Exceeding the Eddington Limit

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Abstract. We review the current theory describing the existence of steady state super-Eddington atmospheres. The key to the understanding of these atmospheres is the existence of a porous layer responsible for a reduced effective opacity. We show how porosity arises from radiative-hydrodynamic instabilities and why the ensuing inhomogeneities reduce the effective opacity. We then discuss the appearance of these atmospheres. In particular, one of their fundamental characteristic is the continuum driven acceleration of a thick wind from regions where the inhomogeneities become transparent. We end by discussing the role that these atmospheres play in the evolution of massive stars.

1. Introduction

The notion that astrophysical objects can remain in a super-Eddington steady state for considerable amounts of time, contradicts the common wisdom usually invoked in which the classical Eddington limit, $\mathcal{L}_{Edd}$, cannot be exceeded in a steady state because no hydrostatic solution can then exist. In other words, if objects do pass $\mathcal{L}_{Edd}$, they are expected to be highly dynamic. A huge mass loss should ensue since large parts of their atmospheres are then gravitationally unbound and should therefore be expelled.

η Carinae, and classical novae for that matter, stand in contrast to the above notion. η Car was super-Eddington (hereafter SED) during its 20 year long eruption (e.g., Davidson & Humphreys 1997). Yet, its observed mass loss and velocity are inconsistent with any standard solution (Shaviv 2000). Basically, the sonic point obtained from the observed conditions necessarily has to reside too high in the atmosphere, at an optical depth of only ~ 1 to ~ 300, while the critical point in a homogeneous atmosphere necessarily has to reside at significantly deeper optical depths. The inconsistency arises because the sonic and critical points have to coincide in a steady state solution.

We review here the proposed solution in which the atmosphere of η Car is inhomogeneous, or porous (Shaviv 2000). This inhomogeneity is a natural result of the instabilities of atmospheres that are close to the Eddington luminosity (Shaviv 2001a, §2 here). The inhomogeneities, or "porosity", reduce the effective opacity and increase the effective Eddington luminosity (Shaviv 1998, §3 here). This porous layer allows the existence of SED states and the acceleration of a continuum driven wind (Shaviv 2000, 2001b; Owocki *et al.* 2004, §4-5 here).

Stellar structure and evolution of massive stars should therefore consider the existence of SED states. With respect to novae, it was shown that a SED branch to the Core-Mass-Luminosity relation can arise, and it adequately describes the behavior of novae (Shaviv 2002). The situation in massive stars is considerably more complicated, but some interesting points can be made (§6).

2. Radiative Hydrodynamic Instabilities in Luminous atmospheres

The first point to note is that as atmospheres approach the Eddington limit, atmospheres become radiative-hydrodynamically unstable. In fact, it is already quite some time since Spiegel (1976, 1977) speculated that atmospheres supported by radiation pressure would likely exhibit instabilities not unlike that of Rayleigh-Taylor, associated with the support of a heavy fluid by a lighter one, leading to formation of "photon bubbles".

Recent quantitative stability analyses by Shaviv (2001a) do lead to the conclusion that even a simple case of a pure "Thomson atmosphere"—i.e., supported by Thomson scattering of radiation by free electrons—would be subject to intrinsic instabilities for development of lateral inhomogeneities. The analysis by Shaviv (2001a) suggests in particular that these instabilities share many similar properties to the excitation of strange mode pulsations (e.g., Glatzel 1994; Papaloizou *et al.* 1997). For example, they are favored when radiation pressure dominates over gas pressure. Both arise when the temperature perturbation term in the effective equation of state for the gas becomes non-local. In strange mode instabilities, the term arises because the temperature in the diffusion limit depends on the radial gradient of the opacity perturbations. In the lateral instability, the term depend on the lateral radiative flux which arises from non-radial structure on a scale of the vertical scale height.

Note that when conditions of a pure Thomson atmosphere are alleviated, more instabilities exist. There are of course the aforementioned strange mode instabilities which require a non-Thomson opacity. If magnetic fields are introduced, even more instabilities can play a role (Arons 1992; Gammie 1998; Begelman 2002; Blaes & Socrates 2003). We stress however that the physical origin of the instabilities is not important to our discussion here. The only critical point is that as atmospheres approach the Eddington limit, non-radial instabilities do exist to make the atmospheres inhomogeneous, while the typical length scale expected is that of the vertical scale height.

