The renormalization group (RG) procedure retains the relevant degrees of freedom and iteratively integrates out the irrelevant ones. We argue that relevant degrees of freedom can be identified using information-theoretic, model-independent principle based on the mutual information between spatially separated regions. We propose an unsupervised machine learning algorithm implementing this principle, based on applying the "information bottleneck" method of compression theory to real space. We substantiate our findings with proof-of-principle numerical results on 2D Ising and dimer models.

Mutual Information (MI) and lossy compression

Intuition: relevant degrees of freedom carry most information about the system at large.

Quantity of interest: mutual information (MI).

The Information Bottleneck Method [2]: compress data so the code retains most MI with random variable of interest.

Key Idea: coarse-graining in an RG scheme can be phrased as retaining d.o.f.s maximizing MI with the rest of the system [3].

Unsupervised machine-learning algorithm for MI

We use a Restricted Boltzmann Machine (RBM) setup, where the joint probability distribution for all variables has thermal form and H only couples to X.

An RBM identifying d.o.f.s in X carrying the most mutual information with environment E is constructed.

The algorithm uses two auxiliary contrastive-divergence RBMs to estimate the data probability distributions P(X,E) and P(X),

And a Monte-Carlo subroutine for estimating the MI between subregions.

A buffer is used to filter out short-range contributions to MI.

The information about relevant degrees of freedom is contained in the weights (filters) of the trained network.

The test case: dimer model

Classical dimer model with "spectator" spin degrees of freedom.

The "spectator" spins are FM-bound and frozen. They are irrelevant physically, but form a strong pattern.

The correct low-energy degrees of freedom are not apparent.

The low-energy d.o.f.s are low-momentum components of electrical fields in x and y directions.

They are exposed by mapping to height model and electrical fields.

Numerical results (Theano/Numpy):

Filters for 4 output neurons couple to low momentum electric fields:

Filters for 2 output neurons:

Generic RBMs couple to irrelevant spectator spins:

And vanishing electric fields:

Conclusions:

RG procedure can be phrased in information-theoretic terms (MI).

MI based description of RG lends itself to machine learning implementation.

Numerical tests on Dimer and Ising models confirm the picture.

Generic Neural Networks do not perform RG when applied to physical systems, contradicting Ref. [1].