will clearly increase the effective Eddington limit. Luminous systems however will become inhomogeneous because these were found to be unstable even before they reach the Eddington limit, even if the opacity is constant as is the case in Thomson atmospheres (Shaviv, 2001a).

Two different dynamic instabilities arise in the constant opacity case. The first instability depends on the actual boundary conditions below the atmosphere (i.e., if it is located on top of a convection zone, degenerate material etc.) and is therefore less general. These unstable modes do not propagate, and have the appearance of the opening of "chimneys" through which it is easier for the radiation to escape. The second instability is more general and it always appears at about 85% of the Eddington flux. It has the appearance of "anti-photon-bubbles". Namely, the phase structure propagates downward. This instability is mathematically similar to the instability of s-modes. First, both have e-folding growth times of order the oscillation period. That is, they are dynamic and therefore significantly different than various Carnot type instabilities, such as the κ-mechanism. Also, each mode has a conjugate that is damped at the same rate. Moreover, as a control parameter is changed (e.g., T_{eff} in s-modes or Γ here), the conjugate pair arises when two modes with different real frequencies merge together. Second, both types of instabilities appear only in systems in which the radiation pressure dominates. Last, both instabilities disappear when the system is driven out of the two fluid limit. Namely, both require that the radiation diffusion time scale be shorter than the dynamical time scale. Nevertheless, there are several noticeable differences: (1) s-modes arise in systems with special opacity laws while the anti-photon bubbles arises in the simplest Thomson atmospheres (which is used to define the Eddington limit). (2) Unlike s-mode instability, the anti-photon bubbles instability is an intrinsically non-radial phenomenon. The differences arise from the different physical origin to the instabilities. In both cases, the instability arises from the non-local relation between the pressure and density, which increases the differential order of the equations of motion. In the s-mode instability, it arises from the coupled behavior of the radiation and the opacity. In the anti-photon bubbles instability, it arises from the non-radial radiative flux arising from non-radial perturbations (Shaviv 2001a). Both the chimney and the anti-photon bubble instabilities are depicted in figure 1.

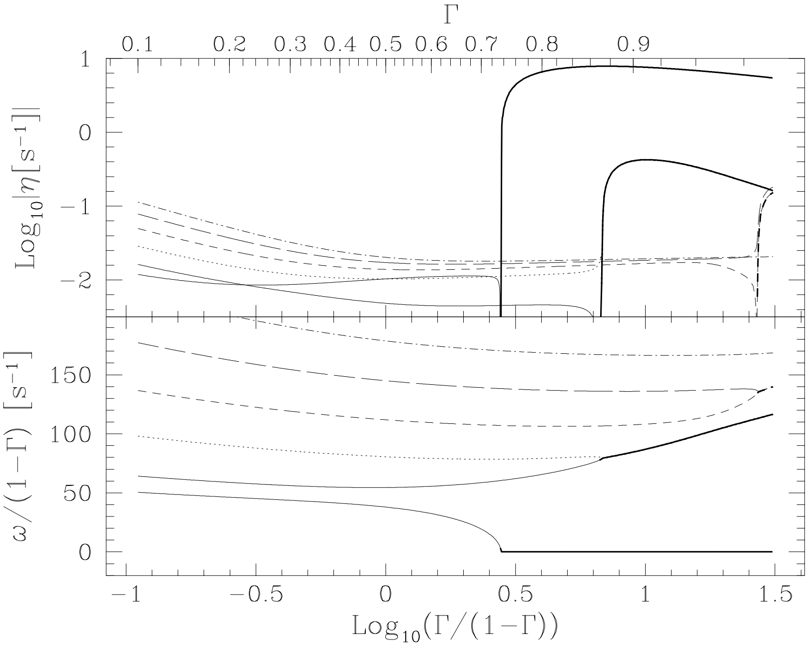

Figure 1. The eigen frequencies of the lowest modes in a luminous but sub-Eddington atmosphere as a function of the Eddington parameter Γ. k_x is kept fixed at $\left\langle l_p^{-1} \right\rangle$ (with l_p being the pressure scale height). Also, $g = 10^8 cm\ s^{-2}$, the absorption to scattering ratio is $\kappa/\chi = 1/100$ and a fixed temperature is imposed at a bottom, placed at $\tau_b = 10$ while the top is at $\tau = 0.3$. The bottom panel describe the real part of the eigenvalues ω while the top panel gives the imaginary part η, which is heavy weighted if negative. ω is multiplied by $1/(1-\Gamma)$ to normalize it to the changing dynamical time scale as the atmosphere puffs up when Γ is increased. As the flux is increased, this particular atmosphere first becomes unstable to the chimney instability (with a purely imaginary mode). As Γ approaches the Eddington limit, more and more modes become unstable to the anti-photon bubble instability (with propagating modes).