3. Inhomogeneities and a reduced effective opacity

The next point to note is that once instabilities are excited in an atmosphere, the unstable modes will grow to become nonlinear. These inhomogeneities play an important role because, once introduced, they change the ratio between the radiative *flux* through the system and the radiative *force.*

In the presence of inhomogeneities, the flux through the system can be written as the volume average of the flux: $F_{\rm avr} = \langle F \rangle_v$. On the other hand, the force exerted by this flux is $f_{\rm avr} = \langle F \kappa_v \rangle_v$, thus, an effective opacity can be defined as (Shaviv 1998):

$$\kappa_{\rm eff} \equiv \frac{f_{\rm avr}}{F_{\rm avr}} = \frac{\langle F \kappa_V \rangle_V}{\langle F \rangle_V}. \tag{1}$$

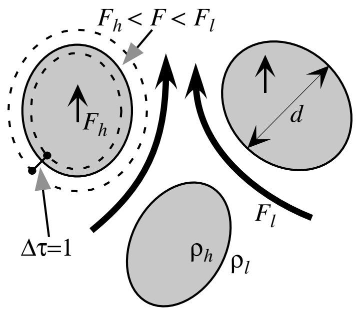

Figure 1. Radiation in a "porous" medium. In the presence of clumps (with a typical size d of order the atmospheric scale height), radiation will tend to pass through the paths of least resistance, giving rise to an anti-correlation between the radiative flux F and the extinction (opacity per unit volume) κ_v. This reduces the effective opacity $\kappa_{\mathrm{eff}} \equiv \langle F \kappa_v \rangle / (\langle F \rangle$. In the Intermediate region, where the density is not too high, the finite optical thickness of the clumps implies that around them a finite region of optical depth $\Delta\tau \sim 1$ exists, where the anti-correlation between the density and flux is degraded. These regions increase the effective opacity. Once κ_{eff} is not small enough to keep the system effectively "sub-Eddington", the net force becomes outwards and a continuum driven wind initiates.

The situation is very similar to the case were inhomogeneities are present in *frequency* space, i.e., to non-gray atmospheres. In such a case, the Rosseland mean is used to calculate the radiative flux through the system, while the force mean of the opacity, which is the flux weighted opacity, is used for the net force. The two cases are similar, with inhomogeneities in either frequency or in real space.

For a few unique opacity laws, the effective opacity can increase. More generally, however, as is the special case of Thomson scattering, the effective opacity is reduced once any inhomogeneities are present.

One example, where κ_{eff} can be calculated, is the limit of small isotropic perturbations in the optically thick limit of a Thomson-scattering atmosphere having a negligible gas heat capacity, such that $\nabla \cdot F = 0$. This limit corresponds to the top layers of an atmosphere of a luminous object (yet deep enough for the inhomogeneities to remain opaque). In this case (Shaviv 1998),

$$\kappa_{\mathrm{eff}} = \kappa_0 \left[1 - \left(\frac{d-1}{d} \right) \sigma^2 \right], \tag{2}$$

where σ is the normalized standard deviation of ρ, and d is the dimension of the system. This result demonstrates that inhomogeneities reduce the effective opacity, but the reduction does not take place in a 1-D system. In other words, the porosity effect is intrinsically *non-radial.*

4. The Super-Eddington State

Joss *et al.* (1973) have shown that deep inside the star, convection will always be excited as the Eddington limit is approached. Convection, however, can only remain efficient as long as the density is high enough, such that convection can carry enough of the flux to keep the leftover carried by radiation at a sub-Eddington level. This breaks when convective motions approach sonic velocities. The density at this radius is relatively high. The high radiation field implies that a homogeneous atmosphere would be unbound above this point. The mass loss from the top of this convection layer would be given by:

$$\dot{M}_{conv} \approx L / v_s^2, \tag{3}$$

which is enormous.

To solve existence problem of SED states, one therefore requires moving the critical point higher in the atmosphere to where the density (and ensuing mass loss) is much lower, as demonstrated by Shaviv (2000). The existence of a porous layer can serve this role—the "missing link" between the top of the convection zone and the critical point where a wind initiates. It therefore allows the construction of a SED steady state, the main elements of which are the following:

Region A: Convection Zone. In the innermost region, where the density is sufficiently high any excess flux above the Eddington luminosity is necessarily advected through convection, which is always excited before the Eddington limit is reached (Joss *et al.* 1973). Therefore, the radiative luminosity remains in this region below the classical Eddington limit: $L_{\rm rad} < \mathcal{L}_{\rm Edd} < L_{\rm tot}$. Note that the convection layer need not penetrate all the way to the interior if the source for the high luminosity is not in the core. We will return to this point in §6.

Region B: The Porous Atmosphere. Once the density decreases sufficiently, as one moves outwards, convection becomes inefficient. Instabilities will necessarily render the atmosphere inhomogeneous, thus facilitating the transfer of flux without exerting as much force. The effective Eddington luminosity is larger than the classical Eddington luminosity: $\mathcal{L}_{\rm Edd} < L_{\rm rad} = L_{\rm tot} < \mathcal{L}_{\rm eff}$. η Car has shown us that the existence of this region allows for the steady state outflow during its 20 year long eruption (Shaviv 2000).

Region C: Optically Thick Wind. When perturbations arising from the instabilities, which are expected to be of order the atmospheric scale height, lose their opaqueness, the effective opacity tends to the microscopic value and the effective Eddington limit tends to the classical value. At the transition between regions (B) and (C), the effective Eddington luminosity is equal to the total luminosity. This critical point is also the sonic point of a continuum driven steady-state wind. Above the transition surface, $L_{\rm tot} > \mathcal{L}_{\rm eff} \rightarrow \mathcal{L}_{\rm Edd}$ and we have an optically thick super-sonic wind.

Region D: Optically Thin Wind. At large enough radii, the geometrical dilution makes the wind transparent. The boundary between the regions is the photosphere of the object, which resides in the wind—the object itself is obscured.

5. Winds from Super-Eddington Atmospheres

The atmosphere can remain effectively sub-Eddington while being classically SED, only as long as the inhomogeneities comprising the atmosphere are optically thick. Clearly however, this assumption should break at some point where the density is low enough. From that radius outwards, the radiative force overcomes the gravitational pull and a wind is generated.

$\dot{M}$ can be obtained by identifying the sonic point of a steady state wind with the critical point, which is the radius where the radiative and gravitational forces balance each other (Shaviv 2001b). We then have $\dot{M} = 4\pi R^2 \rho_{critical} v_s$. Furthermore, this can generally be reduced to the form of

$$\dot{M} = \frac{\mathcal{W}(\Gamma)(L - \mathcal{L}_{\rm Edd})}{c v_s}, \tag{4}$$

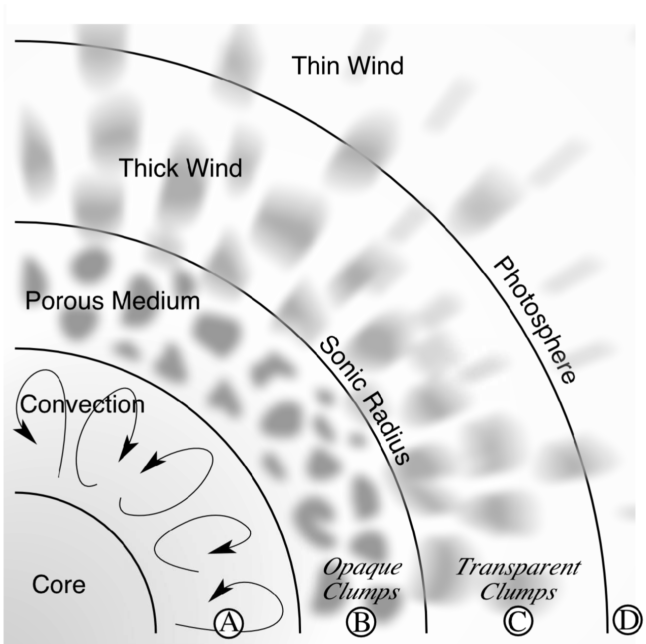

Figure 2. The structure of a massive star in the super-Eddington state. See explanation in the text.

where $\mathcal{W}$ is a dimensionless wind "function". In principle, $\mathcal{W}$ can be calculated from first principles only after the nonlinear state of the inhomogeneities is fully understood. This however is still lacking as it requires elaborate 3D numerical simulations of the nonlinear steady state. Nevertheless, it can be done in several phenomenological models which only depend on geometrical parameters such as the average size of the inhomogeneities in units of the scale height ($\beta \equiv d/l_p$), the average ratio between the surface area and volume of the blobs in units of the blob size (Ξ), and the volume filling factor α of the dense blobs. For example, in the limit where the blobs are optically thick, one obtains that $\mathcal{W} = 3\Xi/32\sqrt{\nu}\alpha\beta(1-\alpha)^2$ Here ν is the ratio between the effective and adiabatic speeds of sound. Thus, $\mathcal{W}$ depends only on geometrical factors. It does not depend explicitly on the Eddington parameter Γ. Moreover, typical values of $\mathcal{W} \sim$ 1-10 are obtained.