3. The Super-Eddington Steady State

As atmospheres approach the Eddington limit, the following takes place:
1. The atmospheres become unstable at $\Gamma \sim 0.85$ or perhaps less if the "chimney" instability arises. The instabilities operate in the region where the diffusion time scale is shorter than the dynamical time scale and the typical scale height in the atmosphere is optically thick.
2. Once instability arises, the atmosphere becomes inhomogeneous.
3. The inhomogeneous atmosphere has a reduced effective opacity such that more radiation can escape without blowing away the atmosphere (even if the flux is super-Eddington).
4. Before the Eddington limit is reached, convection is always excited. Thus, the inner parts of an object where the density is high enough such that convection can be efficient, will never witness a super-Eddington flux. (For $p_{\mathrm{rad}} \approx p_{\mathrm{gas}}$ or $\Gamma \approx 0.5$, this roughly corresponds to the regions where the diffusion time scale is longer than the dynamical time scale).

Combining the above together yields the following structure, which is also described in detail in figure 2. From the inside out, it has a convection layer (which is sub-Eddington), a porous atmosphere that is super-Eddington but effectively sub-Eddington (due to a reduced effective opacity), and a wind generated from the regions where the effective opacity is not reduced enough, such that the system is again both nominally and effectively super-Eddington.

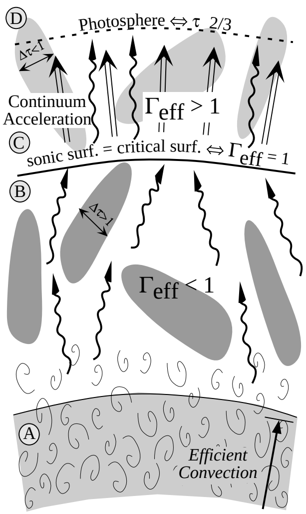

Figure 2. The proposed structure of a super-Eddington atmosphere (one with $L_{\mathrm{tot}} > \mathcal{L}_{\mathrm{Edd}}$) and the wind that it generates. The four regions are: Region (A) A convective envelope in which the radiation is sub-Eddington – $L_{\mathrm{rad}} < \mathcal{L}_{\mathrm{Edd}} < L_{\mathrm{tot}}$. Region (B) A porous atmosphere in which the effective Eddington luminosity is larger than the classical Eddington luminosity: $\mathcal{L}_{\mathrm{Edd}} < L_{\mathrm{rad}} = L_{\mathrm{tot}} < \mathcal{L}_{\mathrm{eff}}$. Region (C) of an optically thick, continuum driven wind, where perturbations are optically thin and the effective Eddington limit tends to the classical value. Region (D) of a photosphere and above.

4. Super-Eddington Winds (SEW)

A key feature in the atmospheres described above is the acceleration of a continuum driven wind. Although for the most part of the atmospheres, the luminosity is effectively sub-Eddington, acceleration is possible because the luminosity is in fact super-Eddington, such that in the regions where the opacity is not sufficiently reduced, the net forces are upward. This takes place where the inhomogeneities that reduce the effective opacity become optically thin, such that they cannot funnel around the radiation anymore, i.e., where the radiation cannot "see" the inhomogeneities and the opacity returns back to its microscopic value.

In a steady state transonic wind, the location of the sonic point has to coincide with the critical point. By properly identifying this point, the mass loss rate can be found using $\dot{M} = 4\pi R^2 \rho_{critical} v_{sonic}$. It can generally be reduced to:

$$\dot{M} = \frac{\mathcal{W}(\Gamma)(L - \mathcal{L}_{\rm Edd})}{c v_s}, \tag{2}$$

where $\mathcal{W}$ is a dimensionless wind "function". In principle, $\mathcal{W}$ can be calculated from first principles only after the nonlinear state of the inhomogeneities is understood. This however is still lacking as it requires elaborate 3D numerical simulations of the nonlinear steady state. Nevertheless, it can be done in several phenomenological toy models.

In one particular toy model, one assumes that the inhomogeneities obtain a two phase form and that they are vertically elongated. This allows the calculation of the effective opacity as a function of the average size of the inhomogeneities in units of the scale height ($\beta \equiv d/l_p$), the average density (ρ_0), the average ratio between the surface area and volume of the blobs in units of the blob size (Ξ), and the volume filling factor α of the dense blobs. In the limit in which the blobs are optically thick, one finally obtains that $\mathcal{W} = 3\Xi/32\sqrt{\nu}\alpha\beta(1-\alpha)^2$ (Shaviv 2001b), with ν being the ratio between the effective speed of sound in the atmosphere to the adiabatic one. Thus, $\mathcal{W}$ depends only on geometrical factors. It does not depend explicitly on the Eddington parameter Γ.

Other toy models can be constructed (e.g., assuming a two phase mixture with Markovian statistics), but they yield similar results. Namely, the unknown physics is hidden in the wind function $\mathcal{W}$. It should be of order unity and it depends only on the geometrical properties of the inhomogeneities. Thus, $\mathcal{W}$ is a function of the Eddington parameter Γ only to the extent that the geometrical properties are a function of Γ. Without any prior knowledge of how the nonlinearities saturate, we assume as a first step that the geometrical properties are not a function of Γ.

This predicted SEW can be applied to actual objects. If we are to test the mass loss prediction, we require objects for which the basic parameters are known – Mass, luminosity and mass loss, such that their super-Eddington nature can be reliably verified. The few objects that satisfy these conditions are the massive star η-Carinae, which had several times the Eddington luminosity for at least 20 years during its great eruption 150 years ago, and the few novae which have good bolometric magnitude and distance measurements.

Since $\mathcal{W}$ is a free parameter in the theory as long as the knowledge of the actual nonlinear steady state is lacking, we can calculate its range of values that satisfy the observations for each object. If the theory is consistent, the same $\mathcal{W}$ should explain both novae and η-Car, even though these are markably different objects. From its geometrical nature, its value should be ~ 1 to ~ 10.

The results for $\mathcal{W}$ are described in figure 3. Clearly, the results from the three objects are consistent with each other. They are somewhat larger than the theoretical prediction using nominal values ($\beta = 1$, $\alpha = 1/2$) in eq. **??** (or values derived from the second toy model developed assuming a Markovian mixture),

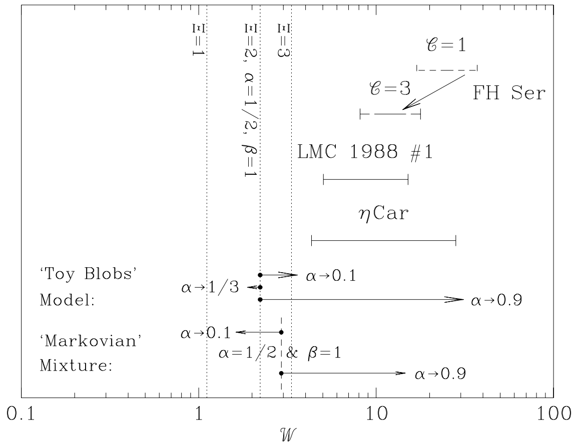

Figure 3. $\mathcal{W}$ obtained from three independent objects. In the case of FH Ser, the result $\mathcal{C}^{0.67}\mathcal{W}$ constitutes an upper limit since any clumpiness in the wind will mimic a larger inferred value for $\mathcal{W}$ if homogeneity (i.e., $\mathcal{C} \equiv \sqrt{\langle\rho^2\rangle / \langle\rho\rangle^2} = 1$ is assumed). The value of $\mathcal{C} = 3$ is a guesstimate from the clumpiness observed in WR winds. Since the wind is expected to be clumpy, the larger value obtained is a good indication that the analysis and the model are consistent. The vertical dotted lines describe the theoretical prediction and its uncertainties using the simple 'blob' model, with nominal values for the unknown geometrical parameters ($\alpha = 1/2$, $\beta = 1$ and $\Xi = 2$). The two additional dotted lines represent the uncertainties arising from the geometrical parameter Ξ describing the surface to volume ratio in units of the size of the blobs, which could reasonable vary from $\Xi \sim 1$ to $\Xi \sim 3$. The arrows represent the change in the predicted $\mathcal{W}$ when changing the volume fraction of the dense regions. When large volume asymmetry is present, the value of $\mathcal{W}$ increases. It is minimized for $\alpha = 1/3$. β can shift $\mathcal{W}$ in both directions. The dashed vertical line represents the prediction using a Markovian model.

but they do agree with the predictions using geometrical parameters well within reasonable bounds (e.g., $\alpha \sim 0.8$ or larger). For more information on the models, see Shaviv (2001b). The agreement can also be seen in figure 4, where the evolution of nova FH Ser is depicted. One finds that the predicted temperature using the SEW theory is in agreement with the observed temperature evolution.