The mass loss predicted by the SED theory (eq. 4) was compared with observations of SED objects which have good observational data. These were two novae which are not very fast and which have the best determined *absolute* bolometric evolution: FH-Ser and LMC 1988 #1. The theory was also applied to the Luminous Blue Variable star η Car.

For the two novae, we find that the predicted mass loss rates agree with their observations if $\mathcal{W} \approx 10 \pm 5$, which is clearly consistent with the theoretical estimate for $\mathcal{W}$. The agreement is also with the temporal evolution of the velocities, if those are taken to be the primary absorption line component.

Using $\mathcal{W} \approx 10 \pm 5$, the mass loss equation can also be applied to η Car, which is an entirely different object from novae (in mass, mass loss rate and duration, photospheric size, etc.). The predicted mass loss is in agreement with older determinations of 1-2$M_\odot$ of ejecta (e.g., Davidson & Humphreys 1997), while the terminal velocity is consistently predicted as well.

Recent observations of η-Car reveal a larger than previously estimated amount of ejected mass (Smith *et al.* 2003). If the inferred ejected mass is indeed as high as $10M_\odot$, it could indicate that the clumps in the continuum driven winds have a power law scaling law. This was shown by Owocki *et al.* (2004) to lead to a modified mass loss luminosity relation, with larger mass loss rates:

$$\dot{M} = \frac{\mathcal{W}\mathcal{L}_{\mathrm{Edd}}(\Gamma)(\Gamma-1)^p}{c v_s}, \tag{5}$$

where the power p is related to the power law α_p in the truncated distribution of "clump" optical depths:

$$\frac{df}{d\ln\tau} = \frac{1}{\Gamma[\alpha_p]}\left(\frac{\tau}{\tau_o}\right)^{\alpha_p} e^{-\tau/\tau_o}, \tag{6}$$

through $p = \min(1, 1/\alpha_p)$. It can better explain the larger mass loss rates from η Car.

6. Super-Eddington States of Massive Stars

The above theory explains how a SED state can exist, and what are its characteristics once it is in this state. The theory does not explain why a given object would "choose" to have a high luminosity to begin with. The answer to this question requires coupling the SED theory to the evolution and stellar structure equations of a particular object.

For example, classical novae eruptions are good examples of SED objects. The standard lore is that these objects are expected to shine near the Eddington luminosity, with a luminosity given by the core-mass-luminosity relation (Paczyński 1970). Once the standard stellar structure equations describing the shell burning states of novae, a SED branch to the CMLR emerges (Shaviv 2002). This state describes the observed behavior of novae.

In the case of massive stars, the situation is more complicated than that of shell burning. In particular, the state of classical novae is a real thermal and hydrostatic steady state, with the evolution determined by mass loss and nuclear burning only. In stars, the state could be out of thermal equilibrium. Moreover, the source for the SED luminosity could be an instability inside the star, but not necessarily the core itself.

Suppose, for example, that η Car could be described as a polytrope of index n, its binding energy would be

$$-E_{tot} = \frac{GM^2}{R}\left(\frac{n-3}{5-n}\right) \sim 10^{51}\mathrm{erg}, \tag{7}$$

where we have taken an intermediate n between gas and radiation dominated. Its radius should be of order $R \sim 10R_{\odot}$, considering that the wind velocity today is of order 2000 km/s. On the other hand, the largest estimate for the energy in the eruption is $\sim 10^{50}$erg (Smith *et al.* 2003, based on the recent determinations of the ejected mass). This implies that it is probably only a large outer layer of the star which participated in the eruption, since the Kelvin-Helmoltz time scale for the whole star is notably longer than the duration of eruption. This explains why beneath the convection layer, there could be a lower luminosity region without convection (see fig. 2), and why the system is not in thermal equilibrium.

The physical origin for the instability responsible for "rearrangement" of the outer layer and release of a large amount of energy is not understood at this moment, nor is it known whether it is the same phenomenon responsible of the variability of less luminous LBVs. It is evident, however, that this phenomenon plays an important role in the evolution of very massive stars, since each time it arises, a huge amount of mass is ejected.