5. An updated Core-Mass Luminosity Relation

We have seen that the observed super-Eddington luminosities necessarily imply a wind which agrees with observations. The next stage is to show why objects become super-Eddington to begin with. In the case of novae, for example, the steady state shell burnings should be given by the CMLR, which is sub-Eddington. Why is it then observed to be super-Eddington?

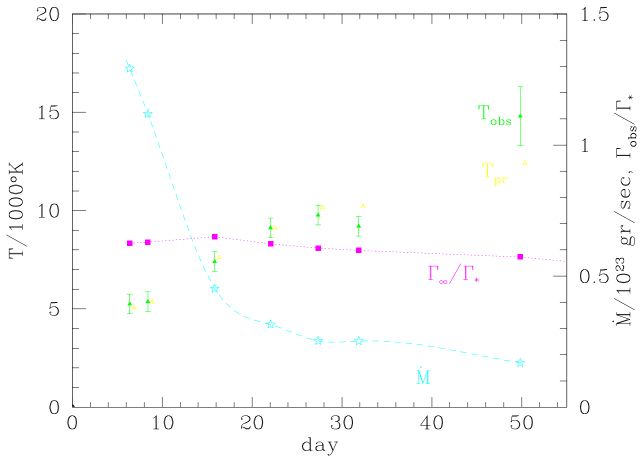

Figure 4. The observed color temperature behavior $T_{\rm obs}$ of Nova FH Ser (filled triangles, taken from Friedjung 1987) as compared with the predicted color temperature behavior $T_{\rm pr}$ from the observed luminosity and wind model (open triangles, slightly offsetted to the right), for the best fit model assuming $M_{\rm WD} = 0.8M_\odot$ and $X = 0.70$. The additional plots are of the mass loss (the stars and dashed line). The mass loss integrates to $8 \times 10^{-5} M_\odot$. Finally we plot as filled squares the ratio of Γ_∞ to Γ_0. Clumpiness will reduce this fraction.

To answer this question, we look for solutions to the stellar structure equations for systems with shell burning. This should describe the steady state of novae after they undergo a TNR. Unlike the standard derivation of the CMLR, we allow the atmospheres to be inhomogeneous. Namely, if the Eddington parameter is larger than a threshold (taken to be 0.85) the opacity obtains a lower value. At this point we do not know yet the exact behavior of $\kappa_{\rm eff}$ as a function of Γ. This could later be obtained by comparing the steady state obtained to the actual observations of novae. At this point however, we want to show that using a reasonable behavior of $\kappa_{\rm eff}(\Gamma)$, a super-Eddington steady state does exist.

We take $\kappa_{\rm eff}(\Gamma > \Gamma_{\rm crit} = 0.85) = \kappa_0 \Gamma_{\rm crit}/\Gamma$ and $\mathcal{W} = 10$ (somewhat different choices do not change the conclusions). We find that the CMLR obtains a super-Eddington branch. The main difference between the structure in this branch and the structure in the sub-Eddington branch is the following: (1) The super-Eddington branch has a SEW at the top of it, with the photosphere located in the wind. The sub-Eddington branch has no wind (though it could have one if a non-Thomson opacity is considered). Since the wind is "heavy" the actual luminosity at the photosphere could be sub-Eddington. (2) The super-Eddington branch has a convective layer that penetrates into the burning shell (most of the energy is actually released in the convection zone). In the sub-Eddington branch, the burning shell is all radiative. (3) The luminosity in the sub-Eddington branch is not a function of the luminosity. It is a function of the luminosity in the super-Eddington branch.

The next step, which is still the subject of on going research, is to compare the obtained steady state to the observed properties of novae at their visual maximum. Using $L(t_3)$, $M(t_3)$ and $v_\infty(t_3)$, we can obtain $\kappa(\Gamma)$ and $\mathcal{W}$. Then, we can also calculate $T_{\rm color}(t_3)$ and $M_{\rm env}(t_3)$ and compare those to their observations. Preliminary results show that the predictions and observations are consistent. Namely, not only can we derive the expressions for $\kappa(\Gamma)$ and $\mathcal{W}$, we have enough redundancy to use the extra variables as a test.