The evolution probably takes place near and along the limit where the above instabilities take place. Each time the limit is crossed, enough mass is shed to bring the star down below the limit. This limit could be the Humphreys-Davidson limit.

During the eruptions, the outer layers of the star probably expand (given that the wind velocity in η Car decreases 3-fold). The large mass loss implies that the photosphere of the winds sits at very large radii. If the opacity were Thomson, the radius of the photosphere during η Car's great eruption would be given by:

$$R \sim \dot{M}\sigma_T/(4\pi v_{\infty} m_p) \sim 500 \text{ AU}. \tag{8}$$

On the other hand, the effective temperature at this radius should be only $\sim 1500°$K. This implies that the photosphere actually resided at $R \sim 100$ AU where Hydrogen recombination, at $\sim 3000°$K takes place.

Owocki *et al.* (2004) have shown that the terminal velocity of porous continuum driven winds with Γ of a few, is close to the escape velocity. Thus, the expansion velocity of η Car's homunculus nebula ($v_{\infty} \sim 650$ km/s), implies that the sonic radius was roughly at:

$$R \sim 2GM_{\star}/v_{\infty}^2 \sim 0.2 \text{ AU}. \tag{9}$$

This was the actual size of the "static" part of η Car. Given however the uncertainty in $v_{\infty}/v_{esc} \approx 0.8 - 1.2$ (Owocki *et al.* 2004), there is a factor of 2 uncertainly.

7. Summary

The following coherent picture was presented to explain the observed existence of SED atmospheres:

- Homogeneous atmospheres becomes inhomogeneous as the the radiative flux approaches the Eddington limit. This is due to a plethora of instabilities. The particular governing instability depends on the details of the atmosphere.

- Once inhomogeneities are preset, the effective opacity is then reduced as it is easier for the radiation to escape; consequently, the effective Eddington limit increases.
- Super-Eddington configurations are now possible because the bulk of the atmosphere is effectively sub-Eddington. Very deep layers advect the excess total luminosity above Eddington by convection. Higher in the atmosphere, where convection is inefficient, the Eddington limit is effectively increased due to the reduced effective opacity.
- The top part of the atmosphere, where perturbations of order of the scale height become optically thin, has however to remain SED. Thus, these layers are pushed off by a continuum driven wind. By identifying the location of the critical point of the outflow, one can obtain a mass-loss luminosity relation. The relation has a universal dimensionless parameter which should be of order or somewhat larger than unity.

This theory explains the behavior of markably different objects (novae, very massive stars) which differ by orders of magnitude in their masses, mass loss rates, and luminosities. The wind model presented, however, is by no means a complete theory since it cannot predict *ab initio* the luminosity at the base of the wind. To obtain the latter, one needs to solve for the complete stellar evolution equations while taking into account the porosity and lowered opacity in these objects' atmospheres.

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References

Arons, J. 1992, ApJ, **388**, 561
Begelman, M. C. 2002, ApJ, **568**, L97
Blaes, O., & Socrates, A. 2003, ApJ, **596**, 509
Davidson, K., & Humphreys, R. M. 1997, ARA&A, **35**, 1
Gammie, C. F. 1998, MNRAS, **297**, 929
Glatzel, W. 1994, MNRAS, **271**, 66
Joss, P. C., Salpeter, E. E., & Ostriker, J. P. 1973, ApJ, **181**, 429
Owocki, S. P., Gayley, K. G., & Shaviv, N. J. 2004, *submitted to ApJ*
Paczyński, B. 1970, *Acta Astronomica*, **20**, 47
Papaloizou, J., Alberts, F., Pringle, J.E., & Savonije, G.J. 1997, MNRAS, **284**, 821
Shaviv, N. J. 1998, ApJ, **494**, L193
Shaviv, N. J. 2000. ApJ, **532**, L137
Shaviv, N. J. 2001a. ApJ, **549**, 1093
Shaviv, N. J. 2001b, MNRAS, **326**, 126
Shaviv, N. J. 2002, *AIP Conf. Proc. 637: Classical Nova Explosions*, p. 259
Smith, N., Gehrz, R. D., Hinz, P. M., Hoffmann, W. F., Hora, J. L., Mamajek, E. E., & Meyer, M. R. 2003, AJ, **125**, 1458
Spiegel, E. A. 1976, *Physique des Mouvements dans les Atmospheres*, p. 19
Spiegel, E. A. 1977, *IAU Colloq. 38: Problems of Stellar Convection*, p. 267