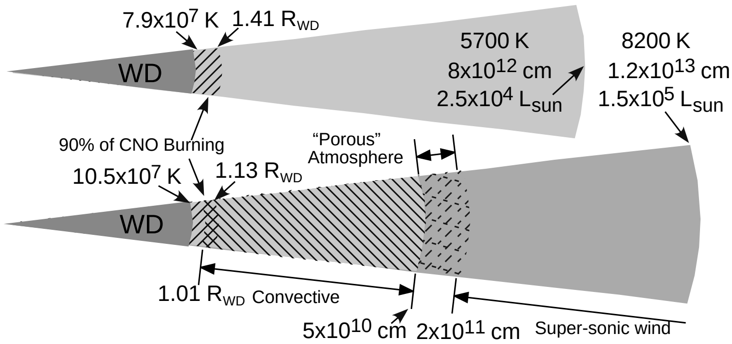

Figure 5. The shell structure of a CNO shell burning white dwarf with a $10^{-4}M_\odot$ envelope. Unlike the top panel, the bottom panel describes an atmosphere in which the radiative instability is taken into account, such that it can become porous. This allows the occurrence of the described steady state. The luminosity is super-Eddington. A convection zone arises and its inner boundary is well within the nuclear burning zone. A continuum driven wind is launched such that the photosphere is in a wind.

6. Discussion

To summarize, several luminous systems have been shown to pose a considerable problem to the naive picture described by the Eddington limit. By studying them more carefully, we have seen that the inconsistency between the naive picture and observations of these objects can be reduced to two questions. *How can an atmosphere have a super-Eddington flux pass through it while exerting a force smaller than g?* And, *why would parts of the atmosphere reach a state in which this takes place?* The answer to the first question is that porous atmospheres naturally have a reduced opacity as the radiation bypasses the optical obstacles without transferring as much momentum. The answer to the second question is that luminous atmospheres were found to be dynamically unstable as they approach the Eddington limit, even if their opacity is only that of Thomson scattering. Thus, inhomogeneities which reduce the effective opacity are the natural state of luminous atmospheres.

We have also seen that a natural consequence of the super-Eddington state is a wind from the point where the inhomogeneities become optically thin. This wind is significantly more "modest" than that naively expected in the presence of a super-Eddington flux. This wind however is dense enough to be optically thick. It nicely predicts for example the mass loss observed in η-Carinae or the observed evolution of nova eruptions.

Last, we have seen that by allowing atmospheres to become inhomogeneous, new stellar structure configurations are obtained. In particular, by studying the Hydrogen shell burning state, a super-Eddington branch can be constructed to the core-mass luminosity relation. Moreover, because atmospheres which are luminous enough are intrinsically unstable, the sub-Eddington branch disappears above a critical core mass such that the super-Eddington branch is the only viable solution. By comparing the obtained steady state to the apparent steady states of novae derived from observations, the nonlinear state of porous atmospheres can be parameterized (using $\mathcal{W}(\Gamma)$ and $\kappa_{\text{eff}}(\Gamma)$). Since more observables exist to those used to obtain the parameterized variables, the theory can be further tested.

All this was obtained without actually solving for the actual nonlinear steady state. Instead, its nonlinearities were simply parameterized. Because this problem is intrinsically nonlinear and radiative-hydrodynamic in nature, its full understanding requires full 3D numerical radiative hydrodynamic simulations, which have only just began. This would allow us to calculate the relevant variables required to describe the nonlinear state without the use of observations to normalize the phenomenological toy models.

Many more specific questions naturally arise. For example, what happens when too heavy a wind is predicted, heavier than the luminosity can push to infinity? Or, are the super and sub-Eddington branches of the CMLR branches continuous? Namely, it was found that the branches are separate, however, this assumed that the sub-Eddington branch has no wind and that the opacity is that of Thomson scattering. More complicated opacity laws can result with a sub-Eddington wind (e.g., through line driving), in which case, the sub and super-Eddington branches could be continuous.

The results described here were applied to a few objects for which good enough observations exist. This allowed us to verify the predictions of the SEW theory, which should now be applied to more objects.

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